

Core 2 June 2006

$$1 \quad (3x-2)^4 = (3x)^4 + \binom{4}{1}(3x)^3(-2) + \binom{4}{2}(3x)^2(-2)^2 + \binom{4}{3}3x(-2)^3 + \binom{4}{4}(-2)^4$$

$$= 81x^4 + 4 \times 27x^3 \times -2 + 6 \times 9x^2 \times 4 + 4 \times 3x \times -8 + 16$$

$$= 81x^4 - 216x^3 + 216x^2 - 96x + 16$$

2 $u_1 = 2 \quad u_2 = 1 - u_1 = 1 - 2 = -1 \quad u_3 = 1 - u_2 = 1 + 1 = 2 \quad u_4 = 1 - u_3 = -1$

$u_1 = 2$

$u_2 = -1$

$u_3 = 2$

$u_4 = -1$

$\sum_{n=1}^{100} u_n = 50 \times 2 + 50 \times (-1) = 50$

3. $\frac{dy}{dx} = 2x^{-1/2} \quad y = \frac{2x^{1/2}}{1/2} + k = 4x^{1/2} + k \quad y = 4\sqrt{x} + k \quad x=4 \quad y=5$

$5 = 4 \times \sqrt{4} + k \quad 5 = 8 + k \quad k = -3 \quad y = 4\sqrt{x} - 3$

4. $4 - x^2 = x + 2 \quad x^2 + x - 2 = 0 \quad (x+2)(x-1) = 0 \quad x = -2, x = 1$

$$\int_{-2}^1 (4 - x^2) dx = \left[4x - \frac{x^3}{3} \right]_{-2}^1 = \left(4 - \frac{1}{3} \right) - \left(-8 + \frac{8}{3} \right) = 12 - 3 = 9$$

area triangle $x=1 \quad y=3 \quad x=-2 \quad y=0 \quad \frac{1}{2} \times 3 \times 3 = 4\frac{1}{2}$

OR $\int_{-2}^1 (x+2) dx = \left[\frac{x^2}{2} + 2x \right]_{-2}^1 = \left(\frac{1}{2} + 2 \right) - \left(\frac{4}{2} - 4 \right) = \frac{5}{2} + 2 = 4\frac{1}{2}$

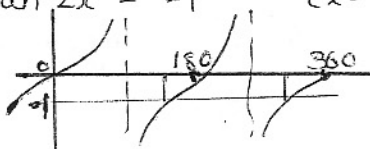
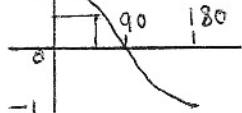
Area shaded region $9 - 4\frac{1}{2} = 4\frac{1}{2}$ units²

5. $2 \sin^2 x = 1 + \cos x \quad 2(1 - \cos^2 x) = 1 + \cos x \quad 2 - 2\cos^2 x - 1 - \cos x = 0$

$2\cos^2 x + \cos x - 1 = 0 \quad (2\cos x - 1)(\cos x + 1) = 0$

$\cos x = \frac{1}{2} \quad \cos x = -1 \quad x = 60^\circ, 180^\circ$

$\sin 2x = -\cos 2x \quad \tan 2x = -1 \quad 2x = 135^\circ, 315^\circ \quad x = 67.5^\circ, 157.5^\circ$



6. 100 105 110 115

240 payments

$a = 100 \quad d = 5 \quad n = 240 \quad 240\text{th } a + (n-1)d \quad 100 + (239 \times 5) = 1295$

$S_n = S_{240} = \frac{240}{2} (P \times 100 + (239 \times 5)) = 120(200 + 1195) = \pounds 167400$

