C2 Specimen (MA)

Q1) $(2+3x)^6 = (2)^6 + \binom{6}{1}(2^5)(3x)^1 + \binom{6}{2}(2)^4(3x)^2 + \ldots$

$= 64 + 192(3x) + 240(9x^2) + \ldots$

$= \boxed{64 + 576x + 2160x^2}$

Q2) $(x-3)^2 + (y-4)^2 = r^2$

radius = distance from $(3,4)$ to $(8,-8)$

$= \sqrt{(3-8)^2 + (4+8)^2} = 13$

$\therefore \quad \boxed{(x-3)^2 + (y-4)^2 = 169} \qquad (13^2 = 169)$

Q3a)

x	2.5	3
y	4.077	5.292

$\sqrt{2.5^3+1} = 4.077$

$\sqrt{3^3+1} = 5.292$

b) Area $\simeq \frac{1}{2}h\left[1.414 + 5.292 + 2(4.077 + 3 + 2.092)\right]$

$\left[h = \frac{b-a}{n} = \frac{3-1}{4} = \frac{1}{2}\right] \quad \simeq \frac{1}{4}(25.044) = \boxed{6.261}$

Q4) $3(1-\cos^2 x) = 1 + \cos x$

$3 - 3\cos^2 x = 1 + \cos x$

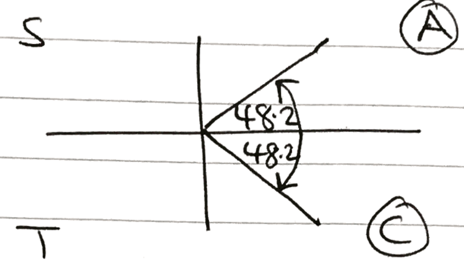
$3\cos^2 x + \cos x - 2 = 0$

$(3\cos x - 2)(\cos x + 1) = 0$

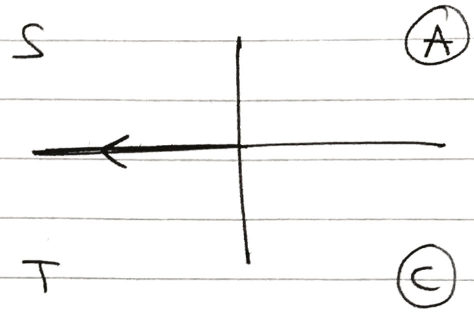
$3\cos x - 2 = 0$	$\cos x + 1 = 0$
$\cos x = \frac{2}{3}$	$\cos x = -1$

$0 \leq x < 360°$

$x = \cos^{-1}\left(\frac{2}{3}\right) = 48.2°$



$x = \cos^{-1}(-1) = 180°$



$x = 48.2°, (360 - 48.2°)$

$\boxed{x = 180°}$

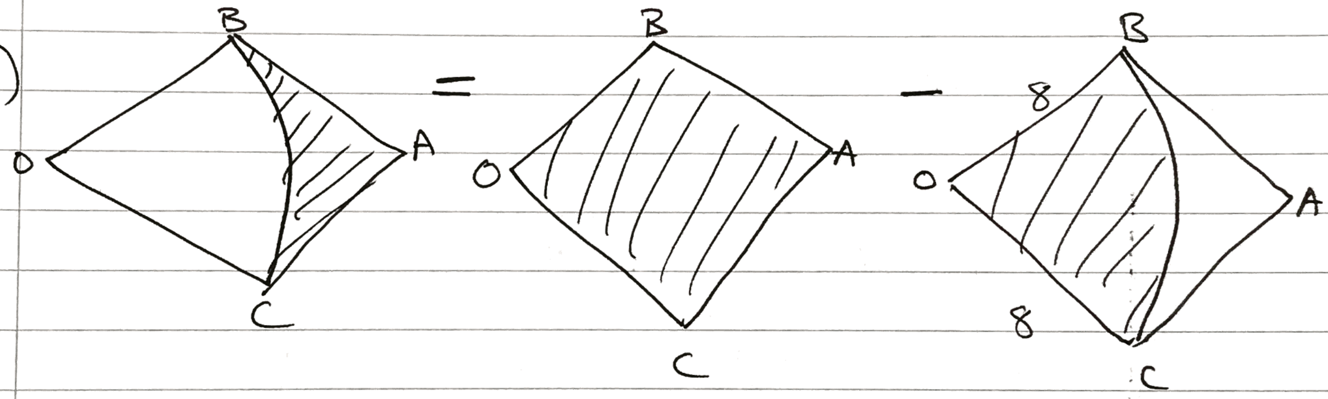
$\boxed{x = 48°, 312°}$

5a) arc BC length $= r\theta = 8 \times \angle BOC = 8 \times \angle BAC = 8 \times 0.9$

$= 7.2\text{mm}$

$\therefore$ Perimeter $= 8 + 8 + 7.2 = \boxed{23.2\text{mm}}$

b)



Badge area $=$ OBAC $-$ OBC

$= \text{Area}_{OBC} \times 2 - \frac{1}{2}(8)^2(0.9)$

$= 2 \times \frac{1}{2}(8)(8)\sin(0.9) - 28.8$

$= 50.133 - 28.8$

$= \boxed{21.3\ \text{mm}^2}$

6a) 2000 2001 2002

a ar ar^2

$a = 15000$, $r = 0.8$ } ar^2 = year 2002 $= 15000(0.8^2) = \boxed{£9600}$

b) $15000(0.8^N) < 500$ where N is the no. of years after 2000.

