

C2 Jan 2014 (IAL) (MA)

$$\text{Q1) } (1+px)^{12} = (1)^{12} + \binom{12}{1}(1)^{11} \cdot (px)^1 + \binom{12}{2}(1)^{10} (px)^2$$

$$(1+px)^{12} = 1 + 12px + 66p^2x^2 //$$

$$\therefore 18 = 12p$$

$$\Rightarrow p = \frac{18}{12} = \boxed{\frac{3}{2}}$$

$$66p^2 = q = 66 \left(\frac{3}{2}\right)^2 = \boxed{\frac{297}{2} = q}$$

$$2a) f(x) = 2x^3 + x^2 + ax + b$$

$$\underline{f(2) = 25}: f(2) = 2(8) + 4 + 2a + b = 25$$

$$\Rightarrow 20 + 2a + b = 25$$

$$\Rightarrow 2a + b = 5 //$$

b)

$$x+3 \overline{) \begin{array}{r} 2x^2 - 5x + (a+15) \\ 2x^3 + x^2 + ax + b \end{array}}$$

$$\underline{2x^3 + 6x^2}$$

$$\ominus -5x^2 + ax$$

$$\underline{-5x^2 - 15x}$$

$$\ominus (a+15)x + b$$

$$\underline{(a+15)x + 3a + 45}$$

$$\ominus b - 3a - 45 // \leftarrow \text{remainder}$$

so because  $(x+3)$  is a factor, the remainder should be 0.

$$\therefore b - 3a - 45 = 0 //$$

now solve simultaneously with eqn. from a,

$$\text{from a, } b = 5 - 2a$$

$$\therefore 5 - 2a - 3a = 45$$

$$-5a = 40 \quad \therefore \boxed{a = -8}$$

$$\text{so } b = 5 - 2(-8) = \boxed{21 = b}$$

alt:  $(x+3)$  is a factor so  $f(-3) = 0$ .

sub  $x = -3$  into  $f(x)$  and equate to 0.

$$3 \text{ ai) } y = 2x^{\frac{1}{2}} + 18x^{-\frac{1}{2}} - 1$$

$$\frac{dy}{dx} = x^{-\frac{1}{2}} - 9x^{-\frac{3}{2}}$$

$$\text{ii) } \frac{d^2y}{dx^2} = -\frac{1}{2}x^{-\frac{3}{2}} + \frac{27}{2}x^{-\frac{5}{2}}$$

$$\text{b) } \frac{dy}{dx} = 0 : \frac{1}{x^{\frac{1}{2}}} - \frac{9}{x^{\frac{3}{2}}} = 0$$

$$\Rightarrow \frac{1}{x^{\frac{1}{2}}} = \frac{9}{x^{\frac{3}{2}}}$$

taking reciprocal of both sides:  $x^{\frac{1}{2}} = \frac{1}{9}x^{\frac{3}{2}}$

$$\Rightarrow x^{\frac{1}{2}} - \frac{1}{9}x^{\frac{3}{2}} = 0$$

$$\Rightarrow x^{\frac{1}{2}} \left( 1 - \frac{1}{9}x \right) = 0 //$$

$[x > 0 \therefore \text{reject } x = 0]$

$$1 - \frac{x}{9} = 0 \Rightarrow \boxed{x = 9}$$

$$\text{at } x=9, y = 2\sqrt{9} + \frac{18}{\sqrt{9}} - 1 = 11.$$

$$\therefore \text{ point is } \boxed{(9, 11)}.$$

$$\text{c) from (a)ii), } \frac{d^2y}{dx^2} = \frac{-\frac{1}{2}}{x^{3/2}} + \frac{27}{2x^{5/2}}$$

$$\text{at } x=9, \frac{d^2y}{dx^2} = \frac{-1}{54} + \frac{1}{18} = \frac{1}{27} > 0 //$$

$\therefore$  its a minimum point.

$$\text{4ai) } \left. \begin{array}{l} a=5 \\ r=1.2 \end{array} \right\} 20^{\text{th}} \text{ term} = ar^{19} = 5 \times 1.2^{19} = \boxed{159.7}$$

$$\text{ii) } S_{20} = \frac{5(1 - (1.2^{20}))}{1 - 1.2} = \boxed{933.4}$$

$$\text{b) } S_n = \frac{5(1 - (1.2)^n)}{1 - 1.2} > 3000$$

$$\frac{5(1 - (1.2)^n)}{-0.2} > 3000$$

$$(x - 0.2) \quad 5(1 - (1.2)^n) - < -600 \quad \left. \begin{array}{l} \text{when 'x' by a} \\ \text{-ve number} \\ \text{the signs flip.} \end{array} \right\}$$

$$\Rightarrow 1 - (1.2)^n < -120$$

$$\Rightarrow 1.2^n > 121$$

$$\log(1.20^n) > \log(121)$$

$$n \log(1.2) > \log(121)$$

$$n > \frac{\log(121)}{\log(1.2)}$$

$$n > 26.3 \dots$$

$$\therefore \boxed{n_{\min} = 27}$$

5a)  $\underline{t=1}$  :  $H = 10 + 5 \sin \frac{\pi}{6} = 10 + \frac{5}{2} = \boxed{12.5 \text{ m}}$

