JUNE 2006 C2

1)

$$(2+x)^6 = 2^6 + \binom{6}{1}2^5x + \binom{6}{2}2^4x^2$$

$$= \underline{\underline{64 + 192x + 240x^2}}$$

2) $\int_1^2 \left(3x^2 + 5 + \frac{4}{x^2}\right)dx$

$$= \int_1^2 (3x^2 + 5 + 4x^{-2})\,dx$$

$$= \left[x^3 + 5x - \frac{4}{x}\right]_1^2$$

$$= 16 - 2 = \underline{\underline{14}}$$

3i) $\log_6 36 = \underline{2}$

ii) $2\log_a 3 + \log_a 11$

$$= \log_a 3^2 + \log_a 11$$

$$= \log_a(3^2 \times 11) = \log_a(9\times 11) = \log_a(99)$$

4a)

$$\begin{array}{r|l} & 2x^2 - x - 27 \\ x+2 & 2x^3 + 3x^2 - 29x - 60 \\ & 2x^3 + 4x^2 \\ & \quad -1x^2 - 29x \\ & \quad -1x^2 - 2x \\ & \qquad -27x - 60 \\ & \qquad -27x - 54 \end{array}$$

remainder = ~~66~~ $\underline{-6}$

b) if $x+3$ is a factor

$f(-3)$ must $= 0$

$f(-3) = 2(-3)^3 + 3(-3)^2 - 29(-3) - 60$
$= -54 + 27 + 87 - 60 = \underline{\underline{0}}$

$\therefore$ $(x+3)$ is a factor

c)

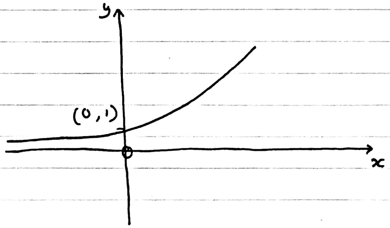
$$\begin{array}{r l}
 & 2x^2 - 3x - 20 \\
x+3 \,\big) & 2x^3 + 3x^2 - 29x - 60 \\
 & 2x^3 + 6x^2 \\
 & \quad -3x^2 - 29x \\
 & \quad -3x^2 - 9x \\
 & \qquad -20x - 60 \\
 & \qquad -20x - 60 \\
 & \qquad\qquad 0
\end{array}$$

$(x+3)(2x^2 - 3x - 20)$

$\to (x^2 - 3x - 40)$
$(x-8)(x+5)$
$(x-4)(2x+5)$

$\boxed{= (x+3)(x-4)(2x+5)}$

5a) $y = 3^x$ when $x=0$ $y = 3^0 = 1$



b)

x	0	0.2	0.4	0.6	0.8	1
3^x	1	1.246	1.552	1.933	2.408	3.000

c) $\frac{1}{2} \times 0.2 \left[(1+3) + 2(1.246 + 1.552 + 1.933 + 2.408)\right]$

$0.1(4 + 14.278) = 1.8278$

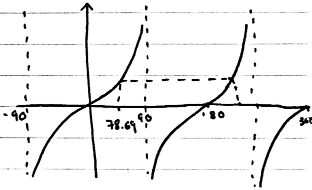
6a) $\sin\theta = 5\cos\theta \Rightarrow \frac{\sin\theta}{\cos\theta} = 5$

$\underline{\tan\theta = 5}$

b) $\theta = \tan^{-1}(5) = 78.69$

$180 + 78.69 = 258.69...$

$\theta = 78.7, 258.7$ (1.d.p)



7a) PQ is perpendicular to y $\therefore m = -\frac{1}{3}$

$y = 3x - 4$

$\frac{dy}{dx} = 3$

PQ: $y - y_1 = m(x - x_1)$

$y - 2 = -\frac{1}{3}(x - 2)$

$y = -\frac{1}{3}x + \frac{2}{3} + 2$

$y = -\frac{1}{3}x + \frac{8}{3}$

b) $y = -\frac{1}{3}x + \frac{8}{3}$

when $y = 1$

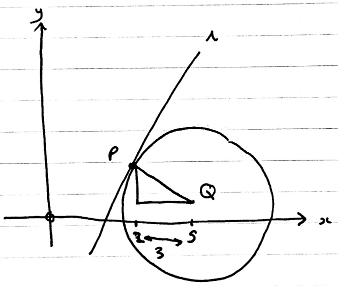
$1 - \frac{8}{3} = -\frac{1}{3}x$

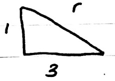
$-\frac{5}{3} = -\frac{1}{3}x \quad \times -1$

