C2 Jan 2012 (MA)

Q1a) 20^{th} term $= ar^{19} = 360 \times \left(\frac{7}{8}\right)^{19} = \boxed{28.5}$

b) $S_{20} = \dfrac{a(1-r^n)}{1-r} = \dfrac{360\left(1-\left(\frac{7}{8}\right)^{20}\right)}{1-\frac{7}{8}} = \boxed{2680}$

c) $S_{\infty} = \dfrac{a}{1-r} = \dfrac{360}{1-\frac{7}{8}} = \boxed{2880}$

Q2) radius = distance from $(-1,7)$ to $(0,0)$

$= \sqrt{1^2+7^2} = 5\sqrt{2}$ //

$\therefore \boxed{(x+1)^2+(y-7)^2 = 50}$ $\quad [5\sqrt{2}]^2 = 50$

Q3a) $\left(1+\frac{x}{4}\right)^8 = 1 + 2x + \frac{8(7)}{2}\left(\frac{x}{4}\right)^2 + \frac{8(7)(6)}{6}\left(\frac{x}{4}\right)^3$

$\begin{bmatrix} n=8 \\ x=\frac{x}{4} \end{bmatrix} = 1 + 2x + \frac{7}{4}x^2 + \frac{7}{8}x^3$

b) $1+\frac{x}{4} = 1.025 \quad \therefore \frac{x}{4} = 0.025 \quad \therefore x = 0.1$ //

using $x = 0.1$: $(1.025)^8 \simeq 1 + 2(0.1) + \frac{7}{4}(0.1)^2 + \frac{7}{8}(0.1)^3$

$\simeq \boxed{1.2184}$

Q4a) $\log_3(y) = \log_3(3x^2)$ (take "logs" of both sides)

$\log_3(y) = \log_3(3) + \log_3(x^2)$

$\log_3(y) = 1 + 2\log_3(x)$

b) $1 = \log_3(28x-9) - \log_3(x^2)$

$1 = \log_3\left(\frac{28x-9}{x^2}\right)$

$3^1 = \frac{28x-9}{x^2}$

$3x^2 = 28x - 9$

$3x^2 - 28x + 9 = 0$

$(3x - 1)(x - 9) = 0$

$x = \frac{1}{3}$ and $x = 9$

Q5a) $f(-2) = 7$: $f(-2) = (-2)^3 + a(-2)^2 - 2b + 3 = 7$

$\Rightarrow -8 + 4a + 3 - 2b = 7$

$\Rightarrow 4a - 2b = 12$

$\div 2 \Rightarrow 2a - b = 6$ — ①

b) $f(1) = 4$: $f(1) = 1 + a + b + 3 = 4$

$a + b = 0$

$\therefore a = -b$

$\hookrightarrow$ ① : $2a - (-a) = 6$

$3a = 6$ $\therefore \boxed{a = 2}$

and $\boxed{b = -2}$

Q6a)

x	2	2.5
y	4	2.31

b) $h = \frac{b-a}{n} = \frac{4-1}{6} = \frac{1}{2}$

$$\text{Area} \simeq \frac{1}{2} \times \frac{1}{2}\left[16.5 + 0 + 2(7.361 + 4 + 2.31 + 1.278 + 0.556)\right]$$

$$\simeq \boxed{11.88}$$

c) $$R = \int_1^4 \left[16x^{-2} - \frac{1}{2}x + 1\right] dx$$

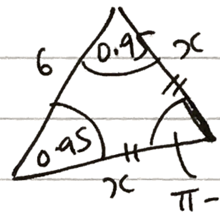
$$= \left[-\frac{16}{x} - \frac{x^2}{4} + x\right]_1^4 = \left[-4 - 4 + 4\right] - \left[-16 - \frac{1}{4} + 1\right]$$

$$= \left[-4 + \frac{61}{4}\right] = \boxed{\frac{45}{4}}$$

Q7a) $BC = r\theta = 6 \times 0.95 = \boxed{5.7\text{cm}}$

b) $\text{Area} = \frac{1}{2}r^2\theta = \frac{1}{2}(6^2)(0.95) = \boxed{17.1\text{cm}^2}$

c)



