June 2005 C2

1) $y = 2x^2 - 12x$

$\frac{dy}{dx} = 4x - 12$

when $\frac{dy}{dx} = 0$ $\quad$ $4x = 12$

$\underline{x = 3}$

when $x = 3$

$y = 2(3)^2 - 12(3)$

$y = -18$

$\boxed{(3, -18)}$

2) a) $5^x = 8$

$\log_5 8 = x = 1.29$ (3.s.f)

b) $\log_2(x+1) - \log_2 x = \log_2 7$

~~$\log_2 x \times \log_2 1 - \log_2 x = \log_2 7$~~

$\log_2\left(\frac{x+1}{x}\right) = \log_2 7$

$\frac{x+1}{x} = 7$

$x+1 = 7x$

$1 = 6x$

$\boxed{x = \frac{1}{6}}$

a) if $(x+4)$ is a factor
$f(-4)$ must equal 0

$2(-4)^3 + (-4)^2 - 25(-4) + 12$
$= -128 + 16 + 100 + 12 \quad = 0 \quad \therefore (x+4)$ is a factor

b)
$$\begin{array}{r} 2x^2 - 7x + 3 \\ x+4 \overline{)\, 2x^3 + x^2 - 25x + 12} \\ 2x^3 + 8x^2 \\ -7x^2 - 25x \\ -7x^2 - 28x \\ 3x + 12 \\ 3x + 12 \\ \hline 0 \end{array}$$

$(x+4)(2x^2 - 7x + 3)$

$x^2 - 7x + 6$
$(x-1)(x-6)$
$(2x-1)(x-3)$

$= (x+4)(2x-1)(x-3)$

4a) $(1+px)^{12} = 1 + 12(px) + \frac{12(12-1)}{2!}(px)^2$

$= 1 + 12px + 66p^2x^2$

b) coeff of $x = -q \quad \therefore 12p = -q \quad \ldots (1)$

coeff of $x^2 = 11q \quad \therefore 66p^2 = 11q \quad \ldots (2)$
from (1) : $-12p = q$
from (2) : $6p^2 = q$

equate : $-12p = 6p^2 \Rightarrow \frac{-12}{6} = p \quad \underline{p = -2}$

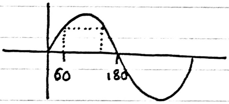
sub $\boxed{p = -2}$
into ①

$12(-2) = -q$
$-24 = -q$
$\boxed{q = 24}$

5
a) $\sin(x+10) = \frac{\sqrt{3}}{2}$

$x+10 = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60$

$x = 60 - 10 = 50$

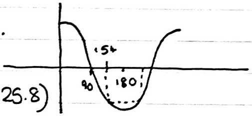


$180 - 60 = 120$
$\therefore x+10 = 120$
$x = 110$

$\boxed{x = \cancel{60}\ 50,\ 110}$

b) $\cos 2x = -0.9$
$2x = \cos^{-1}(-0.9) \simeq 154.158\ldots$

$180 - 154\ldots = 25.84\ldots$



$2x = 154.2\ldots, (180 + 25.8)$

$2x = 154.2, 205.8)$
$\boxed{x = 77.1, 102.9}$

$y = \frac{4}{10} x\sqrt{(20-x)}$

6a)

x	0	4	8	12	16	20
y	0	1.600	2.771	3.394	3.200	0

$h = 4 - 0$ or $8 - 4 = \underline{\underline{4}}$

$= \frac{1}{2}4\left[(0+0) + 2(1.6 + 2.771 + 3.394 + 3.2)\right]$

$= 2 \times 21.93 = 43.86$

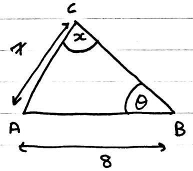
b)

$V = A \times 2 \times 60 \leftarrow$ 60 because in a min <u>not</u> 1 sec

$V = 43.86 \times 2 \times 60$

$= 5263.2 \approx 5260$ (3.s.f)

7)



