

Edexcel Maths Core 2

Mark Scheme Pack

2005–2013

GCE

Edexcel GCE

Core Mathematics C2 (6664)

Summer 2005

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Mark Scheme (Results)

Edexcel GCE
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June 2005
6664 Core Mathematics C2
Mark Scheme

Question number	Scheme	Marks
1.	$\frac{dy}{dx} = 4x - 12$ $4x - 12 = 0 \quad x = 3$ $y = -18$	B1 M1 A1ft A1 (4) 4
	<p>M1: Equate $\frac{dy}{dx}$ (not just y) to zero and proceed to $x = \dots$ A1ft: Follow through only from a linear equation in x.</p> <p><u>Alternative:</u> $y = 2x(x - 6) \Rightarrow$ Curve crosses x-axis at 0 and 6 B1 (By symmetry) $x = 3$ M1 A1ft $y = -18$ A1</p> <p><u>Alternative:</u> $(x - 3)^2$ B1 for $(x - 3)^2$ seen somewhere $y = 2(x^2 - 6x) = 2\{(x - 3)^2 - 9\} \quad x = 3$ M1 for attempt to complete square and deduce $x = \dots$ A1ft $[(x - a)^2 \Rightarrow x = a]$ $y = -18$ A1</p>	

Question number	Scheme	Marks
2.	<p>(a) $x \log 5 = \log 8$, $x = \frac{\log 8}{\log 5}$, $= 1.29$</p> <p>(b) $\log_2 \frac{x+1}{x}$ (or $\log_2 7x$)</p> <p>$\frac{x+1}{x} = 7$ $x = \dots$, $\frac{1}{6}$ (Allow 0.167 or better)</p>	<p>M1, A1, A1 (3)</p> <p>B1</p> <p>M1, A1 (3)</p> <p>6</p>
	<p>(a) Answer only 1.29 : Full marks.</p> <p>Answer only, which rounds to 1.29 (e.g. 1.292): M1 A1 A0</p> <p>Answer only, which rounds to 1.3 : M1 A0 A0</p> <p>Trial and improvement: Award marks as for “answer only”.</p> <p>(b) M1: Form (by legitimate log work) and solve an equation in x.</p> <p>Answer only: No marks unless verified (then full marks are available).</p>	

Question number	Scheme	Marks
3.	<p>(a) Attempt to evaluate $f(-4)$ or $f(4)$</p> $f(-4) = 2(-4)^3 + (-4)^2 - 25(-4) + 12 \quad (= 128 + 16 + 100 + 12) = 0,$ <p style="text-align: center;">so is a factor.</p> <p>(b) $(x + 4)(2x^2 - 7x + 3)$</p> <p style="text-align: center;">.....$(2x - 1)(x - 3)$</p>	<p>M1</p> <p>A1 (2)</p> <p>M1 A1</p> <p>M1 A1 (4)</p> <p>6</p>
	<p>(b) First M requires $(2x^2 + ax + b)$, $a \neq 0, b \neq 0$.</p> <p>Second M for the attempt to factorise the quadratic.</p> <p><u>Alternative:</u></p> <p>$(x + 4)(2x^2 + ax + b) = 2x^3 + (8 + a)x^2 + (4a + b)x + 4b = 0$, then compare coefficients to find <u>values</u> of a and b. [M1]</p> <p style="text-align: center;">$a = -7, b = 3$ [A1]</p> <p><u>Alternative:</u></p> <p>Factor theorem: Finding that $f\left(\frac{1}{2}\right) = 0, \therefore (2x - 1)$ is a factor [M1, A1]</p> <p>n.b. Finding that $f\left(\frac{1}{2}\right) = 0, \therefore (x - \frac{1}{2})$ is a factor scores M1, A0, unless the factor 2 subsequently appears.</p> <p style="text-align: center;">Finding that $f(3) = 0, \therefore (x - 3)$ is a factor [M1, A1]</p>	

Question number	Scheme	Marks
4.	<p>(a) $1 + 12px + \frac{12 \times 11}{2}(px)^2$</p> <p>(b) $12p(x) = -q(x) \quad 66p^2(x^2) = 11q(x^2) \quad (\text{Equate terms, or coefficients})$</p> <p>$\Rightarrow 66p^2 = -132p \quad (\text{Eqn. in } p \text{ or } q \text{ only})$</p> <p>$p = -2, \quad q = 24$</p>	<p>B1, B1 (2)</p> <p>M1</p> <p>M1</p> <p>A1, A1 (4)</p> <p>6</p>
	<p>(a) Terms can be listed rather than added. First B1: Simplified form must be seen, but may be in (b).</p> <p>(b) First M: May still have $\binom{12}{2}$ or ${}^{12}C_2$</p> <p>Second M: <u>Not</u> with $\binom{12}{2}$ or ${}^{12}C_2$. Dependent upon having p's in each term.</p> <p>Zero solutions must be rejected for the final A mark.</p>	

Question number	Scheme	Marks
5.	<p>(a) $(x + 10 =) \quad 60 \quad \alpha$ 120 (M: $180 - \alpha$ or $\pi - \alpha$) $x = 50 \quad x = 110$ (or 50.0 and 110.0) (M: Subtract 10)</p> <p>(b) $(2x =) \quad 154.2 \quad \beta$ Allow a.w.r.t. 154 or a.w.r.t. 2.69 (radians) 205.8 (M: $360 - \beta$ or $2\pi - \beta$) $x = 77.1 \quad x = 102.9$ (M: Divide by 2)</p>	<p>B1 M1 M1 A1 (4)</p> <p>B1 M1 M1 A1 (4)</p> <p>8</p>
	<p>(a) First M: Must be subtracting from 180 <u>before</u> subtracting 10. (b) First M: Must be subtracting from 360 <u>before</u> dividing by 2, <u>or</u> dividing by 2 then subtracting from 180.</p> <p>In each part: Extra solutions outside 0 to 180 : Ignore. Extra solutions between 0 and 180 : A0.</p> <p><u>Alternative for (b): (double angle formula)</u></p> <p>$1 - 2\sin^2 x = -0.9$ $2\sin^2 x = 1.9$ B1 $\sin x = \sqrt{0.95}$ M1 $x = 77.1$ $x = 180 - 77.1 = 102.9$ M1 A1</p>	

Question number	Scheme	Marks
6.	<p>(a) Missing y values: 1.6(00) 3.2(00) 3.394</p> <p>(b) $(A =) \frac{1}{2} \times 4, \{(0 + 0) + 2(1.6 + 2.771 + 3.394 + 3.2)\}$ = 43.86 (or a more accurate value) (or 43.9, or 44)</p> <p>(c) Volume = $A \times 2 \times 60$ = 5260 (m³) (or 5270, or 5280)</p>	<p>B1 B1 (2)</p> <p>B1, M1 A1ft A1 (4)</p> <p>M1 A1 (2)</p> <p>8</p>
	<p>(b) Answer only: No marks.</p> <p>(c) Answer only: Allow. (The M mark in this part can be “implied”).</p>	

Question number	Scheme	Marks
7.	<p>(a) $\frac{\sin x}{8} = \frac{\sin 0.5}{7}$ or $\frac{8}{\sin x} = \frac{7}{\sin 0.5}$, $\sin x = \frac{8 \sin 0.5}{7}$ $\sin x = 0.548$</p> <p>(b) $x = 0.58$ (α) (This mark may be earned in (a)). $\pi - \alpha = 2.56$</p>	<p>M1 A1ft A1 (3)</p> <p>B1 M1 A1ft (3)</p> <p>6</p>
	<p>(a) M: Sine rule attempt (sides/angles possibly the “wrong way round”). A1ft: follow through from sides/angles are the “wrong way round”.</p> <p><u>Too many d.p. given:</u> Maximum 1 mark penalty in the complete question. (Deduct on first occurrence).</p>	

Question number	Scheme	Marks
8.	<p>(a) Centre (5, 0) (or $x = 5, y = 0$)</p> <p>(b) $(x \pm a)^2 \pm b \pm 9 + (y \pm c)^2 = 0 \Rightarrow r^2 = \dots$ or $r = \dots$, Radius = 4</p> <p>(c) (1, 0), (9, 0) Allow just $x = 1, x = 9$</p> <p>(d) Gradient of $AT = -\frac{2}{7}$ $y = -\frac{2}{7}(x - 5)$</p>	<p>B1 B1 (2)</p> <p>M1, A1 (2)</p> <p>B1ft, B1ft (2)</p> <p>B1</p> <p>M1 A1ft (3)</p> <p>9</p>
	<p>(a) (0, 5) scores B1 B0.</p> <p>(d) M1: Equation of straight line through centre, <u>any</u> gradient (except 0 or ∞) (The equation can be in any form). A1ft: Follow through from centre, but gradient must be $-\frac{2}{7}$.</p>	

Question number	Scheme	Marks
9.	<p>(a) ($S =$) $a + ar + \dots + ar^{n-1}$ “$S =$” not required. Addition required.</p> <p>($rS =$) $ar + ar^2 + \dots + ar^n$ “$rS =$” not required (M: Multiply by r)</p> <p>$S(1 - r) = a(1 - r^n)$ $S = \frac{a(1 - r^n)}{1 - r}$ (M: Subtract and factorise) (*)</p> <p>(b) $ar^{n-1} = 35000 \times 1.04^3 = 39400$ (M: Correct a and r, with $n = 3, 4$ or 5).</p> <p>(c) $n = 20$ (Seen or implied)</p> <p>$S_{20} = \frac{35000(1 - 1.04^{20})}{(1 - 1.04)}$</p> <p>(M1: Needs <u>any</u> r value, $a = 35000$, $n = 19, 20$ or 21).</p> <p>(A1ft: ft from $n = 19$ or $n = 21$, but r must be 1.04).</p> <p>$= 1\,042\,000$</p>	<p>B1</p> <p>M1</p> <p>M1 A1cso (4)</p> <p>M1 A1 (2)</p> <p>B1</p> <p>M1 A1ft</p> <p>A1 (4)</p> <p>10</p>
	<p>(a) B1: At least the 3 terms shown above, and no extra terms. A1: Requires a completely correct solution. <u>Alternative for the 2 M marks:</u> M1: Multiply numerator and denominator by $1 - r$. M1: Multiply out numerator convincingly, and factorise.</p> <p>(b) M1 can also be scored by a “year by year” method. <u>Answer only:</u> 39 400 scores full marks, 39 370 scores M1 A0.</p> <p>(c) M1 can also be scored by a “year by year” method, <u>with terms added</u>. In this case the B1 will be scored if the correct number of years is considered. <u>Answer only:</u> Special case: 1 042 000 scores 2 B marks, scored as 1, 0, 0, 1 (Other answers score no marks).</p> <p><u>Failure to round correctly in (b) and (c):</u> Penalise once only (first occurrence).</p>	

Question number	Scheme	Marks
10.	<p>(a) $\int (2x + 8x^{-2} - 5)dx = x^2 + \frac{8x^{-1}}{-1} - 5x$</p> $\left[x^2 + \frac{8x^{-1}}{-1} - 5x \right]_1^4 = (16 - 2 - 20) - (1 - 8 - 5) \quad (= 6)$ <p>$x = 1: y = 5$ and $x = 4: y = 3.5$</p> <p>Area of trapezium = $\frac{1}{2}(5 + 3.5)(4 - 1) \quad (= 12.75)$</p> <p>Shaded area = $12.75 - 6 = 6.75$ (M: Subtract either way round)</p> <p>(b) $\frac{dy}{dx} = 2 - 16x^{-3}$</p> <p>(Increasing where) $\frac{dy}{dx} > 0$; For $x > 2, \frac{16}{x^3} < 2, \therefore \frac{dy}{dx} > 0$ (Allow \geq)</p>	<p>M1 A1 A1</p> <p>M1</p> <p>B1</p> <p>M1</p> <p>M1 A1 (8)</p> <p>M1 A1</p> <p>dM1; A1 (4)</p> <p>12</p>
	<p>(a) Integration: One term wrong M1 A1 A0; two terms wrong M1 A0 A0. Limits: M1 for substituting limits 4 and 1 into a changed function, and subtracting the right way round.</p> <p><u>Alternative:</u></p> <p>$x = 1: y = 5$ and $x = 4: y = 3.5$</p> <p>Equation of line: $y - 5 = -\frac{1}{2}(x - 1) \quad y = \frac{11}{2} - \frac{1}{2}x$, subsequently used in integration with limits.</p> $\left(\frac{11}{2} - \frac{1}{2}x \right) - \left(2x + \frac{8}{x^2} - 5 \right) \quad (\text{M: Subtract either way round})$ $\int \left(\frac{21}{2} - \frac{5x}{2} - 8x^{-2} \right) dx = \frac{21x}{2} - \frac{5x^2}{4} - \frac{8x^{-1}}{-1}$ <p>(Penalise integration mistakes, not algebra for the ft marks)</p> $\left[\frac{21x}{2} - \frac{5x^2}{4} - \frac{8x^{-1}}{-1} \right]_1^4 = (42 - 20 + 2) - \left(\frac{21}{2} - \frac{5}{4} + 8 \right) \quad (\text{M: Right way round})$ <p>Shaded area = 6.75</p> <p>(The follow through marks are for the subtracted version, and again deduct an accuracy mark for a wrong term: One wrong M1 A1 A0; two wrong M1 A0 A0.)</p> <p><u>Alternative for the last 2 marks in (b):</u></p> <p>M1: Show that $x = 2$ is a minimum, using, e.g., 2nd derivative.</p> <p>A1: Conclusion showing understanding of “increasing”, with accurate working.</p>	<p>B1</p> <p>3rd M1</p> <p>4th M1</p> <p>1st M1 A1ft A1ft</p> <p>2nd M1</p> <p>A1</p>

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1.	<p>(a) $2+1-5 + c = 0$ or $-2 + c = 0$</p> <p style="text-align: center;"><u>$c = 2$</u></p> <p>(b) $f(x) = (x-1)(2x^2 + 3x - 2)$ $(x-1)$</p> <p style="text-align: right;">division</p> <p style="text-align: center;">$= \dots \underline{(2x-1)(x+2)}$</p> <p>(c) $f\left(\frac{3}{2}\right) = 2 \times \frac{27}{8} + \frac{9}{4} - \frac{15}{2} + c$</p> <p>Remainder $= c + 1.5 = \underline{3.5}$ ft their c</p>	<p>M1</p> <p>A1 (2)</p> <p>B1</p> <p>M1</p> <p>M1 A1 (4)</p> <p>M1</p> <p>A1ft (2)</p> <p style="text-align: right;">8</p>
	<p>(a) M1 for evidence of substituting $x = 1$ leading to linear equation in c</p> <p>(b) B1 for identifying $(x-1)$ as a factor</p> <p>1st M1 for attempting to divide.</p> <p style="text-align: center;">Other factor must be at least $(2x^2 + \text{one other term})$</p> <p>2nd M1 for attempting to factorise a quadratic resulting from attempted division</p> <p>A1 for just $(2x-1)(x+2)$.</p> <p>(c) M1 for attempting $f(\pm\frac{3}{2})$. If not implied by $1.5 + c$, we must see some substitution of $\pm\frac{3}{2}$.</p> <p>A1 follow through their c only, but it must be a number.</p>	

Question number	Scheme	Marks
2.	<p>(a) $(1+px)^9 = 1+9px; +\binom{9}{2}(px)^2$</p> <p>(b) $9p = 36$, so $p = 4$</p> <p>$q = \frac{9 \times 8}{2} p^2$ or $36p^2$ or $36p$ if that follows from their (a)</p> <p>So $q = 576$</p>	<p>B1 B1 (2)</p> <p>M1 A1</p> <p>M1</p> <p>A1cao (4)</p> <p>6</p>
N.B.	<p>(a) 2nd B1 for $\binom{9}{2}(px)^2$ or better. Condone “,” not “+”.</p> <p>(b) 1st M1 for a linear equation for p.</p> <p>2nd M1 for either printed expression, follow through their p.</p> <p>1+9px+36px² leading to $p = 4$, $q = 144$ scores B1B0 M1A1M1A0 i.e 4/6</p>	
3.	<p>(a) $(AB)^2 = (4-3)^2 + (5)^2$ [= 26]</p> <p>$AB = \sqrt{26}$</p> <p>(b) $p = \left(\frac{4+3}{2}, \frac{5}{2}\right)$</p> <p>$= \left(\frac{7}{2}, \frac{5}{2}\right)$</p> <p>(c) $(x-x_p)^2 + (y-y_p)^2 = \left(\frac{AB}{2}\right)^2$</p> <p>$(x-3.5)^2 + (y-2.5)^2 = 6.5$</p>	<p>M1</p> <p>A1 (2)</p> <p>M1</p> <p>A1 (2)</p> <p>LHS M1</p> <p>RHS M1</p> <p>oe A1 c.a.o (3)</p> <p>7</p>
	<p>(a) M1 for an expression for AB or AB^2 N.B. $(x_1 + x_2)^2 + \dots$ is M0</p> <p>(b) M1 for a full method for x_p</p> <p>(c) 1st M1 for using their x_p and y_p in LHS</p> <p>2nd M1 for using their AB in RHS</p> <p>N.B. $x^2 + y^2 - 7x - 5y + 12 = 0$ scores, of course, 3/3 for part (c).</p> <p>Condone use of calculator approximations that lead to correct answer given.</p>	

Question number	Scheme	Marks
4.	<p>(a) $\frac{a}{1-r} = 480$</p> <p>$\frac{120}{1-r} = 480 \Rightarrow 120 = 480(1-r)$</p> <p>$1-r = \frac{1}{4} \Rightarrow \underline{r = \frac{3}{4}} \quad *$</p> <p>(b) $u_5 = 120 \times \left(\frac{3}{4}\right)^4 [= 37.96875]$</p> <p>$u_6 = 120 \times \left(\frac{3}{4}\right)^5 [= 28.4765625]$</p> <p>Difference = <u>9.49</u></p> <p>(c) $S_7 = \frac{120(1-(0.75)^7)}{1-0.75}$</p> <p>$= 415.9277...$</p> <p>(d) $\frac{120(1-(0.75)^n)}{1-0.75} > 300$</p> <p>$1-(0.75)^n > \frac{300}{480}$</p> <p>$n > \frac{\log(0.375)}{\log(0.75)}$</p> <p>$\underline{n = 4}$</p>	<p>M1</p> <p>M1</p> <p>A1cso (3)</p> <p>either M1</p> <p>(allow \pm) A1 (2)</p> <p>M1</p> <p>(AWRT) <u>416</u> A1 (2)</p> <p>M1</p> <p>(or better) A1</p> <p>(=3.409...) M1</p> <p>A1cso (4)</p> <p>11</p>
Trial & Imp.	<p>(a) 1st M1 for use of S_∞</p> <p>2nd M1 substituting for a and moving $(1-r)$ to form linear equation in r.</p> <p>(b) M1 for some correct use of ar^{n-1}. [$120\left(\frac{3}{4}\right)^5 - 120\left(\frac{3}{4}\right)^6$ is M0]</p> <p>(c) M1 for a correct expression (need use of a and r)</p> <p>(d) 1st M1 for attempting $S_n > 300$ [or = 300] (need use of a and some use of r)</p> <p>2nd M1 for valid attempt to solve $r^n = p(r, p < 1)$, must give linear eqn in n. Any correct log form will do.</p> <p>1st M1 for attempting at least 2 values of S_n, one $n < 4$ and one $n \geq 4$.</p> <p>2nd M1 for attempting S_3 and S_4.</p> <p>1st A1 for both values correct to 2 s.f. or better.</p> <p>2nd A1 for $n = 4$.</p>	<p><u>For Information</u></p> <p>$u_1 = 120$</p> <p>$u_2 = 90$</p> <p>$u_3 = 67.5$</p> <p>$u_4 = 50.625$</p> <p>$S_2 = 210$</p> <p>$S_3 = 277.5$</p> <p>$S_4 = 328.125$</p> <p>$S_5 = 366.09...$</p>

Question number	Scheme	Marks
5.	<p>(a) $\cos \hat{AOB} = \frac{5^2 + 5^2 - 6^2}{2 \times 5 \times 5}$ or</p> <p>$\sin \theta = \frac{3}{5}$ with use of $\cos 2\theta = 1 - 2\sin^2 \theta$ attempted</p> <p>$= \frac{7}{25}$ *</p> <p>(b) $\hat{AOB} = 1.2870022\dots$ radians 1.287 or better</p> <p>(c) Sector $= \frac{1}{2} \times 5^2 \times (b)$, $= 16.087\dots$ (AWRT) <u>16.1</u></p> <p>(d) Triangle $= \frac{1}{2} \times 5^2 \times \sin(b)$ or $\frac{1}{2} \times 6 \times \sqrt{5^2 - 3^2}$</p> <p>Segment = (their sector) – their triangle</p> <p>= (sector from c) – 12 = (AWRT)<u>4.1</u> (ft their part(c))</p>	<p>M1</p> <p>A1cso (2)</p> <p>B1 (1)</p> <p>M1 A1 (2)</p> <p>M1</p> <p>dM1</p> <p>A1ft (3)</p> <p>8</p>
	<p>(a) M1 for a full method leading to $\cos \hat{AOB}$ [N.B. Use of calculator is M0] (usual rules about quoting formulae)</p> <p>(b) Use of (b) in degrees is M0</p> <p>(d) 1st M1 for full method for the area of triangle AOB</p> <p>2nd M1 for their sector – their triangle. Dependent on 1st M1 in part (d).</p> <p>A1ft for their sector from part (c) – 12 [or 4.1 following a correct restart].</p>	

Question number	Scheme	Marks
6.	<p>(a) $t = 15 \quad 25 \quad 30$ $v = \underline{3.80 \quad 9.72 \quad 15.37}$</p> <p>(b) $S \approx \frac{1}{2} \times 5; [0 + 15.37 + 2(1.22 + 2.28 + 3.80 + 6.11 + 9.72)]$ $= \frac{5}{2} [61.63] = 154.075 = \text{AWRT } \underline{154}$</p>	<p>B1 B1 B1 (3)</p> <p>B1 [M1]</p> <p>A1 (3)</p> <p>6</p>
	<p>(a) S.C. Penalise AWRT these values <u>once</u> at first offence, thus the following marks could be AWRT 2 dp (Max 2/3)</p>	

