# C2 Jan 2008 (MA)

Q1(a) $f(3) = (3)^3 - 2(3)^2 - 4(3) + 8 = \boxed{5}$

ii) $f(-2) = (-2)^3 - 2(-2)^2 - 4(-2) + 8 = \boxed{0}$

b) $x+2$ is a factor as $f(-2) = 0$

$$
\begin{array}{r|l}
 & x^2 - 4x + 4 \\
x+2 & x^3 - 2x^2 - 4x + 8 \\
 & x^3 + 2x^2 \\
 & 0 - 4x^2 - 4x \\
 & \quad -4x^2 - 8x \\
 & \quad\quad 0 + 4x + 8 \\
 & \quad\quad\quad 4x + 8 \\
 & \quad\quad\quad 0 \quad 0
\end{array}
$$

$\therefore f(x) = (x+2)(x^2 - 4x + 4) = 0$

but $x^2 - 4x + 4 = (x-2)(x-2)$

$\therefore f(x) = (x+2)(x-2)^2 = 0$

so $\boxed{\begin{array}{l} x = -2 \\ x = 2 \end{array}}$

Q2a) $ar^3$ $\quad$ $ar^6$

$10$ $\quad$ $80$

$$\frac{ar^{6}}{ar^3} = r^3 = \frac{80}{10} = 8$$

$$\therefore r = \sqrt[3]{8} = \boxed{2}$$

b) $ar^3 = 10 \quad \therefore \quad a = \frac{10}{r^3} = \frac{10}{8} = \boxed{1.25}$

c) $S_{20} = \frac{a(1-r^{20})}{1-r} = \frac{1.25(1-(2)^{20})}{1-2} = \boxed{1310719}$

Q3a) $\left(1+\frac{x}{2}\right)^{10} \simeq (1)^{10} + \binom{10}{1}(1)^9\left(\frac{x}{2}\right)^1 + \binom{10}{2}(1)^8\left(\frac{x}{2}\right)^2$

$$+ \binom{10}{3}(1)^7\left(\frac{x}{2}\right)^3$$

$$\left(1+\frac{x}{2}\right)^{10} \simeq \boxed{1 + 5x + \frac{45}{4}x^2 + 15x^3}$$

b) $\left(1+\frac{x}{2}\right) = 1.005$

$\frac{x}{2} = 0.005 \quad \therefore x = 0.01$

<u>using $x = 0.01$</u>: $(1.005)^{10} \simeq 1 + 5(0.01) + \frac{45}{4}(0.01)^2 + 15(0.01)^3$

$\simeq \boxed{1.05114}$

Q4a) $3\sin^2\theta - 2(1-\sin^2\theta) = 1$

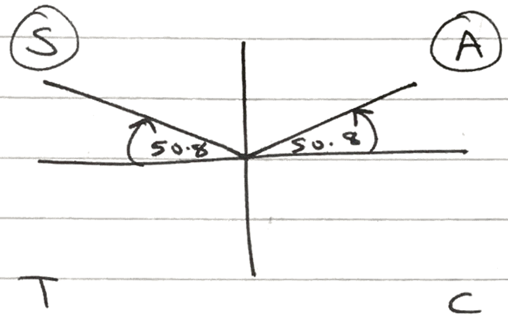
$3\sin^2\theta - 2 + 2\sin^2\theta - 1 = 0$

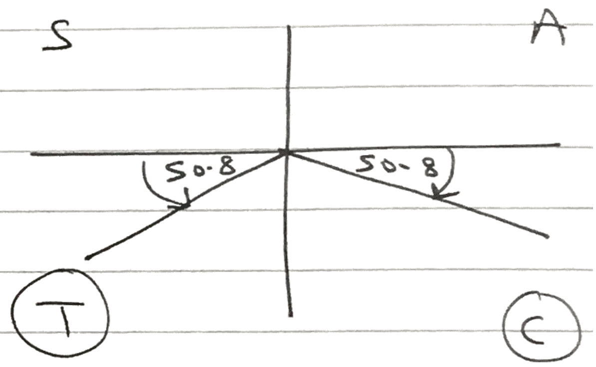
$5\sin^2\theta - 3 = 0$ // $\therefore \boxed{5\sin^2\theta = 3}$

b) $\sin^2\theta = \frac{3}{5}$

$\sin\theta = \pm\sqrt{\frac{3}{5}}$ so we have two sets of solutions

| | |
|---|---|
| $\sin\theta = \sqrt{\frac{3}{5}}$ | $\sin\theta = -\sqrt{\frac{3}{5}}$ |
| $\theta = \sin^{-1}\left(\sqrt{\frac{3}{5}}\right) = 50.77°$ | $\theta = \sin^{-1}\left(-\sqrt{\frac{3}{5}}\right) = -50.77°$ |




$0 \leq \theta < 360°$

| | |
|---|---|
| $\theta = 50.8°, (180 - 50.8°)$ | $\theta = (180 + 50.8°), (360 - 50.8°)$ |
| $\boxed{\theta = 50.8°, 129.2°}$ | $\boxed{\theta = 230.8°, 309.2°}$ |

Q5) $a = 3b$ — ①

$\log_3 a + \log_3 b = 2$ — ②

using ② : $\log_3(ab) = 2$

$3^2 = ab = 9$

from ① : $a = 3b$ $\therefore$ $3b^2 = 9$

$b^2 = 3$ $\therefore$ $\boxed{b = \sqrt{3}}$

$a = 3b = \boxed{3\sqrt{3}}$

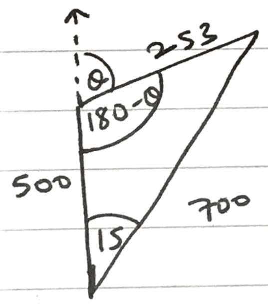
Q6a) cosine rule : $a^2 = b^2 + c^2 - 2bc\cos A$.

