## Core-2 January 2010

| Question | Solution | Comments |
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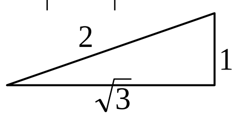
| 1.(i) Show that the equation $2\sin^2 x = 5\cos x - 1$ can be expressed in the form $2\cos^2 x + 5\cos x - 3 = 0$. | Since $\sin^2 + \cos^2 = 1$<br>$LHS = 2(1-\cos^2 x) = 2 - 2\cos^2 x$<br>Hence:<br>$2 - 2\cos^2 x = 5\cos x - 1 ......... re-arranged$<br>$0 = 2\cos^2 x + 5\cos x - 3$ | *as reqd.* |
| 1.(ii) Hence solve the equation $2\sin^2 x = 5\cos x - 1$, giving all values of $x$ between 0° and 360°. | Let $c = \cos x$<br>Thus; $0 = 2c^2 + 5c - 3$<br>$0 = (2c - 1)(c + 3)$<br>$c = \frac{1}{2}$ or $-3$ discard $\lvert \cos x \rvert > 1$<br>Hence $\cos x = \frac{1}{2}$<br><br>Ans: **x=60° or 300°** | *Use any convenient letter.*<br>*Recognise 30°,60° triangle & sketch it*<br>*Need 300° or lose a mark* |
| 2. The gradient of a curve is given by $\frac{dy}{dx} = 6x - 4$<br>2.(i) Find the equation of the curve. | By Integration $y = \frac{6x^2}{2} - 4x + k$<br>Use $(2,5)$ to determine $k$<br>$\Rightarrow 5 = 3\times 4 - 4\times 2 + k$ $k = 1$<br>Hence equation: $y = 3x^2 - 4x + 1$ | *Don't forget the constant !*<br>*Write down the final answer !* |
| 2.(ii) Find the value of $p$. | Substitute $(p,5)$<br>$\Rightarrow$ $5 = 3p^2 - 4p + 1$, $0 = (3p + 2)(p - 2)$ $\rightarrow p = -\frac{2}{3}$ or $p = 2$<br>$p = 2$ has already been used Ans: $p = -\frac{2}{3}$ | *Say why discarding p=2* |
| 3.(i) Find and simplify the first four terms in the expansion of $(2 - x)^7$ in ascending powers of $x$. | $(2-x)^7 = {}^7_0C.2^7.(^-x)^0 + {}^7_1C.2^6.(^-x)^1 + {}^7_2C.2^5.(^-x)^2 \; {}^7_3C.2^4.(^-x)^3$<br>$= 1\times 128\times x^0 - 7\times 64x^1 + 21\times 32x^2 - 35\times 16x^3$<br>$= 128 - 448x + 672x^2 - 560x^3$ | *Nice to keep this lot in a clear tabular form. Easier to work with & easier to mark* |
| 3.(ii) Hence find the coefficient of $w^6$ in the expansion of $(2 - \frac{1}{4}w^2)^7$ | Replace $x \Leftrightarrow (\frac{1}{4}w^2)$<br>Gives $\rightarrow w^6 is ...... x^3$ or<br>$-560\times (\frac{1}{4}w^2)^3 \rightarrow \frac{^-35}{4}w^6$ Ans: $^-35/4$ | *Notice that the negative sign does NOT need substituting.* |
| 4.(i) Use the trapezium rule, with 4 strips each of width 0.5, to find an approximate value for $\int_3^5 \log_{10}(2 + x)\,dx$ | $x$: 3, 3.5, 4, 4.5, 5<br>log3, log3.5, log4, log4.5, log5 $horiz.diff = 0.5$<br>0.6990, 0.7404, 0.7782, 0.8129, 0.8451 $\times \frac{0.5}{2}$<br>×1, ×2, ×2, ×2, ×1<br>**Ans: 1.55** | *As above, it's easier to see and work through in tabular form.* |
| 4.(ii) $\int_3^5 \log_{10}\sqrt{2 + x}\,dx$ | Let $A\log(2+x) = \log\sqrt{2+x} = \log(2+x)^{\frac{1}{2}}$<br>$\Rightarrow A = \frac{1}{2}$ **Ans: 0.776** | *Simple rules-of-logs* |

