C2 Jan 2005 (MA)

Q1) $(3+2x)^5 \simeq (3)^5 + \binom{5}{1}(3)^4(2x)^1 + \binom{5}{2}(3)^3(2x)^2 + \dots$

$\simeq 243 + 405(2)x + 1080x^2 + \dots$

$\simeq \boxed{243 + 810x + 1080x^2}$

Q2a) midpoint $= \left(\frac{5+13}{2}, \frac{11-1}{2}\right) = (9, 5)$

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

b) length = AB = 2 x radius

$|AB| = \sqrt{(13-5)^2+(11--1)^2} = 4\sqrt{13}$ $\therefore$ radius $= 2\sqrt{13}$

$\therefore r^2 = 4 \times 13 = 52$

midpoint of AB = centre = $(9, 5)$

$\Rightarrow \boxed{(x-9)^2 + (y-5)^2 = 52}$

Q3a) $3^x = 5$

$\log(3^x) = \log(5)$

$\therefore x \log 3 = \log 5$

so $x = \frac{\log 5}{\log 3} = \boxed{1.46}$

b) $\log_2(2x+1) - \log_2(x) = 2$

$\log_2\left(\frac{2x+1}{x}\right) = 2$

Q3b cont.) $\log_2\left(2+\frac{1}{x}\right) = 2$

$\therefore\ 2^2 = 2 + \frac{1}{x} = 4$

$2 = \frac{1}{x} \longrightarrow 2x = 1 \longrightarrow \boxed{x = \frac{1}{2}}$

Q4a) $5(1-\sin^2 x) = 3(1+\sin x)$

$5 - 5\sin^2 x = 3 + 3\sin x$

$5\sin^2 x + 3\sin x + 3 - 5 = 0$

$5\sin^2 x + 3\sin x - 2 = 0$ // $\square$

b) using result from (a), $\boxed{0 \leq x < 360°}$

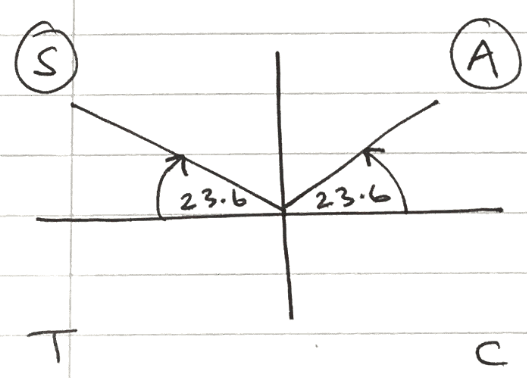
$5\sin^2 x + 3\sin x - 2 = 0$

$(5\sin x - 2)(\sin x + 1) = 0$

$5\sin x - 2 = 0$

$\sin x = \frac{2}{5}$

$x = \sin^{-1}\left(\frac{2}{3}\right) = 23.6°$



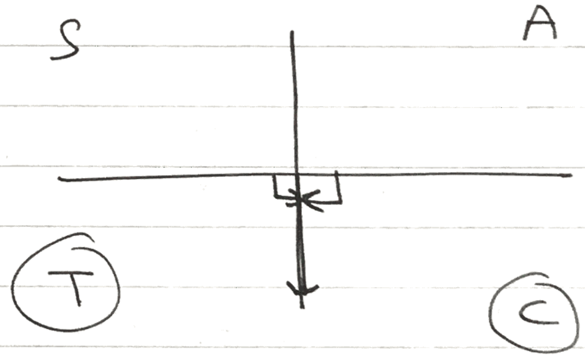
$\Rightarrow x = 23.6°, (180 - 23.6°)$

$\Rightarrow \boxed{x = 23.6°, 156.4°}$

$\sin x + 1 = 0$

$\sin x = -1$

$x = \sin^{-1}(-1) = -90°$



$\Rightarrow \boxed{x = 270°}$

Q5a) $f(x) = x^3 - 2x^2 + ax + b$

$\underline{f(2) = 1}$: $f(2) = (2)^3 - 2(4) + 2a + b = 1$

$\Rightarrow \quad 2a + b = 1 \quad — \ 1$

$\underline{f(-1) = 28}$: $f(-1) = -1 - 2 - a + b = 28$

$\Rightarrow \quad b - a = 31 \ — \ 2$

$\underline{2 - 1}$:

$$\begin{array}{r} [b - a = 31] \\ -[b + 2a = 1] \\ \hline 0 - 3a = 31 - 1 \end{array}$$