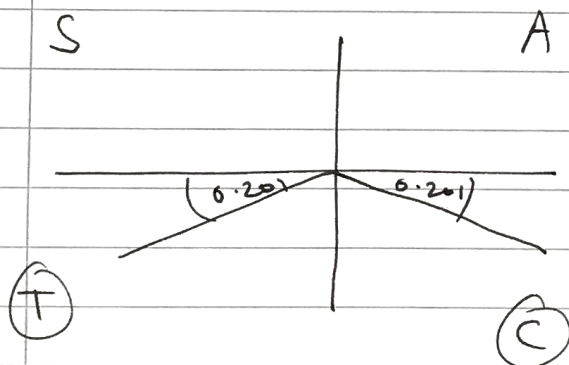
b)  $\underline{H=9}$  :  $9 = 10 + 5 \sin\left(\frac{\pi t}{6}\right)$

$$-\frac{1}{5} = \sin\left(\frac{\pi t}{6}\right)$$

$$\therefore \sin^{-1}\left(-\frac{1}{5}\right) = \frac{\pi t}{6} = \dots -0.20^\circ //$$

Before midday means that  $t < 12$ .

$\therefore$  our range is :  $0 \leq \frac{\pi t}{6} < 2\pi$



$$\frac{\pi t}{6} = \pi + 0.20^\circ, (2\pi - 0.20^\circ)$$

$$\frac{\pi t}{6} = 3.34, 6.08$$

$$t = 6.38, 11.61$$

$$6.38 \text{ hours} \rightarrow \boxed{06:23}$$

$$11.61 \text{ hours} \rightarrow \boxed{11:37} \text{ AM.}$$

$$6) \log_x(7y+1) - \log_x(2y) = 1.$$

$$\log_x\left(\frac{7y+1}{2y}\right) = 1$$

$$x^1 = \frac{7y+1}{2y}$$

$$2xy = 7y+1$$

$$2xy - 7y = 1$$

$$y(2x-7) = 1$$

$$\boxed{y = \frac{1}{2x-7}}$$

$$7a) \frac{dy}{dx} = 3x^2 - 12x + 9$$

$$\text{at } P, \frac{dy}{dx} = 3(4^2) - 12(4) + 9 = 9 //$$

$$\therefore \text{gradient of normal at } P = -\frac{1}{9} //$$

$$\left(-\frac{1}{9} \times 9 = -1\right)$$

$$\Rightarrow y - 9 = -\frac{1}{9}(x - 4)$$

$$\Rightarrow y = -\frac{1}{9}x + \frac{4}{9} + 9$$

$$\Rightarrow y = -\frac{1}{9}x + \frac{85}{9}$$

$$\textcircled{\times 9} : 9y = -x + 85 \quad \therefore \boxed{x + 9y = 85}$$

b)

$$R = \int_0^4 (y_2 - y_1) dx$$

$$= \int_0^4 \left[ -\frac{1}{9}x + \frac{85}{9} - x^3 + 6x^2 - 9x - 5 \right] dx$$

$$= \int_0^4 \left[ -x^3 + 6x^2 - \frac{82}{9}x + \frac{40}{9} \right] dx$$

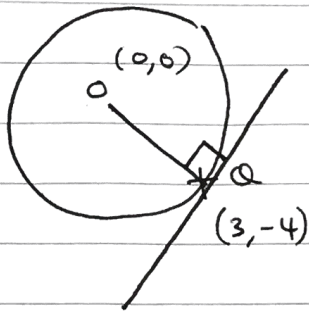
$$= \left[ -\frac{x^4}{4} + 2x^3 - \frac{82x^2}{18} + \frac{40}{9}x \right]_0^4$$

$$= \left[ -64 + 128 - \frac{656}{9} + \frac{160}{9} \right] = \boxed{\frac{80}{9}}$$



8a)  $x^2 + y^2 = 25$ .

b)