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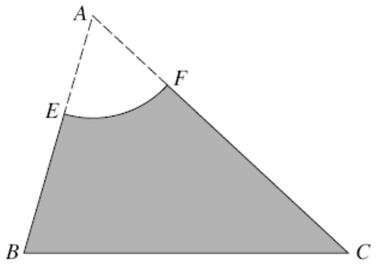
| 5. $y = x^2 + 1$ and $y = 11 - \frac{9}{x^2}$ intersect at (1, 2) and (3, 10)<br><br>exact area | Use:- $\int_1^3 \left\{\left(11-\frac{9}{x^2}\right)-(x^2+1)\right\}dx$<br>$\Rightarrow \int_1^3 (10-9x^{-2}-x^2)\,dx = \left[10x+\frac{9}{x}-\frac{x^3}{3}\right]_1^3$<br>$\Rightarrow 24-19+\frac{1}{3}=5\frac{1}{3}$ Ans: **$5\frac{1}{3}$** | *Identify limits $x=1,3$*<br><br>*Identify the "upper" and "lower" fns.* |
| 6.(i) $f(x) = 2x^3 + ax^2 + bx + 15$<br>Find the values of $a$ and $b$. | Since $(x+3)$ is a factor, use $x=^{-}3$<br>$\rightarrow 0 = 2\times(-3)^3 + a(-3)^2 + b(-3)+15$<br>$\Rightarrow 0 = -54+9a-3b+15 = 9a-3b-39$<br>$\Rightarrow 39 = 9a-3b$<br>$\Rightarrow 13 = 3a-b$<br><br>Similarly, use $x=2$<br>$\rightarrow 35 = 16+4a+2b+15$<br>$\Rightarrow 4 = 4a+2b$<br>$\Rightarrow 2 = 2a+b$<br>$15 = 5a$<br>$a=3 \quad b=-4$ | *Remainder-Theorem*<br><br>*Solve the simult.eqns in any manner* |
| 6.(ii) Using these values of $a$ and $b$, divide $f(x)$ by $(x+3)$ | Substituting for a & b<br>$x+3 \,\overline{)\, 2x^3 + 3x^2 - 4x + 15}$ quotient $2x^2 - 3x + 5$<br>$2x^3 + 6x^2$<br>$-3x^2 - 4x$<br>$-3x^2 - 9x + 15$<br>$5x + 15$<br>Ans: $2x^2-3x+5$ | *Try to keep $x^2$ in column, and similarly x, and numbers*<br><br>*Clearly state the answer (don't leave it hidden in working)* |
| 7.(i) <br>Show that angle $CAB$ is 1.10 radians | Using "cosine-rule"<br>$\hat{A} = \cos^{-1}\left(\frac{10^2+14^2-13^2}{2\times10\times14}\right) = \cos^{-1}\left(\frac{127}{280}\right)$<br>$= \cos^{-1}(0.45357.......) = 1.100028^{c}$ | *Remember that the sign for degrees is °, and for radians is $^{c}$* |
| 7.(ii) Find the perimeter of the shaded region $EBCF$ | Perim =<br>$arc(EF)+6+13+10$<br>$\rightarrow r.\Theta+29$<br>$\rightarrow 4\times1.100+29$<br>Ans: **Perim = 33.4** | *Lay out your working and explain each part*<br><br>*Stating r.Θ is crucial* |
| 7.(iii) Find the area of the shaded region $EBCF$ | Area= Triangle – Sector<br>$\frac{1}{2}ab\sin c - \frac{r^2\Theta}{2} \rightarrow 62.3854 - 9.6805 \rightarrow 53.5848$<br>Ans: **53.6 to 3sf** | *Stating $\frac{1}{2}r^2\Theta$ is crucial*<br><br>*In these questions the area sine-rule is VERY common !* |

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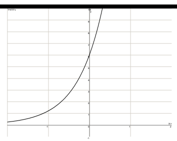
| 8.(i) | $u_1 = 8$ and $u_{n+1} = u_n + 3$<br>Show that $u_5 = 20$ | Using $u_n = a + (n-1)d$ where a=8, n=5, d=3<br>$u_5 = 8 + (5-1) \times 3 \quad = 8 + 4 \times 3 \quad = 8 + 12 = 20$ | *Use formulae given in tables* |
| 8.(ii) | $u_n = pn + q$. State the values of $p$ and $q$ | p=common difference **= 3**; q = zero'th term **= 5** | |
| 8.(iii) | type of sequence | Arithmetic | *Simple statement will do* |
| 8.(iv) | $N$ such that $\sum_{n=1}^{2N} u_n - \sum_{n=1}^{N} u_n = 1256$. | Use $s_n = \frac{1}{2} n(First + Last)$ for each case<br>$for.2N : -.... first = 8;....last = 3(2N) + 5 = 6N + 5$<br>$for.N : -...... first = 8; ....last = 3(N) + 5 = 3N + 5$<br>$1256 = \frac{1}{2} 2N[8 + (6N+5)] - \frac{1}{2} N[8 + (3N+5)]$<br>$2512 = 2N[6N + 13] - N[3N + 13]$<br>$2512 = 12N^2 + 26N - 3N^2 - 13N$<br>$0 = 9N^2 + 13N - 2516 \qquad \rightarrow N = 16..or.. - 17.4$<br>Negative cannot be valid: Ans: **N=16** | *Don't try to do too much in one go. List all the terms, then use them.*<br><br>*This is a slightly weird quadratic, but apply the formula, or simply use your calculator. (This is a „find" question, so any valid method will do.* |
| 9.(i) | Sketch the curve $y = 6 \times 5^x$ | <br>y-intercept at 6; asymptotic to y=0 | *Exponential, through (0,6)* |
| 9.(ii) | $y = 9^x$ has y-coordinate equal to 150 | $150 = 9^x$<br>$\log(150) = \log(9^x) = x\log(9)$ Ans: **x = 2.28**<br>$x = \frac{\log(150)}{\log(9)} = 2.28094$ | *„classic" log-both-sides style of question* |
| 9.(iii) | $y = 6 \times 5^x$ and $y = 9^x$ | Let $6 \times 5^x = 9^x$<br>$\log(6 \times 5^x) = \log(9^x)$<br>$\log 6 + x\log 5 = x\log 9$<br>$(\log 2 + \log 3) + x\log 5 = x\log 9$<br>Now using base-3 wherever appropriate:-<br>$\log_3 3 = 1$ and $\log_3 9 = 2$<br>Hence: L.H.S.= $\log_3 2 + 1 + x\log_3 5$ &<br>R.H.S.= $2x$<br><br>overall by dividing:<br>$[\log_3 2] + 1 = 2x - x[\log_3 5] = x(2 - \log_3 5)$<br>$\frac{1 + \log_3 2}{2 - \log_3 5} = x$ | *„classic" log-both-sides style of question;*<br><br>*Extract any multiple of 3*<br><br>*Simple re-arranging algebra from here-on.*<br><br>*As required* |