C2 Mock (MA)

Q1a) $f(1) = 0$: $2 - 1 + p + 6 = 0$

$$p + 7 = 0$$

$$\therefore \boxed{p = -7}$$

b) $2x + 1 = 0$

$x = -\frac{1}{2}$: $f\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)^3 - \left(-\frac{1}{2}\right)^2 - \frac{1}{2}(-7) + 6$

$$= -\frac{1}{4} - \frac{1}{4} + \frac{19}{2} = \boxed{9}$$

2a) $\int \left[3 + 4x^3 + - 2x^{-2}\right] dx = \left[3x + x^4 + 2x^{-1}\right] + c$

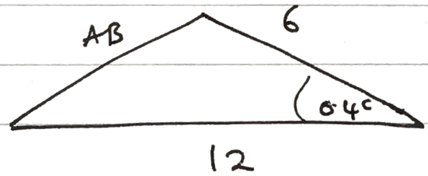
$$= \boxed{3x + x^4 + \frac{2}{x} + c}$$

b) $\left[3x + x^4 + \frac{2}{x}\right]_1^2 = \left[6 + 16 + 1\right] - \left[3 + 1 + 2\right]$

$$= 23 - 6 = \boxed{17}$$

3a) BD length $= r\theta = 6 \times 0.4 = \boxed{2.4\text{cm}}$

b)



cosine rule

$AB^2 = 6^2 + 12^2 - 2(6)(12)\cos(0.4^c)$

$= 47.37\ldots$

$\therefore AB = \boxed{6.88\text{cm}}$

c) $P = 6.88 + 6 + 2.4 = \boxed{15.3\text{cm}}$

4) $\log_3(x^2) - \log_3(x-2) = 2$

$$\log_3\left(\frac{x^2}{x-2}\right) = 2$$

$$3^2 = \frac{x^2}{x-2} = 9$$

$$\therefore x^2 = 9(x-2)$$

$$x^2 = 9x - 18$$
$$x^2 - 9x + 18 = 0$$
$$(x-6)(x-3) = 0$$
$$\boxed{x=6,\ x=3}$$

5a) $a \quad ar \quad ar^2 \quad ar^3 \quad ar^4$

$$\left.\begin{array}{l} ar = 9 \\ ar^4 = 1.125 \end{array}\right\} \frac{ar^4}{ar} = r^3 = \frac{1.125}{9}$$

$$\therefore r = \sqrt[3]{\frac{1.125}{9}} = \sqrt[3]{\frac{1}{8}} = \boxed{\frac{1}{2}}$$

b) $a = \frac{9}{r} = \frac{9}{\frac{1}{2}} = \boxed{18}$

c) $S_\infty = \frac{a}{1-r} = \frac{18}{1-\frac{1}{2}} = \frac{18}{\frac{1}{2}} = \boxed{36}$

6a) completing square: $(x-3)^2 - 9 + (y+2)^2 - 4 - 12 = 0$

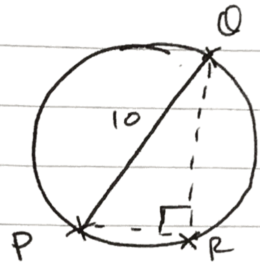
$$\therefore (x-3)^2 + (y+2)^2 = 25$$

So $A(3,-2)$

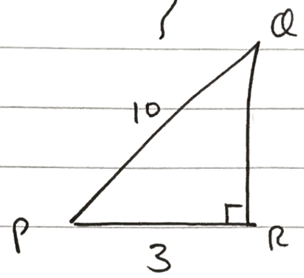
b) $r = \sqrt{25} = \boxed{5}$ [from a]

c)

c)



PQ length 10 means that PQ is a diameter. (as $r = 5$).



Angle PRQ will be 90° due to circle theorem (angle in semicircle)

$\therefore QR = \sqrt{10^2 - 3^2} = \sqrt{91} = \boxed{9.5}$

(Pythagoras).

7a) $(1+kx)^n = 1^n + \binom{n}{1}(1)^{n-1}(kx)^1 + \binom{n}{2}(1)^{n-2}(kx)^2 + \binom{n}{3}(1)^{n-3}(kx)^3$

$(1+kx)^n = 1 + nkx + \frac{n(n-1)}{2}k^2x^2 + \frac{n(n-1)(n-2)}{6}k^3x^3$

$\boxed{\binom{n}{1} = n}$

$\boxed{\binom{n}{2} = \frac{n(n-1)}{2!}}$

$\boxed{\binom{n}{3} = \frac{n(n-1)(n-2)}{3!}}$

$\frac{n(n-1)}{2}k^2 = \frac{n(n-1)(n-2)}{6}k^3$

$\boxed{\div n(n-1)k^2}$: $\frac{1}{2} = \frac{(n-2)}{6}k$.

$\times 6$: $\therefore 3 = (n-2)k$

b) $\underline{A=4}$: $4 = nk$.

from a, $k = \frac{3}{n-2}$: $4 = \frac{3n}{n-2}$

$\Rightarrow 4n - 8 = 3n$

$\Rightarrow \boxed{n = 8}$ $\therefore k = \frac{3}{8-2} = \boxed{\frac{1}{2}}$

$\underline{0 \leq x < 360^\circ}$

8a) $\cos(x - 20^\circ) = -0.437$

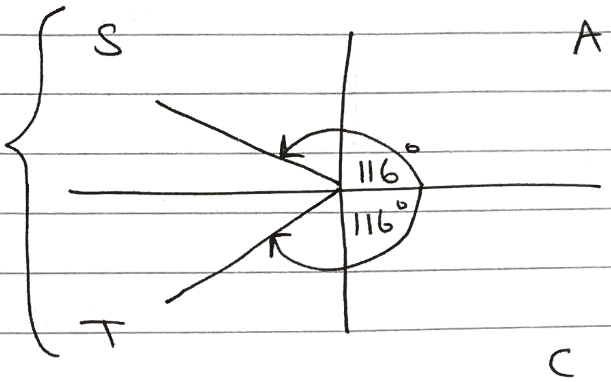
$\cos^{-1}(-0.437) = x - 20 = 115.9^\circ$

