JUNE 2013 C2

1a)

$a = 18 \quad ar = 12 \quad ar^2 = p$

$18r = 12$

$r = \frac{2}{3}$

b) $p = 18\left(\frac{2}{3}\right)^2 = 8$

c) $S_{15} = \frac{a(1-r^{15})}{1-r} = \frac{18(1-\frac{2}{3}^{15})}{1-\frac{2}{3}} = \underline{53.877}$

2a)

$(2+3x)^4$

$= 2^4 + \binom{4}{1}2^3(3x) + \binom{4}{2}2^2(3x)^2 + \binom{4}{3}2(3x)^3 + \binom{4}{4}(3x)^4$

$= 16 + 96x + 216x^2 + 216x^3 + 81x^4$

b) $16 - 96x + 216x^2 - 216x^3 + 81x^4$

3a) $(x-3)$ is a factor $\therefore f(3) = 0$ let $\underline{x = 3}$

$2(3)^3 - 5(3)^2 + ax + 18 = 0$

$ax = -27$

$3a = -27$

$\underline{a = -9}$

b)

$$
\begin{array}{r|l}
 & 2x^2 + x - 6 \\
x-3 & 2x^3 - 5x^2 - 9x + 18 \\
 & 2x^3 - 6x^2 \\
 & \quad x^2 - 9x \\
 & \quad x^2 - 3x \\
 & \quad\quad -6x + 18 \\
 & \quad\quad -6x + 18 \\
 & \quad\quad\quad 0
\end{array}
$$

$(x-3)(2x^2+x-6)$

$= (x-3)(2x-3)(x+2)$

c) from equating coeffs

$$x = 3^y$$

$x = 3$ or $\frac{3}{2}$ or -2

$3 = 3^y$ $\quad \underline{y = 1}$

$\frac{3}{2} = 3^y \Rightarrow \log_3 \frac{3}{2} = y \quad = \underline{0.37}$

~~to~~

$\cancel{-2 = 3^y \Rightarrow \log_3 -2 = y}$

4a) $y = \dfrac{5}{x^2+1}$

when $x = 1.5$ $\quad y = \underline{1.538}$

b) $\frac{1}{2} 0.5 \left[(5+0.5) + 2(4+2.5+1.538+1+0.690)\right]$

$= \underline{6.239}$

c) $\displaystyle\int_0^3 4 + \frac{5}{x^2+1}\,dx$

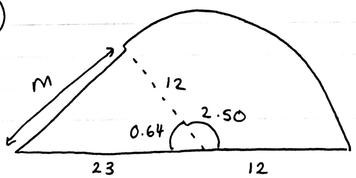
$= \displaystyle\int_0^3 4\,dx + \int_0^3 \frac{5}{x^2+1}\,dx$

$\left[4x\right]_0^3 + 6.239$

$(12-0) + 6.239$

$= 18.239 = \underline{18.24}$ (2 dp)

5)



a) Area of sector $= \frac{1}{2}r^2\theta = \frac{1}{2}(12)^2(2.50) \simeq 180.11$

area of triangle $= \frac{1}{2}ab\sin c = \frac{1}{2}(23)(12)\sin(0.64)$

$= 82.41$

$180.11 + 82.41 = \underline{262.5\ m^2}$ (1dp)

b) length of arc $= r\theta \quad = 12(2.5) = \cancel{30}\ 30$

$30 + 12 + 23 + M$

$m^2 = b^2 + c^2 - 2bc\cos M$

$m^2 = 23^2 + 12^2 - 2(12)(23)\cos(0.64)$

$m^2 = 230.2431416\ldots$

$m = 15.17 \simeq 15.2$

perimeter $= 30 + 12 + 23 + 15.2 \quad = \underline{80.2}$ (1 dp)

6) $y = x(x+4)(x-2) \quad = x(x^2 + 2x - 8)$

$0 = x \qquad x = -4 \qquad x = 2$

$A\ x = -4 \qquad B\ x = 2$

b) $\displaystyle\int_{-4}^{0} x(x+4)(x-2)\,dx + \int_{0}^{2} x(x+4)(x-2)\,dx$

$\displaystyle\int_{-4}^{0} x^3 + 2x^2 - 8x\,dx + \int_{0}^{2} x^3 + 2x^2 - 8x\,dx$