Question number	Scheme	Marks
7.	<p>(a) $\frac{dy}{dx} = 6x^2 - 10x - 4$</p> <p>(b) $6x^2 - 10x - 4 = 0$ $2(3x + 1)(x - 2) [=0]$ $\underline{x = 2 \text{ or } -\frac{1}{3}}$ (both x values)</p> <p>Points are $(2, -10)$ and $(-\frac{1}{3}, 2\frac{19}{27} \text{ or } \frac{73}{27} \text{ or } 2.70 \text{ or better})$ (both y values)</p> <p>(c) $\frac{d^2y}{dx^2} = 12x - 10$</p> <p>(d) $x = 2 \Rightarrow \frac{d^2y}{dx^2} (=14) \geq 0 \therefore [(2, -10)]$ is a <u>Min</u></p> <p>$x = -\frac{1}{3} \Rightarrow \frac{d^2y}{dx^2} (= -14) \leq 0 \therefore [(-\frac{1}{3}, \frac{73}{27})]$ is a <u>Max</u></p>	<p>M1 A1 (2)</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1 (4)</p> <p>M1 A1 (2)</p> <p>M1</p> <p>A1 (2)</p> <p>10</p>
	<p>(a) M1 for some correct attempt to differentiate $x^n \rightarrow x^{n-1}$</p> <p>(b) 1st M1 for setting their $\frac{dy}{dx} = 0$</p> <p>2nd M1 for attempting to solve 3TQ but it must be based on their $\frac{dy}{dx}$.</p> <p>NO marks for answers only in part (b)</p> <p>(c) M1 for attempting to differentiate their $\frac{dy}{dx}$</p> <p>(d) M1 for one correct use of their second derivative or a full method to determine the nature of one of their stationary points</p> <p>A1 both correct (=14 and = -14) are not required</p>	

Question number	Scheme	Marks
8.	<p>(a) $\sin(\theta + 30) = \frac{3}{5}$ ($\frac{3}{5}$ on RHS)</p> <p style="text-align: center;">$\theta + 30 = 36.9$ ($\alpha = \text{AWRT } 37$)</p> <p>or $\theta = 143.1$ ($180 - \alpha$)</p> <p style="text-align: center;"><u>$\theta = 6.9, 113.1$</u> A1cao</p> <p>(b) $\tan \theta = \pm 2$ or $\sin \theta = \pm \frac{2}{\sqrt{5}}$ or $\cos \theta = \pm \frac{1}{\sqrt{5}}$ B1</p> <p>($\tan \theta = 2 \Rightarrow$) $\theta = \underline{63.4}$ ($\beta = \text{AWRT } 63.4$) B1</p> <p>or $\underline{243.4}$ ($180 + \beta$) M1</p> <p>($\tan \theta = -2 \Rightarrow$) $\theta = \underline{116.6}$ ($180 - \beta$) M1</p> <p>or $\underline{296.6}$ ($180 + \text{their } 116.6$) M1</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>(4)</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>(5)</p> <p>9</p>
	<p>(a) M1 for $180 -$ their first solution. Must be at the correct stage i.e. for $\theta + 30$</p> <p>(b) ALL M marks in (b) must be for $\theta = \dots$</p> <p>1st M1 for $180 +$ their first solution</p> <p>2nd M1 for $180 -$ their first solution</p> <p>3rd M1 for $180 +$ their 116.6 or $360 -$ their first solution</p> <p><u>Answers Only</u> can score full marks in both parts</p> <p><u>Not 1 d.p.:</u> loses A1 in part (a). In (b) all answers are AWRT.</p> <p>Ignore extra solutions outside range</p> <p><u>Radians</u> Allow M marks for consistent work with radians only, but all A and B marks for angles must be in degrees. Mixing degrees and radians is M0.</p>	

Question number	Scheme	Marks
9.	<p>(a) $\frac{3}{2} = -2x^2 + 4x$</p> <p>$4x^2 - 8x + 3 (= 0)$</p> <p>$(2x-1)(2x-3) = 0$</p> <p><u>$x = \frac{1}{2}, \frac{3}{2}$</u></p> <p>(b) Area of $R = \int_{\frac{1}{2}}^{\frac{3}{2}} (-2x^2 + 4x) \, dx - \frac{3}{2}$ (for $-\frac{3}{2}$)</p> <p>$\int (-2x^2 + 4x) \, dx = \left[-\frac{2}{3}x^3 + 2x^2 \right]$ (Allow $\pm[\quad]$, accept $\frac{4}{2}x^2$)</p> <p>$\int_{\frac{1}{2}}^{\frac{3}{2}} (-2x^2 + 4x) \, dx = \left(-\frac{2}{3} \times \frac{3^3}{2^3} + 2 \times \frac{3^2}{2^2} \right) - \left(-\frac{2}{3} \times \frac{1}{2^3} + 2 \times \frac{1}{2^2} \right)$</p> <p>$\left(= \frac{11}{6} \right)$</p> <p>Area of $R = \frac{11}{6} - \frac{3}{2} = \frac{1}{3}$ (Accept exact equivalent but not 0.33...)</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1 (4)</p> <p>B1</p> <p>M1 [A1]</p> <p>M1 M1</p> <p>A1cao (6)</p> <p>10</p>
Special Case	<p>(a) 1st M1 for forming a correct equation</p> <p>1st A1 for a correct 3TQ (condone missing =0 but must have all terms on one side)</p> <p>2nd M1 for attempting to solve appropriate 3TQ</p> <p>(b) B1 for subtraction of $\frac{3}{2}$. Either “curve – line” or “integral – rectangle”</p> <p>1st M1 for some correct attempt at integration ($x^n \rightarrow x^{n+1}$)</p> <p>1st A1 for $-\frac{2}{3}x^3 + 2x^2$ only i.e. can ignore $-\frac{3}{2}x$</p> <p>2nd M1 for some correct use of their $\frac{3}{2}$ as a limit in integral</p> <p>3rd M1 for some correct use of their $\frac{1}{2}$ as a limit in integral and subtraction either way round</p> <p><u>Line – curve</u> gets B0 but can have the other A marks provided final answer is $+\frac{1}{3}$.</p>	

GENERAL PRINCIPLES FOR C1 & C2 MARKING

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm p)^2 \pm q \pm c$, $p \neq 0$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but will be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may gain no credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Misreads

A misread must be consistent for the whole question to be interpreted as such.

These are not common. In clear cases, please deduct the first 2 A (or B) marks which would have been lost by following the scheme. (Note that 2 marks is the maximum misread penalty, but that misreads which alter the nature or difficulty of the question cannot be treated so generously and it will usually be necessary here to follow the scheme as written).

Sometimes following the scheme as written is more generous to the candidate than applying the misread rule, so in this case use the scheme as written.

If in doubt please send to review or refer to Team Leader.

GCE

Edexcel GCE

Core Mathematics C2 (6664)

June 2006

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Mark Scheme
(Results)

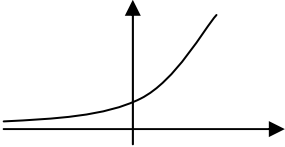
June 2006
6664 Core Mathematics C2
Mark Scheme

Question number	Scheme	Marks
1.	$(2+x)^6 = 64 \dots$ $+ (6 \times 2^5 \times x) + \left(\frac{6 \times 5}{2} \times 2^4 \times x^2 \right), \quad +192x, \quad +240x^2$	B1 M1, A1, A1 (4) 4
	<p>The terms can be 'listed' rather than added.</p> <p>M1: Requires correct structure: 'binomial coefficients' (perhaps from Pascal's triangle), increasing powers of one term, decreasing powers of the other term (this may be 1 if factor 2 has been taken out). Allow 'slips'.</p> <p>$\binom{6}{1}$ and $\binom{6}{2}$ or equivalent are acceptable, or even $\binom{6}{1}$ and $\binom{6}{2}$.</p> <p><u>Decreasing powers of x:</u> Can score only the M mark.</p> <p>64(1 +), even if all terms in the bracket are correct, scores max. B1M1A0A0.</p>	

Question number	Scheme	Marks
2.	$\int (3x^2 + 5 + 4x^{-2}) dx = \frac{3x^3}{3} + 5x + \frac{4x^{-1}}{-1} \quad (= x^3 + 5x - 4x^{-1})$ $[x^3 + 5x - 4x^{-1}]_1^2 = (8 + 10 - 2) - (1 + 5 - 4), = 14$	M1 A1 A1 M1, A1 (5) 5
	<p><u>Integration:</u></p> <p>Accept any correct version, simplified or not.</p> <p>All 3 terms correct: M1 A1 A1, Two terms correct: M1 A1 A0, One power correct: M1 A0 A0.</p> <p>The <u>given</u> function must be integrated to score M1, and not e.g. $3x^4 + 5x^2 + 4$.</p> <p><u>Limits:</u></p> <p>M1: Substituting 2 and 1 into a ‘changed function’ and subtracting, either way round.</p>	

Question number	Scheme	Marks
3.	(i) 2 (ii) $2\log 3 = \log 3^2$ (or $2\log p = \log p^2$) $\log_a p + \log_a 11 = \log_a 11p, \quad = \log_a 99$ (Allow e.g. $\log_a (3^2 \times 11)$)	B1 (1) B1 M1, A1 (3) 4
	(ii) Ignore 'missing base' or wrong base. The correct answer with no working scores full marks. $\log_a 9 \times \log_a 11 = \log_a 99$, or similar mistakes, score M0 A0.	

Question number	Scheme	Marks
4.	<p>(a) $f(-2) = 2(-2)^3 + 3(-2)^2 - 29(-2) - 60$ M: Attempt $f(2)$ or $f(-2)$ $= -16 + 12 + 58 - 60 = -6$ A1 (2)</p> <p>(b) $f(-3) = 2(-3)^3 + 3(-3)^2 - 29(-3) - 60$ M: Attempt $f(3)$ or $f(-3)$ $(= -54 + 27 + 87 - 60) = 0 \therefore (x + 3)$ is a factor A1 (2)</p> <p>(c) $(x + 3)(2x^2 - 3x - 20)$ M1 A1 $= (x + 3)(2x + 5)(x - 4)$ M1 A1 (4)</p>	8
	<p>(a) <u>Alternative (long division):</u> Divide by $(x + 2)$ to get $(2x^2 + ax + b)$, $a \neq 0, b \neq 0$. [M1] $(2x^2 - x - 27)$, remainder $= -6$ [A1]</p> <p>(b) A1 requires zero and a simple conclusion (even just a tick, or Q.E.D.).</p> <p>(c) First M requires division by $(x + 3)$ to get $(2x^2 + ax + b)$, $a \neq 0, b \neq 0$. Second M for the attempt to factorise their quadratic. Usual rule: $(2x^2 + ax + b) = (2x + c)(x + d)$, where $cd = b$.</p> <p><u>Alternative (first 2 marks):</u> $(x + 3)(2x^2 + ax + b) = 2x^3 + (6 + a)x^2 + (3a + b)x + 3b = 0$, then compare coefficients to find <u>values</u> of a and b. [M1] $a = -3, b = -20$ [A1]</p> <p><u>Alternative:</u> Factor theorem: Finding that $f\left(-\frac{5}{2}\right) = 0 \therefore$ factor is, $(2x + 5)$ [M1, A1] Finding that $f(4) = 0 \therefore$ factor is, $(x - 4)$ [M1, A1] “Combining” all 3 factors is <u>not</u> required. If just one of these is found, score the <u>first 2 marks</u> M1 A1 M0 A0. <u>Losing a factor of 2:</u> $(x + 3)\left(x + \frac{5}{2}\right)(x - 4)$ scores M1 A1 M1 A0. <u>Answer only, one sign wrong:</u> e.g. $(x + 3)(2x - 5)(x - 4)$ scores M1 A1 M1 A0.</p>	

Question number	Scheme	Marks
5.	<p>(a) </p> <p style="text-align: right;">Shape</p> <p>(0, 1), or just 1 on the y-axis, or seen in table for (b)</p> <p>(b) Missing values: 1.933, 2.408 (Accept awrt)</p> <p>(c) $\frac{1}{2} \times 0.2, \{(1+3) + 2(1.246 + 1.552 + 1.933 + 2.408)\}$ $= 1.8278$ (awrt 1.83)</p>	<p>B1</p> <p>B1 (2)</p> <p>B1, B1 (2)</p> <p>B1, M1 A1ft</p> <p>A1 (4)</p> <p>8</p>
	<p>Beware the order of marks!</p> <p>(a) Must be a curve (not a straight line). Curve must extend to the left of the y-axis, and must be increasing. Curve can 'touch' the x-axis, but must not go below it. Otherwise, be generous in cases of doubt.</p> <p>The B1 for (0, 1) is independent of the sketch.</p> <p>(c) Bracketing mistake: i.e. $\frac{1}{2} \times 0.2(1+3) + 2(1.246 + 1.552 + 1.933 + 2.408)$</p> <p>scores B1 M1 A0 A0 <u>unless</u> the final answer implies that the calculation has been done correctly (then full marks can be given).</p>	

Question number	Scheme	Marks
6.	<p>(a) $\tan \theta = 5$</p> <p>(b) $\tan \theta = k$ $\left(\theta = \tan^{-1} k \right)$</p> <p>$\theta = 78.7, \quad 258.7$ (Accept awrt)</p>	<p>B1 (1)</p> <p>M1</p> <p>A1, A1ft (3)</p> <p>4</p>
	<p>(a) Must be seen explicitly, e.g. $\tan \theta = \tan^{-1} 5 = 78.7$ or equiv. is B0, unless $\tan \theta = 5$ is also seen.</p> <p>(b) The M mark may be implied by working in (a).</p> <p>A1ft for $180 + \alpha$. ($\alpha \neq k$).</p> <p>Answers in radians would lose both the A marks.</p> <p>Extra answers between 0 and 360: Deduct the final mark.</p> <p><u>Alternative:</u></p> <p>Using $\cos^2 \theta = 1 - \sin^2 \theta$ (or equiv.) and proceeding to $\sin \theta = k$ (or equiv.): M1 then A marks as in main scheme.</p>	

Question number	Scheme	Marks
7.	<p>(a) Gradient of PQ is $-\frac{1}{3}$</p> $y - 2 = -\frac{1}{3}(x - 2) \quad (3y + x = 8)$ <p>(b) $y = 1: \quad 3 + x = 8 \quad x = 5 \quad (*)$</p> <p>(c) $(5 - 2)^2 + (1 - 2)^2 \quad \text{M: Attempt } PQ^2 \text{ or } PQ$</p> $(x - 5)^2 + (y - 1)^2 = 10 \quad \text{M: } (x \pm a)^2 + (y \pm b)^2 = k$	<p>B1</p> <p>M1 A1 (3)</p> <p>B1 (1)</p> <p>M1 A1</p> <p>M1 A1 (4)</p> <p>8</p>
	<p>(a) M1: eqn. of a straight line through (2, 2) with any gradient except 3, 0 or ∞. <u>Alternative:</u> Using (2, 2) in $y = mx + c$ to find a value of c scores M1, but an equation (general or specific) must be seen. If the given value $x = 5$ is used to find the gradient of PQ, maximum marks are (a) B0 M1 A1 (b) B0.</p> <p>(c) For the first M1, condone <u>one</u> slip, numerical or sign, <u>inside</u> a bracket. The first M1 can be scored if <u>their</u> x-coord. is used instead of 5. For the second M1, allow any equation in this form, with non-zero a, b and k.</p>	

Question number	Scheme	Marks
8.	<p>(a) $r\theta = 2.12 \times 0.65$ 1.38 (m)</p> <p>(b) $\frac{1}{2}r^2\theta = \frac{1}{2} \times 2.12^2 \times 0.65$ 1.46 (m²)</p> <p>(c) $\frac{\pi}{2} - 0.65$ 0.92 (radians) (α)</p> <p>(d) $\Delta ACD : \frac{1}{2}(2.12)(1.86)\sin \alpha$ (With the value of α from part (c))</p> <p>Area = “1.46” + “1.57”, 3.03 (m²)</p>	<p>M1 A1 (2)</p> <p>M1 A1 (2)</p> <p>M1 A1 (2)</p> <p>M1</p> <p>M1 A1 (3)</p> <p>9</p>
	<p>(a) M1: Use of $r\theta$ with $r = 2.12$ or 1.86, and $\theta = 0.65$, or equiv. method for the angle changed to degrees (allow awrt 37°).</p> <p>(b) M1: Use of $\frac{1}{2}r^2\theta$ with $r = 2.12$ or 1.86, and $\theta = 0.65$, or equiv. method for the angle changed to degrees (allow awrt 37°).</p> <p>(c) M1: Subtracting 0.65 from $\frac{\pi}{2}$, or subtracting awrt 37 from 90 (degrees), (perhaps implied by awrt 53).</p> <p><u>Angle changed to degrees wrongly and used throughout (a), (b) and (c):</u> Penalise ‘method’ only once, so could score M0A0, M1A0, M1A0.</p> <p>(d) First M1: Other area methods must be fully correct. Second M1: Adding answer to (b) to their ΔACD.</p> <p><u>Failure to round to 2 d.p:</u> Penalise only once, on the first occurrence, then accept awrt.</p> <p><u>If 0.65 is taken as degrees throughout:</u> Only award marks in part (d).</p>	

Question number	Scheme	Marks
9.	<p>(a) $ar = 4$, $\frac{a}{1-r} = 25$ (These can be seen elsewhere)</p> <p>$a = 25(1-r)$ $25r(1-r) = 4$ M: Eliminate a</p> <p>$25r^2 - 25r + 4 = 0$ (*)</p> <p>(b) $(5r-1)(5r-4) = 0$ $r = \dots$, $\frac{1}{5}$ or $\frac{4}{5}$</p> <p>(c) $r = \dots \Rightarrow a = \dots$, 20 or 5</p> <p>(d) $S_n = \frac{a(1-r^n)}{1-r}$, but $\frac{a}{1-r} = 25$, so $S_n = 25(1-r^n)$ (*)</p> <p>(e) $25(1-0.8^n) > 24$ and proceed to $n = \dots$ (or $>$, or $<$) with no unsound algebra.</p> <p>$\left(n > \frac{\log 0.04}{\log 0.8} \right. (= 14.425\dots) \left. \right)$ $n = 15$</p>	<p>B1, B1</p> <p>M1</p> <p>A1cso (4)</p> <p>M1, A1 (2)</p> <p>M1, A1 (2)</p> <p>B1 (1)</p> <p>M1</p> <p>A1 (2)</p> <p>11</p>
	<p>(a) The M mark is not dependent, but both expressions must contain both a and r.</p> <p>(b) <u>Special case</u>: One correct r value given, with no method (or perhaps trial and error): B1 B0.</p> <p>(c) M1: Substitute one r value back to find a value of a.</p> <p>(d) Sufficient here to verify with just one pair of values of a and r.</p> <p>(e) Accept “=” rather than inequalities throughout, and also allow the <u>wrong</u> inequality to be used at any stage. M1 requires use of <u>their</u> larger value of r. A correct answer with no working scores both marks. For “trial and error” methods, to score M1, a value of n between 12 and 18 (inclusive) must be tried.</p>	

Question number	Scheme	Marks
10.	<p>(a) $\frac{dy}{dx} = 3x^2 - 16x + 20$ $3x^2 - 16x + 20 = 0 \quad (3x - 10)(x - 2) = 0 \quad x = \dots, \quad \frac{10}{3} \text{ and } 2$</p> <p>(b) $\frac{d^2y}{dx^2} = 6x - 16 \quad \text{At } x = 2, \frac{d^2y}{dx^2} = \dots$ $-4 \quad (\text{or } < 0, \text{ or both}), \text{ therefore maximum}$</p> <p>(c) $\int (x^3 - 8x^2 + 20x) dx = \frac{x^4}{4} - \frac{8x^3}{3} + \frac{20x^2}{2} (+C)$</p> <p>(d) $4 - \frac{64}{3} + 40 \quad \left(= \frac{68}{3} \right)$ $A: x = 2: \quad y = 8 - 32 + 40 = 16 \quad (\text{May be scored elsewhere})$ $\text{Area of } \Delta = \frac{1}{2} \left(\frac{10}{3} - 2 \right) \times 16 \quad \left(\frac{1}{2} (x_B - x_A) \times y_A \right) \quad \left(= \frac{32}{3} \right)$ $\text{Shaded area} = \frac{68}{3} + \frac{32}{3} = \frac{100}{3} \quad \left(= 33\frac{1}{3} \right)$</p>	<p>M1 A1 dM1, A1 (4) M1 A1ft (2) M1 A1 A1 (3) M1 B1 M1 M1 A1 (5) 14</p>
	<p>(a) The second M is dependent on the first, and requires an attempt to solve a 3 term quadratic.</p> <p>(b) M1: Attempt second differentiation and substitution of one of the x values. A1ft: Requires correct second derivative and negative value of the second derivative, but ft from their x value.</p> <p>(c) All 3 terms correct: M1 A1 A1, Two terms correct: M1 A1 A0, One power correct: M1 A0 A0.</p> <p>(d) Limits M1: Substituting their lower x value into a 'changed' expression.</p> <p>Area of triangle M1: Fully correct method. Alternative for the triangle (finding an equation for the straight line then integrating) requires a fully correct method to score the M mark.</p> <p>Final M1: Fully correct method (beware valid alternatives!)</p>	

GENERAL PRINCIPLES FOR C2 MARKING**Method mark for solving 3 term quadratic:**1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm p)^2 \pm q \pm c$, $p \neq 0$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

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Misreads

(See the next sheet for a simple example).

A misread must be consistent for the whole question to be interpreted as such.

These are not common. In clear cases, please deduct the first 2 A (or B) marks which would have been lost by following the scheme. (Note that 2 marks is the maximum misread penalty, but that misreads which alter the nature or difficulty of the question cannot be treated so generously and it will usually be necessary here to follow the scheme as written).

Sometimes following the scheme as written is more generous to the candidate than applying the misread rule, so in this case use the scheme as written.

