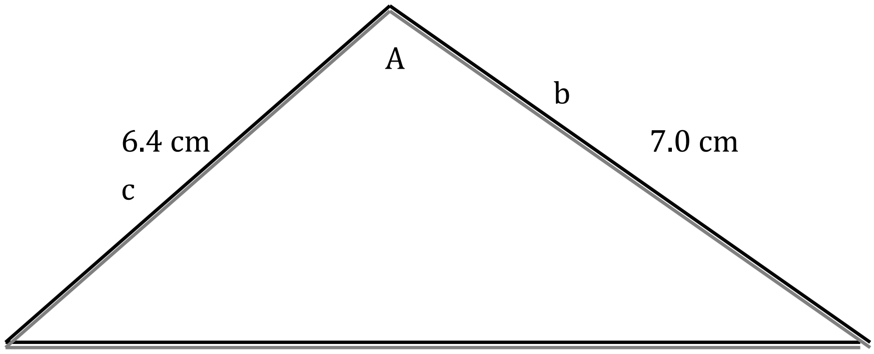
Core2 Answers June2009.doc

1 (i) Use $a^2 = b^2 + c^2 - 2bc\cos A$,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$ Learn

Make A the angle you need opposite biggest side



$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$= \frac{6.4^2 + 7.0^2 - 11.3^2}{2 \times 6.4 \times 7.0}$$
$$= -0.421\ldots$$
$$A = 115°$$

(ii) Area $= \frac{1}{2} bc\sin A$

$$= \frac{1}{2} \times 6.4 \times 7.0 \times \sin 115$$
$$= 20.3\,\text{cm}^2$$

2 (i) Use $U_n = a + (n-1)d$: $U_{10} = a + 9d$ $U_4 = a + 3d$

But $U_{10} = 2U_4$ so

$a + 9d = 2a + 6d$ $U_{20} = a + 19d = 44$

$a = 3d$ $a = 44 - 19d$

$44 - 19d = 3d$

$22d = 44$

$d = 2$ $a = 6$

(ii) Use $S_n = \frac{1}{2} n\left(2a + (n-1)d\right)$

$$S_{50} = 25\left(12 + 49 \times 2\right)$$
$$= 2750$$

3 Take logs $\log 7^{x} = \log 2^{x+1}$

$$x\log 7 = (x+1)\log 2$$

$$x\log 7 = x\log 2 + \log 2$$

$$x(\log 7 - \log 2) = \log 2$$

$$x = \frac{\log 2}{\log 7 - \log 2}$$

$$= 0.5532\ldots$$

x = 0.553 (3 sf)

4(i) Coefficients are 1 3 3 1 so the expansion is:

$$(x^2)^3 + 3(x^2)^2(-5) + 3x^2(-5)^2 + (-5)^3 = x^6 - 15x^4 + 75x^2 - 125$$

(ii)

$$\int (x^2-5)^3\,dx = \int (x^6 - 15x^4 + 75x^2 - 125)\,dx$$

$$= \tfrac{1}{7}x^7 - 3x^5 + 25x^3 - 125x + c$$

5 (i) sin 2x = 0.5

let y = 2x

siny = 0.5

y = 30° or 150⁰ from sine graph

x = 15° and x = 75° as x = $\frac{1}{2}$y

(ii)

$$2\sin^2 x = 2 - \sqrt{3}\cos x$$

$$2(1-\cos^2 x) = 2 - \sqrt{3}\cos x$$

$$2\cos^2 x = \sqrt{3}\cos x$$

$$\cos x(2\cos x - \sqrt{3}) = 0$$

$\cos x = 0$ or $\cos x = \dfrac{\sqrt{3}}{2}$ x = 90^0 or 30^0

x = 90^0 or x = 30°

6 $\frac{dy}{dx} = 3x^2 + a$ integrating $y = x^3 + ax + c$

When x = -1, y = 2   2 = –1 –a + c   c = a + 3

When x = 2, y = 17   17 = 8 + 2a + c   9 = 3a + 3   a = 2 c = 5

Equation is $y = x^3 + 2x + 5$

7 (i) Remainder theorem subst x = -2

$$f(-2) = 2(-2)^3 + 9(-2)^2 + 11(-2) - 8$$
$$= -16 + 36 - 22 - 8$$
$$= -10$$

Remainder is -10

(ii) $f\left(\tfrac{1}{2}\right) = 2\left(\tfrac{1}{2}\right)^3 + 9\left(\tfrac{1}{2}\right)^2 + 11\left(\tfrac{1}{2}\right) - 8$
$$= 2 \times \tfrac{1}{8} + 9 \times \tfrac{1}{4} + 11 \times \tfrac{1}{2} - 8$$
$$= \tfrac{1}{4} + 2\tfrac{1}{4} + 5\tfrac{1}{2} - 8$$
$$= 0$$

(iii) $f(x) = (2x - 1)(x^2 + 5x + 8)$
Do long division to get the quadratic factor

(iv) There is one real root when $x = \frac{1}{2}$ as shown.