$\displaystyle\left[\frac{1}{4}x^4 + \frac{2}{3}x^3 - 4x^2\right]_{-4}^{0} + \left[\frac{1}{4}x^4 + \frac{2}{3}x^3 - 4x^2\right]_{0}^{2}$

$\displaystyle\left|0 - -\frac{128}{3}\right| + \left|-\frac{20}{3} - 0\right|$

$\displaystyle = \frac{128}{3} + \frac{20}{3} = \cancel{36}\ \frac{148}{3}$

7i) $\log_2 2x = \log_2 (5x+4) - 3$

$$3 = \log_2 (5x+4) - \log_2 2x$$

$$3 = \log_2 \frac{5x+4}{2x}$$

$$2^3 = \frac{5x+4}{2x}$$

$$8(2x) = 5x+4$$

$$11x = 4$$

$$x = \frac{4}{11}$$

ii) $\log_a y + 3\log_a 2 = 5$

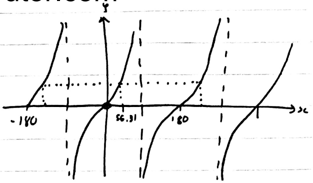
$$\log_a y + \log_a 8 = 5$$

$$\log_a 8y = 5$$

$$a^5 = 8y$$

$$y = \frac{a^5}{8}$$

8 i) $-180 \le x < 180$



$\tan(x-40) = 1.5$

$x - 40 = 56.31$

$180 - 56.31 = 123.69$

$x - 40 = 56.31, -123.69$

$x = 96.31, -83.69$

$\underline{x = 96.3, -83.7}$

ii) $\sin\theta \tan\theta = 3\cos\theta + 2$

$\sin\theta \dfrac{\sin\theta}{\cos\theta} = 3\cos\theta + 2$

$\sin^2\theta = 3\cos^2\theta + 2\cos\theta$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$1 - \cos^2\theta = 3\cos^2\theta + 2\cos\theta$

$= 4\cos^2\theta + 2\cos\theta - 1 = 0$

b) from (iia), the equation we need to solve is: $[4\cos^2\theta + 2\cos\theta - 1 = 0]$

let $m = \cos\theta$; $4m^2 + 2m - 1 = 0$

By Quadratic formula...

