C2 Jan 2010 (MA)

Q1) $(3-x)^6 \simeq (3)^6 + \binom{6}{1}(3)^5(-x)^1 + \binom{6}{2}(3)^4(-x)^2 + \dots$

$\simeq \boxed{729 - 1458x + 1215x^2}$

Q2a) $5\sin x = 1 + 2(1-\sin^2 x)$

$5\sin x = 1 + 2 - 2\sin^2 x$

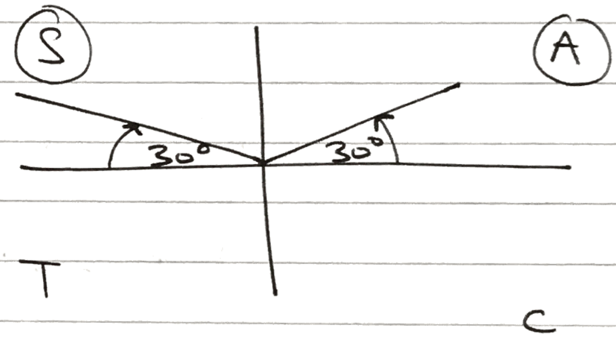
$2\sin^2 x + 5\sin x - 3 = 0$ // □

b) $(2\sin x - 1)(\sin x + 3) = 0$

$2\sin x - 1 = 0$	$\sin x + 3 = 0$
$\sin x = \frac{1}{2}$	$\sin x = -3$ × no valid solutions

$x = \sin^{-1}\left(\frac{1}{2}\right) = 30°$.

$\boxed{x = 30°, 150°}$



Q3a) $2x - 1 = 0 \rightarrow x = \frac{1}{2}$ $\therefore f\left(\frac{1}{2}\right) = -5$ // ~ ①

$x + 2 = 0 \rightarrow x = -2$ $\therefore f(-2) = 0$ // ~ ②

① : $f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \frac{1}{4}a + \frac{1}{2}b - 6 = -5$

$\frac{a}{4} + \frac{b}{2} = \frac{3}{4}$ $\therefore a + 2b = 3$ //

② : $f(-2) = 2(-8) + 4a - 2b - 6 = 0$

$4a - 2b = 22$

$\therefore 2a - b = 11$ //

$a = 3 - 2b$ and $2a - b = 11$ //

$\therefore 2(3-2b) - b = 11$

$6 - 4b - b = 11$

$6 - 11 = 5b = -5 \quad \therefore \boxed{b = -1}$

$\therefore a = 3 - 2(-1) = \boxed{5 = a}$

b)

$$
\begin{array}{r|l}
 & 2x^2 + x - 3 \\
x+2 & 2x^3 + 5x^2 - x - 6 \\
 & 2x^3 + 4x^2 \\
 & 0 \quad\quad x^2 - x \\
 & \quad\quad x^2 + 2x \\
 & \quad\quad 0 \quad -3x - 6 \\
 & \quad\quad\quad -3x - 6 \\
 & \quad\quad\quad 0 \quad 0
\end{array}
$$

//

$\therefore f(x) = (x+2)(2x^2 + x - 3) = 0$

but $2x^2 + x - 3 = (2x+3)(x-1)$

$\therefore f(x) = \boxed{(x+2)(2x+3)(x-1)}$

Q4a)

sine rule : $\dfrac{\sin 0.6}{4} = \dfrac{\sin \angle BCA}{5}$

$\therefore \sin(\angle BCA) = \dfrac{5}{4}\sin(0.6) = \overset{0.706}{\cancel{0.678}}\ldots$

$\therefore \angle BCA = \sin^{-1}(\overset{0.706}{\cancel{0.678}}\ldots) = 0.784$ //

