Mathematics in Education and Industry

MEI STRUCTURED MATHEMATICS

CONCEPTS FOR ADVANCED MATHEMATICS, C2

Practice Paper C2-C

Additional materials: Answer booklet/paper
Graph paper
MEI Examination formulae and tables (MF12)

TIME 1 hour 30 minutes

INSTRUCTIONS

- Write your Name on each sheet of paper used or the front of the booklet used.
- Answer **all** the questions.
- You **may** use a graphical calculator in this paper.

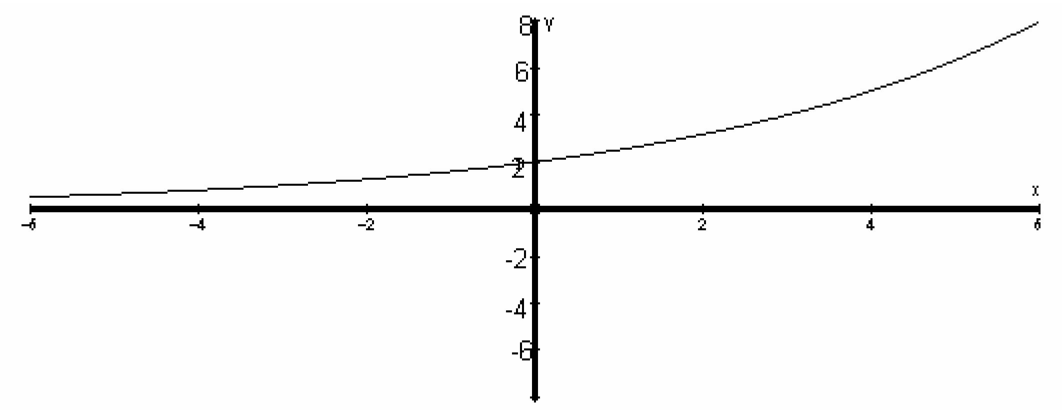
INFORMATION

- The number of marks is given in brackets [] at the end of each question or part-question.
- You are advised that you may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.

Section A (36 marks)

1 The common ratio of a geometric progression is -0.5. The sum of its first three terms is 15. Find the first term.
Find also the sum to infinity. [5]

2 The diagram shows the graph of $y = \mathrm{f}(x)$. The graph passes through the point with coordinates (0, 2).



On separate diagrams sketch the graphs of the following functions, indicating clearly the point of intersection with the y axis.

(i) $y = -\mathrm{f}(x)$ [1]

(ii) $y = \mathrm{f}(3x)$ [2]

3 Given that A is the obtuse angle such that $\sin A = \frac{1}{5}$, find the exact value of $\cos A$. [4]

4 You are given that $y = x^3 - 12x$.

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(ii) Hence find the coordinates of the turning points of the curve. [3]

5 A sequence is defined by $a_k = 5k + 1$, for $k = 1, 2, 3\ldots$

(i) Write down the first three terms of the sequence. [1]

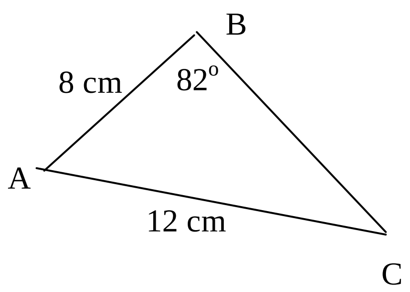
(ii) Evaluate $\sum_{k=1}^{100} a_k$. [4]

6 Find the solution to this equation, correct to 3 significant figures.

$$(2^x)(2^{x+1}) = 10. \quad [5]$$

7 The gradient of a curve $y = \mathrm{f}(x)$ is given by $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 10x + 6$. The curve passes through the point (2,3) Find the equation of the curve. [5]

8 In the triangle ABC shown, AB = 8 cm. AC = 12 cm and angle ABC = 82°. Find θ correct to 3 significant figures. [4]



Section B (36 marks)

9 Fig. 9 shows

P The line $y = x$

Q The curve $y = \sqrt{\frac{1}{2}\left(x + x^2\right)}$

R The curve $y = \sqrt{x}$.

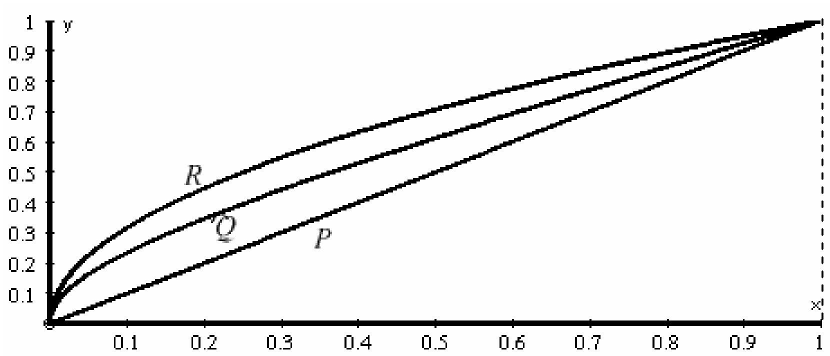


Fig. 9

(i) Write down the area of the triangle formed by the line $y = x$, the line $x = 1$ and the x-axis. [1]

(ii) Show that the area of the region formed by the curve $y = \sqrt{x}$, the line $x = 1$ and the x-axis is $\frac{2}{3}$. [3]

An estimate is required of the Area, A, of the region formed by the curve $y = \sqrt{\frac{1}{2}\left(x + x^2\right)}$, the line $x = 1$ and the x-axis.

(iii) Use results to parts (i) and (ii) to complete the statement

……….. $< A <$ ……………. [1]

(iv) Use the Trapezium Rule with 4 strips to find an estimate for A. [4]

(v) Draw a sketch of Fig. 9. Use it to illustrate the area found as the trapezium rule estimate for A.
Explain how your diagram shows that the trapezium rule estimate must be:

consistent with the answer to part (iv);

an under-estimate for A. [3]

10 A culture of bacteria is observed during an experiment. The number of bacteria is denoted by N and the time in hours after the start of the experiment by t.
The table gives observations of t and N.

Time (t hours)	1	2	3	4	5
Number of bacteria (N)	120	170	250	370	530

(i) Plot the points (t, N) on graph paper and join them with a smooth curve. [2]

(ii) Explain why the curve suggests why the relationship connecting t and N may be of the form $N = ab^t$. [1]

(iii) Explain how, by using logarithms, the curve given by plotting N against t can be transformed into a straight line.
State the gradient of this straight line and its intercept with the vertical axis in terms of a and b. [3]

(iv) Complete a table of values for $\log_{10} N$ and plot the points $(t, \log_{10} N)$ on graph paper.
Draw the best fit line through the points and use it to estimate the values of a and b. [6]

11 The sequence $a_1, a_2, a_3,$ ……where a_1 is a given real number is defined by $a_{n+1} = 1 - \frac{1}{a_n}$.

(i) For the case when $a_1 = 2$, find a_2, a_3 and a_4. Describe the behaviour of this sequence [3]

(ii) For the case when $a_1 = k$, where k is an integer greater than 1, find a_2 in terms of k as a single fraction.
Find also a_3 in its simplest form and hence deduce that $a_4 = k$. [4]

(iii) Show that $a_2 a_3 a_4 = -1$ for any integer k. [2]

(iv) When $a_1 = 2$ evaluate $\sum_{i=1}^{99} a_i$. [3]