$$m = \frac{-2 \pm \sqrt{2^2 - 4(4)(-1)}}{8} = \frac{-2 \pm 2\sqrt{5}}{8}$$

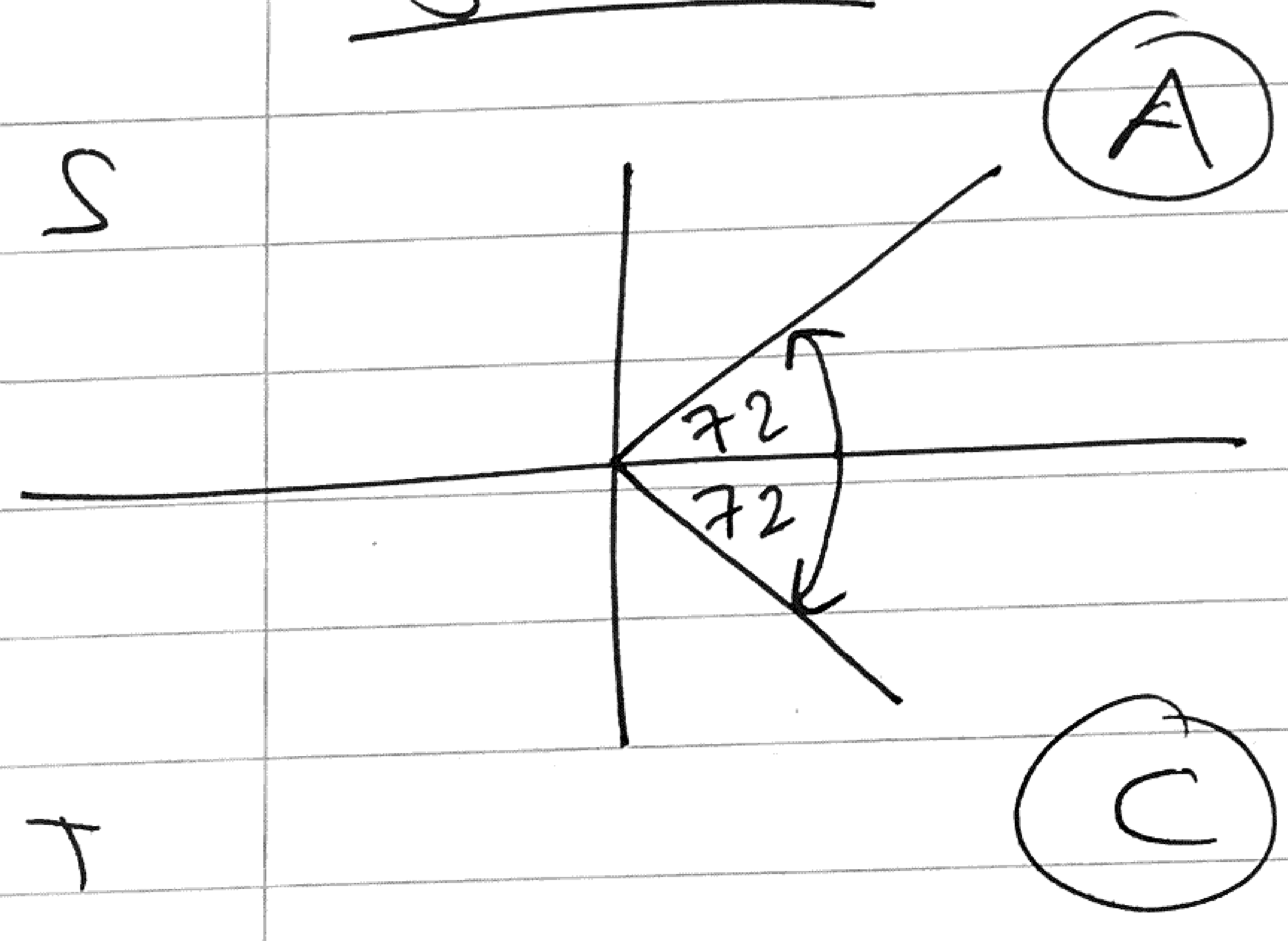
$$\therefore \quad m = \frac{-1 \pm \sqrt{5}}{4}$$

so we acquire two 'sets' of solutions:

