C2 S14

1.

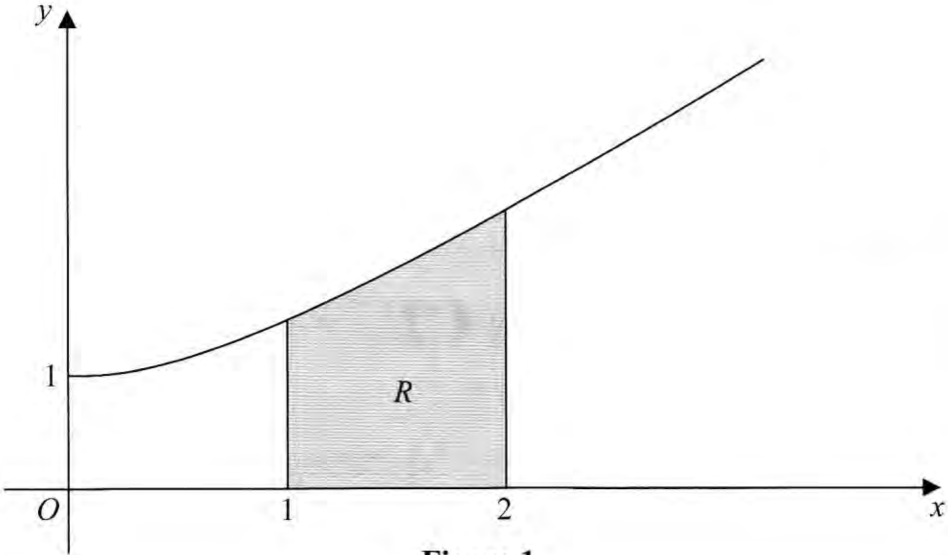


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \sqrt{(x^2 + 1)}$, $x \geqslant 0$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$

The table below shows corresponding values for x and y for $y = \sqrt{(x^2 + 1)}$.

$h = 0.25$

x	1	1.25	1.5	1.75	2
y	1.414	1.601	1.803	2.016	2.236

(a) Complete the table above, giving the missing value of y to 3 decimal places.

(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to find an approximate value for the area of R, giving your answer to 2 decimal places.

(4)

Area $\simeq \frac{1}{2}(0.25)\left[1.414 + 2(1.601 + 1.803 + 2.016) + 2.236\right]$

$\simeq 1.81125$ $\qquad \underline{1.81}$ (2dp)

2. $$f(x) = 2x^3 - 7x^2 + 4x + 4$$

(a) Use the factor theorem to show that $(x - 2)$ is a factor of $f(x)$.

(2)

(b) Factorise $f(x)$ completely.

(4)

a) $f(2) = 2(8) - 7(4) + 4(2) + 4 = 16 - 28 + 8 + 4 = 0$

$\therefore (x-2)$ is a factor

[NOTE - YOU NEED THIS!]

b)

$\times$	$2x^2$	$-3x$	-2
x	$2x^3$	$-3x^2$	$-2x$
-2	$-4x^2$	$+6x$	$+4$

$r = 0$

$(x-2)(x-2)(2x+1)$

3. (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2-3x)^6$$

giving each term in its simplest form.

(4)

(b) Hence, or otherwise, find the first 3 terms, in ascending powers of x, of the expansion of

$$\left(1+\frac{x}{2}\right)(2-3x)^6$$

(3)

$(a-b)^6 = a^6 - 6a^5b + 15a^4b^2$

$(2-3x)^6 = 2^6 - 6(2)^5(3x) + 15(2)^4(3x)^2$

$(2-3x)^6 = 64 - 576x + 2160x^2$

1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

b)

$\times$	64	$-576x$	$+2160x^2$
1	64	$-576x$	$2160x^2$
$+\frac{1}{2}x$	$32x$	$-288x^2$	$1080x^3$

$64 - 544x + 1872x^2$

4. Use integration to find

$$\int_1^{\sqrt{3}} \left(\frac{x^3}{6} + \frac{1}{3x^2} \right) \mathrm{d}x$$

giving your answer in the form $a + b\sqrt{3}$, where a and b are constants to be determined.

(5)

$$\int_1^{\sqrt{3}} \frac{1}{6}x^3 + \frac{1}{3}x^{-2}\,dx = \left[\frac{1}{24}x^4 - \frac{1}{3}x^{-1}\right]_1^{\sqrt{3}}$$

$$= \left(\frac{3}{8} - \frac{1}{9}\sqrt{3}\right) - \left(\frac{1}{24} - \frac{1}{3}\right) = \frac{2}{3} - \frac{1}{9}\sqrt{3}$$

5.