MISREADS

Question 1. $(2 + x)^6$ misread as $(2 + x)^8$

1.	$(2 + x)^8 = 256 \dots$	B0
	$+ (8 \times 2^7 \times x) + \left(\frac{8 \times 7}{2} \times 2^6 \times x^2 \right),$	$+ 1024x, + 1792x^2$ M1, A0, A1

Mark Scheme (Results)

January 2007

GCE

GCE Mathematics

Core Mathematics C2 (6664)

January 2007
6664 Core Mathematics C2
Mark Scheme

Question Number	Scheme	Marks
1. (a)	$f'(x) = 3x^2 + 6x$ $f''(x) = 6x + 6$	B1 M1, A1cao (3)

Notes cao = correct answer only

1(a)	
Acceptable alternatives include $3x^2 + 6x^1$; $3x^2 + 3 \times 2x$; $3x^2 + 6x + 0$ Ignore LHS (e.g. use [whether correct or not] of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$) $3x^2 + 6x + c$ or $3x^2 + 6x + \text{constant}$ (i.e. the written word constant) is B0	B1
M1 Attempt to differentiate their $f'(x)$; $x^n \rightarrow x^{n-1}$. $x^n \rightarrow x^{n-1}$ seen in at least one of the terms. Coefficient of x^{\dots} ignored for the method mark. $x^2 \rightarrow x^1$ and $x \rightarrow x^0$ are acceptable.	M1
Acceptable alternatives include $6x^1 + 6x^0$; $3 \times 2x + 3 \times 2$ $6x + 6 + c$ or $6x + 6 + \text{constant}$ is A0	A1 cao

Examples

1(a)	$f''(x) = 3x^2 + 6x$	B1 M0 A0	1(a)	$f'(x) = x^2 + 3x$ $f''(x) = x + 3$	B0 M1 A0
1(a)	$f'(x) = 3x^2 + 6x$ $f''(x) = 6x$	B1 M1 A0	1(a)	$x^3 + 3x^2 + 5$ $= 3x^2 + 6x$ $= 6x + 6$	B1 M1 A1
1(a)	$y = x^3 + 3x^2 + 5$		1(a)	$f'(x) = 3x^2 + 6x$ $+ 5$	B0
	$\frac{dy}{dx} = 3x^2 + 3x$	B0		$f''(x) = 6x + 6$	M1 A1
	$\frac{d^2y}{dx^2} = 6x + 3$	M1 A0	1(a)	$f'(x) = 3x^2 + 6x$ $f''(x) = 6x + 6 + c$	B1 M1 A0
1(a)	$f'(x) = 3x^2 + 6x + c$ $f''(x) = 6x + 6$	B0 M1 A1			

Question Number	Scheme	Marks
1. (b)	$\int (x^3 + 3x^2 + 5) dx = \frac{x^4}{4} + \frac{3x^3}{3} + 5x$ $\left[\frac{x^4}{4} + x^3 + 5x \right]_1^2 = 4 + 8 + 10 - \left(\frac{1}{4} + 1 + 5 \right)$ $= 15\frac{3}{4} \text{ o.e.}$	M1, A1 M1 A1 (4) (7)

Notes o.e. = or equivalent

1(b)	
Attempt to integrate $f(x)$; $x^n \rightarrow x^{n+1}$ Ignore incorrect notation (e.g. inclusion of integral sign)	M1
o.e. Acceptable alternatives include $\frac{x^4}{4} + x^3 + 5x$; $\frac{x^4}{4} + \frac{3x^3}{3} + 5x^1$; $\frac{x^4}{4} + \frac{3x^3}{3} + 5x + c$; $\int \frac{x^4}{4} + \frac{3x^3}{3} + 5x$ N.B. If the candidate has written the integral (either $\frac{x^4}{4} + \frac{3x^3}{3} + 5x$ or what they think is the integral) in part (a), it may not be rewritten in (b), but the marks may be awarded if the integral is used in (b).	A1
Substituting 2 and 1 into any function other than $x^3 + 3x^2 + 5$ and subtracting either way round. So using their $f'(x)$ or $f''(x)$ or \int their $f'(x) dx$ or \int their $f''(x) dx$ will gain the M mark (because none of these will give $x^3 + 3x^2 + 5$). Must substitute for all x s but could make a slip. $4 + 8 + 10 - \frac{1}{4} + 1 + 5$ (for example) is acceptable for evidence of subtraction ('invisible' brackets).	M1
o.e. (e.g. $15\frac{3}{4}$, 15.75, $\frac{63}{4}$) Must be a single number (so $22 - 6\frac{1}{4}$ is A0).	A1
Answer only is M0A0M0A0	

Examples

1(b) $\frac{x^4}{4} + x^3 + 5x + c$	M1 A1	1(b) $\frac{x^4}{4} + x^3 + 5x + c$	M1 A1
$4 + 8 + 10 + c - (\frac{1}{4} + 1 + 5 + c)$	M1	$x = 2, \quad 22 + c$	
$= 15\frac{3}{4}$	A1	$x = 1, \quad 6\frac{1}{4} + c$	M0 A0
		(no subtraction)	
1(b) $\int_1^2 f(x) dx = 2^3 + 3 \times 2^2 + 5 - (1 + 3 + 5)$	M0 A0, M0		
$= 25 - 9$			
$= 16$	A0		
(Substituting 2 and 1 into $x^3 + 3x^2 + 5$, so 2nd M0)			

$$1(b) \int_1^2 (6x+6) \, dx = [3x^2 + 6x]_1^2 \quad \text{M0 A0}$$

$$= 12 + 12 - (3 + 6) \quad \text{M1 A0}$$

$$1(b) \int_1^2 (3x^2 + 6x) \, dx = [x^3 + 3x^2]_1^2 \quad \text{M0 A0}$$

$$= 8 + 12 - (1 + 3) \quad \text{M1 A0}$$

$$1(b) \quad \frac{x^4}{4} + x^3 + 5x \quad \text{M1 A1}$$

$$\frac{2^4}{4} + 2^3 + 5 \times 2 - \frac{1^4}{4} + 1^3 + 5 \quad \text{M1}$$

(one negative sign is sufficient for evidence of subtraction)

$$= 22 - 6\frac{1}{4} = 15\frac{3}{4} \quad \text{A1}$$

(allow 'recovery', implying student was using 'invisible brackets')

$$1(a) \quad f(x) = x^3 + 3x^2 + 5$$

$$f''(x) = \frac{x^4}{4} + x^3 + 5x \quad \text{B0 M0 A0}$$

$$(b) \quad \frac{2^4}{4} + 2^3 + 5 \times 2 - \frac{1^4}{4} - 1^3 - 5 \quad \text{M1 A1 M1}$$

$$= 15\frac{3}{4} \quad \text{A1}$$

The candidate has written the integral in part (a). It is not rewritten in (b), but the marks may be awarded as the integral is used in (b).

Question Number	Scheme	Marks
2. (a)	$(1-2x)^5 = 1 + 5 \times (-2x) + \frac{5 \times 4}{2!} (-2x)^2 + \frac{5 \times 4 \times 3}{3!} (-2x)^3 + \dots$ $= 1 - 10x + 40x^2 - 80x^3 + \dots$	B1, M1, A1, A1 (4)
(b)	$(1+x)(1-2x)^5 = (1+x)(1-10x + \dots)$ $= 1 + x - 10x + \dots$ $\approx 1 - 9x \quad (*)$	M1 A1 (2) (6)

Notes

2(a)	
1 - 10x 1 - 10x must be seen in this simplified form in (a).	B1
Correct structure: 'binomial coefficients' (perhaps from Pascal's triangle), increasing powers of x . Allow slips. Accept other forms: 5C_1 , $\binom{5}{1}$, also condone $\left(\frac{5}{1}\right)$ but must be attempting to use 5. Condone use of invisible brackets and using $2x$ instead of $-2x$. Powers of x : at least 2 powers of the type $(2x)^a$ or $2x^a$ seen for $a \geq 1$.	M1
$40x^2$ (1st A1)	A1
$-80x^3$ (2nd A1)	A1
Allow commas between terms. Terms may be listed rather than added Allow 'recovery' from invisible brackets, so $1^5 + \binom{5}{1}1^4 - 2x + \binom{5}{2}1^3 - 2x^2 + \binom{5}{3}1^2 - 2x^3$ $= 1 - 10x + 40x^2 - 80x^3 + \dots$ gains full marks. $1 + 5 \times (2x) + \frac{5 \times 4}{2!} (2x)^2 + \frac{5 \times 4 \times 3}{3!} (2x)^3 + \dots = 1 + 10x + 40x^2 + 80x^3 + \dots$ gains B0M1A1A0	
Misread: first 4 terms, descending terms: if correct, would score B0, M1, 1st A1: one of $40x^2$ and $-80x^3$ correct; 2nd A1: both $40x^2$ and $-80x^3$ correct.	

2(a) Long multiplication	
$(1-2x)^2 = 1 - 4x + 4x^2$, $(1-2x)^3 = 1 - 6x + 12x^2 - 8x^3$, $(1-2x)^4 = 1 - 8x + 24x^2 - 32x^3 \{+ 16x^4\}$ $(1-2x)^5 = 1 - 10x + 40x^2 + 80x^3 + \dots$	
1 - 10x 1 - 10x must be seen in this simplified form in (a).	B1
Attempt repeated multiplication up to and including $(1-2x)^5$	M1

$40x^2$ (1st A1)	A1
$-80x^3$ (2nd A1)	A1
Misread: first 4 terms, descending terms: if correct, would score B0, M1, 1st A1: one of $40x^2$ and $-80x^3$ correct; 2nd A1: both $40x^2$ and $-80x^3$ correct.	

2(b)	
<p>Use their (a) and attempt to multiply out; terms (whether correct or incorrect) in x^2 or higher can be ignored.</p> <p>If their (a) is correct an attempt to multiply out can be implied from the correct answer, so $(1+x)(1-10x) = 1-9x$ will gain M1 A1.</p> <p>If their (a) is correct, the 2nd bracket must contain at least $(1-10x)$ and an attempt to multiply out for the M mark. An attempt to multiply out is an attempt at 2 out of the 3 relevant terms (N.B. the 2 terms in x^1 may be combined – but this will still count as 2 terms).</p> <p>If their (a) is incorrect their 2nd bracket must contain all the terms in x^0 and x^1 from their (a) AND an attempt to multiply all terms that produce terms in x^0 and x^1.</p> <p>N.B. $(1+x)(1-2x)^5 = (1+x)(1-2x)$ [where $1-2x + \dots$ is NOT the candidate's answer to (a)]</p> <p style="text-align: center;">$= 1 - x$</p> <p>i.e. candidate has ignored the power of 5: M0</p> <p>N.B. The candidate may start again with the binomial expansion for $(1-2x)^5$ in (b). If correct (only needs $1-10x$) may gain M1 A1 even if candidate did not gain B1 in part (a).</p>	M1
N.B. Answer given in question.	A1

Example

Answer in (a) is $= 1 + 10x + 40x^2 - 80x^3 + \dots$

$$\begin{array}{ll}
 \text{(b) } (1+x)(1+10x) = 1 + 10x + x & \text{M1} \\
 = 1 + 11x & \text{A0}
 \end{array}$$

Question Number	Scheme	Marks
3.	<p>Centre $\left(\frac{-1+3}{2}, \frac{6+4}{2}\right)$, i.e. (1, 5)</p> <p>$r = \frac{\sqrt{(3-(-1))^2 + (6-4)^2}}{2}$</p> <p>or $r^2 = (1-(-1))^2 + (5-4)^2$ or $r^2 = (3-1)^2 + (6-5)^2$ o.e.</p> <p>$(x-1)^2 + (y-5)^2 = 5$</p>	<p>M1, A1</p> <p>M1</p> <p>M1,A1,A1 (6)</p>

Notes

Some use of correct formula in x or y coordinate. Can be implied. Use of $\left(\frac{1}{2}(x_A - x_B), \frac{1}{2}(y_A - y_B)\right) \rightarrow (-2, -1)$ or $(2, 1)$ is M0 A0 but watch out for use of $x_A + \frac{1}{2}(x_A - x_B)$ etc which is okay.	M1
(1, 5) (5, 1) gains M1 A0.	A1
Correct method to find r or r^2 using given points or f.t. from their centre. Does not need to be simplified. Attempting radius = $\sqrt{\frac{(\text{diameter})^2}{2}}$ is an incorrect method, so M0. N.B. Be careful of labelling: candidates may not use d for diameter and r for radius. Labelling should be ignored. Simplification may be incorrect – mark awarded for correct method. Use of $\sqrt{(x_1 - x_2)^2 - (y_1 - y_2)^2}$ is M0.	M1
Write down $(x \pm a)^2 + (y \pm b)^2 = \text{any constant}$ (a letter or a number). Numbers do not have to be substituted for a, b and if they are they can be wrong.	M1
LHS is $(x-1)^2 + (y-5)^2$. Ignore RHS.	A1
RHS is 5.	A1
Ignore subsequent working. Condone use of decimals that leads to exact 5.	
Or correct equivalents, e.g. $x^2 + y^2 - 2x - 10y + 21 = 0$.	

Alternative – note the order of the marks needed for ePEN.	
As above.	M1
As above.	A1
$x^2 + y^2 + (\text{constant})x + (\text{constant})y + \text{constant} = 0$. Numbers do not have to be substituted for the constants and if they are they can be wrong.	3rd M1
Attempt an appropriate substitution of the coordinates of their centre (i.e. working with coefficient of x and coefficient of y in equation of circle) and substitute $(-1, 4)$ or $(3, 6)$ into equation of circle.	2nd M1
$-2x - 10y$ part of the equation $x^2 + y^2 - 2x - 10y + 21 = 0$.	A1
$+21 = 0$ part of the equation $x^2 + y^2 - 2x - 10y + 21 = 0$.	A1
Or correct equivalents, e.g. $(x-1)^2 + (y-5)^2 = 5$.	

Question Number	Scheme	Marks
4.	$x \log 5 = \log 17$ or $x = \log_5 17$ $x = \frac{\log 17}{\log 5}$ $= 1.76$	M1 A1 A1 (3)

Notes N.B. It is never possible to award an A mark after giving M0. If M0 is given then the marks will be M0 A0 A0.

4	
Acceptable alternatives include $x \log 5 = \log 17$; $x \log_{10} 5 = \log_{10} 17$; $x \log_e 5 = \log_e 17$; $x \ln 5 = \ln 17$; $x = \log_5 17$ Can be implied by a correct exact expression as shown on the first A1 mark	1st M1
An exact expression for x that can be evaluated on a calculator. Acceptable alternatives include $x = \frac{\log 17}{\log 5}$; $x = \frac{\log_{10} 17}{\log_{10} 5}$; $x = \frac{\log_e 17}{\log_e 5}$; $x = \frac{\ln 17}{\ln 5}$; $x = \frac{\log_q 17}{\log_q 5}$ where q is a number This may not be seen (as, for example, $\log_5 17$ can be worked out directly on many calculators) so this A mark can be implied by the correct final answer or the right answer corrected to or truncated to a greater accuracy than 3 significant figures or 1.8 Alternative: $x = \frac{\text{a number}}{\text{a number}}$ where this fraction, when worked out as a decimal rounds to 1.76. (N.B. remember that this A mark cannot be awarded without the M mark). If the line for the M mark is missing but this line is seen (with or without the $x =$) and is <u>correct</u> the method can be assumed and M1 1st A1 given.	1st A1
1.76 cao	2nd A1
N.B. $\sqrt[5]{17} = 1.76$ and $x^5 = 17$, $\therefore x = 1.76$ are both M0 A0 A0	
Answer only 1.76: full marks (M1 A1 A1) Answer only to a greater accuracy but which rounds to 1.76: M1 A1 A0 (e.g. 1.760, 1.7603, 1.7604, 1.76037 etc) Answer only 1.8: M1 A1 A0 Trial and improvement: award marks as for “answer only”.	

Examples

4. $x = \log 5^{17}$ M0 A0
 $= 1.76$ A0
 Working seen, so scheme applied

4. $5^{1.76} = 17$ M1 A1 A1
 Answer only but clear that $x = 1.76$

4. $5^{1.8} = 17$ M1 A1 A0
 Answer only but clear that $x = 1.8$

4. $5^{1.76}$ M0 A0 A0

4. $\log_5 17 = x$ M1
 $x = 1.760$ A1 A0

4. $\log_5 17 = x$ M1
 $x = 1.76$ A1 A1

4. $x \log 5 = \log 17$ M1
 $x = \frac{1.2304...}{0.69897...}$ A1
 $x = 1.76$ A1

4. $x \ln 5 = \ln 17$ M1
 $x = \frac{2.833212...}{1.609437...}$ A1
 $x = 1.76$ A1

4. $x \log 5 = \log 17$ M1
 $x = \frac{2.57890}{1.46497}$ A1
 $x = 1.83$ A0

4. $\log_{17} 5 = x$ M0
 $x = \frac{\log 5}{\log 17}$ A0
 $x = 0.568$ A0

4. $5^{1.8} = 18.1, 5^{1.75} = 16.7$
 $5^{1.761} = 17$ M1 A1 A0

4. $x = 5^{1.76}$ M0 A0 A0

4. $x \log 5 = \log 17$ M1
 $x = 1.8$ A1 A0

4. $x = \frac{\log 17}{\log 5}$ M1 A1
 $x = 1.8$ A0

N.B.

4. $x^5 = 17$ M0 A0
 $x = 1.76$ A0

4. $\sqrt[5]{17}$ M0 A0
 $= 1.76$ A0

Question Number	Scheme	Marks
5. (a)	$f(-2) = (-2)^3 + 4(-2)^2 + (-2) - 6$ $\{ = -8 + 16 - 2 - 6 \}$ $= 0, \therefore x + 2 \text{ is a factor}$	M1 A1 (2)
(b)	$x^3 + 4x^2 + x - 6 = (x + 2)(x^2 + 2x - 3)$ $= (x + 2)(x + 3)(x - 1)$	M1, A1 M1, A1 (4)
(c)	-3, -2, 1	B1 (1) (7)

Notes Line in mark scheme in { } does not need to be seen.

5(a)	
Attempting $f(\pm 2)$: No x s; allow invisible brackets for M mark Long division: M0 A0.	M1
$= 0$ and minimal conclusion (e.g. factor, hence result, QED, \checkmark , \square). If result is stated first [i.e. If $x + 2$ is a factor, $f(-2) = 0$] conclusion is not needed. Invisible brackets used as brackets can get M1 A1, so $f(-2) = -2^3 + 4 \times -2^2 + -2 - 6 \{ = -8 + 16 - 2 - 6 \} = 0, \therefore x + 2 \text{ is a factor}$ M1 A1, but $f(-2) = -2^3 + 4 \times -2^2 + -2 - 6 = -8 - 16 - 2 - 6 = 0, \therefore x + 2 \text{ is a factor}$ M1 A0 Acceptable alternatives include: $x = -2$ is a factor, $f(-2)$ is a factor.	A1

5(b)	
1st M1 requires division by $(x + 2)$ to get $x^2 + ax + b$ where $a \neq 0$ and $b \neq 0$ or equivalent with division by $(x + 3)$ or $(x - 1)$.	M1
$(x + 2)(x^2 + 2x - 3)$ or $(x + 3)(x^2 + x - 2)$ or $(x - 1)(x^2 + 5x + 6)$ [If long division has been done in (a), minimum seen in (b) to get first M1 A1 is to make some reference to their quotient $x^2 + ax + b$.]	A1
Attempt to factorise their quadratic (usual rules).	M1
“Combining” all 3 factors is not required.	A1
Answer only: Correct M1 A1 M1 A1 Answer only with one sign slip: $(x + 2)(x + 3)(x + 1)$ scores 1st M1 1st A1 2nd M0 2nd A0 $(x + 2)(x - 3)(x - 1)$ scores 1st M0 1st A0 2nd M1 2nd A1	
Answer to (b) can be seen in (c).	

5(b) Alternative comparing coefficients	
$(x + 2)(x^2 + ax + b) = x^3 + (2 + a)x^2 + (2a + b)x + 2b$ Attempt to compare coefficients of two terms to find values of a and b $a = 2, b = -3$	M1 A1
Or $(x + 2)(ax^2 + bx + c) = ax^3 + (2a + b)x^2 + (2b + c)x + 2c$ Attempt to compare coefficients of three terms to find values of a, b and c . $a = 1, b = 2, c = -3$	M1 A1
Then apply scheme as above	

5(b) Alternative using factor theorem	
Show $f(-3) = 0$; allow invisible brackets	M1
$\therefore x + 3$ is a factor	A1
Show $f(1) = 0$	M1
$\therefore x - 1$ is a factor	A1
5(c)	
$-3, -2, 1$ or $(-3, 0), (-2, 0), (1, 0)$ only. Do not ignore subsequent working. Ignore any working in previous parts of the question. Can be seen in (b)	B1

Question Number	Scheme	Marks
6.	$2(1 - \sin^2 x) + 1 = 5 \sin x$ $2 \sin^2 x + 5 \sin x - 3 = 0$ $(2 \sin x - 1)(\sin x + 3) = 0$ $\sin x = \frac{1}{2}$ $x = \frac{\pi}{6}, \frac{5\pi}{6}$	M1 M1, A1 M1, M1, A1cso (6)

Notes

Use of $\cos^2 x = 1 - \sin^2 x$. Condone invisible brackets in first line if $2 - 2 \sin^2 x$ is present (or implied) in a subsequent line. Must be using $\cos^2 x = 1 - \sin^2 x$. Using $\cos^2 x = 1 + \sin^2 x$ is M0.	M1
Attempt to solve a 2 or 3 term quadratic in $\sin x$ up to $\sin x = \dots$ Usual rules for solving quadratics. Method may be factorising, formula or completing the square	M1
Correct factorising for correct quadratic and $\sin x = \frac{1}{2}$. So, e.g. $(\sin x + 3)$ as a factor $\rightarrow \sin x = 3$ can be ignored.	A1
Method for finding any angle in any range consistent with (either of) their trig. equation(s) in degrees or radians (even if x not exact). [Generous M mark] Generous mark. Solving any trig. equation that comes from minimal working (however bad). So $x = \sin^{-1}/\cos^{-1}/\tan^{-1}(\text{number}) \rightarrow$ answer in degrees or radians correct for their equation (in any range)	M1
Method for finding second angle consistent with (either of) their trig. equation(s) in radians. Must be in range $0 \leq x < 2\pi$. Must involve using π (e.g. $\pi \pm \dots, 2\pi - \dots$) but \dots can be inexact. Must be using the same equation as they used to attempt the 3rd M mark. Use of π must be consistent with the trig. equation they are using (e.g. if using \cos^{-1} then must be using $2\pi - \dots$) If finding both angles in degrees: method for finding 2nd angle equivalent to method above in degrees and an attempt to change both angles to radians.	M1
$\frac{\pi}{6}, \frac{5\pi}{6}$ c.s.o. Recurring decimals are okay (instead of $\frac{1}{6}$ and $\frac{5}{6}$). Correct decimal values (corrected or truncated) before the final answer of $\frac{\pi}{6}, \frac{5\pi}{6}$ is acceptable.	A1 cso
Ignore extra solutions outside range; deduct final A mark for extra solutions in range.	
Special case Answer only $\frac{\pi}{6}, \frac{5\pi}{6}$ M0, M0, A0, M1, M1 A1 Answer only $\frac{\pi}{6}$ M0, M0, A0, M1, M0 A0	