$\frac{5}{3} = \frac{1}{3}x \quad \times 3$

$\underline{5 = x}$

c) centre $= (5, 1)$





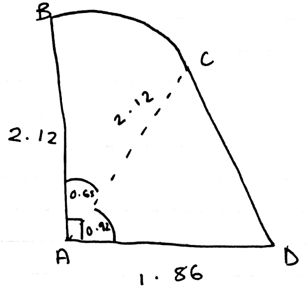
$r = \sqrt{1^2 + 3^2}$

$= \sqrt{10}$

$\therefore r^2 = \sqrt{10}^2 = 10$

$$\boxed{(x-5)^2 + (y-1)^2 = 10}$$

8)



a) arc length $= r\theta = 2.12 \times 0.65$
$= \underline{1.38\text{m}}$

b) area of sector $= \frac{1}{2}r^2\theta = \frac{1}{2}(2.12)^2 \times 0.65$

$= \underline{1.46\text{m}^2}$

c) $\frac{\pi}{2} - 0.65 = \underline{0.92}$ rad

d) Area of triangle $= \frac{1}{2}ab\sin c = \frac{1}{2} \times 2.12 \times 1.86 \times \sin(0.92)$

$= 1.5696\ldots$

total $= 1.5696 + 1.46 = \underline{3.03\text{m}^2}$

9a) $ar = 4$

$a = \frac{4}{r} \quad \dots (1)$

$S_\infty = \frac{a}{1-r} = 25$

$a = 25(1-r)$

$a = 25 - 25r \dots (2)$

equate (1) and (2)

$\frac{4}{r} = 25 - 25r$

$4 = 25r - 25r^2$

$\therefore \quad 25r^2 - 25r + 4 = 0$

b)

$25r^2 - 25r + 4 = 0$

$r^2 - 25r + 100 = 0$

$(r - 20)(r - 5)$

$(r - \frac{4}{5})(25r - 5)$

$r = \frac{4}{5} \quad \text{or} \quad r = \frac{1}{5}$

c) $a = 4 \div \frac{4}{5} = 5 \quad \text{or} \quad a = 4 \div \frac{1}{5} = 20$

$\boxed{a = 5 \text{ or } 20}$

d) $S_\infty = \frac{a}{1-r} = 25$

$S_n = \frac{a(1-r^n)}{1-r} = \frac{a}{1-r} \times (1-r^n) = 25(1-r^n)$

e) $r = \frac{4}{5} \quad \therefore \quad a = 5$

$25(1-0.8^n) > 24$

$-0.8^n > -0.04$

$0.8^n < 0.04$

$\log_{0.8} 0.04 < n$

$14.425\ldots < n$

$\therefore n = 15$ ← next integer

10 a) $y = x^3 - 8x^2 + 20x$

stationary $\therefore \frac{dy}{dx} = 0$

$$\frac{dy}{dx} = 3x^2 - 16x + 20 = 0$$

$$x^2 - 16x + 60 = 0$$

$$(x-6)(x-10)$$

$$(x-2)(3x-10)$$

$$x = 2 \quad \text{or} \quad x = \frac{10}{3}$$

$A = 2 \qquad B = \frac{10}{3}$

b) $\frac{d^2y}{dx^2} = 6x - 16$

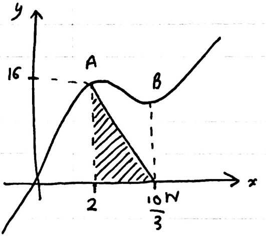
when $x = 2 \qquad 12 - 16 = -4$

$\therefore \frac{d^2y}{dx^2} < 0 \quad \therefore$ A is a maximum

c) $\int x^3 - 8x^2 + 20x \; dx$

$= \frac{1}{4}x^4 - \frac{8}{3}x^3 + 10x^2 + c$

d) when $x = 2 \qquad y = 16 \qquad A(2, 16)$



R = area of triangle + $\int_0^2$

$= \frac{1}{2} \times 16 \times \frac{4}{3} + {}_0^2\left[\frac{1}{4}x^4 - \frac{8}{3}x^3 + 10x^2\right]$

$= \frac{32}{3} + \frac{68}{3} = \boxed{\frac{100}{3}}$