$\cos\theta = \frac{-1+\sqrt{5}}{4}$

$\theta = \cos^{-1}\left(\frac{-1+\sqrt{5}}{4}\right) = 72°$

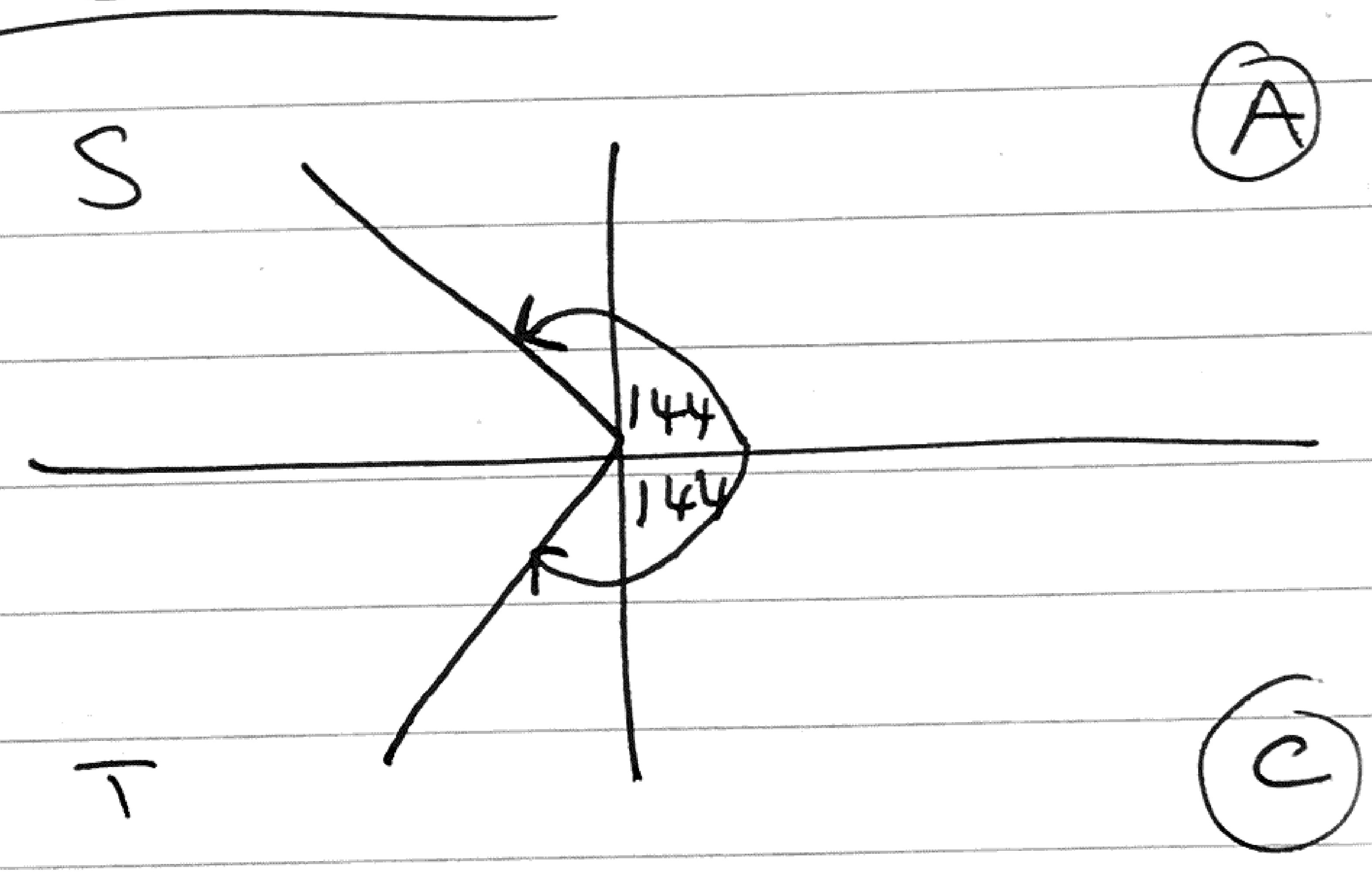
By CAST



$\cos\theta = \frac{-1-\sqrt{5}}{4}$

$\theta = \cos^{-1}\left(\frac{-1-\sqrt{5}}{4}\right) = 144°$

By CAST



$\theta = 72°, (360 - 72)°$ $\qquad$ $\theta = 144°, (360 - 144)°$

$$\Rightarrow \theta = \boxed{72°, 144°, 216°, 288°}$$

a graphical method may also be employed to find the solutions.

9a) $y = x^2 - 32x^{\frac{1}{2}} + 20$

$\therefore \frac{dy}{dx} = 2x - 32\left(\frac{1}{2}\right)x^{-\frac{1}{2}}$

P is a stationary point $\therefore \frac{dy}{dx} = 0$ at P.

$\Rightarrow \quad 2x - 16x^{-\frac{1}{2}} = 0$

$\Rightarrow \quad 2x = \frac{16}{\sqrt{x}} \longrightarrow x = \frac{8}{\sqrt{x}}$

$\Rightarrow \quad x^{\frac{3}{2}} = 8 \qquad \therefore \sqrt[\frac{3}{2}]{8} = x = (\sqrt{8})^3 = 4$ //

Substitute $x = 4$ into given eqn : $y = -28$ //

$\therefore P(4, -28)$

b) $\frac{dy}{dx} = 2x - 16x^{-1/2}$

$\therefore \frac{d^2y}{dx^2} = 2 + 8x^{-3/2} = 2 + \frac{8}{x^{3/2}} > 0$

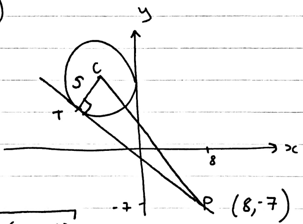
$\therefore \frac{d^2y}{dx^2} > 0 \quad \therefore$ P is a minimum.

10 a) centre $= (-5, 9)$

$r = 5$

$(x+5)^2 + (y-9)^2 = 25$

b) $P(-5, 9)$



$PC = \sqrt{(8+5)^2 + (-7-9)^2}$

$= 5\sqrt{17}$

$PT = \sqrt{(5\sqrt{17})^2 - 5^2} \quad = \underline{20}$