$\pi - 0.95 - 0.95$
$= 1.242$

sine rule : $\frac{x}{\sin(0.95)} = \frac{6}{\sin(1.242)}$

$\therefore x = \frac{6\sin(0.95)}{\sin(1.242)} = \boxed{5.16}$

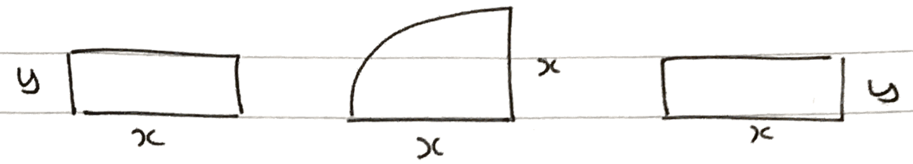
d) Length $AD = x = 5.16$ $\therefore R_p = \underset{BC}{5.7} + \underset{CD}{(6 - 5.16)} + \underset{BD}{5.16}$

$= \underline{\underline{11.7\text{cm}}}$

e) $\text{Area } \Delta ABD = \frac{1}{2}(5.16)(6)\sin(0.95) = 12.592$

$\therefore R = 17.1 - 12.592 = \boxed{4.51\text{cm}^2}$

Q8a)



Area $= xy + \frac{\pi x^2}{4} + xy = 2xy + \frac{\pi x^2}{4}$

$\therefore 2xy + \frac{\pi x^2}{4} = 4$

$2xy = 4 - \frac{\pi x^2}{4}$

$y = \frac{4 - \frac{\pi x^2}{4}}{2x} \xrightarrow{\times 4} = \frac{16 - \pi x^2}{8x}$ //

to all terms

b) $P = \frac{2\pi x}{4} + 4y + 2x = \frac{\pi x}{2} + 2x + 4y$

$\therefore P = \frac{\pi x}{2} + 2x + 4\left(\frac{16 - \pi x^2}{8x}\right)$

$P = \frac{\pi x}{2} + 2x + \frac{16 - \pi x^2}{2x}$

$P = \cancel{\frac{\pi x}{2}} + 2x + \frac{8}{x} - \cancel{\frac{\pi x}{2}}$

$P = 2x + \frac{8}{x}$ //

c) $\frac{dP}{dx} = 2 - 8x^{-2} = 0$ // $\therefore 2 = \frac{8}{x^2}$

$\Rightarrow x^2 = 4 \quad \therefore x = 2$ //

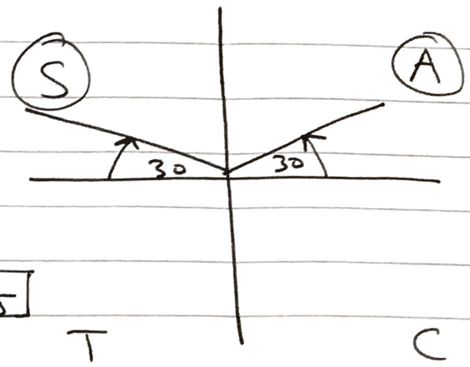
at $x = 2$: $P_{min} = 2(2) + \frac{8}{2} = \boxed{8m}$

To check P is minimum,

$\frac{d^2P}{dx^2} = \frac{16}{x^3} > 0$ //

d) from (a), $y = \frac{16 - \pi(2^2)}{8(2)} = 0.2146m \approx \boxed{21cm}$

Q9a) $\sin(3x-15) = \frac{1}{2}$



$\sin^{-1}\left(\frac{1}{2}\right) = 3x-15 = 30°$

solving in : $\boxed{-15 \leq 3x-15 \leq 525}$

$3x-15 = 30°, 150°, 390°, 510°$
$3x = 45°, 165°, 405°, 525°$
$x = 15°, 55°, 135°, 175°$

ii) $y = \sin(ax-b)$ passes through $\left(\frac{\pi}{10}, 0\right)$ $\left(\frac{3\pi}{5}, 0\right)$

substituting : $0 = \sin\left(a\frac{\pi}{10} - b\right)$

$\frac{\pi a}{10} - b = \sin^{-1}(0) = 0$

$\therefore \frac{\pi a}{10} = b \sim ①$

For the first root we want the first solution for $\boxed{\sin^{-1}(0)}$ ie 0

For the second root we want the 2nd solution of $\sin^{-1}(0)$ (ie π)

$0 = \sin\left(\frac{3\pi}{5}(a) - b\right)$

$\frac{3\pi a}{5} - b = \sin^{-1}(0) = \pi$

$\therefore b = \frac{3\pi a}{5} - \pi \sim ②$

① = ② : $\frac{3\pi a}{5} - \pi = \frac{\pi a}{10}$

$\frac{a}{2} = 1 \quad \therefore \boxed{a = 2}$

sub into ① : $b = \frac{\pi a}{10} = \frac{\pi(2)}{10} = \boxed{\frac{\pi}{5}}$