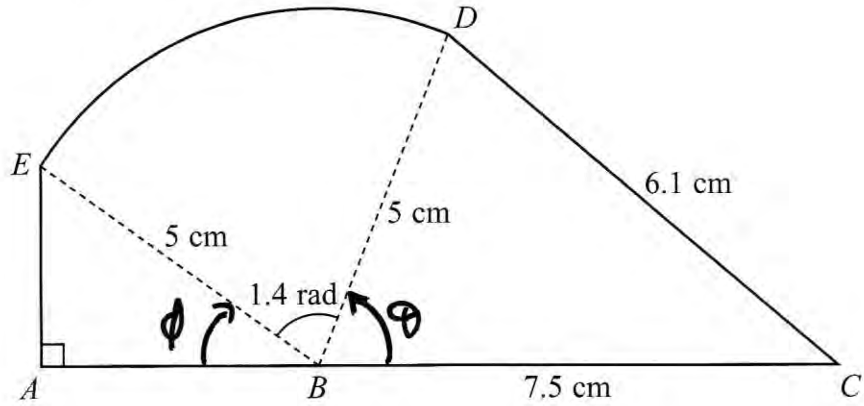


Figure 2

The shape *ABCDEA*, as shown in Figure 2, consists of a right-angled triangle *EAB* and a triangle *DBC* joined to a sector *BDE* of a circle with radius 5 cm and centre *B*.

The points *A*, *B* and *C* lie on a straight line with $BC = 7.5$ cm.

Angle $EAB = \dfrac{\pi}{2}$ radians, angle $EBD = 1.4$ radians and $CD = 6.1$ cm.

(a) Find, in cm^2, the area of the sector *BDE*.

(2)

(b) Find the size of the angle *DBC*, giving your answer in radians to 3 decimal places.

(2)

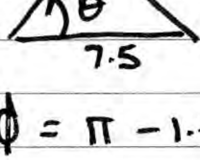
(c) Find, in cm^2, the area of the shape *ABCDEA*, giving your answer to 3 significant figures.

(5)

a) area $= \frac{1}{2}(5)^2(1.4) = 17.5\text{ cm}^2$

b) $\cos\theta = \dfrac{5^2 + 7.5^2 - 6.1^2}{2 \times 5 \times 7.5} \Rightarrow \theta = 0.943^c$

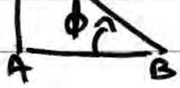
c)



area $= \frac{1}{2}(5)(7.5)\sin 0.943\ldots$

area $= 15.177\ldots$

$\phi = \pi - 1.4 - 0.943\ldots = 0.79839\ldots^c$



$AB = 5\cos 0.798\ldots$

$AB = 3.489\ldots$

$\therefore$ Total Area $= 38.9\text{ cm}^2$

area $= \frac{1}{2}(5)(3.489)\sin 0.798 = 6.2478\ldots$

6. The first term of a geometric series is 20 and the common ratio is $\frac{7}{8}$

The sum to infinity of the series is S_∞

(a) Find the value of S_∞

(2)

The sum to N terms of the series is S_N

(b) Find, to 1 decimal place, the value of S_{12}

(2)

(c) Find the smallest value of N, for which

$$S_\infty - S_N < 0.5$$

(4)

a) $S_\infty = \frac{a}{1-r} = \frac{20}{\left(\frac{1}{8}\right)} = 160$

b) $S_n = \frac{a(1-r^n)}{1-r} = \frac{20\left(1-\left(\frac{7}{8}\right)^{12}\right)}{\frac{1}{8}} = 127.7$

c) $160 - S_N < 0.5 \Rightarrow S_N > 159.5$

$\therefore \frac{20\left(1-\left(\frac{7}{8}\right)^N\right)}{\frac{1}{8}} > 159.5 \Rightarrow 1-\left(\frac{7}{8}\right)^N > 0.996875$

$\Rightarrow \left(\frac{7}{8}\right)^N < \frac{1}{320} \Rightarrow \log\left(\frac{7}{8}\right)^N < \log\left(\frac{1}{320}\right)$

$\Rightarrow N \times \log\left(\frac{7}{8}\right) < \log\left(\frac{1}{320}\right) \Rightarrow N > \frac{\log\left(\frac{1}{320}\right)}{\log\left(\frac{7}{8}\right)}$

$\therefore N > 43.198\ldots \quad \therefore N = 44$

7. (i) Solve, for $0 \leqslant \theta < 360°$, the equation

$$9\sin(\theta + 60°) = 4$$

giving your answers to 1 decimal place.
You must show each step of your working.