Find the gradient of line OQ

$$M_{OQ} = \frac{-4-0}{3-0} = -\frac{4}{3} //$$

$$\therefore \text{gradient of tangent} = \frac{3}{4} //$$

(tangent at Q will be perp to OQ)

$$y+4 = \frac{3}{4}(x-3)$$

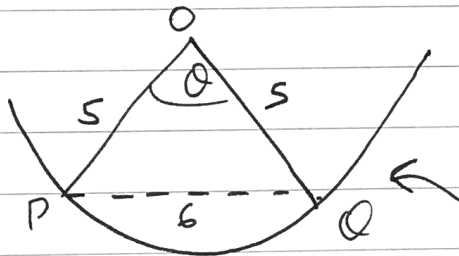
$$y = \frac{3}{4}x - \frac{9}{4} - 4$$

$$y = \frac{3}{4}x - \frac{25}{4}$$

(x4):  $4y = 3x - 25$

$$\boxed{3x - 4y = 25}$$

c)



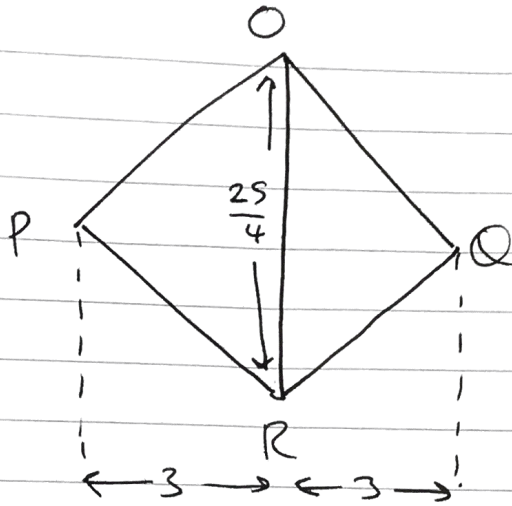
$$\text{length } PQ = 3 - (-3) = 6$$

cosine rule

$$\cos POQ = \frac{s^2 + s^2 - 6^2}{2(s)(s)} = \frac{7}{25} //$$

$$\therefore \angle POQ = \cos^{-1}\left(\frac{7}{25}\right) = \boxed{1.287^\circ}$$

d)



using eqn from (b),

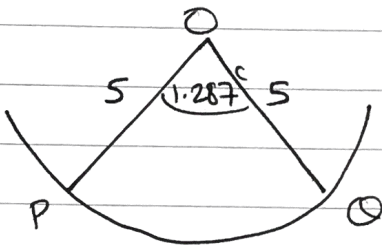
$$3x - 4y = 25$$

$$x=0; y = -\frac{25}{4}$$

$$\therefore |OR| = \frac{25}{4} //$$

$$\therefore \text{Area}_{OPR} = \text{Area}_{OQR} = \frac{1}{2} \times \frac{25}{4} \times 3 = \frac{75}{8}$$

$$\therefore \text{Area}_{OPRQ} = 2 \times \frac{75}{8} = \frac{75}{4} //$$



$$\begin{aligned} \text{Area}_{OPQ} &= \frac{1}{2} (5^2) (1.287^\circ) \\ &= 16.0875 // \end{aligned}$$

$$\text{Area required} = \frac{75}{4} - 16.0875 = \boxed{2.66 \text{ units}^2}$$

$$9a) 5\sin 2x - (1 - \sin^2 x) + 2\sin^2 x = 1$$

$$3\sin^2 x + 5\sin x - 1 = 1$$

$$3\sin^2 x + 5\sin x - 2 = 0$$

$$b) 3\sin^2 2\theta + 5\sin 2\theta - 2 = 0$$

$$\hookrightarrow (3\sin 2\theta - 1)(\sin 2\theta + 2) = 0$$

$$3\sin 2\theta - 1 = 0 \rightarrow \sin 2\theta = \frac{1}{3} //$$

$$2\theta = \sin^{-1}\left(\frac{1}{3}\right) = 19.471^\circ //$$

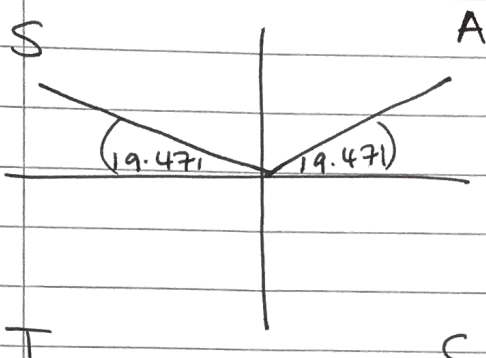
$$\sin 2\theta + 2 = 0$$

$$\sin 2\theta = -2 \quad \times$$

reject: no valid solutions



Solving in:  $\boxed{-360 \leq 2\theta < 360}$



$$2\theta = 19.471^\circ, 180 - 19.471^\circ, \\ -(180 + 19.471^\circ), -(360 - 19.471^\circ)$$

$$2\theta = 19.47^\circ, 160.53^\circ, -199.47^\circ, \\ 340.53^\circ$$

$$\boxed{\theta = 9.74^\circ, 80.26^\circ, \\ -99.74^\circ, -170.26^\circ}$$