

MEI STRUCTURED MATHEMATICS**CONCEPTS FOR ADVANCED MATHEMATICS, C2****Practice Paper C2-D**

Additional materials: Answer booklet/paper
Graph paper
MEI Examination formulae and tables (MF12)

TIME 1 hour 30 minutes

INSTRUCTIONS

- Write your Name on each sheet of paper used or the front of the booklet used.
- Answer **all** the questions.
- You **may** use a graphical calculator in this paper.

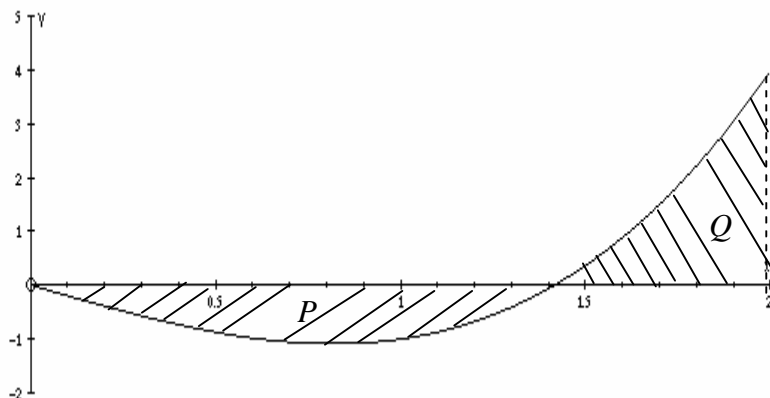
INFORMATION

- The number of marks is given in brackets [] at the end of each question or part-question.
- You are advised that you may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.

Section A (36 marks)

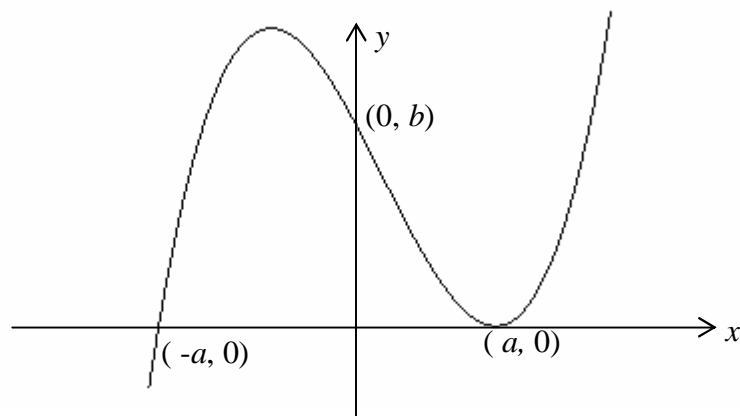
- 1 (i)** Find $\int (x^3 - 2x) dx$. [2]

The graph below shows part of the curve $y = x^3 - 2x$ for $0 \leq x \leq 2$.



- (ii)** Show that the area of the shaded region P is the same as the area of the shaded region Q . [3]
- 2** The growth in population P of a certain town after time t years can be modelled by the equation $P = 11\,000 \times 10^{kt}$ where k is a constant.
- (i)** State the initial population of the town. [1]
- (ii)** After three years the population of the town is 24 000. Use this information to find the value of k correct to two decimal places. [3]
- 3 (i)** Write $\log_{10}(x+4) - 2 \log_{10}x + \log_{10}(x+16)$ as a single logarithm. [2]
- (ii)** Without using your calculator, verify that $x = 4$ is a root of the equation $\log_{10}(x+4) - 2 \log_{10}x + \log_{10}(x+16) = 1$. [2]
- 4** Find the values of θ such that $0^\circ \leq \theta \leq 360^\circ$ which satisfy the equation $\cos\theta \tan\theta = \frac{\sqrt{3}}{2}$. [3]

- 5 The diagram shows the curve $y = f(x)$ where a is a positive constant.

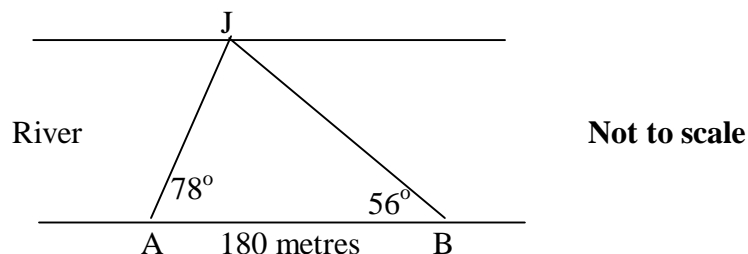


Sketch the following curves on separate diagrams, in each case stating the coordinates of points where they meet the x - and y -axes.