$\Rightarrow \quad -3a = 30$

$\therefore \quad a = \boxed{-10}$

from 2 : $b - a = 31 \quad \therefore \ b = 31 + a = 31 - 10$

$= \boxed{21}$

b) if $(x-3)$ is a factor of $f(x)$ then $f(+3) = 0$

$f(3) = (3)^3 - 2(3)^2 + 3(-10) + 21$

$= 27 - 18 - 30 + 21 = 48 - 48 = \boxed{0}$

$\therefore (x-3)$ is a factor of $f(x)$

Q6a) $a \quad ar \quad ar^2 \quad ar^3$

$7.2 \qquad 5.832$

$$\frac{ar^3}{ar} = \frac{5.832}{7.2} = r^2$$

$$\therefore r = \sqrt{\frac{5.832}{7.2}} = \sqrt{0.81} = \boxed{0.9}$$

b) $ar = 7.2$

$$\therefore a = \frac{7.2}{r} = \frac{7.2}{0.9} = \boxed{8}$$

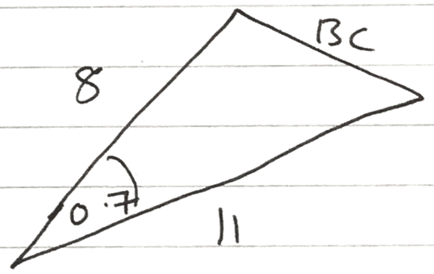
c) $$S_{50} = \frac{a(1-r^{50})}{1-r} = \frac{8(1-(0.9^{50}))}{1-0.9} = \boxed{79.588}$$

d) $$S_\infty = \frac{a}{1-r} = \frac{8}{1-0.9} = \underline{\underline{80}}$$

Difference $= 80 - 79.588 = \boxed{0.412}$

Q7a) $BD = r\theta = 8 \times 0.7 = \boxed{5.6cm}$

b) $CD = 11 - 8 = 3cm$



<u>cosine rule to find BC</u>

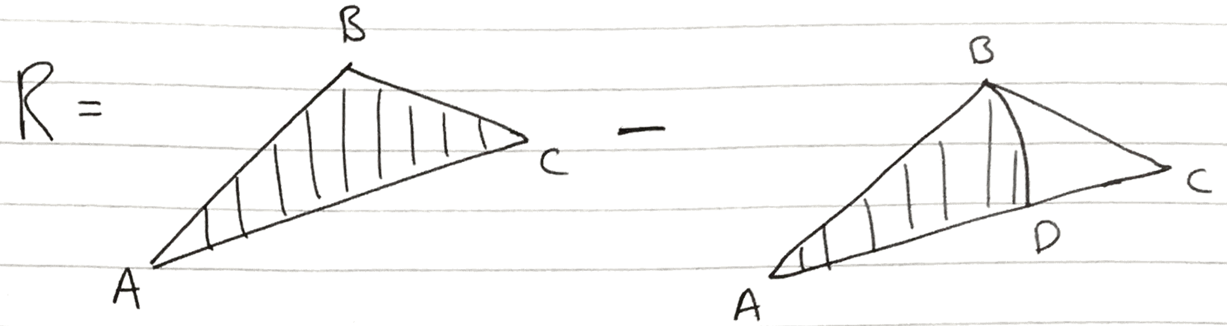
$BC^2 = 8^2 + 11^2 - 2(8)(11)\cos 0.7$

$BC^2 = 50.39...$

$BC \simeq 7.1cm$

$\therefore$ Perimeter of R $= \underset{BD}{5.6} + \underset{CD}{3} + \underset{BC}{7.1}$