$\underline{\text{new range}}$: $-20^\circ \leq x - 20 < 340$

$x - 20 = 116^\circ, 360 - 116^\circ$

$x - 20 = 116^\circ, 244^\circ$

$\boxed{x = 136^\circ, 264^\circ}$



b) $\dfrac{3\sin\theta}{\cos\theta} = 2\cos\theta$

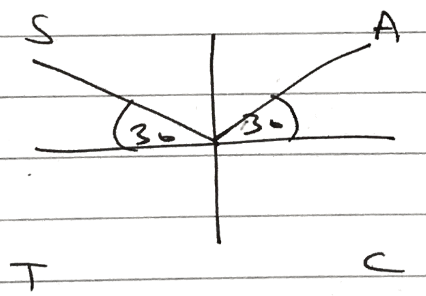
$\times\cos\theta$: $3\sin\theta = 2\cos^2\theta$

$3\sin\theta = 2 - 2\sin^2\theta$

$2\sin^2\theta + 3\sin\theta - 2 = 0$

$(2\sin\theta - 1)(\sin\theta + 2) = 0$

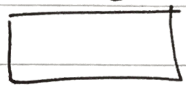
$\sin\theta = \frac{1}{2} \therefore \theta = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$ // | $\sin\theta = -2$ × reject. no valid solutions.



$\Rightarrow \boxed{\theta = 30^\circ, 150^\circ}$

9a) $V = \pi r^2 h$

$S.A = \underbrace{\pi r^2}_{\text{base}} + \underbrace{2\pi r h}_{\text{'rectangle bit'}} = 250$



$\therefore \quad 2\pi r h = 250 - \pi r^2$

$$h = \frac{250 - \pi r^2}{2\pi r}$$

<u>substitute into V eqn</u>: $V = \pi r^2 \left(\frac{250 - \pi r^2}{2\pi r} \right)$

$$\therefore V = \frac{(250 - \pi r^2) r}{2} = 125r - \frac{\pi r^3}{2}$$

b) $\frac{dV}{dr} = 125 - \frac{3\pi r^2}{2} = 0$

$$125 = \frac{3\pi r^2}{2}$$

$$\Rightarrow \frac{250}{3\pi} = r^2 \quad \therefore r = \sqrt{\frac{250}{3\pi}} = 5.15 \text{ cm}$$

c) $\frac{d^2V}{dr^2} = -3\pi r < 0$ (always since $r > 0$)

$\therefore$ the value of r we found gives a <u>max</u> value for V.

d) <u>$r = 5.15$</u>: $V = 125(5.15) - \frac{\pi}{2}(5.15)^3 = \boxed{429 \text{ cm}^3}$

10a) $\underline{x=4}$: $y = 9 - 2(4) - 1 = 0$ //

hence $b = 4$.

b) $\frac{dy}{dx} = -2 + x^{-3/2}$

$\underline{\text{at } x=1}$: $\frac{dy}{dx} = -2 + 1 = -1$ //

$\therefore \quad y - 5 = -1(x-1)$

$y = -x + 1 + 5$

$y = -x + 6$

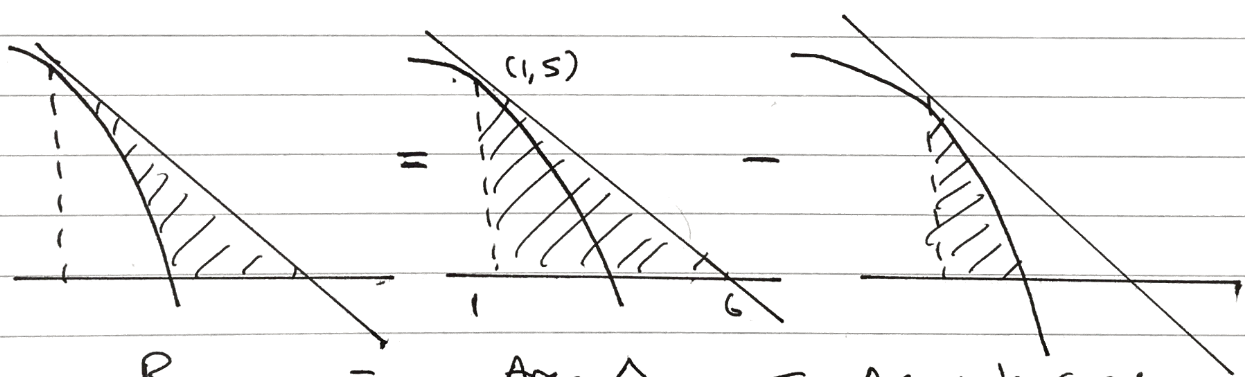
$\therefore \quad y + x = 6$

□

c) $\underline{y=0}$: $x = 6$ (from b).

$\therefore D(6, 0)$

d)



R = Area △ − Area under curve

$$R = \frac{1}{2}(6-1)(5) - \int_1^4 \left[9 - 2x - 2x^{-\frac{1}{2}}\right] dx$$

$$R = \frac{25}{2} - \left[9x - x^2 - 4x^{\frac{1}{2}}\right]_1^4 = \frac{25}{2} - [12] + [4]$$

$$= \boxed{4.5}$$