so $\angle ABC = \pi - 0.784 - 0.6 = \boxed{1.76^c}$ //

b) Area emblem = Area ΔABC + BCD sector

$= \frac{1}{2}(5)(4)\sin(1.76^c) + \frac{1}{2}(4^2)(\pi - 1.76)$

$= 9.82 + 11.05 = \boxed{20.9\text{cm}^2}$

Q5a) $\log_x 64 = 2$

$x^2 = 64$

$x = \pm\sqrt{64} \rightarrow x = \pm 8$

$\therefore \boxed{x = 8} \quad (x > 0)$

b) $\log_2(11-6x) = \log_2[(x-1)^2] + 3$

$\log_2(11-6x) - \log_2[(x-1)^2] = 3$

$\log_2\left(\frac{11-6x}{(x-1)^2}\right) = 3$

$2^3 = \frac{11-6x}{(x-1)^2}$

$\therefore 8[x-1]^2 = 11-6x$

$\Rightarrow 8(x^2-2x+1) = 11-6x$

$\Rightarrow 8x^2 - 16x + 8 + 6x - 11 = 0$

$\Rightarrow 8x^2 - 10x - 3 = 0$

$\Rightarrow (4x+1)(2x-3) = 0$

$\boxed{x = -\frac{1}{4}} \quad \boxed{x = \frac{3}{2}}$

	purchased	1 year on	2 years on	3 years on
Q6a)	a	ar	ar^2	ar^3

$$ar^3 = 18000 \times (0.8^3) = \boxed{£9216}$$

b) $ar^N < 1000$

(taking logs) $\log(18000(0.8^N)) < \log(1000)$

$\Rightarrow \log(18000) + \log(0.8^N) < \log(1000)$

$\Rightarrow \log(0.8^N) < \log(1000) - \log(18000)$

$\Rightarrow N\log(0.8) < \log\left(\frac{1000}{18000}\right)$

$\Rightarrow N > \dfrac{\log\left(\frac{1}{18}\right)}{\log(0.8)}$ ← signs change when ÷ or × by a negative number.

$\Rightarrow N > 12.953\ldots \rightarrow N \geq 13$

so $N_{min} = \boxed{13}$

c)

a	ar	ar^2
200	200×1.12	200×1.12^2

5^{th} year $= ar^4 = 200 \times (1.12)^4 = \boxed{£314.70}$

d) $S_{15} = \dfrac{a(1-r^{15})}{1-r} = \dfrac{200(1-(1.12)^{15})}{1-1.12} = \boxed{£7455.94}$

Q7a) $\underline{y=0}$: $(x-4)(x-1)=0$

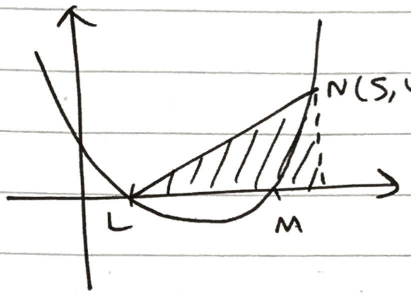
$x=4$ & $x=1$

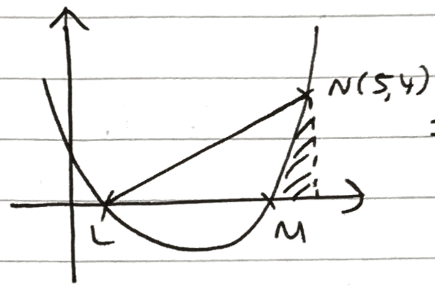
$\therefore L(1,0)$ and $M(4,0)$

b) $\underline{x=5}$: $y = (5)^2 - 5(5) + 4 = 4$ //

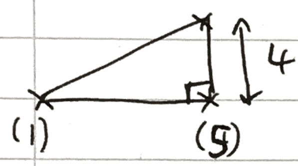
$\therefore (5,4)$ lies on C

c) $\int [x^2 - 5x + 4]\,dx = \left[\frac{x^3}{3} - \frac{5}{2}x^2 + 4x\right] + c$

d) 

$-$ 

$= R$



$\left[\frac{1}{2} \times 4 \times 4\right] - \left[\int_4^5 (x^2 - 5x + 4)\,dx\right] = R$

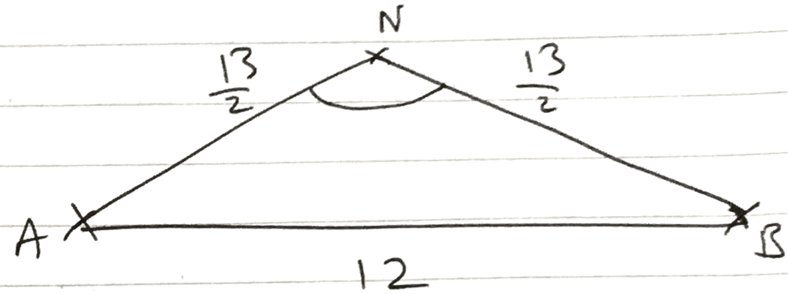
$[8] - \left[\frac{x^3}{3} - \frac{5x^2}{2} + 4x\right]_4^5 = R$