Finding answers by trying different values (e.g. trying multiples of π) in $2\cos^2 x + 1 = 5\sin x$: as for answer only.	
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Question Number	Scheme	Marks
7.	$y = x(x^2 - 6x + 5)$ $= x^3 - 6x^2 + 5x$ $\int (x^3 - 6x^2 + 5x) \, dx = \frac{x^4}{4} - \frac{6x^3}{3} + \frac{5x^2}{2}$ $\left[\frac{x^4}{4} - 2x^3 + \frac{5x^2}{2} \right]_0^1 = \left(\frac{1}{4} - 2 + \frac{5}{2} \right) - 0 = \frac{3}{4}$ $\left[\frac{x^4}{4} - 2x^3 + \frac{5x^2}{2} \right]_1^2 = (4 - 16 + 10) - \frac{3}{4} = -\frac{11}{4}$ $\therefore \text{total area} = \frac{3}{4} + \frac{11}{4}$ $= \frac{7}{2} \quad \text{o.e.}$	<p>M1, A1</p> <p>M1, A1ft</p> <p>M1</p> <p>M1, A1(both)</p> <p>M1</p> <p>A1cso</p> <p>(9)</p>

Notes

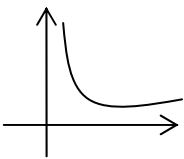
Attempt to multiply out, must be a cubic.	M1
Award A mark for their final version of expansion (but final version does not need to have like terms collected).	A1
Attempt to integrate; $x^n \rightarrow x^{n+1}$. Generous mark for some use of integration, so e.g. $\int x(x-1)(x-5) dx = \frac{x^2}{2} \left(\frac{x^2}{2} - x \right) \left(\frac{x^2}{2} - 5x \right)$ would gain method mark.	M1
Ft on their final version of expansion provided it is in the form $ax^p + bx^q + \dots$. Integrand must have at least two terms and all terms must be integrated correctly. If they integrate twice (e.g. \int_0^1 and \int_1^2) and get different answers, take the better of the two.	A1ft
Substitutes and subtracts (either way round) for one integral. Integral must be a 'changed' function. Either 1 and 0, 2 and 1 or 2 and 0. For $\left[\right]_0^1$: - 0 for bottom limit can be implied (provided that it is 0).	M1
M1 Substitutes and subtracts (either way round) for two integrals. Integral must be a 'changed' function. Must have 1 and 0 and 2 and 1 (or 1 and 2). The two integrals do not need to be the same, but they must have come from attempts to integrate the same function.	M1
$\frac{3}{4}$ and $-\frac{11}{4}$ o.e. (if using $\int_1^2 f(x)$) or $\frac{3}{4}$ and $\frac{11}{4}$ o.e. (if using $\int_2^1 f(x)$ or $-\int_1^2 f(x)$ or $\int_1^2 -f(x)$) where $f(x) = \frac{x^4}{4} - 2x^3 + \frac{5x^2}{2}$. The answer must be consistent with the integral they are using (so $\int_1^2 f(x) = \frac{11}{4}$ loses this A and the final A). $-\frac{11}{4}$ may not be seen explicitly. Can be implied by a subsequent line of working.	A1
5th M1 their value for $\left[\right]_0^1$ + their value for $\left[\right]_1^2$ Dependent on at least one of the values coming from integration (other may come from e.g. trapezium rules). This can be awarded even if both values already positive.	M1
$\frac{7}{2}$ o.e. N.B. c.s.o.	A1 cso

Question Number	Scheme	Marks
8. (a)	$\frac{dC}{dv} = -1400v^{-2} + \frac{2}{7}$ $-1400v^{-2} + \frac{2}{7} = 0$ $v^2 = 4900$ $v = 70$	M1, A1 M1 dM1 A1cso (5)
(b)	$\frac{d^2C}{dv^2} = 2800v^{-3}$ $v = 70, \frac{d^2C}{dv^2} > 0 \quad \{\Rightarrow \text{minimum}\}$ $\text{or } v = 70, \frac{d^2C}{dv^2} = 2800 \times 70^{-3} \quad \{= \frac{2}{245} = 0.00816...\} \quad \{\Rightarrow \text{minimum}\}$	M1 A1ft (2)
(c)	$v = 70, C = \frac{1400}{70} + \frac{2 \times 70}{7}$ $C = 40$	M1 A1 (2) (9)

Notes

8(a)	
Attempt to differentiate $v^n \rightarrow v^{n-1}$. Must be seen and marked in part (a) not part (b). Must be differentiating a function of the form $av^{-1} + bv$.	M1
o.e. $(-1400v^{-2} + \frac{2}{7} + c \text{ is A0})$	A1
Their $\frac{dC}{dv} = 0$. Can be implied by their $\frac{dC}{dv} = P + Q \rightarrow P = \pm Q$.	M1
Dependent on both of the previous Ms. Attempt to rearrange their $\frac{dC}{dv}$ into the form $v^n = \text{number}$ or $v^n - \text{number} = 0, n \neq 0$.	dM1
$v = 70$ cso but allow $v = \pm 70$. $v = 70$ km per h also acceptable.	A1cso
Answer only is 0 out of 5.	
Method of completing the square: send to review.	

8(a) Trial and improvement	$f(v) = \frac{1400}{v} + \frac{2v}{7}$	
Attempts to evaluate $f(v)$ for 3 values a, b, c where (i) $a < 70, b = 70$ and $c > 70$ or (ii) $a, b < 70$ and $c > 70$ or (iii) $a < 70$ and $b, c > 70$.		M1
All 3 correct and states $v = 70$ (exact)		A1
Then 2nd M0, 3rd M0, 2nd A0.		

8(a) Graph		
 <p>Correct shape (ignore anything drawn for $v < 0$).</p>		M1
$v = 70$ (exact)		A1
Then 2nd M0, 3rd M0, 2nd A0.		

8(b)		
Attempt to differentiate their $\frac{dC}{dv}$; $v^n \rightarrow v^{n-1}$ (including $v^0 \rightarrow 0$).		M1
$\frac{d^2C}{dv^2}$ must be correct. Ft only from their value of v and provided their value of v is +ve. Must be some (minimal) indication that their value of v is being used. Statement: "When $v =$ their value of $v, \frac{d^2C}{dv^2} > 0$ " is sufficient provided $2800v^{-3} > 0$ for their value of v . If substitution of their v seen: correct substitution of their v into $2800v^{-3}$, but, provided evaluation is +ve, ignore incorrect evaluation. N.B. Parts in mark scheme in { } do not need to be seen.		A1ft

8(c)		
Substitute their value of v that they think will give C_{\min} (independent of the method of obtaining this value of v and independent of which part of the question it comes from).		M1
40 or £40		A1
Must have part (a) completely correct (i.e. all 5 marks) to gain this A1.		
Answer only gains M1A1 provided part (a) is completely correct..		

Examples 8(b)

$$8(b) \quad \frac{d^2C}{dv^2} = 2800v^{-3} \quad \text{M1}$$

$$v = 70, \frac{d^2C}{dv^2} > 0 \quad \text{A1}$$

$$8(b) \quad \frac{d^2C}{dv^2} = 2800v^{-3} \quad \text{M1}$$

$$> 0 \quad \text{A0 (no indication that a value of } v \text{ is being used)}$$

$$8(b) \quad \text{Answer from (a): } v = 30$$

$$\frac{d^2C}{dv^2} = 2800v^{-3} \quad \text{M1}$$

$$v = 30, \frac{d^2C}{dv^2} > 0 \quad \text{A1ft}$$

$$8(b) \quad \frac{d^2C}{dv^2} = 2800v^{-3} \quad \text{M1}$$

$$v = 70, \frac{d^2C}{dv^2} = 2800 \times 70^{-3}$$

$$= 8.16 \quad \text{A1 (correct substitution of 70 seen, evaluation wrong but positive)}$$

$$8(b) \quad \frac{d^2C}{dv^2} = 2800v^{-3} \quad \text{M1}$$

$$v = 70, \frac{d^2C}{dv^2} = 0.00408 \quad \text{A0 (correct substitution of 70 not seen)}$$

Question Number	Scheme	Marks
9. (a)	$\cos PQR = \frac{6^2 + 6^2 - (6\sqrt{3})^2}{2 \times 6 \times 6} \left\{ = -\frac{1}{2} \right\}$ $PQR = \frac{2\pi}{3}$	M1, A1 A1 (3)
(b)	$\text{Area} = \frac{1}{2} \times 6^2 \times \frac{2\pi}{3} \text{ m}^2$ $= 12\pi \text{ m}^2 (*)$	M1 A1cso (2)
(c)	$\text{Area of } \Delta = \frac{1}{2} \times 6 \times 6 \times \sin \frac{2\pi}{3} \text{ m}^2$ $= 9\sqrt{3} \text{ m}^2$	M1 A1cso (2)
(d)	$\text{Area of segment} = 12\pi - 9\sqrt{3} \text{ m}^2$ $= 22.1 \text{ m}^2$	M1 A1 (2)
(e)	$\text{Perimeter} = 6 + 6 + \left[6 \times \frac{2\pi}{3} \right] \text{ m}$ $= 24.6 \text{ m}$	M1 A1ft (2) (11)

Notes

9(a) N.B. $a^2 = b^2 + c^2 - 2bc \cos A$ is in the formulae book.	
Use of cosine rule for $\cos PQR$. Allow A, θ or other symbol for angle. (i) $(6\sqrt{3})^2 = 6^2 + 6^2 - 2.6.6 \cos PQR$: Apply usual rules for formulae: (a) formula not stated, must be correct, (b) correct formula stated, allow one sign slip when substituting. or (ii) $\cos PQR = \frac{\pm 6^2 \pm 6^2 \pm (6\sqrt{3})^2}{\pm 2 \times 6 \times 6}$ Also allow invisible brackets [so allow $6\sqrt{3}^2$] in (i) or (ii)	M1
Correct expression $\frac{6^2 + 6^2 - (6\sqrt{3})^2}{2 \times 6 \times 6}$ o.e. (e.g. $-\frac{36}{72}$ or $-\frac{1}{2}$)	A1
$\frac{2\pi}{3}$	A1

9(a) Alternative	
$\sin \theta = \frac{a\sqrt{3}}{6}$ where θ is any symbol and $a < 6$.	M1
$\sin \theta = \frac{3\sqrt{3}}{6}$ where θ is any symbol.	A1
$\frac{2\pi}{3}$	A1

9(b)	
Use of $\frac{1}{2}r^2\theta$ with $r = 6$ and $\theta =$ their (a). For M mark θ does not have to be exact. M0 if using degrees.	M1
12π c.s.o. (\Rightarrow (a) correct exact or decimal value) N.B. Answer given in question	A1
Special case: Can come from an inexact value in (a) $PQR = 2.09 \rightarrow \text{Area} = \frac{1}{2} \times 6^2 \times 2.09 = 37.6$ (or 37.7) $= 12\pi$ (no errors seen, assume full values used on calculator) gets M1 A1. $PQR = 2.09 \rightarrow \text{Area} = \frac{1}{2} \times 6^2 \times 2.09 = 37.6$ (or 37.7) $= 11.97\pi = 12\pi$ gets M1 A0.	

9(c)	
Use of $\frac{1}{2}r^2\sin \theta$ with $r = 6$ and their (a). $\theta = \cos^{-1}$ (their PQR) in degrees or radians Method can be implied by correct decimal provided decimal is correct (corrected or truncated to at least 3 decimal places). 15.58845727	M1
$9\sqrt{3}$ c.s.o. Must be exact, but correct approx. followed by $9\sqrt{3}$ is okay (e.g. ... = 15.58845 = $9\sqrt{3}$)	A1cso

9(c) Alternative (using $\frac{1}{2}bh$)	
Attempt to find h using trig. or Pythagoras and use this h in $\frac{1}{2}bh$ form to find the area of triangle PQR	M1
$9\sqrt{3}$ c.s.o. Must be exact, but correct approx. followed by $9\sqrt{3}$ is okay (e.g. ... = 15.58845 = $9\sqrt{3}$)	A1cso

9(d)	
Use of area of sector – area of Δ or use of $\frac{1}{2}r^2(\theta - \sin \theta)$.	M1
Any value to 1 decimal place or more which rounds to 22.1	A1

9(e)	
$6 + 6 + [6 \times \text{their (a)}]$.	M1
Correct for their (a) to 1 decimal place or more	A1 ft

Question Number	Scheme	Marks
10. (a)	$\{S_n = \} a + ar + \dots + ar^{n-1}$ $\{rS_n = \} ar + ar^2 + \dots + ar^n$ $(1-r)S_n = a(1-r^n)$ $S_n = \frac{a(1-r^n)}{1-r} \quad (*)$	B1 M1 dM1 A1cso (4)
(b)	$a = 200, r = 2, n = 10, \quad S_{10} = \frac{200(1-2^{10})}{1-2}$ $= 204,600$	M1, A1 A1 (3)
(c)	$a = \frac{5}{6}, r = \frac{1}{3}$ $S_\infty = \frac{a}{1-r}, \quad S_\infty = \frac{\frac{5}{6}}{1-\frac{1}{3}}$ $= \frac{5}{4} \text{ o.e.}$	B1 M1 A1 (3)
(d)	$-1 < r < 1 \quad (\text{or } r < 1)$	B1 (1) (11)

Notes

10(a)	
S_n not required. The following must be seen: at least one + sign, a , ar^{n-1} and one other intermediate term. No extra terms (usually ar^n).	B1
Multiply by r ; rS_n not required. At least 2 of their terms on RHS correctly multiplied by r .	M1
Subtract both sides: LHS must be $\pm(1-r)S_n$, RHS must be in the form $\pm a(1-r^{n+q})$. Only award this mark if the line for $S_n = \dots$ or the line for $rS_n = \dots$ contains a term of the form ar^{cn+d} Method mark, so may contain a slip but not awarded if last term of their $S_n =$ last term of their rS_n .	dM1
Completion c.s.o. N.B. Answer given in question	A1 cso

10(a)	
S_n not required. The following must be seen: at least one + sign, a , ar^{n-1} and one other intermediate term. No extra terms (usually ar^n).	B1
On RHS, multiply by $\frac{1-r}{1-r}$ Or Multiply LHS and RHS by $(1-r)$	M1

Multiply by $(1 - r)$ convincingly (RHS) and take out factor of a . Method mark, so may contain a slip.	dM1
Completion c.s.o. N.B. Answer given in question	A1 cso

10(b)	
Substitute $r = 2$ with $a = 100$ or 200 and $n = 9$ or 10 into formula for S_n .	M1
$\frac{200(1 - 2^{10})}{1 - 2}$ or equivalent.	A1
204,600	A1

10(b) Alternative method: adding 10 terms	
(i) Answer only: full marks. (M1 A1 A1)	
(ii) $200 + 400 + 800 + \dots \{+ 102,400\} = 204,600$ or $100(2 + 4 + 8 + \dots \{+ 1,024\}) = 204,600$ M1 for two correct terms (as above o.e.) and an indication that the sum is needed (e.g. + sign or the word sum).	M1
102,400 o.e. as final term. Can be implied by a correct final answer.	A1
204,600.	A1

10(c) N.B. $S_{\infty} = \frac{a}{1-r}$ is in the formulae book.	
$r = \frac{1}{3}$ seen or implied anywhere.	B1
Substitute $a = \frac{5}{6}$ and their r into $\frac{a}{1-r}$. Usual rules about quoting formula.	M1
$\frac{5}{4}$ o.e.	A1

10(d) N.B. $S_{\infty} = \frac{a}{1-r}$ for $ r < 1$ is in the formulae book.	
$-1 < r < 1$ or $ r < 1$ In words or symbols. Take symbols if words and symbols are contradictory. Must be $<$ not \leq .	B1

Mark Scheme (Results)

Summer 2007

GCE

GCE Mathematics

Core Mathematics C2 (6664)

6664 Core Mathematics C2 2
June 2007 Advanced Subsidiary/Advanced Level in GCE Mathematics

Question number	Scheme	Marks
2.	<p>(a) $f(2) = 24 - 20 - 32 + 12 = -16$ (M: Attempt $f(2)$ or $f(-2)$)</p> <p>(If continues to say 'remainder = 16', isw)</p> <p>Answer must be seen in part (a), not part (b).</p> <p>(b) $(x + 2)(3x^2 - 11x + 6)$</p> <p>$(x + 2)(3x - 2)(x - 3)$</p> <p>(If continues to 'solve an equation', isw)</p>	<p>M1 A1 (2)</p> <p>M1 A1</p> <p>M1 A1 (4)</p> <p>6</p>
	<p>(a) Answer only (if correct) scores both marks. (16 as 'answer only' is M0 A0).</p> <p><u>Alternative (long division):</u></p> <p>Divide by $(x - 2)$ to get $(3x^2 + ax + b)$, $a \neq 0, b \neq 0$. [M1]</p> <p>$(3x^2 + x - 14)$, and -16 seen. [A1]</p> <p>(If continues to say 'remainder = 16', isw)</p> <p>(b) First M requires division by $(x + 2)$ to get $(3x^2 + ax + b)$, $a \neq 0, b \neq 0$.</p> <p>Second M for attempt to factorise <u>their</u> quadratic, even if wrongly obtained, perhaps with a remainder from their division.</p> <p>Usual rule: $(kx^2 + ax + b) = (px + c)(qx + d)$, where $pq = k$ and $cd = b$.</p> <p>Just solving their quadratic by the formula is M0.</p> <p>"Combining" all 3 factors is <u>not</u> required.</p> <p><u>Alternative (first 2 marks):</u></p> <p>$(x + 2)(3x^2 + ax + b) = 3x^3 + (6 + a)x^2 + (2a + b)x + 2b = 0$, then compare coefficients to find <u>values</u> of a and b. [M1]</p> <p>$a = -11, b = 6$ [A1]</p> <p><u>Alternative:</u></p> <p>Factor theorem: Finding that $f(3) = 0 \therefore$ factor is, $(x - 3)$ [M1, A1]</p> <p>Finding that $f\left(\frac{2}{3}\right) = 0 \therefore$ factor is, $(3x - 2)$ [M1, A1]</p> <p>If just one of these is found, score the <u>first 2 marks</u> M1 A1 M0 A0.</p> <p><u>Losing a factor of 3:</u> $(x + 2)\left(x - \frac{2}{3}\right)(x - 3)$ scores M1 A1 M1 A0.</p> <p><u>Answer only, one sign wrong:</u> e.g. $(x + 2)(3x - 2)(x + 3)$ scores M1 A1 M1 A0.</p>	

Question number	Scheme	Marks
3.	<p>(a) $1 + 6kx$ [Allow unsimplified versions, e.g. $1^6 + 6(1^5)kx$, ${}^6C_0 + {}^6C_1kx$] $+ \frac{6 \times 5}{2}(kx)^2 + \frac{6 \times 5 \times 4}{3 \times 2}(kx)^3$ [See below for acceptable versions] N.B. THIS NEED NOT BE SIMPLIFIED FOR THE A1 (isw is applied)</p> <p>(b) $6k = 15k^2$ $k = \frac{2}{5}$ (or equiv. fraction, or 0.4) (Ignore $k = 0$, if seen)</p> <p>(c) $c = \frac{6 \times 5 \times 4}{3 \times 2} \left(\frac{2}{5}\right)^3 = \frac{32}{25}$ (or equiv. fraction, or 1.28) (Ignore x^3, so $\frac{32}{25}x^3$ is fine)</p>	<p>B1</p> <p>M1 A1 (3)</p> <p>M1 A1cso (2)</p> <p>A1cso (1)</p> <p>6</p>
	<p>(a) The terms can be 'listed' rather than added.</p> <p>M1: Requires correct structure: 'binomial coefficients' (perhaps from Pascal's triangle), increasing powers of x. Allow a 'slip' or 'slips' such as: $+ \frac{6 \times 5}{2}kx^2 + \frac{6 \times 5 \times 4}{3 \times 2}kx^3$, $+ \frac{6 \times 5}{2}(kx)^2 + \frac{6 \times 5}{3 \times 2}(kx)^3$ $+ \frac{5 \times 4}{2}kx^2 + \frac{5 \times 4 \times 3}{3 \times 2}kx^3$, $+ \frac{6 \times 5}{2}x^2 + \frac{6 \times 5 \times 4}{3 \times 2}x^3$ <u>But</u>: $15 + k^2x^2 + 20 + k^3x^3$ or similar is M0. Both x^2 and x^3 terms must be seen. $\binom{6}{2}$ and $\binom{6}{3}$ or equivalent such as 6C_2 and 6C_3 are acceptable, and even $\left(\frac{6}{2}\right)$ and $\left(\frac{6}{3}\right)$ are acceptable for the method mark.</p> <p>A1: Any correct (possibly unsimplified) version of these 2 terms. $\binom{6}{2}$ and $\binom{6}{3}$ or equivalent such as 6C_2 and 6C_3 are acceptable.</p> <p><u>Descending powers of x:</u> Can score the M mark if the required first 4 terms are not seen.</p> <p><u>Multiplying out</u> $(1 + kx)(1 + kx)(1 + kx)(1 + kx)(1 + kx)(1 + kx)$: M1: A full attempt to multiply out (power 6) B1 and A1 as on the main scheme.</p> <p>(b) M: Equating the coefficients of x and x^2 (even if trivial, e.g. $6k = 15k$). Allow this mark also for the 'misread': equating the coefficients of x^2 and x^3. An equation in k alone is required for this M mark, although... ...condone $6kx = 15k^2x^2 \Rightarrow (6k = 15k^2 \Rightarrow) k = \frac{2}{5}$.</p>	

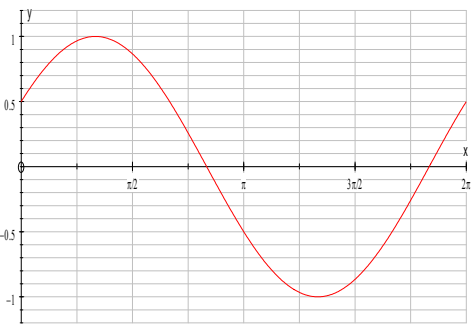
Question number	Scheme	Marks
4.	<p>(a) $4^2 = 5^2 + 6^2 - (2 \times 5 \times 6 \cos \theta)$</p> $\cos \theta = \frac{5^2 + 6^2 - 4^2}{2 \times 5 \times 6}$ $\left(= \frac{45}{60} \right) = \frac{3}{4} \quad (*)$ <p>(b) $\sin^2 A + \left(\frac{3}{4} \right)^2 = 1$ (or equiv. Pythag. method)</p> $\left(\sin^2 A = \frac{7}{16} \right) \quad \sin A = \frac{1}{4} \sqrt{7} \quad \text{or equivalent exact form, e.g. } \sqrt{\frac{7}{16}}, \sqrt{0.4375}$	<p>M1</p> <p>A1</p> <p>A1cso (3)</p> <p>M1</p> <p>A1 (2)</p> <p>5</p>
	<p>(a) M: Is also scored for $5^2 = 4^2 + 6^2 - (2 \times 4 \times 6 \cos \theta)$ or $6^2 = 5^2 + 4^2 - (2 \times 5 \times 4 \cos \theta)$ or $\cos \theta = \frac{4^2 + 6^2 - 5^2}{2 \times 4 \times 6}$ or $\cos \theta = \frac{5^2 + 4^2 - 6^2}{2 \times 5 \times 4}$.</p> <p>1st A: Rearranged correctly and numerically correct (possibly unsimplified), in the form $\cos \theta = \dots$ or $60 \cos \theta = 45$ (or equiv. in the form $p \cos \theta = q$).</p> <p><u>Alternative</u> (verification):</p> $4^2 = 5^2 + 6^2 - \left(2 \times 5 \times 6 \times \frac{3}{4} \right) \quad [\text{M1}]$ <p>Evaluate correctly, at least to $16 = 25 + 36 - 45$ [A1] Conclusion (perhaps as simple as a tick). [A1cso] (Just achieving $16 = 16$ is insufficient without at least a tick).</p> <p>(b) M: Using a correct method to find an equation in $\sin^2 A$ or $\sin A$ which would give an exact value.</p> <p><u>Correct answer without working</u> (or with unclear working or decimals): Still scores both marks.</p>	

