C2 June 2010 (MA)

Q1a)

x	0	0.2	0.4	0.6	0.8	1
y	1	1.65	2.35	3.13	4.01	5

b) $h = \frac{b-a}{n} = \frac{1-0}{5} = \frac{1}{5}$

$$\text{Area} \simeq \frac{1}{2} \times \frac{1}{5}\left[1+5+2(1.65+2.35+3.13+4.01)\right]$$

$$\simeq \boxed{2.83}$$

Q2a) $f(3) = 3(3^3) - 5(3^2) - 58(3) + 40$

$$= 36 + 40 - 174 = \boxed{-98}$$

b)

$$\begin{array}{r} 3x^2 + 10x - 8 \\ x-5 \overline{\smash{)}\,3x^3 - 5x^2 - 58x + 40} \\ \underline{3x^3 - 15x^2} \\ 0 \quad 10x^2 - 58x \\ \underline{10x^2 - 50x} \\ 0 \quad -8x + 40 \\ \underline{-8x + 40} \\ 0 \quad 0 \end{array}$$

$\therefore f(x) = (x-5)(3x^2+10x-8)$

but $3x^2+10x-8 = (3x-2)(x+4)$

$\therefore f(x) = (x-5)(3x^2+10x-8) = 0$

$= \boxed{(x-5)(3x-2)(x+4)} = 0$

$x-5=0 \rightarrow \boxed{x=5}$

$3x-2=0 \rightarrow \boxed{x=\frac{2}{3}}$

$x+4=0 \rightarrow \boxed{x=-4}$

3a) $y = x^2 - kx^{\frac{1}{2}}$

$\frac{dy}{dx} = 2x - \frac{1}{2}kx^{-\frac{1}{2}}$

b) at $x = 4$, $\frac{dy}{dx} < 0$

$\left(\frac{dy}{dx}\right)_{x=4} = 8 - \frac{1}{2}k\left(4^{-\frac{1}{2}}\right) = 8 - \frac{1}{4}k$

$\therefore \quad 8 - \frac{k}{4} < 0$

$\frac{k}{4} > 8 \quad \therefore \boxed{k > 32}$

4a) $(1+ax)^7 \simeq (1)^7 + \binom{7}{1}(1)^6(ax) + \binom{7}{2}(1)^5(ax)^2 + \binom{7}{3}(1)^4(ax)^3$

$(1+ax)^7 \simeq \underline{1 + 7ax + 21a^2x^2 + 35a^3x^3}$

b) $21a^2 = 525$

$a^2 = \frac{525}{21} = 25$

$a = \pm\sqrt{25} = 5, -5$

$\boxed{\begin{array}{l} a = 5 \\ a = -5 \end{array}}$

5a) $5\sin\theta = 2\cos\theta$

$\div\cos\theta$: $5\tan\theta = 2$

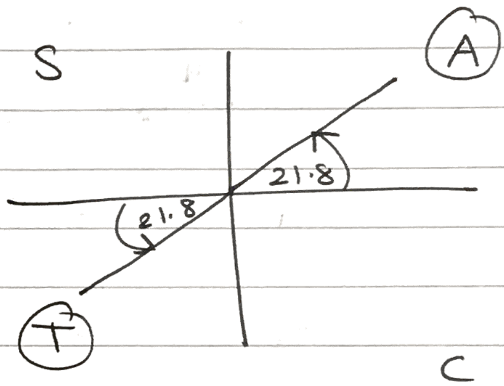
$\boxed{\tan\theta = \frac{2}{5}}$

b)