$\theta = 0.5$ rad

$\frac{\sin x}{8} = \frac{\sin(0.5)}{7}$

$\sin x = \frac{8 \times \sin 0.5}{7}$

$\boxed{\sin x = 0.548}$ rad

b) $x = \sin^{-1}(0.548) = \underline{0.58}$

or $\pi - 0.58\ldots = \underline{2.56}$

8) $x^2 + y^2 - 10x + 9 = 0$

a) $(x - \frac{10}{2})^2 - (\frac{10}{2})^2 + 9 + (y-0)^2 = 0$

$(x-5)^2 - 25 + 9 + (y-0)^2 = 0$

$(x-5)^2 + (y-0)^2 = 16$

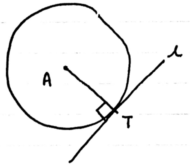
$\boxed{A\ (5,0)}$ (centre)

b) $r = \sqrt{16} = \underline{\underline{4}}$

c) $(x-5)^2 + y^2 = 16$
when $y = 0$

$(x-5)(x-5) = 16$
$x^2 - 10x + 25 - 16 = 0$
$x^2 - 10x + 9 = 0$
$(x-9)(x-1)$
$\boxed{x = 9 \quad \text{or} \quad x = 1}$

d)



AT = perpendicular
$\therefore m = -\frac{2}{7}$

$y - y_1 = m(x - x_1)$

$y = -\frac{2}{7}(x - 5)$

$y = -\frac{2x}{7} + \frac{10}{7}$

9a) prove $S_n = \frac{a(1-r^n)}{1-r}$

$S_n = a + ar + ar^2 \dots ar^{n-2} + ar^{n-1}$

$rS_n = ar + ar^2 + ar^3 \dots ar^{n-1} + ar^n$

$S_n - rS_n = S_n(1-r) = a + 0 + 0 \dots + 0 - ar^n$

$= a - ar^n = a(1-r^n)$

$S_n = \frac{S_n(1-r)}{1-r} = \frac{a(1-r^n)}{1-r}$

b) 2005 → 2008 = 4 years

$u_4 = 35\,000 \times 1.04^{(4-1)} = 35000 \times 1.04^3$

$= 39370.24$

to nearest £100 = $\underline{\underline{39400}}$

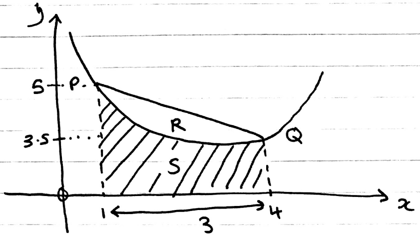
c) 2005 → 2024 = 20 years

$S_{20} = \frac{35000(1-1.04^{20})}{1-1.04} = 1042232.75$

to nearest £1000

$= \underline{\underline{1,042,000}}$

10)
a)



$y = 2x + \frac{8}{x^2} - 5$

when $x = 1$ $\quad y = 5 \quad \therefore \quad P(1, 5)$
when $x = 4$ $\quad y = 3.5 \quad \therefore \quad Q(4, 3.5)$

$\int_1^4 2x + 8x^{-2} - 5 \, dx$ = Area S

$= \left[x^2 - \frac{8}{x} - 5x \right]_1^4 = -6 - (-12) = \underline{6}$

* area R = trapezium − area S

$= \frac{1}{2}(5 + 3.5) \times 3 - 6$

~~= 17 − 6 = 11 units²~~

$= 12.75 - 6 = \underline{\underline{6.75 \text{ units}^2}}$

b) increasing when $\frac{dy}{dx} > 0$

$\frac{dy}{dx} = 2 - 16x^{-3}$

$2 - \frac{16}{x^3} > 0 \quad \therefore \quad 2 > \frac{16}{x^3}$

as 2 will be bigger than $\frac{16}{x^3}$ for any x value > 2
$2 - \frac{16}{x^3}$ will be positive
$\therefore \frac{dy}{dx} > 0$

$\therefore$ for all value $x > 2$ $\quad \frac{dy}{dx} > 0$