Question number	Scheme	Marks
5.	<p>(a) 1.414 (allow also exact answer $\sqrt{2}$), 3.137 Allow awrt</p> <p>(b) $\frac{1}{2}(0.5) \dots$</p> <p>$\dots \{0 + 6 + 2(0.530 + 1.414 + 3.137)\}$</p> <p>$= 4.04$ (Must be 3 s.f.)</p> <p>(c) Area of triangle $= \frac{1}{2}(2 \times 6)$</p> <p>(Could also be found by integration, or even by the trapezium rule on $y = 3x$)</p> <p>Area required = Area of triangle – Answer to (b) (Subtract <u>either way round</u>)</p> <p>$6 - 4.04 = 1.96$ Allow awrt</p> <p>(ft from (b), dependent on the B1, and on answer to (b) <u>less than 6</u>)</p>	<p>B1, B1 (2)</p> <p>B1</p> <p>M1 A1ft</p> <p>A1 (4)</p> <p>B1</p> <p>M1</p> <p>A1ft (3)</p> <p>9</p>
	<p>(a) If answers are given to only 2 d.p. (1.41 and 3.14), this is B0 B0, but full marks can be given in part (b) if 4.04 is achieved.</p> <p>(b) Bracketing mistake: i.e. $\frac{1}{2}(0.5)(0 + 6) + 2(0.530 + 1.414 + 3.137)$</p> <p>scores B1 M1 A0 A0 <u>unless</u> the final answer implies that the calculation has been done correctly (then full marks can be given).</p> <p><u>Alternative</u> (finding and adding separate areas):</p> <p>$\frac{1}{2} \times \frac{1}{2}$ (Triangle/trapezium formulae, and height of triangle/trapezium)[B1]</p> <p>Fully correct method for total area, with values from table. [M1, A1ft]</p> <p>4.04 [A1]</p> <p>(c) B1: Can be given for 6 with no working, but should <u>not</u> be given for 6 obtained from <u>wrong</u> working.</p> <p>A1ft: This is a dependent follow-through: the B1 for 6 must have been scored, and the answer to (b) must be less than 6.</p>	

Question number	Scheme	Marks
6.	<p>(a) $x = \frac{\log 0.8}{\log 8}$ or $\log_8 0.8$, $= -0.107$ Allow awrt</p> <p>(b) $2\log x = \log x^2$</p> <p>$\log x^2 - \log 7x = \log \frac{x^2}{7x}$</p> <p>“Remove logs” to form equation in x, using the base correctly: $\frac{x^2}{7x} = 3$</p> <p>$x = 21$ (Ignore $x = 0$, if seen)</p>	<p>M1, A1 (2)</p> <p>B1</p> <p>M1</p> <p>M1</p> <p>A1cso (4)</p> <p>6</p>
	<p>(a) Allow also the ‘implicit’ answer $8^{-0.107}$ (M1 A1).</p> <p>Answer only: -0.107 or awrt: Full marks.</p> <p>Answer only: -0.11 or awrt (insufficient accuracy): M1 A0</p> <p>Trial and improvement: Award marks as for “answer only”.</p> <p>(b) <u>Alternative:</u></p> <p>$2\log x = \log x^2$ B1</p> <p>$\log 7x + 1 = \log 7x + \log 3 = \log 21x$ M1</p> <p>“Remove logs” to form equation in x: $x^2 = 21x$ M1</p> <p>$x = 21$ (Ignore $x = 0$, if seen) A1</p> <p><u>Alternative:</u></p> <p>$\log 7x = \log 7 + \log x$ B1</p> <p>$2\log x - (\log 7 + \log x) = 1$</p> <p>$\log_3 x = 1 + \log_3 7$ M1</p> <p>$x = 3^{(1+\log_3 7)} (= 3^{2.771\dots})$ or $\log_3 x = \log_3 3 + \log_3 7$ M1</p> <p>$x = 21$ A1</p> <p>Attempts using change of base will usually require the same steps as in the main scheme or alternatives, so can be marked equivalently.</p> <p><u>A common mistake:</u></p> <p>$\log x^2 - \log 7x = \frac{\log x^2}{\log 7x}$ B1 M0</p> <p>$\frac{x^2}{7x} = 3$ $x = 21$ M1(‘Recovery’), but A0</p>	

Question number	Scheme	Marks
7.	<p>(a) Gradient of AM: $\frac{1 - (-2)}{3 - 1} = \frac{3}{2}$ or $\frac{-3}{-2}$</p> <p>Gradient of l: $= -\frac{2}{3}$ M: use of $m_1 m_2 = -1$, or equiv.</p> <p>$y - 1 = -\frac{2}{3}(x - 3)$ or $\frac{y - 1}{x - 3} = -\frac{2}{3}$ $[3y = -2x + 9]$ (Any equiv. form)</p> <p>(b) $x = 6$: $3y = -12 + 9 = -3$ $y = -1$ (or show that for $y = -1$, $x = 6$) (*) (A conclusion is <u>not</u> required).</p> <p>(c) $(r^2 =)$ $(6 - 1)^2 + (-1 - (-2))^2$ M: Attempt r^2 or r</p> <p>N.B. Simplification is <u>not</u> required to score M1 A1</p> <p>$(x \pm 6)^2 + (y \pm 1)^2 = k$, $k \neq 0$ (<u>Value</u> for k not needed, could be r^2 or r)</p> <p>$(x - 6)^2 + (y + 1)^2 = 26$ (or equiv.)</p> <p>Allow $(\sqrt{26})^2$ or other exact equivalents for 26. (But... $(x - 6)^2 + (y - 1)^2 = 26$ scores M1 A0)</p> <p>(Correct answer with no working scores full marks)</p>	<p>B1</p> <p>M1</p> <p>M1 A1 (4)</p> <p>B1 (1)</p> <p>M1 A1</p> <p>M1</p> <p>A1 (4)</p> <p>9</p>
	<p>(a) 2nd M1: eqn. of a straight line through (3, 1) with any gradient except 0 or ∞. <u>Alternative</u>: Using (3, 1) in $y = mx + c$ to find a value of c scores M1, but an equation (general or specific) must be seen. Having coords the <u>wrong way round</u>, e.g. $y - 3 = -\frac{2}{3}(x - 1)$, loses the 2nd M mark <u>unless</u> a correct general formula is seen, e.g. $y - y_1 = m(x - x_1)$. If the point $P(6, -1)$ is used to find the gradient of MP, maximum marks are (a) B0 M0 M1 A1 (b) B0.</p> <p>(c) 1st M1: Condone <u>one</u> slip, numerical or sign, <u>inside</u> a bracket. Must be attempting to use points $P(6, -1)$ and $A(1, -2)$, or perhaps P and B. (Correct coordinates for B are (5, 4)). 1st M alternative is to use a complete Pythag. method on triangle MAP, n.b. $MP = MA = \sqrt{13}$.</p> <p><u>Special case</u>: If candidate persists in using <u>their</u> value for the y-coordinate of P instead of the given -1, allow the M marks in part (c) if earned.</p>	

Question number	Scheme	Marks
8.	<p>(a) $50\,000r^{n-1}$ (or equiv.) (Allow ar^{n-1} if $50\,000r^{n-1}$ is seen in (b))</p> <p>(b) $50\,000r^{n-1} > 200\,000$ (Using answer to (a), which must include r and n, and 200 000) (Allow equals sign or the wrong inequality sign) (Condone 'slips' such as omitting a zero)</p> <p>$r^{n-1} > 4 \Rightarrow (n-1)\log r > \log 4$ (Introducing logs and dealing correctly with the power) (Allow equals sign or the wrong inequality sign)</p> <p>$n > \frac{\log 4}{\log r} + 1$ (*)</p> <p>(c) $r = 1.09$: $n > \frac{\log 4}{\log 1.09} + 1$ or $n - 1 > \frac{\log 4}{\log 1.09}$ ($n > 17.086\dots$) (Allow equality)</p> <p>Year 18 or 2023 (If one of these is correct, ignore the other)</p> <p>(d) $S_n = \frac{a(1-r^n)}{1-r} = \frac{50000(1-1.09^{10})}{1-1.09}$ $\pounds 760\,000$ (Must be this answer... nearest $\pounds 100\,000$)</p>	<p>B1 (1)</p> <p>M1</p> <p>M1</p> <p>A1cso (3)</p> <p>M1</p> <p>A1 (2)</p> <p>M1 A1</p> <p>A1 (3)</p> <p>9</p>
	<p>(b) <u>Incorrect</u> inequality sign at any stage loses the A mark. Condone missing brackets if otherwise correct, e.g. $n - 1 \log r > \log 4$.</p> <p><u>A common mistake:</u> $50\,000r^{n-1} > 200\,000$ M1 $(n-1)\log 50\,000r > \log 200\,000$ M0 ('Recovery' from here is not possible).</p> <p>(c) Correct answer with no working scores full marks. Year 17 (or 2022) with no working scores M1 A0. Treat other methods (e.g. "year by year" calculation) as if there is no working.</p> <p>(d) M1: Use of the correct formula with $a = 50\,000$, 5000 or 500000, and $n = 9, 10, 11$ or 15.</p> <p>M1 can also be scored by a "year by year" method, <u>with terms added</u>. (Allow the M mark if there is evidence of adding 9, 10, 11 or 15 terms). 1st A1 is scored if 10 correct terms have been added (allow "nearest $\pounds 100$"). (50000, 54500, 59405, 64751, 70579, 76931, 83855, 91402, 99628, 108595)</p> <p><u>No</u> working shown: Special case: 760 000 scores 1 mark, scored as 1, 0, 0. (Other answers with no working score no marks).</p>	

Question number	Scheme	Marks
9.	<p>(a) </p> <p>Sine wave (anywhere) with at least 2 turning points.</p> <p>Starting on positive y-axis, going up to a max., then min. below x-axis, no further turning points in range, finishing above x-axis at $x = 2\pi$ or 360°. There must be <u>some</u> indication of scale on the y-axis... (e.g. 1, -1 or 0.5)</p> <p>Ignore parts of graph outside 0 to 2π.</p> <p>n.b. Give credit if necessary for what is seen on an initial sketch (before any transformation has been performed).</p> <p>(b) $\left(0, \frac{1}{2}\right), \left(\frac{5\pi}{6}, 0\right), \left(\frac{11\pi}{6}, 0\right)$ (Ignore any extra solutions) (<u>Not</u> $150^\circ, 330^\circ$)</p> <p>$\left(\pi - \frac{\pi}{6}\right)$ and $\left(2\pi - \frac{\pi}{6}\right)$ are insufficient, but if <u>both</u> are seen allow B1 B0.</p> <p>(c) awrt 0.71 radians (0.70758...), or awrt 40.5° (40.5416...) (α)</p> <p>$(\pi - \alpha)$ (2.43...) or $(180 - \alpha)$ <u>if α is in degrees</u>. <u>NOT</u> $\pi - \left(\alpha - \frac{\pi}{6}\right)$</p> <p>Subtract $\frac{\pi}{6}$ from α (or from $(\pi - \alpha)$)... or subtract 30 <u>if α is in degrees</u></p> <p>0.18 (or 0.06π), 1.91 (or 0.61π) Allow awrt</p> <p>(The 1st A mark is dependent on just the 2nd M mark)</p>	<p>M1</p> <p>A1 (2)</p> <p>B1, B1, B1 (3)</p> <p>B1</p> <p>M1</p> <p>M1</p> <p>A1, A1 (5)</p> <p>10</p>
	<p>(b) The zeros are not required, i.e. allow 0.5, etc. (and also allow coordinates the wrong way round).</p> <p>These marks are also awarded if the exact intercept values are seen in part (a), but if values in (b) and (a) are contradictory, (b) takes precedence.</p> <p>(c) B1: If the required value of α is <u>not seen</u>, this mark can be given by implication if a final answer rounding to 0.18 or 0.19 (or a final answer rounding to 1.91 or 1.90) is achieved. (Also see premature approx. note*).</p> <div style="border: 1px solid black; padding: 10px; margin: 10px 0;"> <p><u>Special case:</u> $\sin\left(x + \frac{\pi}{6}\right) = 0.65 \Rightarrow \sin x + \sin \frac{\pi}{6} = 0.65 \Rightarrow \sin x = 0.15$</p> <p>$x = \arcsin 0.15 = 0.15056...$ and $x = \pi - 0.15056 = 2.99$ (B0 M1 M0 A0 A0)</p> <p>(This special case mark is also available for degrees... $180 - 8.62...$)</p> </div> <p>Extra solutions outside 0 to 2π : Ignore.</p> <p>Extra solutions between 0 and 2π : Loses the final A mark.</p> <p>*<u>Premature approximation</u> in part (c):</p> <p>e.g. $\alpha = 41^\circ$, $180 - 41 = 139$, $41 - 30 = 11$ and $139 - 30 = 109$</p> <p>Changing to radians: 0.19 and 1.90</p> <p>This would score B1 (required value of α not seen, but there is a final answer 0.19 (or 1.90)), M1 M1 A0 A0.</p>	

Question number	Scheme	Marks
10.	<p>(a) $4x^2 + 6xy = 600$</p> $V = 2x^2y = 2x^2\left(\frac{600 - 4x^2}{6x}\right) \quad V = 200x - \frac{4x^3}{3} \quad (*)$ <p>(b) $\frac{dV}{dx} = 200 - 4x^2$</p> <p>Equate their $\frac{dV}{dx}$ to 0 and solve for x^2 or x: $x^2 = 50$ or $x = \sqrt{50}$ (7.07...)</p> <p>Evaluate V: $V = 200(\sqrt{50}) - \frac{4}{3}(50\sqrt{50}) = 943 \text{ cm}^3$ Allow awrt</p> <p>(c) $\frac{d^2V}{dx^2} = -8x$ Negative, \therefore Maximum</p>	<p>M1 A1</p> <p>M1 A1cso (4)</p> <p>B1</p> <p>M1 A1</p> <p>M1 A1 (5)</p> <p>M1, A1ft (2)</p> <p>11</p>
	<p>(a) 1st M: Attempting an expression in terms of x and y for the total surface area (the expression should be dimensionally correct).</p> <p>1st A: Correct expression (not necessarily simplified), equated to 600.</p> <p>2nd M: Substituting their y into $2x^2y$ to form an expression in terms of x only. (Or substituting y from $2x^2y$ into their area equation).</p> <p>(b) 1st A: Ignore $x = -\sqrt{50}$, if seen.</p> <p>The 2nd M mark (for substituting their x value into the given expression for V) is dependent on the 1st M.</p> <p>Final A: Allow also exact value $\frac{400\sqrt{50}}{3}$ or $\frac{2000\sqrt{2}}{3}$ or equiv. <u>single term</u>.</p> <p>(c) Allow marks if the work for (c) is seen in (b) (or vice-versa).</p> <p>M: Find second derivative <u>and consider its sign</u>.</p> <p>A: Second derivative following through correctly from their $\frac{dV}{dx}$, and correct reason/conclusion (it must be a maximum, not a minimum). An actual value of x does not have to be used... this mark can still be awarded if no x value has been found or if a wrong x value is used.</p> <p><u>Alternative:</u></p> <p>M: Find <u>value</u> of $\frac{dV}{dx}$ on each side of "$x = \sqrt{50}$" and consider sign.</p> <p>A: Indicate sign change of positive to negative for $\frac{dV}{dx}$, and conclude max.</p> <p><u>Alternative:</u></p> <p>M: Find <u>value</u> of V on each side of "$x = \sqrt{50}$" and compare with "943".</p> <p>A: Indicate that both values are less than 943, and conclude max.</p>	

Mark Scheme (Results)

January 2008

GCE

GCE Mathematics (6664/01)

PhysicsAndMathsTutor.com
January 2008
6664 Core Mathematics C2
Mark Scheme

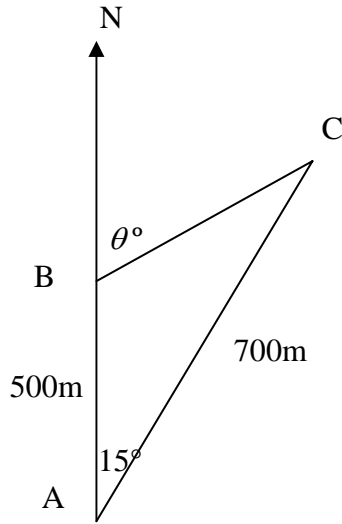
Question Number	Scheme	Marks
1.	<p>a)i) $f(3) = 3^3 - 2 \times 3^2 - 4 \times 3 + 8 \quad ; = 5$</p> <p>ii) $f(-2) = (-8 - 8 + 8 + 8) = 0$ (B1 on Epen, but A1 in fact) M1 is for attempt at either $f(3)$ or $f(-3)$ in (i) or $f(-2)$ or $f(2)$ in (ii).</p> <p>(b) $\frac{[(x+2)](x^2 - 4x + 4)}{(x+2)(x-2)^2} \quad (= 0 \text{ not required}) \text{ [must be seen or used in (b)]}$ $\quad \quad \quad (= 0) \quad \quad \quad (\text{can imply previous 2 marks})$</p> <p>Solutions: $x = 2$ or -2 (both) or $(-2, 2, 2)$ A1 (4)</p>	<p>M1; A1</p> <p>A1 (3)</p> <p>M1 A1 M1</p> <p>[7]</p>
Notes: (a)	<p>No working seen: Both answers correct scores full marks One correct ;M1 then A1B0 or A0B1, whichever appropriate.</p> <p><u>Alternative (Long division)</u> Divide by $(x-3)$ OR $(x+2)$ to get $x^2 + ax + b$, a may be zero [M1] $x^2 + x - 1$ and $+5$ seen i.s.w. (or "remainder = 5") [A1] $x^2 - 4x + 4$ and 0 seen (or "no remainder") [B1]</p> <p>(b) First M1 requires division by a found factor ; e.g $(x+2)$, $(x-2)$ or what candidate thinks is a factor to get $(x^2 + ax + b)$, a may be zero. First A1 for $[(x+2)](x^2 - 4x + 4)$ or $(x-2)(x^2 - 4)$ Second M1: attempt to factorise their found quadratic. (or use formula correctly) [Usual rule: $x^2 + ax + b = (x+c)(x+d)$, where $cd = b$.] N.B. Second A1 is for solutions, not factors <u>Alternative (first two marks)</u> $(x+2)(x^2 + bx + c) = x^3 + (2+b)x^2 + (2b+c)x + 2c = 0$ and then compare with $x^3 - 2x^2 - 4x + 8 = 0$ to find b and c. [M1] $b = -4$, $c = 4$ [A1]</p> <p><u>Method of grouping</u> $x^3 - 2x^2 - 4x + 8 = x^2(x-2) + 4(x-2)$ M1; $= x^2(x-2) - 4(x-2)$ A1 $[= (x^2 - 4)(x-2)] = (x+2)(x-2)^2$ M1 Solutions: $x=2$, $x=-2$ both A1</p>	
2.	<p>(a) Complete method, using terms of form ar^k, to find r [e.g. Dividing $ar^6 = 80$ by $ar^3 = 10$ to find r; $r^6 - r^3 = 8$ is M0] $r = 2$</p> <p>(b) Complete method for finding a [e.g. Substituting value for r into equation of form $ar^k = 10$ or 80 and finding a value for a.]</p>	<p>M1</p> <p>A1 (2)</p> <p>M1</p>

(c)	$(8a = 10) \quad a = \frac{5}{4} = 1\frac{1}{4} \quad (\text{equivalent single fraction or } 1.25)$ A1 (2) Substituting their values of a and r into correct formula for sum. M1 $S = \frac{a(r^n - 1)}{r - 1} = \frac{5}{4}(2^{20} - 1) \quad (= 1310718.75) \quad 1\,310\,719 \quad (\text{only this})$ A1 (2) [6]
Notes:	<p>(a) M1: Condone errors in powers, e.g. $ar^4 = 10$ and/or $ar^7 = 80$, A1: For $r = 2$, allow even if $ar^4 = 10$ and $ar^7 = 80$ used (just these) (M mark can be implied from numerical work, if used correctly)</p> <p>(b) M1: Allow for numerical approach: e.g. $\frac{10}{r_c^3} \leftarrow \frac{10}{r_c^2} \leftarrow \frac{10}{r_c} \leftarrow 10$</p> <p>In (a) and (b) correct answer, with no working, allow both marks.</p> <p>(c) Attempt 20 terms of series and add is M1 (correct last term 655360) If formula not quoted, errors in applying their a and/or r is M0 Allow full marks for correct answer with no working seen.</p>
3.	<p>(a) $\left(1 + \frac{1}{2}x\right)^{10} = 1 + \frac{\binom{10}{1}\left(\frac{1}{2}x\right) + \binom{10}{2}\left(\frac{1}{2}x\right)^2 + \binom{10}{3}\left(\frac{1}{2}x\right)^3}{\phantom{1 + 5x; + \frac{45}{4}(or 11.25)x^2 + 15x^3}}$ M1 A1</p> <p>$= 1 + 5x; + \frac{45}{4}(\text{or } 11.25)x^2 + 15x^3$ (coeffs need to be these, i.e, simplified) A1; A1 (4) [Allow A1A0, if totally correct with unsimplified, single fraction coefficients]</p> <p>(b) $\left(1 + \frac{1}{2} \times 0.01\right)^{10} = 1 + 5(0.01) + \frac{45}{4}(\text{or } 11.25)(0.01)^2 + 15(0.01)^3$ M1 A1✓ $= 1 + 0.05 + 0.001125 + 0.000015$ $= 1.05114 \quad \text{cao} \quad \text{A1 (3) [7]}$</p>
Notes:	<p>(a) For M1 first A1: Consider underlined expression only. M1 Requires correct structure for at least two of the three terms: (i) Must be attempt at binomial coefficients. (ii) Must have increasing powers of x, (iii) May be listed, need not be added; <i>this applies for all marks.</i></p> <p>First A1: Requires all three correct terms but need not be simplified, allow 1^{10} etc, $^{10}C_2$ etc, and condone omission of brackets around powers of $\frac{1}{2}x$ Second A1: Consider as B1 for $1 + 5x$</p> <p>(b) For M1: Substituting their (0.01) into their (a) result First A1 (f.t.): Substitution of (0.01) into their 4 termed expression in (a) Answer with no working scores no marks (calculator gives this answer)</p>