(4)

(ii) Solve, for $-\pi \leqslant x < \pi$, the equation

$$2\tan x - 3\sin x = 0$$

giving your answers to 2 decimal places where appropriate.
[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(5)

i) a) $\sin(\theta+60) = \frac{4}{9} \Rightarrow \theta + 60 = \sin^{-1}\left(\frac{4}{9}\right) = 23.39$

$\theta + 60 = 23.39, \ \underset{180-}{153.61}, \ \underset{+360}{386.39}$

$-60 \quad \therefore \theta = 93.6, \ 326.4$

ii) $2\frac{\sin x}{\cos x} - 3\sin x = 0 \quad \times\cos x \quad 2\sin x - 3\sin x\cos x = 0$

$\therefore \sin x(2 - 3\cos x) = 0$

$\Rightarrow \sin x = 0 \qquad \cos x = \frac{2}{3} \ \therefore x = 0.84^c, -0.84^c$

$x = -\pi, 0$

8. (a) Sketch the graph of

$$y = 3^x, \quad x \in \mathbb{R}$$

showing the coordinates of any points at which the graph crosses the axes.

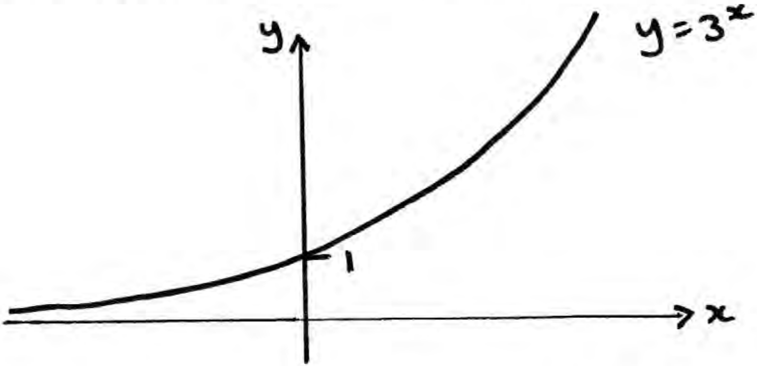
(2)

(b) Use algebra to solve the equation

$$3^{2x} - 9(3^x) + 18 = 0$$

giving your answers to 2 decimal places where appropriate.

(5)



b) $(3^x)^2 - 9(3^x) + 18 = 0 \Rightarrow (3^x - 6)(3^x - 3) = 0$

$\therefore 3^x = 3 \Rightarrow \underline{x = 1}$ $\quad 3^x = 6 \Rightarrow x = \log_3 6 = \underline{1.63}$

9.

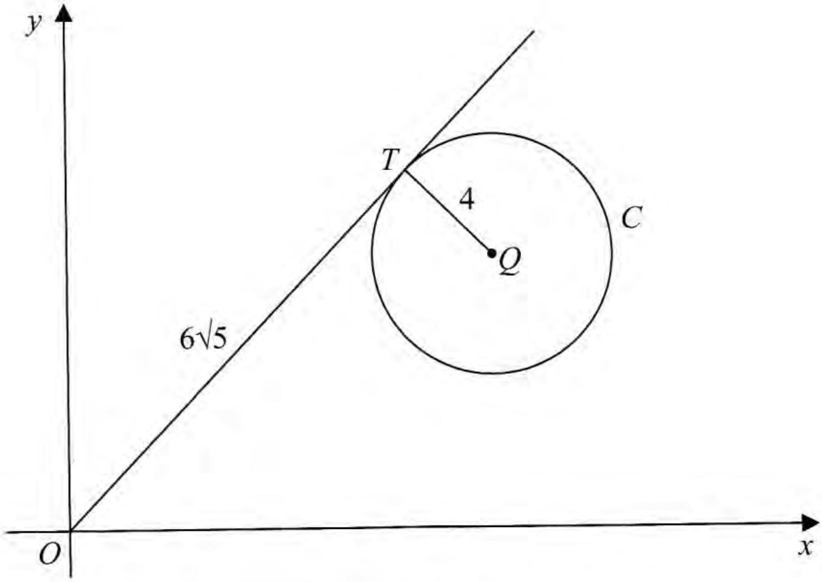


Figure 3

Figure 3 shows a circle C with centre Q and radius 4 and the point T which lies on C.

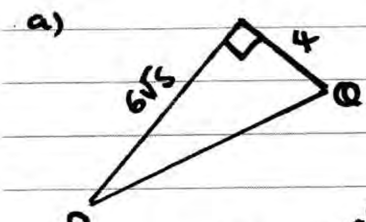
The tangent to C at the point T passes through the origin O and $OT = 6\sqrt{5}$

Given that the coordinates of Q are $(11, k)$, where k is a positive constant,

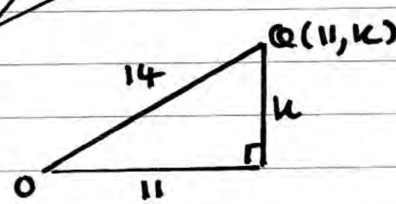
(a) find the exact value of k, **(3)**

(b) find an equation for C. **(2)**

a)



$OQ^2 = 4^2 + (6\sqrt{5})^2 = 196 \quad \therefore OQ = 14$



$\Rightarrow k^2 = 14^2 - 11^2 \Rightarrow k^2 = 75$

$\therefore k = \underline{5\sqrt{3}}$

b) $(x - 11)^2 + (y - 5\sqrt{3})^2 = 16$

10.