$0.8^N < \frac{1}{30}$

$N\log(0.8) < \log(\frac{1}{30})$

$N <> \frac{\log(\frac{1}{30})}{\log(0.8)}$ ← $\div \log(0.8)$ means the signs change as $\log 0.8 < 0$

$N > 15.2\ldots$ ∴ $\boxed{\text{year is 2015}}$

c) a ar ar^2 $a = 1000$

1000 1000(1.05) $1000(1.05^2)$ $r = 1.05$

16th term is when the machine is replaced.

∴ we need to find the sum of the first 16 terms.

$$S_{16} = \frac{1000(1.05^{16} - 1)}{1.05 - 1} = \boxed{£23657}$$

7a) let $f(x) = x^3 - x^2 - 10x - 8$

$f(-1) = -1 - 1 + 10 - 8 = 10 - 10 = 0$ //

$\therefore (x+1)$ is a factor.

b)

$$\begin{array}{r|l} & x^2 - 2x - 8 \\ \hline x+1 & x^3 - x^2 - 10x - 8 \\ & \underline{x^3 + x^2} \\ & 0 - 2x^2 - 10x \\ & \quad \underline{-2x^2 - 2x} \\ & \quad 0 \quad - 8x - 8 \\ & \quad\quad \underline{-8x - 8} \\ & \quad\quad 0 \quad 0 \end{array}$$

$\therefore f(x) = (x+1)(x^2 - 2x - 8)$

$x^2 - 2x - 8 = (x + 2)(x - 4)$

$\therefore f(x) = (x+1)(x+2)(x-4)$.

all solutions : $\boxed{\begin{array}{l} x = -1 \\ x = -2 \\ x = 4 \end{array}}$

c) $\log_2(x^2) + \log_2(x-1) - \log_2(5x+4) = 1$

$\log_2(x^2(x-1)) - \log_2(5x+4) = 1$

$\log_2\left(\dfrac{x^3 - x^2}{5x+4}\right) = 1$

$\therefore 2^1 = \dfrac{x^3 - x^2}{5x+4}$

$\therefore 10x + 8 = x^3 - x^2$

$\Rightarrow \quad x^3 - x^2 - 10x - 8 = 0$

d) $x = 4$ as $\log_a n$ where $n < 0$ is undefined.

[using $x = -1$ or $x = -2$ would yield a negative number inside the "log"].

~~e)~~

8a) $x^2 - 3x + 8 = x + 5$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 3, \; x = 1$

$y = 3 + 5 = 8 \qquad y = 1 + 5 = 6$

$\therefore A(1, 6)$ and $B(3, 8)$.

b) $\displaystyle\int_1^3 (y_2 - y_1)\, dx = \int_1^3 [x + 5 - x^2 + 3x - 8]\, dx$

$= \displaystyle\int_1^3 [-x^2 + 4x - 3]\, dx = \left[\frac{-x^3}{3} + 2x^2 - 3x\right]_1^3$

$= [-9 + 18 - 9] - \left[-\frac{1}{3} + 2 - 3\right] = \boxed{\frac{4}{3}}$

9a) $\text{Area} = \frac{1}{2}ab\sin C = \frac{1}{2}(x+1)(4-x)^2 \times \sin 30$

NOTE: error in diagram.
PQ length = $(x+1)$
not $(x+1)^2$

$= \frac{1}{4}[16-8x+x^2][x+1]$

$= \frac{1}{4}[16x+16-8x^2-8x+x^3+x^2]$

$= \frac{1}{4}[x^3-7x^2+8x+16]$ $\square$

b) $A = \frac{1}{4}(x^3-7x^2+8x+16)$

$\frac{dA}{dx} = \frac{1}{4}(3x^2-14x+8)$

at a max point, $\frac{dA}{dx} = 0$:

$3x^2-14x+8=0$

$(3x-2)(x-4)=0$

$x = \frac{2}{3}$ $\quad x = 4$

to find which is max, find $\frac{d^2A}{dx^2}$

$\frac{d^2A}{dx^2} = \frac{1}{4}(6x-14)$

$x=4$: $\frac{d^2A}{dx^2} = \frac{5}{2} > 0$ $\therefore$ minimum

$x=\frac{2}{3}$: $\frac{d^2A}{dx^2} = -\frac{5}{2} < 0$ $\therefore$ maximum

$\therefore$ Area is max when $x = \frac{2}{3}$.

c) $A_{max} = \frac{1}{4}\left(\left(\frac{2}{3}\right)^3 - 7\left(\frac{2}{3}\right)^2 + 8\left(\frac{2}{3}\right) + 16\right)$

$= \boxed{4.63}$

d) Cosine rule : $QR^2 = \left(\frac{5}{3}\right)^2 + \left(\frac{100}{9}\right)^2 - 2\left(\frac{5}{3}\right)\left(\frac{10}{3}\right)^2 \cos 30$

$QP = \frac{2}{3} + 1 = \frac{5}{3}$

$PR = \left(4 - \frac{2}{3}\right)^2 = \frac{100}{9}$

$QR^2 = 94.16\ldots$

$\therefore \boxed{QR = 9.70}$