(i) $y = -f(x)$ [2]

(ii) $y = f(-x)$ [3]

- 6 A and B are points on the same side of a straight river. A and B are 180 metres apart. The angles made with a jetty J on the opposite side of the river 78° and 56° respectively as shown.



Calculate the width of the river correct to the nearest metre. [5]

- 7 For each of the following sequences, write down sufficient terms of the sequence in order to be able to describe its behaviour as divergent, periodic or convergent. For any convergent sequence, state its limit.

(i) $a_1 = -1$; $a_{k+1} = \frac{4}{a_k}$ [2]

(ii) $a_1 = 1$; $a_k = 2 - 2 \times \left(\frac{1}{2}\right)^k$ [2]

(iii) $a_1 = 0$ $a_{k+1} = (1 + a_k)^2$. [1]

- 8 Fig.8 shows a sector of a circle with centre O and radius 6 cm and a chord AB which subtends an angle of 1.8 radians at O.

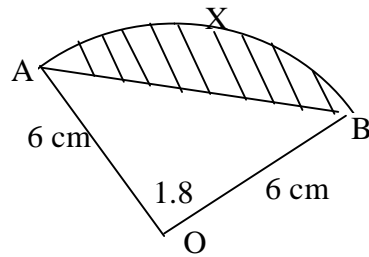


Fig. 8

- (i) Calculate the area of the sector OAXB. [2]
- (ii) Calculate the area of the triangle OAB and hence find the area of the shaded segment AXB. [3]

Section B (36 marks)

- 9 The gradient of a curve is given by $\frac{dy}{dx} = 3x^2 - 12x + 9$. The curve passes through the point (2, -2).
- (i) Find the equation of the curve. [5]
- (ii) Show that the curve touches the x -axis at one point (A) and cuts it at another (B).
State the coordinates of A and B. [4]
- (iii) The curve cuts the y -axis at C. Show that the tangent at C is perpendicular to the normal at B. [3]

- 10 Fig. 10 shows the curve with equation $y = x^2 + \frac{16}{x}$.

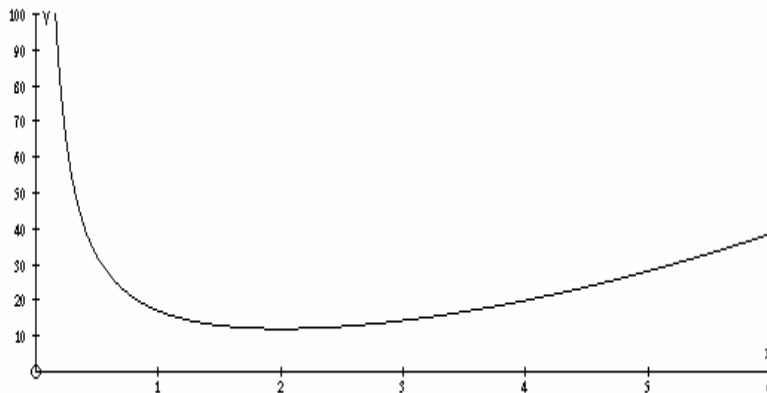


Fig.10

- (i) Find $\frac{dy}{dx}$. [2]
- (ii) Hence calculate the coordinates of the stationary point on the curve. [2]
- (iii) Find $\frac{d^2y}{dx^2}$ and explain why this confirms that the stationary point is a minimum. [3]
- (iv) Using the trapezium rule with 4 intervals, estimate the area between the curve and the x axis between $x = 2$ and $x = 4$. [3]
- (v) State, giving a reason, whether this estimate of the area under-estimates or over-estimates the true area beneath the curve. [2]
- 11 When Fred joined a computer firm his salary was £28 000 per annum. In each subsequent year he received an annual increase of 12% of his previous year's salary.
- (i) State Fred's salary for each of his first 3 years with the company. State also the common ratio of the geometric sequence formed by his salaries. [2]
- (ii) How much did Fred earn in the tenth year? [2]
- (iii) Show that the total amount Fred earned over the ten years was between £400 000 and £500 000. [3]
- (iv) When Fred joined the computer firm, his brother Archie set up a plumbing business. He earned £35000 in his first year and each year earned £ d more than in the previous year. At the end of ten years, he had earned exactly the same total amount as Fred. Calculate the value of d . [5]