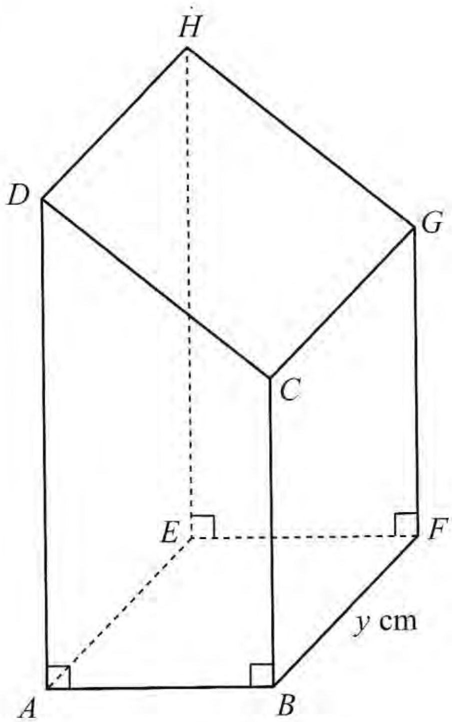


Figure 4

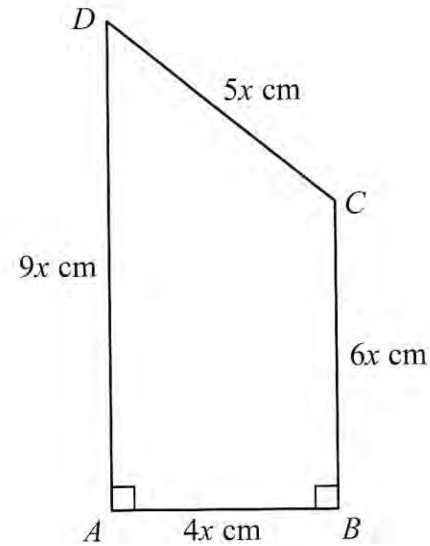


Figure 5

Figure 4 shows a closed letter box *ABFEHGCD*, which is made to be attached to a wall of a house.

The letter box is a right prism of length y cm as shown in Figure 4. The base *ABFE* of the prism is a rectangle. The total surface area of the six faces of the prism is $S\ \text{cm}^2$.

The cross section *ABCD* of the letter box is a trapezium with edges of lengths $DA = 9x$ cm, $AB = 4x$ cm, $BC = 6x$ cm and $CD = 5x$ cm as shown in Figure 5.
The angle $DAB = 90°$ and the angle $ABC = 90°$.

The volume of the letter box is $9600\ \text{cm}^3$.

(a) Show that

$$y = \frac{320}{x^2}$$

(2)

(b) Hence show that the surface area of the letter box, $S\ \text{cm}^2$, is given by

$$S = 60x^2 + \frac{7680}{x}$$

(4)

(c) Use calculus to find the minimum value of S.

(6)

(d) Justify, by further differentiation, that the value of S you have found is a minimum.

(2)

a) Area of trapezium $= \dfrac{(9x+6x)\times 4x}{2} = 15x \times 2x = 30x^2$

Volume $= 30x^2 \times y = 9600 \quad \therefore \quad y = \dfrac{9600}{30x^2} = \dfrac{320}{x^2}$

b) $SA = 2\times(30x^2) + 6xy + 5xy + 9xy + 4xy$

$SA = 60x^2 + 24xy = 60x^2 + 24x\left(\dfrac{320}{x^2}\right) = 60x^2 + \dfrac{7680}{x}$

c) $S = 60x^2 + 7680x^{-1}$ Min S when $S' = 0$

$S' = 120x - 7680x^{-2}$ $\Rightarrow 120x = \dfrac{7680}{x^2} \Rightarrow x^3 = 64$

$S'' = 120 + 15360x^{-3}$ $\therefore x = 4$ cm

$\therefore$ min $S = 60(4)^2 + \dfrac{7680}{4} = 2880\text{cm}^2$

d) when $x=4$ $S'' = 120 + \dfrac{15360}{4^3} = 360$ $\therefore S'' > 0$ $\cup$

$\therefore$ S is at a maximum when $x=4$