4.	<p>(a) $3 \sin^2 \theta - 2 \cos^2 \theta = 1$ $3 \sin^2 \theta - 2(1 - \sin^2 \theta) = 1$ (M1: Use of $\sin^2 \theta + \cos^2 \theta = 1$) $3 \sin^2 \theta - 2 + 2 \sin^2 \theta = 1$ $5 \sin^2 \theta = 3$ cso AG</p> <p>(b) $\sin^2 \theta = \frac{3}{5}$, so $\sin \theta = (\pm)\sqrt{0.6}$ Attempt to solve both $\sin \theta = +..$ and $\sin \theta = -$ (may be implied by later work) M1 $\theta = 50.7685^\circ$ awrt $\theta = 50.8^\circ$ (dependent on first M1 only) A1 $\theta (= 180^\circ - 50.7685^\circ); = 129.23...^\circ$ awrt 129.2° M1; A1 ✓ [f.t. dependent on first M and 3rd M] $\sin \theta = -\sqrt{0.6}$ $\theta = 230.785^\circ$ and 309.23152° awrt $230.8^\circ, 309.2^\circ$ (both) M1A1 (7) [9]</p>	
Notes:	<p>(a) N.B: AG; need to see at least one line of working after substituting $\cos^2 \theta$.</p> <p>(b) First M1: Using $5 \sin^2 \theta = 3$ to find value for $\sin \theta$ or θ Second M1: Considering the $-$ value for $\sin \theta$. (usually later) First A1: Given for awrt 50.8°. Not dependent on second M. Third M1: For $(180 - 50.8)^\circ$, need not see written down Final M1: Dependent on second M (but may be implied by answers) For $(180 + \text{candidate's } 50.8)^\circ$ or $(360 - 50.8)^\circ$ or equiv. Final A1: Requires both values. (no follow through) [Finds $\cos^2 \theta = k$ ($k = 2/5$) and so $\cos \theta = (\pm)...$M1, then mark equivalently]</p>	

5.	<p><u>Method 1</u> (Substituting $a = 3b$ into second equation at some stage)</p> <p>Using a law of logs correctly (anywhere) e.g. $\log_3 ab = 2$ M1</p> <p>Substitution of $3b$ for a (or $a/3$ for b) e.g. $\log_3 3b^2 = 2$ M1</p> <p>Using base correctly on correctly derived $\log_3 p = q$ e.g. $3b^2 = 3^2$ M1</p> <p>First correct value $b = \sqrt{3}$ (allow $3^{1/2}$) A1</p> <p>Correct method to find other value (dep. on at least first M mark) M1</p> <p>Second answer $a = 3b = 3\sqrt{3}$ or $\sqrt{27}$ A1</p> <p><u>Method 2</u> (Working with two equations in $\log_3 a$ and $\log_3 b$)</p> <p>“Taking logs” of first equation and “separating” $\log_3 a = \log_3 3 + \log_3 b$ M1 $(= 1 + \log_3 b)$</p> <p>Solving simultaneous equations to find $\log_3 a$ or $\log_3 b$ M1 $[\log_3 a = 1\frac{1}{2}, \log_3 b = \frac{1}{2}]$</p> <p>Using base correctly to find a or b M1</p> <p>Correct value for a or b $a = 3\sqrt{3}$ or $b = \sqrt{3}$ A1</p> <p>Correct method for second answer, dep. on first M; correct second answer [Ignore negative values] M1;A1[6]</p>	
Notes:	<p>Answers must be exact; decimal answers lose both A marks</p> <p>There are several variations on Method 1, depending on the stage at which $a = 3b$ is used, but they should all mark as in scheme.</p> <p>In this method, the first three method marks on Epen are for</p> <p>(i) First M1: correct use of log law,</p> <p>(ii) Second M1: substitution of $a = 3b$,</p> <p>(iii) Third M1: requires using base correctly on correctly derived $\log_3 p = q$</p>	

6.



$$BC^2 = 700^2 + 500^2 - 2 \times 500 \times 700 \cos 15^\circ$$

$$(\quad = 63851.92 \dots)$$

$$BC = 253 \quad \text{awrt}$$

M1 A1

A1 (3)

(a) $\frac{\sin B}{700} = \frac{\sin 15}{\text{candidate's } BC}$

M1

$$\sin B = \sin 15 \times 700 / 253_c = 0.716.. \quad \text{and giving an obtuse } B \quad (134.2^\circ) \quad \text{dep}$$

M1

(b) $\theta = 180^\circ - \text{candidate's angle } B \quad (\text{Dep. on first M only, } B \text{ can be acute})$

$$\theta = 180 - 134.2 = (0)45.8 \quad (\text{allow } 46 \text{ or awrt } 45.7, 45.8, 45.9)$$

[46 needs to be from correct working]

M1

A1 (4) [7]

Notes:

(a) If use $\cos 15^\circ = \dots$, then A1 not scored until written as $BC^2 = \dots$ correctly

Splitting into 2 triangles BAX and CAX, where X is foot of perp. from B to AC

Finding value for BX and CX and using Pythagoras

M1

$$BC^2 = (500 \sin 15^\circ)^2 + (700 - 500 \cos 15^\circ)^2 \quad \text{A1}$$

$$BC = 253 \quad \text{awrt}$$

A1

(b) Several alternative methods: (Showing the M marks, 3rd M dep. on first M)

(i) $\cos B = \frac{500^2 + \text{candidate's } BC^2 - 700^2}{2 \times 500 \times \text{candidate's } BC}$ or $700^2 = 500^2 + BC_c^2 - 2 \times 500 \times BC_c$ M1

Finding angle B M1, then M1 as above

(ii) 2 triangle approach, as defined in notes for (a)

$$\tan CBX = \frac{700 - \text{value for } AX}{\text{value for } BX} \quad \text{M1}$$

$$\text{Finding value for } \angle CBX \quad (\approx 59^\circ) \quad \text{M1}$$

$$\theta = [180^\circ - (75^\circ + \text{candidate's } \angle CBX)] \quad \text{M1}$$

(iii) Using sine rule (or cos rule) to find C first:

Correct use of sine or cos rule for C M1, Finding value for C M1

Either $B = 180^\circ - (15^\circ + \text{candidate's } C)$ or $\theta = (15^\circ + \text{candidate's } C)$ M1

(iv) $700 \cos 15^\circ = 500 + BC \cos \theta$ M2 {first two Ms earned in this case}

Solving for θ ; $\theta = 45.8$ (allow 46 or 5.7, 45.8, 45.9 M1; A1

7	<p>(a) Either solving $0 = x(6 - x)$ and showing $x = 6$ (and $x = 0$) or showing $(6,0)$ (and $x = 0$) satisfies $y = 6x - x^2$ [allow for showing $x = 6$]</p> <p>(b) Solving $2x = 6x - x^2$ ($x^2 = 4x$) to $x = ..$ $x = 4$ (and $x = 0$) Conclusion: when $x = 4$, $y = 8$ and when $x = 0$, $y = 0$,</p> <p>(c) (Area =) $\int_{(0)}^{(4)} (6x - x^2) dx$ Limits not required Correct integration $3x^2 - \frac{x^3}{3} (+c)$ Correct use of correct limits on their result above (see notes on limits) [" $3x^2 - \frac{x^3}{3}$ "]⁴ - [" $3x^2 - \frac{x^3}{3}$ "]₀ with limits substituted [= $48 - 21\frac{1}{3} = 26\frac{2}{3}$] Area of triangle = $2 \times 8 = 16$ (Can be awarded even if no M scored, i.e. B1) Shaded area = \pm (area under curve - area of triangle) applied correctly (= $26\frac{2}{3} - 16$) = $10\frac{2}{3}$ (awrt 10.7)</p>	<p>B1 (1)</p> <p>M1 A1 A1 (3)</p> <p>M1 A1 M1 A1 M1 A1 (6)[10]</p>
Notes	<p>(b) In scheme first A1: need only give $x = 4$ If <i>verifying approach</i> used: Verifying $(4,8)$ satisfies both the line and the curve M1(attempt at both), Both shown successfully A1 For final A1, $(0,0)$ needs to be mentioned ; accept " clear from diagram"</p> <p>(c) Alternative Using Area = $\pm \int_{(0)}^{(4)} \{(6x - x^2); -2x\} dx$ approach</p> <p>(i) If candidate integrates separately can be marked as main scheme If combine to work with = $\pm \int_{(0)}^{(4)} (4x - x^2) dx$, first M mark and third M mark $= (\pm) [2x^2 - \frac{x^3}{3} (+c)] \quad \text{A1,}$ Correct use of correct limits on their result second M1, Totally correct, unsimplified \pm expression (may be implied by correct ans.) A1 $10\frac{2}{3}$ A1 [Allow this if, having given - $10\frac{2}{3}$, they correct it] M1 for correct use of correct limits. Must substitute correct limits for their strategy into a changed expression and subtract, either way round, e.g $\pm \{ []^4 - []_0 \}$ If a long method is used, e.g, finding three areas, this mark only gained for correct strategy and all limits need to be correct for this strategy.</p> <p>Use of trapezium rule: M0A0MA0,possibleA1for triangle M1(if correct application of trap. rule from $x = 0$ to $x = 4$) A0</p>	

8	<p>(a) $(x-6)^2 + (y-4)^2 = ; 3^2$</p> <p>(b) Complete method for MP: $= \sqrt{(12-6)^2 + (6-4)^2}$ $= \sqrt{40} \quad (= 6.325)$ [These first two marks can be scored if seen as part of solution for (c)] Complete method for $\cos \theta$, $\sin \theta$ or $\tan \theta$ e.g. $\cos \theta = \frac{MT}{MP} = \frac{3}{\text{candidate's } \sqrt{40}} \quad (= 0.4743) \quad (\theta = 61.6835^\circ)$ [If $TP = 6$ is used, then M0] $\theta = 1.0766 \text{ rad} \quad \mathbf{AG}$</p> <p>(c) Complete method for area TMP; e.g. $= \frac{1}{2} \times 3 \times \sqrt{40} \sin \theta$ $= \frac{3}{2} \sqrt{31} \quad (= 8.3516..)$ allow awrt 8.35 Area (sector) $MTQ = 0.5 \times 3^2 \times 1.0766 \quad (= 4.8446..)$ Area $TPQ = \text{candidate's } (8.3516.. - 4.8446..)$ $= 3.507 \quad \text{awrt}$ [Note: 3.51 is A0]</p>	<p>B1; B1 (2)</p> <p>M1 A1</p> <p>M1</p> <p>A1 (4)</p> <p>M1 A1</p> <p>M1</p> <p>M1</p> <p>A1 (5) [11]</p>
Notes	<p>(a) Allow 9 for 3^2.</p> <p>(b) First M1 can be implied by $\sqrt{40}$ For second M1: May find $TP = \sqrt{(\sqrt{40})^2 - 3^2} = \sqrt{31}$, then either $\sin \theta = \frac{TP}{MP} = \frac{\sqrt{31}}{\sqrt{40}} \quad (= 0.8803..)$ or $\tan \theta = \frac{\sqrt{31}}{3} \quad (1.8859..)$ or cos rule NB. Answer is given, but allow final A1 if all previous work is correct.</p> <p>(c) First M1: (alternative) $\frac{1}{2} \times 3 \times \sqrt{40 - 9}$</p>	

9	<p>(a) (Total area) = $3xy + 2x^2$ (Vol:) $x^2y = 100$ $(y = \frac{100}{x^2}, xy = \frac{100}{x})$ Deriving expression for area in terms of x only (Substitution, or clear use of, y or xy into expression for area) (Area =) $\frac{300}{x} + 2x^2$ AG</p> <p>(b) $\frac{dA}{dx} = -\frac{300}{x^2} + 4x$ Setting $\frac{dA}{dx} = 0$ and finding a value for correct power of x, for cand. M1 $[x^3 = 75]$ $x = 4.2172$ awrt 4.22 (allow exact $\sqrt[3]{75}$)</p> <p>(c) $\frac{d^2A}{dx^2} = \frac{600}{x^3} + 4 = \text{positive}$ therefore minimum</p> <p>(d) Substituting found value of x into (a) (Or finding y for found x and substituting both in $3xy + 2x^2$) $[y = \frac{100}{4.2172^2} = 5.6228]$ Area = 106.707 awrt 107</p>	<p>B1 B1 M1 A1 cso (4) M1A1 A1 (4) M1A1 (2) M1 A1 (2) [12]</p>
Notes	<p>(a) First B1: Earned for correct unsimplified expression, isw.</p> <p>(c) For M1: Find $\frac{d^2A}{dx^2}$ and explicitly consider its sign, state > 0 or “positive” A1: Candidate’s $\frac{d^2A}{dx^2}$ must be correct for their $\frac{dA}{dx}$, sign must be + ve and conclusion “so minimum”, (allow QED, \checkmark). (may be wrong x, or even no value of x found)</p> <p><u>Alternative:</u> M1: Find value of $\frac{dA}{dx}$ on either side of “$x = \sqrt[3]{75}$” and consider sign A1: Indicate sign change of negative to positive for $\frac{dA}{dx}$, and conclude minimum.</p> <p>OR M1: Consider values of A on either side of “$x = \sqrt[3]{75}$” and compare with “107” A1: Both values greater than “$x = 107$” and conclude minimum.</p> <p>Allow marks for (c) and (d) where seen; even if part labelling confused.</p>	

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Mark Scheme (Results)

Summer 2008

GCE Mathematics (6664/01)

GCE

June 2008
Core Mathematics C2
Mark Scheme

Question number	Scheme	Marks
1.	<p>(a) Attempt to find $f(-4)$ or $f(4)$. $\left(f(-4) = 2(-4)^3 - 3(-4)^2 - 39(-4) + 20\right)$ $(= -128 - 48 + 156 + 20) = 0$, so $(x + 4)$ is a factor.</p> <p>(b) $2x^3 - 3x^2 - 39x + 20 = (x + 4)(2x^2 - 11x + 5)$ $\dots(2x - 1)(x - 5)$ (The 3 brackets need not be written together) or $\dots\left(x - \frac{1}{2}\right)(2x - 10)$ or equivalent</p>	<p>M1 A1 (2)</p> <p>M1 A1 M1 A1cso (4)</p> <p style="text-align: right;">6</p>
	<p>(a) Long division scores no marks in part (a). The <u>factor theorem</u> is required. However, the first two marks in (b) can be earned from division seen in (a)... ... but if a different long division result is seen in (b), the work seen in (b) takes precedence for marks in (b).</p> <p>A1 requires zero and a simple <u>conclusion</u> (even just a tick, or Q.E.D.), or may be scored by a <u>preamble</u>, e.g. 'If $f(-4) = 0$, $(x + 4)$ is a factor.....'</p> <p>(b) First M requires use of $(x + 4)$ to obtain $(2x^2 + ax + b)$, $a \neq 0, b \neq 0$, even with a remainder. Working need not be seen... this could be done 'by inspection'. Second M for the attempt to factorise their three-term quadratic. Usual rule: $(kx^2 + ax + b) = (px + c)(qx + d)$, where $cd = b$ and $pq = k$. If 'solutions' appear before or after factorisation, ignore... ... but factors must be seen to score the second M mark.</p> <p><u>Alternative (first 2 marks):</u> $(x + 4)(2x^2 + ax + b) = 2x^3 + (8 + a)x^2 + (4a + b)x + 4b = 0$, then compare coefficients to find <u>values</u> of a and b. [M1] $a = -11, b = 5$ [A1]</p> <p><u>Alternative:</u> Factor theorem: Finding that $f\left(\frac{1}{2}\right) = 0 \therefore$ factor is, $(2x - 1)$ [M1, A1] Finding that $f(5) = 0 \therefore$ factor is, $(x - 5)$ [M1, A1] "Combining" all 3 factors is <u>not</u> required. If just one of these is found, score the <u>first 2 marks</u> M1 A1 M0 A0. <u>Losing a factor of 2:</u> $(x + 4)\left(x - \frac{1}{2}\right)(x - 5)$ scores M1 A1 M1 A0. <u>Answer only, one sign wrong:</u> e.g. $(x + 4)(2x - 1)(x + 5)$ scores M1 A1 M1 A0</p>	

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Core Mathematics C2
Mark Scheme

Question number	Scheme	Marks
2.	<p>(a) 1.732, 2.058, 5.196 awrt (One or two correct B1 B0, All correct B1 B1)</p> <p>(b) $\frac{1}{2} \times 0.5 \dots\dots$ $\dots\dots \{(1.732 + 5.196) + 2(2.058 + 2.646 + 3.630)\}$ $= 5.899$ (awrt 5.9, allowed even after minor slips in values)</p>	<p>B1 B1 (2)</p> <p>B1</p> <p>M1 A1ft</p> <p>A1 (4)</p> <p>6</p>
	<p>(a) Accept awrt (but <u>less</u> accuracy loses these marks). Also accept <u>exact</u> answers, e.g. $\sqrt{3}$ at $x = 0$, $\sqrt{27}$ or $3\sqrt{3}$ at $x = 2$.</p> <p>(b) For the M mark, the first bracket must contain the 'first and last' values, and the second bracket must have no additional values. If the only mistake is to <u>omit</u> one of the values from the second bracket, this can be considered as a slip and the M mark can be allowed.</p> <p>Bracketing mistake: i.e. $\frac{1}{2} \times 0.5(1.732 + 5.196) + 2(2.058 + 2.646 + 3.630)$</p> <p>scores B1 M1 A0 A0 <u>unless</u> the final answer implies that the calculation has been done correctly (then full marks can be given).</p> <p><u>x values</u>: M0 if the values used in the brackets are x values instead of y values.</p> <p><u>Alternative</u>: Separate trapezia may be used, and this can be marked equivalently.</p> $\left[\frac{1}{4}(1.732 + 2.058) + \frac{1}{4}(2.058 + 2.646) + \frac{1}{4}(2.646 + 3.630) + \frac{1}{4}(3.630 + 5.196) \right]$	

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Core Mathematics C2
Mark Scheme

Question number	Scheme	Marks
3.	<p>(a) $(1+ax)^{10} = 1+10ax.....$ (<u>Not</u> unsimplified versions) $+ \frac{10 \times 9}{2}(ax)^2 + \frac{10 \times 9 \times 8}{6}(ax)^3$ Evidence from <u>one</u> of these terms is sufficient $+ 45(ax)^2, +120(ax)^3$ or $+45a^2x^2, +120a^3x^3$</p> <p>(b) $120a^3 = 2 \times 45a^2$ $a = \frac{3}{4}$ or equiv. $\left(\text{e.g. } \frac{90}{120}, 0.75 \right)$ Ignore $a = 0$, if seen</p>	<p>B1</p> <p>M1</p> <p>A1, A1 (4)</p> <p>M1 A1 (2)</p> <p>6</p>
	<p>(a) The terms can be ‘listed’ rather than added.</p> <p>M1: Requires correct structure: ‘binomial coefficient’ (perhaps from Pascal’s triangle) and the correct power of x.</p> <p>(The M mark can also be given for an expansion in <u>descending</u> powers of x). Allow ‘slips’ such as: $\frac{10 \times 9}{2}ax^2, \frac{10 \times 9}{3 \times 2}(ax)^3, \frac{10 \times 9}{2}x^2, \frac{9 \times 8 \times 7}{3 \times 2}a^3x^3$ However, $45 + a^2x^2 + 120 + a^3x^3$ or similar is M0.</p> <p>$\binom{10}{2}$ and $\binom{10}{3}$ or equivalent such as ${}^{10}C_2$ and ${}^{10}C_3$ are acceptable, and even $\left(\frac{10}{2}\right)$ and $\left(\frac{10}{3}\right)$ are acceptable for the method mark.</p> <p>1st A1: Correct x^2 term. 2nd A1: Correct x^3 term (These <u>must</u> be simplified). If simplification is not seen in (a), but correct simplified terms are seen in (b), these marks can be awarded. However, if <u>wrong</u> simplification is seen in (a), this takes precedence.</p> <p><u>Special case:</u> If $(ax)^2$ and $(ax)^3$ are seen within the working, but then lost... ... A1 A0 can be given if $45ax^2$ and $120ax^3$ are <u>both</u> achieved.</p> <p>(b) M: Equating their coefficient of x^3 to twice their coefficient of x^2... ... <u>or</u> equating their coefficient of x^2 to twice their coefficient of x^3. (... or coefficients can be <u>correct</u> coefficients rather than their coefficients). Allow this mark even if the equation is trivial, e.g. $120a = 90a$. An equation in a alone is required for this M mark, although... ...condone, e.g. $120a^3x^3 = 90a^2x^2 \Rightarrow (120a^3 = 90a^2 \Rightarrow) a = \frac{3}{4}$.</p> <p><u>Beware:</u> $a = \frac{3}{4}$ following $120a = 90a$, which is A0.</p>	

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Core Mathematics C2
Mark Scheme

Question number	Scheme	Marks
4.	<p>(a) $x = \frac{\log 7}{\log 5}$ or $x = \log_5 7$ (i.e. correct method up to $x = \dots$) 1.21 Must be this answer (3 s.f.)</p> <p>(b) $(5^x - 7)(5^x - 5)$ Or another variable, e.g. $(y - 7)(y - 5)$, even $(x - 7)(x - 5)$ $(5^x = 7 \text{ or } 5^x = 5)$ $x = 1.2$ (awrt) ft from the answer to (a), if used $x = 1$ (allow 1.0 or 1.00 or 1.000)</p>	<p>M1 A1 (2)</p> <p>M1 A1 A1ft B1 (4)</p> <p>6</p>
	<p>(a) 1.21 with no working: M1 A1 (even if it left as $5^{1.21}$). Other answers which round to 1.2 with no working: M1 A0.</p> <p>(b) M: Using the <u>correct</u> quadratic equation, attempt to factorise $(5^x \pm 7)(5^x \pm 5)$, or attempt quadratic formula. Allow $\log_5 7$ or $\frac{\log 7}{\log 5}$ instead of 1.2 for A1ft. No marks for simply substituting a decimal answer from (a) into the given equation (perhaps showing that it gives approximately zero). <u>However</u>, note the following <u>special case</u>: Showing that $5^x = 7$ satisfies the given equation, therefore 1.21 is a solution scores 0, 0, 1, 0 (and could score <u>full marks</u> if the $x = 1$ were also found). e.g. If $5^x = 7$, then $5^{2x} = 49$, and $5^{2x} - 12(5^x) + 35 = 49 - 84 + 35 = 0$, so one solution is $x = 1.21$ ('conclusion' must be seen). To score this special case mark, values substituted into the equation must be <u>exact</u>. Also, the mark would <u>not</u> be scored in the following case: e.g. If $5^x = 7$, $5^{2x} - 84 + 35 = 0 \Rightarrow 5^{2x} = 49 \Rightarrow x = 1.21$ (Showing no appreciation that $5^{2x} = (5^x)^2$) B1: Do not award this mark if $x = 1$ clearly follows from <u>wrong</u> working.</p>	