from (a), $\tan 2x = \frac{2}{5}$

$2x = \tan^{-1}\left(\frac{2}{5}\right) = 21.80°$

<u>solving in</u> : $0 \leq 2x \leq 720°$



$2x = 21.8°, (180 + 21.8), (381.8°), (540 + 21.8°)$

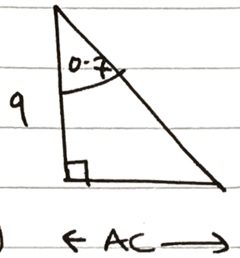
$2x = 21.8°, 201.8°, 381.8°, 561.8°$

$\boxed{x = 10.9°, 100.9°, 190.9°, 280.9°}$

6a) length $= r\theta = 9 \times 0.7 = \boxed{6.3\text{cm}}$

b) Area $= \frac{1}{2}r^2\theta = \frac{1}{2}(9^2)(0.7) = \boxed{28.35\text{cm}^2}$

c) $\tan(0.7) = \frac{AC}{9}$

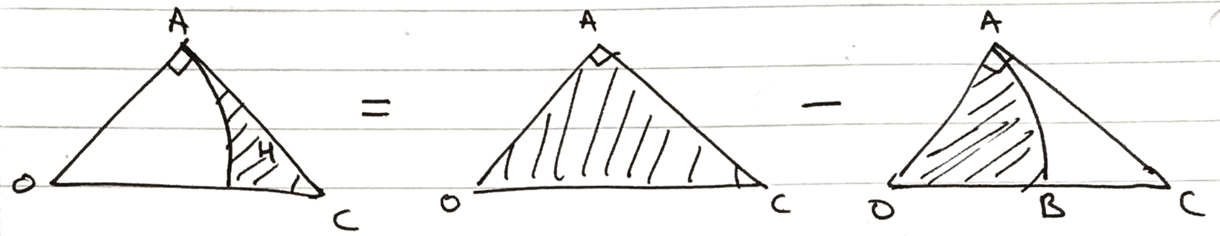


$\tan x = \frac{\text{opposite}}{\text{adjacent}}$.

$\therefore AC = 9\tan(0.7)$

$= \boxed{7.58\text{cm}}$

d)



$H = \text{Area } \Delta OAC - \text{Area } OAB$

$H = \frac{1}{2}(7.58)(9) - 28.35$

$H = 34.11 - 28.35$

$H = \boxed{5.76\text{cm}^2}$

7a) $\log_3[(x-5)^2] - \log_3[2x-13] = 1$

$$\log_3\left[\frac{(x-5)^2}{2x-13}\right] = 1$$

$$3^1 = \frac{(x-5)^2}{2x-13}$$

$$6x - 39 = x^2 - 10x + 25$$

$$x^2 - 16x + 25 + 39 = 0$$

$$x^2 - 16x + 64 = 0$$

$\square$

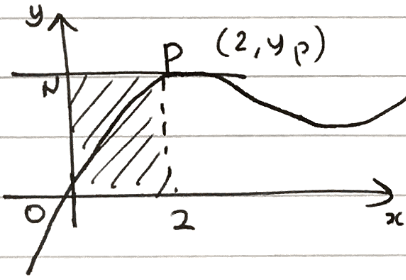
b) $(x-8)^2 = 0$

$$\boxed{x = 8}$$

8a) $\frac{dy}{dx} = 3x^2 - 20x + k = 0$

$\underline{x=2}$: $6(2) - 20(2) + k = 0$

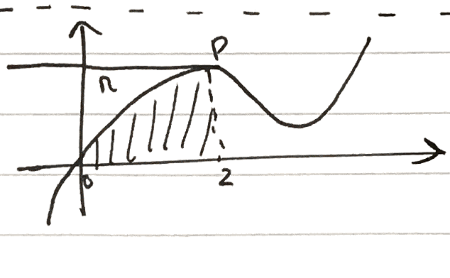
$k = 40 - 12 = \boxed{28}$

b) 

Area of rectangle $= 2 \times y_P$.