To find how many real roots there are in the quadratic, use the discriminant.   Use $b^2 - 4ac$ $a = 1 \;\; b = 5 \;\; c = 8$

$5^2 - 4 \times 1 \times 8 = -7$ which is negative so no real roots

There is one real root of the equation $f(x) = 0$. i.e x = $\frac{1}{2}$

8 (i) $A = \frac{1}{2}r^2\theta$ so Radius $r = \sqrt{\frac{2A}{\theta}} = \sqrt{\frac{120}{1.2}} = 10\ \text{cm}$

Arc length $L = r\theta = 1.2 \times 10 = 12\text{cm}$

Perimeter $= 2 \times 10 + 12 = 32\ \text{cm}$ (3 sf)

(ii) (a) $U_n = U_{n-1} \times \frac{3}{5}$ i.e GP

$U_n = ar^{n-1} = 60 \times \left(\frac{3}{5}\right)^{n-1}$

$U_5 = 60 \times \left(\frac{3}{5}\right)^4 = 7.776 = 7.78$ (3 sf)

(b) $S_n = \frac{a(1-r^n)}{1-r}$

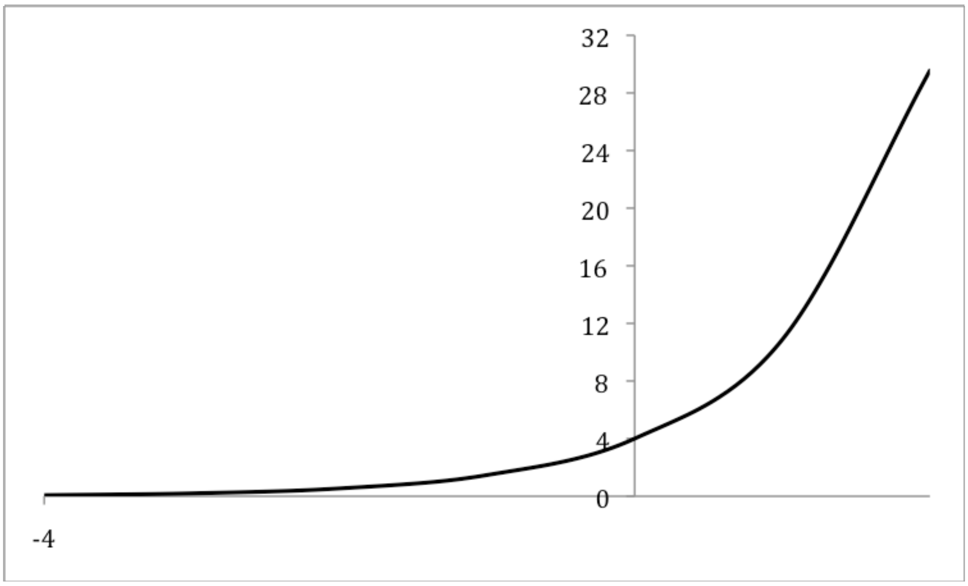
$S_{10} = \frac{60\left(1-\left(\frac{3}{5}\right)^{10}\right)}{1-\frac{3}{5}} = 149.09\ldots = 149\ \text{cm}^2$ (3 sf)

(c) r<1 so r^n tends to 0 as n increases. $S_n = \frac{a}{1-r} = \frac{60}{0.4} = 150$

This means the sum will converge to as $n \Rightarrow \infty$.

9 (i) When x = 0, y = 4.

The x-axis is an asymptote to the curve in the negative x direction



(ii) $20k^2 = 4k^x$

$5k^2 = k^x$

Take logs to the base k

$\log_k(5k^2) = \log_k k^x$

$\log_k 5 + 2\log_k k = x\log_k k$ $\qquad \log_k k = 1$

$x = \log_k 5 + 2$

(iii) (a) Use Area $= \frac{1}{2}h(y_0 + y_n + 2(y_1 + \ldots y_{n-1}))$ $\qquad h = \frac{1}{2}$

$y_0 = 4 \qquad y_1 = 4k^{\frac{1}{2}} \qquad y_2 = 4k$

Area $= \frac{1}{2} \times \frac{1}{2}(4 + 4k + 2 \times 4k^{\frac{1}{2}})$

$= 1 + k + 2\sqrt{k}$

(b) $1 + k + 2\sqrt{k} = 16$

$k + 2\sqrt{k} = 15$

Let $m = \sqrt{k}$ $\qquad m^2 + 2m - 15 = 0$

$(m+5)(m-3) = 0$

$\sqrt{k} = -5$ impossible

or $\sqrt{k} = 3$ $\quad k = 9$