$BC^2 = 500^2 + 700^2 - 2(500)(700)\cos(15°)$

$BC^2 = 63851.9\ldots$

$BC = 252.7 = \boxed{253\text{m}}$

b) sine rule : $\dfrac{\sin(180-\theta)}{700} = \dfrac{\sin(15)}{253}$




$\sin(180-\theta) = \dfrac{700\sin 15}{253}$

$180-\theta = \sin^{-1}\left(\dfrac{700\sin 15}{253}\right) = 45.7°, \ (180-45.7°)$

Remember $(180-\theta)$ is obtuse so it cant be $45.7°$, but $(180-45.7°)$ $= 134.3°$

$\therefore$ $180-\theta = 134.3°$

$\theta = 180 - 134.3$

$\boxed{\theta = 45.7°}$

Q7a) $y = 6x - x^2 = 0$

$x(6-x) = 0$

$\boxed{x=0}$ $\quad 6 - x = 0 \rightarrow \boxed{x = 6}$

b) $6x - x^2 = 2x$

$x^2 + 2x - 6x = 0$

$x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0$ // $\quad x - 4 = 0 \longrightarrow x = 4$ //

$\downarrow \qquad\qquad \downarrow$

$y = 2(0) = 0$ // $\qquad y = 2(4) = 8$ //

$\therefore$ (4, 8) is another point of intersection

$\therefore$ a point of intersection is (0, 0)

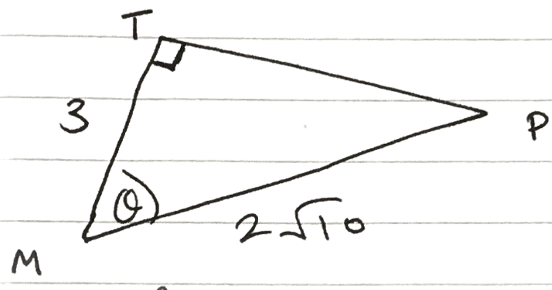
c) $R = \int_0^4 (y_2 - y_1)\,dx = \int_0^4 [6x - x^2 - 2x]\,dx$

$= \int_0^4 [4x - x^2]\,dx = \left[2x^2 - \frac{x^3}{3}\right]_0^4$

$= \left[32 - \frac{64}{3}\right] - [0] = \boxed{\frac{32}{3}}$

Q8a) $\boxed{(x-6)^2+(y-4)^2} = 9 = \boxed{3^2}$

b)




(Tangent will be perp. to MT)

$MP = \sqrt{(12-6)^2+(6-4)^2} = 2\sqrt{10}$ //

$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}} \quad \therefore \cos\theta = \dfrac{3}{2\sqrt{10}}$

so $\theta = \cos^{-1}\left(\dfrac{3}{2\sqrt{10}}\right) = \boxed{1.0766^c}$

c) Area of sector MTQ $= \frac{1}{2}r^2\theta$

$= \frac{1}{2}\cdot 3^2 \cdot 1.0766 = 4.845$ //

Area of $\Delta$PTM $= \frac{1}{2} \times \underbrace{3}_{MT} \times \underbrace{\left(\sqrt{(2\sqrt{10})^2-(3)^2}\right)}_{PT}$

$= \frac{1}{2} \times 3 \times \sqrt{31}$

$= 8.352$ //

$\therefore$ Area TPQ $= 8.352 - 4.845 = \boxed{3.507}$

the top is open

(Q9a) Area = A = Total surface area = $x^2(2) + xy(3)$

$$A = 2x^2 + 3xy$$

but capacity = volume = $x^2y = 100$

$$\therefore y = \frac{100}{x^2}$$

$$\Rightarrow A = 2x^2 + 3x\left(\frac{100}{x^2}\right)$$

$$A = 2x^2 + \frac{300}{x}$$

b) $$\frac{dA}{dx} = 4x - 300x^{-2} = 0$$

$$4x = \frac{300}{x^2}$$

$$4x^3 = 300 \quad \therefore x^3 = \frac{300}{4} = 75$$

$$\therefore x = \sqrt[3]{75} = \boxed{4.217\ldots}$$

c) $$\frac{d^2A}{dx^2} = 4 + 600x^{-3} > 0$$ (for all values of $x$ $x > 0$.)

$\therefore$ this value of $x$ gives a minimum value.

d) $$A_{min} = 2(4.217)^2 + \frac{300}{(4.217)} = \boxed{106.7\text{ cm}^2}$$