$[8] - \left[\left[-\frac{5}{6}\right] - \left[-\frac{8}{3}\right]\right] = R$

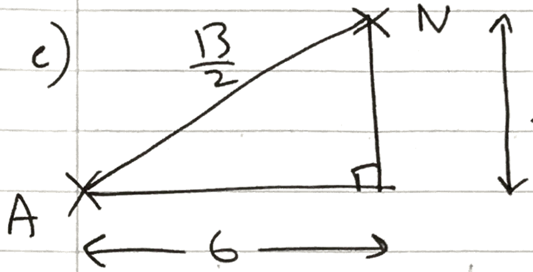
$[8] - \left[\frac{11}{6}\right] = R = \boxed{\frac{37}{6}}$

Q8a) $N(2,-1)$

b) $r = \sqrt{\frac{169}{4}} = \frac{13}{2}$ //



c)



$\sqrt{\left(\frac{13}{2}\right)^2-(6)^2} = \frac{5}{2}$ //

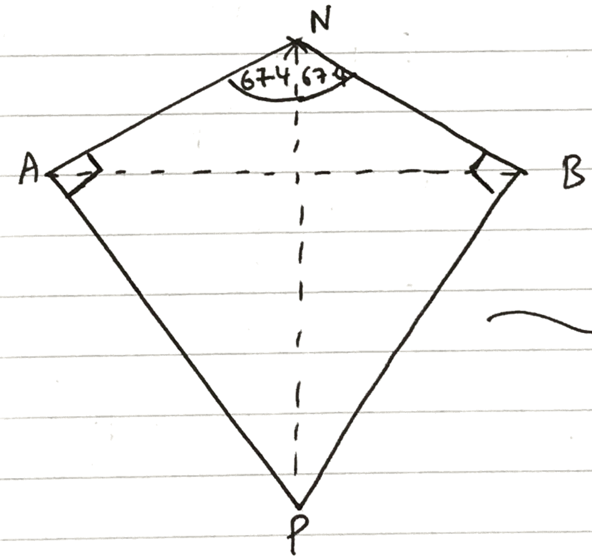
A is 6 units to the left and 2.5 units down from N.

$\therefore$ $A\left(-4, \frac{-7}{2}\right)$ and $B\left(8, \frac{-7}{2}\right)$

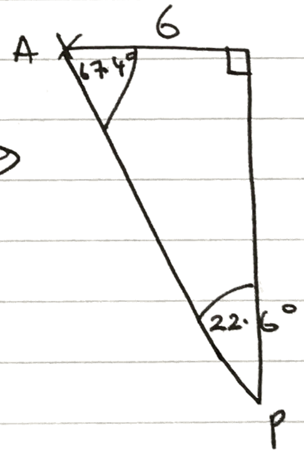
d) cosine rule : $\cos ANB = \dfrac{\left(\frac{13}{2}\right)^2 + \left(\frac{13}{2}\right)^2 - (12)^2}{2\left(\frac{13}{2}\right)\left(\frac{13}{2}\right)} = \dfrac{-119}{169}$

$\angle ANB = \cos^{-1}\left(\frac{-119}{169}\right) = 134.8°$

e)



$90 - 67.4 = 22.6°$ //



$\sin(22.6) = \dfrac{6}{AP}$

$AP = \dfrac{6}{\sin(22.6)}$

$= \boxed{15.6}$

Q9a) $y = 12x^{\frac{1}{2}} - x^{\frac{3}{2}} - 10$

$\frac{dy}{dx} = 6x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} = 0$

$\frac{6}{\sqrt{x}} = \frac{3}{2}\sqrt{x}$

$6 = \frac{3}{2}x \quad \therefore \boxed{x = 4}$

$y = 12\sqrt{4} - (4)^{3/2} - 10 = \boxed{6 = y}$

$\Rightarrow (4, 6)$.

b) $\frac{d^2y}{dx^2} = -\frac{1}{2}(6x^{-\frac{3}{2}}) - \frac{3}{4}x^{-\frac{1}{2}}$

$= -3x^{-\frac{3}{2}} - \frac{3}{4}x^{-\frac{1}{2}}$ //

c) $x = 4$: $-3(4)^{-\frac{3}{2}} - \frac{3}{4}(4^{-\frac{1}{2}}) = -\frac{3}{4} < 0$

$\therefore$ this turning point is a maximum.