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Core Mathematics C2
Mark Scheme

Question number	Scheme	Marks
5.	<p>(a) $(8-3)^2 + (3-1)^2$ or $\sqrt{(8-3)^2 + (3-1)^2}$ $(x \pm 3)^2 + (y \pm 1)^2 = k$ or $(x \pm 1)^2 + (y \pm 3)^2 = k$ (k a positive <u>value</u>) $(x-3)^2 + (y-1)^2 = 29$ (<u>Not</u> $(\sqrt{29})^2$ or 5.39^2)</p> <p>(b) Gradient of radius = $\frac{2}{5}$ (or exact equiv.) Must be seen or used in (b) Gradient of tangent = $-\frac{5}{2}$ (Using perpendicular gradient method) $y-3 = -\frac{5}{2}(x-8)$ (ft gradient of radius, dependent upon <u>both</u> M marks) $5x + 2y - 46 = 0$ (Or equiv., equated to zero, e.g. $92 - 4y - 10x = 0$) (Must have <u>integer</u> coefficients)</p>	<p>M1 A1 M1 A1 (4) B1 M1 M1 A1ft A1 (5) 9</p>
	<p>(a) For the M mark, condone <u>one</u> slip <u>inside</u> a bracket, e.g. $(8-3)^2 + (3+1)^2$, $(8-1)^2 + (1-3)^2$ The first two marks may be gained implicitly from the circle equation.</p> <p>(b) 2nd M: Eqn. of line through (8, 3), in any form, with any grad.(except 0 or ∞). If the 8 and 3 are the 'wrong way round', this M mark is only given if a correct general formula, e.g. $y - y_1 = m(x - x_1)$, is quoted. <u>Alternative:</u> 2nd M: Using (8, 3) and an m value in $y = mx + c$ to find a value of c. A1ft: as in main scheme. (Correct substitution of 8 and 3, then a wrong c value will still score the A1ft)</p> <p>(b) <u>Alternatives for the first 2 marks:</u> (but in these 2 cases the 1st A mark is <u>not</u> ft) (i) Finding gradient of tangent by <u>implicit</u> differentiation $2(x-3) + 2(y-1)\frac{dy}{dx} = 0$ (or equivalent) B1 Subs. $x = 8$ <u>and</u> $y = 3$ into a 'derived' expression to find a value for dy/dx M1 (ii) Finding gradient of tangent by differentiation of $y = 1 + \sqrt{20 + 6x - x^2}$ $\frac{dy}{dx} = \frac{1}{2}(20 + 6x - x^2)^{-\frac{1}{2}}(6 - 2x)$ (or equivalent) B1 Subs. $x = 8$ into a 'derived' expression to find a value for dy/dx M1 <u>Another alternative:</u> Using $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$ $x^2 + y^2 - 6x - 2y - 19 = 0$ B1 $8x + 3y, -3(x+8) - (y+3) - 19 = 0$ M1, M1 A1ft (ft from circle eqn.) $5x + 2y - 46 = 0$ A1</p>	

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Mark Scheme

Question number	Scheme	Marks
6.	<p>(a) $T_{20} = 5 \times \left(\frac{4}{5}\right)^{19} = 0.072$ (Accept awrt) Allow $5 \times \frac{4^{19}}{5}$ for M1</p> <p>(b) $S_{\infty} = \frac{5}{1-0.8} = 25$</p> <p>(c) $\frac{5(1-0.8^k)}{1-0.8} > 24.95$ (Allow with = or <)</p> <p>$1-0.8^k > 0.998$ (or equiv., see below) (Allow with = or <)</p> <p>$k \log 0.8 < \log 0.002$ or $k > \log_{0.8} 0.002$ (Allow with = or <)</p> <p>$k > \frac{\log 0.002}{\log 0.8}$ (*)</p> <p>(d) $k = 28$ (Must be this integer value) <u>Not</u> $k > 27$, or $k < 28$, or $k > 28$</p>	<p>M1 A1 (2)</p> <p>M1 A1 (2)</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1cso (4)</p> <p>B1 (1)</p> <p>9</p>
	<p>(a) and (b): Correct answer without working scores both marks.</p> <p>(a) M: Requires use of the correct formula ar^{n-1}.</p> <p>(b) M: Requires use of the correct formula $\frac{a}{1-r}$</p> <p>(c) 1st M: The sum may have already been 'manipulated' (perhaps wrongly), but this mark can still be allowed.</p> <p>1st A: A 'numerically correct' version that has dealt with $(1-0.8)$ denominator,</p> <p>e.g. $1 - \left(\frac{4}{5}\right)^k > 0.998$, $5(1-0.8^k) > 4.99$, $25(1-0.8^k) > 24.95$,</p> <p>$5 - 5(0.8^k) > 4.99$. In any of these, $\frac{4}{5}$ instead of 0.8 is fine,</p> <p>and condone $\frac{4^k}{5}$ if correctly treated later.</p> <p>2nd M: Introducing logs and using laws of logs correctly (this must include dealing with the power k so that $p^k = k \log p$).</p> <p>2nd A: An <u>incorrect</u> statement (including equalities) at any stage in the working loses this mark (this is often identifiable at the stage $k \log 0.8 > \log 0.002$). (So a fully correct method with inequalities is required.)</p>	

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Mark Scheme

Question number	Scheme	Marks
7.	<p>(a) $r\theta = 7 \times 0.8 = 5.6$ (cm)</p> <p>(b) $\frac{1}{2}r^2\theta = \frac{1}{2} \times 7^2 \times 0.8 = 19.6$ (cm²)</p> <p>(c) $BD^2 = 7^2 + (\text{their } AD)^2 - (2 \times 7 \times (\text{their } AD) \times \cos 0.8)$ $BD^2 = 7^2 + 3.5^2 - (2 \times 7 \times 3.5 \times \cos 0.8)$ (or awrt 46° for the angle) $(BD = 5.21)$ Perimeter = (their DC) + “5.6” + “5.21” = 14.3 (cm) (Accept awrt)</p> <p>(d) $\Delta ABD = \frac{1}{2} \times 7 \times (\text{their } AD) \times \sin 0.8$ (or awrt 46° for the angle) (ft their AD) $(= 8.78\dots)$ (If the correct formula $\frac{1}{2}ab \sin C$ is <u>quoted</u> the use of any two of the sides of ΔABD as a and b scores the M mark). Area = “19.6” – “8.78...” = 10.8 (cm²) (Accept awrt)</p>	<p>M1 A1 (2)</p> <p>M1 A1 (2)</p> <p>M1 A1 M1 A1 (4)</p> <p>M1 A1ft</p> <p>M1 A1 (4)</p> <p>12</p>
	<p>Units (cm or cm²) are not required in any of the answers.</p> <p>(a) and (b): Correct answers without working score both marks.</p> <p>(a) M: Use of $r\theta$ (with θ in radians), or equivalent (could be working in degrees with a correct degrees formula).</p> <p>(b) M: Use of $\frac{1}{2}r^2\theta$ (with θ in radians), or equivalent (could be working in degrees with a correct degrees formula).</p> <p>(c) 1st M: Use of the (correct) cosine rule formula to find BD^2 or BD. Any other methods need to be complete methods to find BD^2 or BD. 2nd M: Adding their DC to their arc BC and their BD. <u>Beware:</u> If 0.8 is used, but calculator is in degree mode, this can still earn M1 A1 (for the required expression), but this gives $BD = 3.50\dots$ so the perimeter may appear as $3.5 + 5.6 + 3.5$ (earning M1 A0).</p> <p>(d) 1st M: Use of the (correct) area formula to find ΔABD. Any other methods need to be complete methods to find ΔABD. 2nd M: Subtracting their ΔABD from their sector ABC. Using segment formula $\frac{1}{2}r^2(\theta - \sin \theta)$ scores no marks in part (d).</p>	

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Mark Scheme

Question number	Scheme	Marks
8.	<p>(a) $\left(\frac{dy}{dx} = \right) 8 + 2x - 3x^2$ (M: $x^n \rightarrow x^{n-1}$ for one of the terms, <u>not</u> just $10 \rightarrow 0$)</p> <p>$3x^2 - 2x - 8 = 0$ $(3x + 4)(x - 2) = 0$ $x = 2$ (Ignore other solution) (*)</p> <p>(b) Area of triangle = $\frac{1}{2} \times 2 \times 22$ (M: Correct method to find area of triangle)</p> <p>(Area = 22 with no working is acceptable)</p> <p>$\int 10 + 8x + x^2 - x^3 dx = 10x + \frac{8x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$ (M: $x^n \rightarrow x^{n+1}$ for one of the terms)</p> <p>Only one term correct: M1 A0 A0 2 or 3 terms correct: M1 A1 A0</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> Integrating the <u>gradient function</u> loses this M mark. </div> <p>$\left[10x + \frac{8x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \right]_0^2 = \dots\dots$ (Substitute limit 2 into a 'changed function')</p> <p>$\left(= 20 + 16 + \frac{8}{3} - 4 \right)$ (This M can be awarded even if the other limit is wrong)</p> <p>Area of R = $34\frac{2}{3} - 22 = \frac{38}{3} \left(= 12\frac{2}{3} \right)$ (Or 12.6)</p> <p>M: <u>Dependent on use of calculus in (b) and correct overall 'strategy':</u> subtract either way round. A: Must be <u>exact</u>, not 12.67 or similar. A negative area at the end, even if subsequently made positive, loses the A mark.</p>	<p>M1 A1</p> <p>A1cso (3)</p> <p>M1 A1</p> <p>M1 A1 A1</p> <p>M1</p> <p>M1 A1 (8)</p> <p>11</p>
	<p>(a) The final mark may also be scored by <u>verifying</u> that $\frac{dy}{dx} = 0$ at $x = 2$.</p> <p>(b) <u>Alternative:</u> Eqn. of line $y = 11x$. (Marks dependent on subsequent use in integration) (M1: Correct method to find equation of line. A1: Simplified form $y = 11x$)</p> <p>$\int 10 + kx + x^2 - x^3 dx = 10x + \frac{kx^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$ (k perhaps -3)</p> <p>$\left[10x + \frac{kx^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \right]_0^2 = \dots\dots$ (Substitute limit 2 into a 'changed function')</p> <p>Area of R = $\left[10x - \frac{3x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \right]_0^2 = 20 - 6 + \frac{8}{3} - 4 = \frac{38}{3} \left(= 12\frac{2}{3} \right)$</p> <p>Final M1 for $\int(\text{curve}) - \int(\text{line})$ or $\int(\text{line}) - \int(\text{curve})$.</p>	<p>M1 A1</p> <p>M1 A1 A1</p> <p>M1</p> <p>M1 A1 (8)</p>

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Question number	Scheme	Marks
9.	<p>(a) 45 (α) (This mark can be implied by an answer 65) $180 - \alpha$, Add 20 (for at least one angle) 65 155</p> <p>(b) 120 or 240 (β): (This mark can be implied by an answer 40 or 80) (Could be achieved by working with 60, using $180 - 60$ and/or $180 + 60$) $360 - \beta$, $360 + \beta$ (or $120 +$ an angle that has been divided by 3) Dividing by 3 (for at least one angle) 40 80 160 200 280 320 First A1: at least 3 correct</p>	<p>B1 M1, M1 A1 (4)</p> <p>B1 M1, M1 M1 A1 A1 (6)</p> <p>10</p>
	<p>(a) Extra solution(s) in range: Loses the A mark. Extra solutions outside range: Ignore (whether correct or not). Common solutions: 65 (only correct solution) will score B1 M0 M1 A0 (2 marks) 65 and 115 will score B1 M0 M1 A0 (2 marks) 44.99 (or similar) for α is B0, and 64.99, 155.01 (or similar) is A0.</p> <p>(b) Extra solution(s) in range: Loses the final A mark. Extra solutions outside range: Ignore (whether correct or not). Common solutions: 40 (only correct solution) will score B1 M0 M0 M1 A0 A0 (2 marks) 40 and 80 (only correct solutions) B1 M1 M0 M1 A0 A0 (3 marks) 40 and 320 (only correct solutions) B1 M0 M0 M1 A0 A0 (2 marks)</p> <p><u>Answers without working:</u> Full marks can be given (in both parts), B and M marks by implication.</p> <p><u>Answers given in radians:</u> Deduct a maximum of 2 marks (misread) from B and A marks. (Deduct these at first and second occurrence.)</p> <p><u>Answers that begin</u> with statements such as $\sin(x - 20) = \sin x - \sin 20$ or $\cos x = -\frac{1}{6}$, then go on to find a value of 'α' or 'β', however badly, <u>can</u> continue to earn the first M mark in either part, but will score <u>no further marks</u>.</p> <p><u>Possible misread:</u> $\cos 3x = \frac{1}{2}$, giving 20, 100, 140, 220, 260, 340 Could score up to 4 marks B0 M1 M1 M1 A0 A1 for the above answers.</p>	

Mark Scheme (Results)

January 2009

GCE

GCE Mathematics (6664/01)

January 2009
6664 Core Mathematics C2
Mark Scheme

Question Number	Scheme	Marks
1	$(3 - 2x)^5 = 243, \dots + 5 \times (3)^4 (-2x) = -810x \dots$ $+ \frac{5 \times 4}{2} (3)^3 (-2x)^2 = +1080x^2$	B1, B1 M1 A1 (4) [4]
Notes	<p>First term must be 243 for B1, writing just 3^5 is B0 (Mark their final answers except in second line of special cases below).</p> <p>Term must be simplified to $-810x$ for B1</p> <p>The x is required for this mark.</p> <p>The method mark (M1) is generous and is awarded for an attempt at Binomial to get the third term.</p> <p>There must be an x^2 (or no x- i.e. not wrong power) and attempt at Binomial Coefficient and at dealing with powers of 3 and 2. The power of 3 should not be one, but the power of 2 may be one (regarded as bracketing slip).</p> <p>So allow $\binom{5}{2}$ or $\binom{5}{3}$ or 5C_2 or 5C_3 or even $\left(\frac{5}{2}\right)$ or $\left(\frac{5}{3}\right)$ or use of '10' (maybe from Pascal's triangle)</p> <p>May see ${}^5C_2(3)^3(-2x)^2$ or ${}^5C_2(3)^3(-2x^2)$ or ${}^5C_2(3)^5(-\frac{2}{3}x^2)$ or $10(3)^3(2x)^2$ which would each score the M1</p> <p>A1 is c.a.o and needs $1080x^2$ (if $1080x^2$ is written with no working this is awarded both marks i.e. M1 A1.)</p>	
Special cases	<p>$243 + 810x + 1080x^2$ is B1B0M1A1 (condone no negative signs)</p> <p>Follows correct answer with $27 - 90x + 120x^2$ can isw here (sp case)– full marks for correct answer</p> <p>Misreads <i>ascending</i> and gives $-32x^5 + 240x^4 - 720x^3$ is marked as B1B0M1A0 special case and must be completely correct. (If any slips could get B0B0M1A0)</p> <p>Ignores 3 and expands $(1 \pm 2x)^5$ is 0/4</p> <p>$243, -810x, 1080x^2$ is full marks but $243, -810, 1080$ is B1,B0,M1,A0</p> <p>NB Alternative method $3^5(1 - \frac{2}{3}x)^5 = 3^5 - 5 \times 3^5 \times (\frac{2}{3}x) + \binom{5}{3} 3^5 (-\frac{2}{3}x)^2 + \dots$ is B0B0M1A0</p> <p>– answers must be simplified to $243 - 810x + 1080x^2$ for full marks (awarded as before)</p> <p>Special case $3(1 - \frac{2}{3}x)^5 = 3 - 5 \times 3 \times (\frac{2}{3}x) + \binom{5}{3} 3(-\frac{2}{3}x)^2 + \dots$ is B0, B0, M1, A0</p> <p>Or $3(1 - 2x)^5$ is B0B0M0A0</p>	

Question Number	Scheme	Marks
2	$y = (1 + x)(4 - x) = 4 + 3x - x^2$ <p style="text-align: right;">M: Expand, giving 3 (or 4) terms</p> $\int (4 + 3x - x^2) dx = 4x + \frac{3x^2}{2} - \frac{x^3}{3}$ <p style="text-align: right;">M: Attempt to integrate</p> $= [\dots\dots\dots]_{-1}^4 = \left(16 + 24 - \frac{64}{3}\right) - \left(-4 + \frac{3}{2} + \frac{1}{3}\right) = \frac{125}{6} \quad \left(= 20\frac{5}{6}\right)$	<p>M1</p> <p>M1 A1</p> <p>M1 A1 (5) [5]</p>
Notes	<p>M1 needs expansion, there may be a slip involving a sign or simple arithmetical error e.g. $1 \times 4 = 5$, but there needs to be a ‘constant’ an ‘x term’ and an ‘x^2 term’. The x terms do not need to be collected. (Need not be seen if next line correct)</p> <p>Attempt to integrate means that $x^n \rightarrow x^{n+1}$ for at least one of the terms, then M1 is awarded (even 4 becoming $4x$ is sufficient) – one correct power sufficient.</p> <p>A1 is for correct answer only, not follow through. But allow $2x^2 - \frac{1}{2}x^2$ or any correct equivalent. Allow + <i>c</i>, and even allow an evaluated extra constant term.</p> <p>M1: Substitute limit 4 and limit -1 into a changed function (must be -1) and indicate subtraction (either way round).</p> <p>A1 must be exact, not 20.83 or similar. If recurring indicated can have the mark. Negative area, even if subsequently positive loses the A mark.</p>	
Special cases	<p>(i) Uses calculator method: M1 for expansion (if seen) M1 for limits if answer correct, so 0 , 1 or 2 marks out of 5 is possible (Most likely M0 M0 A0 M1 A0)</p> <p>(ii) Uses trapezium rule : not exact, no calculus – 0/5 unless expansion mark M1 gained.</p> <p>(iii) Using original method, but then change all signs after expansion is likely to lead to: M1 M1 A0, M1 A0 i.e. 3/5</p>	

Question Number	Scheme	Marks
3	<p>(a) 3.84, 4.14, 4.58 (Any one correct B1 B0. All correct B1 B1)</p> <p>(b) $\frac{1}{2} \times 0.4, \{(3 + 4.58) + 2(3.47 + 3.84 + 4.14 + 4.39)\}$ $= 7.852$ (awrt 7.9)</p>	<p>B1 B1 (2)</p> <p>B1, M1 A1ft A1 (4) [6]</p>
Notes	<p>(a) B1 for one answer correct Second B1 for all three correct</p> <p>Accept awrt ones given or exact answers so $\sqrt{21}$, $\sqrt{\left(\frac{369}{25}\right)}$ or $\frac{3\sqrt{41}}{5}$, and $\sqrt{\left(\frac{429}{25}\right)}$ or $\frac{\sqrt{429}}{5}$, score the marks.</p> <p>(b) B1 is for using 0.2 or $\frac{0.4}{2}$ as $\frac{1}{2}h$.</p> <p>M1 requires first bracket to contain first plus last values and second bracket to include no additional values from those in the table. If the only mistake is to omit one value from 2nd bracket this may be regarded as a slip and can be allowed (An extra repeated term forfeits the M mark however) x values: M0 if values used in brackets are x values instead of y values. Separate trapezia may be used : B1 for 0.2, M1 for $\frac{1}{2}h(a+b)$ used 4 or 5 times (and A1ft all e.g.. $0.2(3 + 3.47) + 0.2(3.47 + 3.84) + 0.2(3.84 + 4.14) + 0.2(4.14 + 4.58)$ is M1 A0 equivalent to missing one term in { } in main scheme A1ft follows their answers to part (a) and is for {correct expression} Final A1 must be correct. (No follow through)</p> <p>Special cases Bracketing mistake: i.e. $\frac{1}{2} \times 0.4(3 + 4.58) + 2(3.47 + 3.84 + 4.14 + 4.39)$ scores B1 M1 A0 A0 <u>unless</u> the final answer implies that the calculation has been done correctly (then full marks can be given). Need to see trapezium rule – answer only (with no working) is 0/4.</p>	

Question Number	Scheme	Marks
4	$2\log_5 x = \log_5(x^2), \quad \log_5(4-x) - \log_5(x^2) = \log_5 \frac{4-x}{x^2}$ $\log\left(\frac{4-x}{x^2}\right) = \log 5 \quad 5x^2 + x - 4 = 0 \text{ or } 5x^2 + x = 4 \text{ o.e.}$ $(5x-4)(x+1) = 0 \quad x = \frac{4}{5} \quad (x = -1)$	B1, M1 M1 A1 dM1 A1 (6) [6]
Notes	<p>B1 is awarded for $2\log x = \log x^2$ anywhere.</p> <p>M1 for correct use of $\log A - \log B = \log \frac{A}{B}$</p> <p>M1 for replacing 1 by $\log_k k$. A1 for correct quadratic</p> <p>$(\log(4-x) - \log x^2 = \log 5 \Rightarrow 4-x-x^2 = 5$ is B1M0M1A0 M0A0)</p> <p>dM1 for attempt to solve quadratic with usual conventions. (Only award if previous two M marks have been awarded)</p> <p>A1 for 4/5 or 0.8 or equivalent (Ignore extra answer).</p>	
Alternative 1	$\log_5(4-x) - 1 = 2\log_5 x \text{ so } \log_5(4-x) - \log_5 5 = 2\log_5 x$ $\log_5 \frac{4-x}{5} = 2\log_5 x$ <p>then could complete solution with $2\log_5 x = \log_5(x^2)$</p> $\left(\frac{4-x}{5}\right) = x^2 \quad 5x^2 + x - 4 = 0$ <p>Then as in first method $(5x-4)(x+1) = 0 \quad x = \frac{4}{5} \quad (x = -1)$</p>	M1 M1 B1 A1 dM1 A1 (6) [6]
Special cases	<p>Complete trial and error yielding 0.8 is M3 and B1 for 0.8</p> <p>A1, A1 awarded for each of two tries evaluated. i.e. 6/6</p> <p>Incomplete trial and error with wrong or no solution is 0/6</p> <p>Just answer 0.8 with no working is B1</p> <p>If log base 10 or base e used throughout - can score B1M1M1A0M1A0</p>	