$= \boxed{15.7cm}$

c) 

$R = \frac{1}{2}ab\sin C - \frac{1}{2}r^2\theta$

$R = \frac{1}{2}\times 8\times 11\times \sin(0.7) - \frac{1}{2}(8^2)(0.7)$

$R = 28.346 - 22.4$

$R = \boxed{5.95\text{ cm}^2}$

Q8a) $3x + 20 = x^2 + 6x + 10$

$x^2 + 3x - 10 = 0$

$(x-2)(x+5) = 0$

$x - 2 = 0$ $\quad$ $x + 5 = 0$

$x = 2$ $\quad$ $x = -5$

$y = 3(2) + 20 = \boxed{26}$ $\quad$ $y = 3(-5) + 20 = \boxed{5}$

A $(-5, 5)$

B $(2, 26)$

b) $S = \int_{-5}^{2} [y_2 - y_1]\,dx = \int_{-5}^{2} \left[3x + 20 - (x^2 + 6x + 10)\right] dx$

doesn't matter which you choose to be y_1 or y_2 as long as you take the absolute value of your final answer.

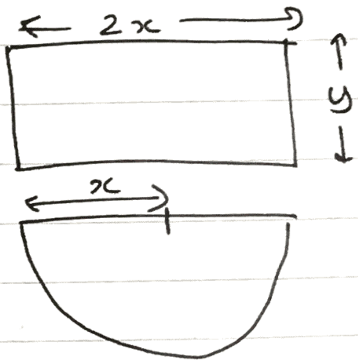
8b cont.) $S = \int_{-5}^{2} \left[3x + 20 - x^2 - 6x - 10\right] dx$

$= \int_{-5}^{2} \left[-x^2 - 3x + 10\right] dx = \left[-\frac{x^3}{3} - \frac{3}{2}x^2 + 10x\right]_{-5}^{2}$

$= \left[\frac{34}{3}\right] - \left[-\frac{275}{6}\right] = \boxed{\frac{343}{6}}$ units^2

Q9a) Area $\square = 2xy$

Area $\cup = \frac{\pi x^2}{2}$



$\therefore A = 2xy + \frac{\pi x^2}{2}$ //

but Perimeter $= 2x + y + y + \pi(x)$ ← circular arc

$\Rightarrow 2x + 2y + \pi x = 80$ // ← we are told $P = 80$.

$\therefore 2y = 80 - 2x - \pi x$

$\Rightarrow y = 40 - x - \frac{\pi x}{2}$ //

$\therefore A = 2x\left(40 - x - \frac{\pi x}{2}\right) + \frac{\pi x^2}{2}$

$A = 80x - 2x^2 - \pi x^2 + \frac{\pi x^2}{2}$

$A = 80x - 2x^2 - \frac{\pi x^2}{2}$

$A = 80x - \left(2 + \frac{\pi}{2}\right)x^2$

b) $A = 80x - \left(2+\frac{\pi}{2}\right)x^2$

$$\frac{dA}{dx} = 80 - 2\left(2+\frac{\pi}{2}\right)x = 0$$

$$80 = (4+\pi)x$$

$$\therefore x = \frac{80}{\pi+4} = \boxed{11.2\,m}$$

c) $\frac{d^2A}{dx^2} = -2\left(2+\frac{\pi}{2}\right) < 0$ $\therefore$ value of x found will give a max value for A.

d) $A = 80\left(\frac{80}{\pi+4}\right) - \left(2+\frac{\pi}{2}\right)\left(\frac{80}{\pi+4}\right)^2$

$$= 448.08..\,m^2 = \boxed{448m^2}$$