$\underline{x=2}$: $y_P = 8 - 40 + 28(2) = 24$

$\therefore$ Area $\square = 2 \times 24 = 48$ units^2



Area under curve $= \int_0^2 [x^3 - 10x^2 + 28x]\,dx$

Area under curve $= \displaystyle\int_0^2 \left[x^3 - 10x^2 + 28x\right] dx$

$$= \left[\frac{x^4}{4} - \frac{10x^3}{3} + 14x^2\right]_0^2 = \left[\frac{100}{3}\right] - \left[0\right]$$

$$= \frac{100}{3}$$

$H =$ Area $\square$ $-$ Area under curve

$H = 48 - \dfrac{100}{3} = \boxed{\dfrac{44}{3}}$ units^2

a	ar	ar^2	...
end 1st	end 2nd	end 3rd	...

9a) $a = 25000$, $r = 1.03$ } $ar = 25000 \times 1.03 = \boxed{25750}$

b) $r = \boxed{1.03}$

c) $ar^{N-1} > 40000$

$25000(1.03)^{N-1} > 40000$

$\div 25000$: $(1.03)^{N-1} > \frac{8}{5}$

$\log(1.03^{N-1}) > \log(\frac{8}{5})$

$(N-1)\log(1.03) > \log(\frac{8}{5})$

d) $N-1 > \frac{\log(8/5)}{\log(1.03)}$ $\therefore N-1 > 15.9$

$N > 16.9\ldots$

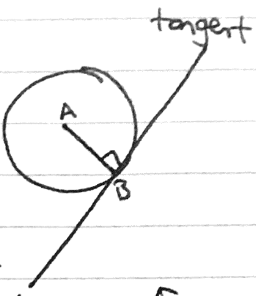
$\therefore \boxed{N = 17}$

e) [sum of first ten terms] $\times 1$ = required value.

$S_{10} = \frac{25000(1-1.03^{10})}{1-1.03} = \boxed{£287000}$ to the nearest £1000

Q10a) radius $= |AB| = \sqrt{(10-2)^2 + (7-1)^2} = 10$

$$\therefore \boxed{(x-2)^2 + (y-1)^2 = 100}$$



b) $M_{AB} = \frac{7-1}{10-2} = \frac{6}{8} = \frac{3}{4}$ //

$\therefore$ gradient of tangent will be $-\frac{4}{3}$ //

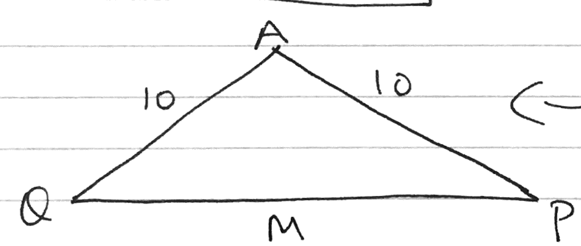
[tangent is perpendicular to AB]

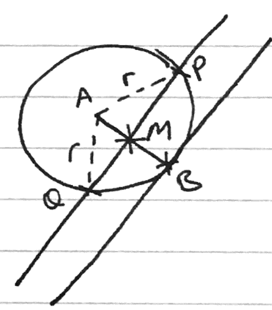
$$y - 7 = -\frac{4}{3}(x - 10)$$

$$y = -\frac{4}{3}x + \frac{40}{3} + 7$$

$$\boxed{y = -\frac{4}{3}x + \frac{61}{3}}$$

c)



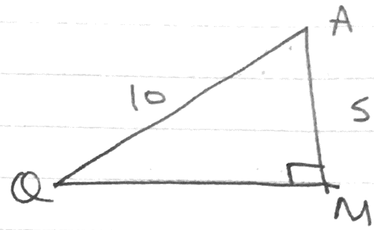


let M be midpoint of AB,

$$M = \left(\frac{2+10}{2}, \frac{1+7}{2}\right) = (6, 4)$$ //

$$\therefore |AM| = \sqrt{(6-2)^2 + (4-1)^2} = 5$$ //

now consider ΔAMQ :



$|AMQ| = \sqrt{(10)^2 - (5)^2} = 5\sqrt{3}$ and $|MQ| = |MP|$

so distance from P to $Q = 2 \times 5\sqrt{3}$

$= \boxed{10\sqrt{3}}$