Question Number	Scheme	Marks
5		
(a)	$PQ: m_1 = \frac{10-2}{9-(-3)} (= \frac{2}{3}) \quad \text{and} \quad QR: m_2 = \frac{10-4}{9-a}$	M1
(b)	$m_1 m_2 = -1: \frac{8}{12} \times \frac{6}{9-a} = -1 \quad a = 13 \quad (*)$	M1 A1 (3)
Alt for (a)	(a) Alternative method (Pythagoras) Finds all three of the following $(9-(-3))^2 + (10-2)^2, (i.e.208), (9-a)^2 + (10-4)^2, (a-(-3))^2 + (4-2)^2$ Using Pythagoras (correct way around) e.g. $a^2 + 6a + 9 = 240 + a^2 - 18a + 81$ to form equation Solve (or verify) for $a, a = 13 (*)$ (b) Centre is at (5, 3) $(r^2 =) (10-3)^2 + (9-5)^2$ or equiv., or $(d^2 =) (13-(-3))^2 + (4-2)^2$ $(x-5)^2 + (y-3)^2 = 65$ or $x^2 + y^2 - 10x - 6y - 31 = 0$	M1 M1 A1 (3) B1 M1 A1 M1 A1 (5)
Alt for (b)	Uses $(x-a)^2 + (y-b)^2 = r^2$ or $x^2 + y^2 + 2gx + 2fy + c = 0$ and substitutes (-3, 2), (9, 10) and (13, 4) then eliminates one unknown Eliminates second unknown Obtains $g = -5, f = -3, c = -31$ or $a = 5, b = 3, r^2 = 65$	M1 M1 A1, A1, B1cao (5) [8]
Notes	(a) M1 -considers gradients of PQ and QR -must be y difference / x difference (or considers three lengths as in alternative method) M1 Substitutes gradients into product = -1 (or lengths into Pythagoras' Theorem the correct way round) A1 Obtains $a = 13$ with no errors by solution or verification. Verification can score 3/3. (b) Geometrical method: B1 for coordinates of centre – can be implied by use in part (b) M1 for attempt to find r^2, d^2, r or d (allow one slip in a bracket). A1 cao. These two marks may be gained implicitly from circle equation M1 for $(x \pm 5)^2 + (y \pm 3)^2 = k^2$ or $(x \pm 3)^2 + (y \pm 5)^2 = k^2$ ft their (5,3) Allow k^2 non numerical. A1 cao for whole equation and rhs must be 65 or $(\sqrt{65})^2$, (similarly B1 must be 65 or $(\sqrt{65})^2$, in alternative method for (b))	

Question Number	Scheme	Marks
Further alternatives	<p>(i) A number of methods find gradient of $PQ = 2/3$ then give perpendicular gradient is $-3/2$ This is M1 They then proceed using equations of lines through point Q or by using gradient QR to obtain equation such as $\frac{4-10}{a-9} = -\frac{3}{2}$ M1 (may still have x in this equation rather than a and there may be a small slip) They then complete to give $(a) = 13$ A1</p> <p>(ii) A long involved method has been seen finding the coordinates of the centre of the circle first. This can be done by a variety of methods Giving centre as $(c, 3)$ and using an equation such as $(c-9)^2 + 7^2 = (c+3)^2 + 1^2$ (equal radii) or $\frac{3-6}{c-3} = -\frac{3}{2}$ M1 (perpendicular from centre to chord bisects chord) Then using $c (= 5)$ to find a is M1 Finally $a = 13$ A1</p> <p>(iii) Vector Method: States PQ · QR = 0, with vectors stated $12\mathbf{i} + 8\mathbf{j}$ and $(9-a)\mathbf{i} + 6\mathbf{j}$ is M1 Evaluates scalar product so $108 - 12a + 48 = 0$ (M1) solves to give $a = 13$ (A1)</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p>

Question Number	Scheme	Marks
6	<p>(a) $f(2) = 16 + 40 + 2a + b$ or $f(-1) = 1 - 5 - a + b$</p> <p>Finds 2nd remainder and equates to 1st $\Rightarrow 16 + 40 + 2a + b = 1 - 5 - a + b$</p> <p>$a = -20$</p> <p>(b) $f(-3) = (-3)^4 + 5(-3)^3 - 3a + b = 0$</p> <p>$81 - 135 + 60 + b = 0$ gives $b = -6$</p>	<p>M1 A1</p> <p>M1 A1</p> <p>A1cso (5)</p> <p>M1 A1ft</p> <p>A1 cso</p> <p>(3)</p> <p>[8]</p>
Alternative for (a)	<p>(a) Uses long division, to get remainders as $b + 2a + 56$ or $b - a - 4$ or correct equivalent</p> <p>Uses second long division as far as remainder term, to get $b + 2a + 56 = b - a - 4$ or correct equivalent</p> <p>$a = -20$</p>	<p>M1 A1</p> <p>M1 A1</p> <p>A1cso (5)</p>
Alternative for (b)	<p>(b) Uses long division of $x^4 + 5x^3 - 20x + b$ by $(x + 3)$ to obtain $x^3 + 2x^2 - 6x + a + 18$ (with their value for a)</p> <p>Giving remainder $b + 6 = 0$ and so $b = -6$</p>	<p>M1 A1ft</p> <p>A1 cso</p> <p>(3)</p> <p>[8]</p>
Notes	<p>(a) M1 : Attempts $f(\pm 2)$ or $f(\pm 1)$</p> <p>A1 is for the answer shown (or simplified with terms collected) for one remainder</p> <p>M1: Attempts other remainder and puts one equal to the other</p> <p>A1: for correct equation in a (and b) then A1 for $a = -20$ cso</p> <p>(b) M1 : Puts $f(\pm 3) = 0$</p> <p>A1 is for $f(-3) = 0$, (where f is original function), with no sign or substitution errors (follow through on 'a' and could still be in terms of a)</p> <p>A1: $b = -6$ is cso.</p>	
Alternatives	<p>(a) M1: Uses long division of $x^4 + 5x^3 + ax + b$ by $(x \pm 2)$ or by $(x \pm 1)$ as far as three term quotient</p> <p>A1: Obtains at least one correct remainder</p> <p>M1: Obtains second remainder and puts two remainders (no x terms) equal</p> <p>A1: correct equation A1: correct answer $a = -20$ following correct work.</p> <p>(b) M1: complete long division as far as constant (ignore remainder)</p> <p>A1ft: needs correct answer for their a</p> <p>A1: correct answer</p>	
<p>Beware: It is possible to get correct answers with wrong working. If remainders are equated to 0 in part (a) both correct answers are obtained fortuitously. This could score M1A1M0A0A0M1A1A0</p>		

Question Number	Scheme	Marks
7	<p>(a) $\frac{1}{2}r^2\theta = \frac{1}{2} \times 6^2 \times 2.2 = 39.6 \text{ (cm}^2\text{)}$</p> <p>(b) $\left(\frac{2\pi - 2.2}{2}\right) \pi - 1.1 = 2.04 \text{ (rad)}$</p> <p>(c) $\Delta DAC = \frac{1}{2} \times 6 \times 4 \sin 2.04 \text{ } (\approx 10.7)$</p> <p>Total area = sector + 2 triangles = 61 $\text{(cm}^2\text{)}$</p>	<p>M1 A1 (2)</p> <p>M1 A1 (2)</p> <p>M1 A1ft</p> <p>M1 A1 (4)</p> <p>[8]</p>
(a)	<p>M1: Needs θ in radians for this formula. Could convert to degrees and use degrees formula.</p> <p>A1: Does not need units. Answer should be 39.6 exactly. Answer with no working is M1 A1. This M1A1 can only be awarded in part (a).</p>	
(b)	<p>M1: Needs full method to give angle in radians</p> <p>A1: Allow answers which round to 2.04 (Just writes 2.04 – no working is 2/2)</p>	
(c)	<p>M1: Use $\frac{1}{2} \times 6 \times 4 \sin A$ (if any other triangle formula e.g. $\frac{1}{2}b \times h$ is used the method must be complete for this mark) (No value needed for A, but should not be using 2.2)</p> <p>A1: fit the value obtained in part (b) – need not be evaluated- could be in degrees</p> <p>M1: Uses Total area = sector + 2 triangles or other complete method</p> <p>A1: Allow answers which round to 61. (Do not need units)</p> <p>Special case degrees: Could get M0A0, M0A0, M1A1M1A0</p> <p>Special case: Use $\Delta BDC - \Delta BAC$ Both areas needed for first M1 Total area = sector + area found is second M1</p> <p>NB Just finding lengths BD, DC, and angle BDC then assuming area BDC is a sector to find area BDC is 0/4</p>	

Question Number	Scheme	Marks
8		
(a)	$4(1 - \cos^2 x) + 9 \cos x - 6 = 0$ $4\cos^2 x - 9\cos x + 2 = 0$ (*)	M1 A1 (2)
(b)	$(4\cos x - 1)(\cos x - 2) = 0$ $\cos x = \dots, \frac{1}{4}$ $x = 75.5$ (α) $360 - \alpha, \quad 360 + \alpha$ or $720 - \alpha$ $284.5, 435.5, 644.5$	M1 A1 B1 M1, M1 A1 (6) [8]
(a)	M1: Uses $\sin^2 x = 1 - \cos^2 x$ (may omit bracket) not $\sin^2 x = \cos^2 x - 1$ A1: Obtains the printed answer without error – must have = 0	
(b)	M1: Solves the quadratic with usual conventions A1: Obtains $\frac{1}{4}$ accurately- ignore extra answer 2 but penalise e.g. -2. B1: allow answers which round to 75.5 M1: $360 - \alpha$ ft their value, M1: $360 + \alpha$ ft their value or $720 - \alpha$ ft A1: Three and only three correct exact answers in the range achieves the mark	
Special cases	In part (b) Error in solving quadratic $(4\cos x - 1)(\cos x + 2)$ Could yield, M1A0B1M1M1A1 losing one mark for the error Works in radians: Complete work in radians :Obtains 1.3 B0 . Then allow M1 M1 for $2\pi - \alpha$, $2\pi + \alpha$ or $4\pi - \alpha$ Then gets 5.0, 7.6, 11.3 A0 so 2/4 Mixed answer 1.3, $360 - 1.3$, $360 + 1.3$, $720 - 1.3$ still gets B0M1M1A0	

Question Number	Scheme	Marks
9	<p>(a) Initial step: Two of: $a = k + 4$, $ar = k$, $ar^2 = 2k - 15$ Or one of: $r = \frac{k}{k+4}$, $r = \frac{2k-15}{k}$, $r^2 = \frac{2k-15}{k+4}$, Or $k = \sqrt{(k+4)(2k-15)}$ or even $k^3 = (k+4)k(2k-15)$ $k^2 = (k+4)(2k-15)$, so $k^2 = 2k^2 + 8k - 15k - 60$ Proceed to $k^2 - 7k - 60 = 0$ (*)</p> <p>(b) $(k-12)(k+5) = 0$ $k = 12$ (*)</p> <p>(c) Common ratio: $\frac{k}{k+4}$ or $\frac{2k-15}{k} = \frac{12}{16} \left(= \frac{3}{4} \text{ or } 0.75 \right)$</p> <p>(d) $\frac{a}{1-r} = \frac{16}{\left(\frac{1}{4}\right)} = 64$</p>	<p>M1 M1, A1 A1 (4)</p> <p>M1 A1 (2)</p> <p>M1 A1 (2)</p> <p>M1 A1 (2) [10]</p>
(a)	<p>M1: The 'initial step', scoring the first M mark, may be implied by next line of proof M1: Eliminates a and r to give valid equation in k only. Can be awarded for equation involving fractions. A1: need some correct expansion and working and answer equivalent to required quadratic but with uncollected terms. Equations involving fractions do not get this mark. (No fractions, no brackets – could be a cubic equation) A1: as answer is printed this mark is for cso (Needs = 0) All four marks must be scored in part (a)</p>	
(b)	<p>M1: Attempt to solve quadratic A1: This is for correct factorisation or solution and $k = 12$. Ignore the extra solution ($k = -5$ or even $k = 5$), if seen. Substitute and verify is M1 A0 Marks must be scored in part (b)</p>	
(c)	<p>M1: Complete method to find r Could have answer in terms of k A1: 0.75 or any correct equivalent Both Marks must be scored in (c)</p>	
(d)	<p>M1: Tries to use $\frac{a}{1-r}$, (even with $r > 1$). Could have an answer still in terms of k. A1: This answer is 64 cao.</p>	

Question Number	Scheme	Marks
10	<p>(a) $2\pi rh + 2\pi r^2 = 800$</p> <p>$h = \frac{400 - \pi r^2}{\pi r}, \quad V = \pi r^2 \left(\frac{400 - \pi r^2}{\pi r} \right) = 400r - \pi r^3 \quad (*)$</p> <p>(b) $\frac{dV}{dr} = 400 - 3\pi r^2$</p> <p>$400 - 3\pi r^2 = 0 \quad r^2 = \dots, \quad r = \sqrt{\frac{400}{3\pi}} \quad (= 6.5 \text{ (2 s.f.)})$</p> <p>$V = 400r - \pi r^3 = 1737 = \frac{800}{3} \sqrt{\frac{400}{3\pi}} \text{ (cm}^3\text{)}$</p> <p>(accept awrt 1737 or exact answer)</p> <p>(c) $\frac{d^2V}{dr^2} = -6\pi r$, Negative, \therefore maximum</p> <p>(Parts (b) and (c) should be considered together when marking)</p>	<p>B1</p> <p>M1, M1 A1 (4)</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1 A1 (6)</p> <p>M1 A1 (2)</p> <p>[12]</p>
Other methods for part (c):	<p><u>Either:</u> M: Find <u>value</u> of $\frac{dV}{dr}$ on each side of "$r = \sqrt{\frac{400}{3\pi}}$" and consider sign.</p> <p>A: Indicate sign change of positive to negative for $\frac{dV}{dr}$, and conclude max.</p> <p><u>Or:</u> M: Find <u>value</u> of V on each side of "$r = \sqrt{\frac{400}{3\pi}}$" and compare with "1737".</p> <p>A: Indicate that both values are less than 1737 or 1737.25, and conclude max.</p>	
Notes	<p>(a) B1: For any correct form of this equation (may be unsimplified, may be implied by 1st M1)</p> <p>M1 : Making h the subject of their three or four term formula</p> <p>M1: Substituting expression for h into $\pi r^2 h$ (independent mark) Must now be expression in r only.</p> <p>A1: cso</p> <p>(b) M1: At least one power of r decreased by 1 A1: cao</p> <p>M1: Setting $\frac{dV}{dr} = 0$ and finding a value for correct power of r for candidate</p> <p>A1 : This mark may be credited if the value of V is correct. Otherwise answers should round to 6.5 (allow ± 6.5) or be exact answer</p> <p>M1: Substitute a positive value of r to give V A1: 1737 or 1737.25..... or exact answer</p>	

<p>(c)</p> <p>Alternative for (a)</p>	<p>M1: needs complete method e.g. attempts differentiation (power reduced) of their first derivative and considers its sign</p> <p>A1(first method) should be $-6\pi r$ (do not need to substitute r and can condone wrong r if found in (b))</p> <p>Need to conclude maximum or indicate by a tick that it is maximum.</p> <p>Throughout allow confused notation such as dy/dx for dV/dr</p> <p>$A = 2\pi r^2 + 2\pi rh$, $\frac{A}{2} \times r = \pi r^3 + \pi r^2 h$ is M1 Equate to $400r$ B1</p> <p>Then $V = 400r - \pi r^3$ is M1 A1</p>
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June 2009
6664 Core Mathematics C2
Mark Scheme

Question Number	Scheme	Marks
Q1	$\int \left(2x + 3x^{\frac{1}{2}} \right) dx = \frac{2x^2}{2} + \frac{3x^{\frac{3}{2}}}{\frac{3}{2}}$ $\int_1^4 \left(2x + 3x^{\frac{1}{2}} \right) dx = \left[x^2 + 2x^{\frac{3}{2}} \right]_1^4 = (16 + 2 \times 8) - (1 + 2)$ $= 29 \quad (29 + C \text{ scores A0})$	<p>M1 A1A1</p> <p>M1</p> <p>A1 (5) [5]</p>
	<p>1st M1 for attempt to integrate $x \rightarrow kx^2$ or $x^{\frac{1}{2}} \rightarrow kx^{\frac{3}{2}}$.</p> <p>1st A1 for $\frac{2x^2}{2}$ or a simplified version.</p> <p>2nd A1 for $\frac{3x^{\frac{3}{2}}}{(\frac{3}{2})}$ or $\frac{3x\sqrt{x}}{(\frac{3}{2})}$ or a simplified version.</p> <p>Ignore + C, if seen, but two correct terms and an <u>extra non-constant</u> term scores M1A1A0.</p> <p>2nd M1 for correct use of correct limits ('top' – 'bottom'). Must be used in a 'changed function', not just the original. (The changed function may have been found by differentiation).</p> <p>Ignore 'poor notation' (e.g. missing integral signs) if the intention is clear.</p> <p><u>No working:</u> The answer 29 with no working scores M0A0A0M1A0 (1 mark).</p>	

Question Number	Scheme	Marks
Q2 (a)	$(7 \times \dots \times x)$ or $(21 \times \dots \times x^2)$ The 7 or 21 can be in 'unsimplified' form. $(2 + kx)^7 = 2^7 + 2^6 \times 7 \times kx + 2^5 \times \binom{7}{2} k^2 x^2$ $= 128; + 448kx, + 672k^2 x^2$ [or $672(kx)^2$] (If $672kx^2$ follows $672(kx)^2$, isw and allow A1)	M1 B1; A1, A1 (4)
(b)	$6 \times 448k = 672k^2$ $k = 4$ (Ignore $k = 0$, if seen)	M1 A1 (2) [6]
(a)	<p>The terms can be 'listed' rather than added. Ignore any extra terms.</p> <p>M1 for <u>either</u> the x term <u>or</u> the x^2 term. Requires <u>correct</u> binomial coefficient in any form <u>with the correct power of x</u>, but the other part of the coefficient (perhaps including powers of 2 and/or k) may be wrong or missing.</p> <p><u>Allow</u> binomial coefficients such as $\binom{7}{1}, \binom{7}{1}, \binom{7}{2}, {}^7C_1, {}^7C_2$.</p> <p>However, $448 + kx$ or similar is M0.</p> <p>B1, A1, A1 for the <u>simplified</u> versions seen above.</p> <p><u>Alternative:</u></p> <p>Note that a factor 2^7 can be taken out first: $2^7 \left(1 + \frac{kx}{2}\right)^7$, but the mark scheme still applies.</p> <p><u>Ignoring subsequent working (isw):</u></p> <p>Isw if necessary after correct working:</p> <p>e.g. $128 + 448kx + 672k^2 x^2$ M1 B1 A1 A1 $= 4 + 14kx + 21k^2 x^2$ isw</p> <p>(Full marks are still available in part (b)).</p>	
(b)	<p>M1 for equating their coefficient of x^2 to 6 times that of $x \dots$ to get an equation in k, \dots <u>or</u> equating their coefficient of x to 6 times that of x^2, to get an equation in k.</p> <p>Allow this M mark even if the equation is trivial, providing their coefficients from part (a) have been used, e.g. $6 \times 448k = 672k$, but beware $k = 4$ following from this, which is A0.</p> <p><u>An equation in k alone</u> is required for this M mark, so...</p> <p>e.g. $6 \times 448kx = 672k^2 x^2 \Rightarrow k = 4$ or similar is M0 A0 (equation in coefficients only is never seen), but ...</p> <p>e.g. $6 \times 448kx = 672k^2 x^2 \Rightarrow 6 \times 448k = 672k^2 \Rightarrow k = 4$ will get M1 A1 (as coefficients rather than terms have now been considered).</p> <p>The mistake $2 \left(1 + \frac{kx}{2}\right)^7$ would give a maximum of 3 marks: M1B0A0A0, M1A1</p>	

Question Number	Scheme	Marks
Q3 (a)	$f(k) = -8$	B1 (1)
(b)	$f(2) = 4 \Rightarrow 4 = (6-2)(2-k) - 8$	M1
	So $k = -1$	A1 (2)
(c)	$f(x) = 3x^2 - (2+3k)x + (2k-8) = 3x^2 + x - 10$	M1
	$= (3x-5)(x+2)$	M1A1 (3)
		[6]
(b)	<p>M1 for substituting $x = 2$ (<u>not</u> $x = -2$) and equating to 4 to form an equation in k. If the expression is expanded in this part, condone 'slips' for this M mark. Treat the omission of the -8 here as a 'slip' and allow the M mark.</p> <p><u>Beware:</u> Substituting $x = -2$ and equating to 0 (M0 A0) also gives $k = -1$.</p> <p><u>Alternative;</u> M1 for dividing by $(x-2)$, to get $3x + (\text{function of } k)$, with remainder as a function of k, and equating the remainder to 4. [Should be $3x + (4-3k)$, remainder $-4k$].</p> <p><u>No working:</u> $k = -1$ with no working scores M0 A0.</p>	
(c)	<p>1st M1 for multiplying out <u>and</u> substituting their (constant) value of k (in either order). The multiplying-out may occur earlier. Condone, for example, sign slips, but if the 4 (from part (b)) is included in the $f(x)$ expression, this is M0. The 2nd M1 is still available.</p> <p>2nd M1 for an attempt to factorise their three term quadratic (3TQ).</p> <p>A1 The correct answer, as a <u>product of factors</u>, is required. Allow $3\left(x - \frac{5}{3}\right)(x+2)$</p> <p>Ignore following work (such as a solution to a quadratic equation). If the 'equation' is solved but factors are never seen, the 2nd M is not scored.</p>	

Question Number	Scheme	Marks
Q4 (a)	$x = 2$ gives 2.236 (allow AWR) Accept $\sqrt{5}$ $x = 2.5$ gives 2.580 (allow AWR) Accept 2.58	B1 B1 (2)
(b)	$\left(\frac{1}{2} \times \frac{1}{2}\right), [(1.414 + 3) + 2(1.554 + 1.732 + 1.957 + 2.236 + 2.580)]$ $= 6.133$ (AWRT 6.13, even following minor slips)	B1, [M1A1ft] A1 (4)
(c)	<p>Overestimate</p> <p>'Since the trapezia lie <u>above the curve</u>', or an equivalent explanation, or sketch of (one or more) trapezia above the curve on a diagram (or on the given diagram, in which case there should be reference to this). (Note that there must be some reference to a trapezium or trapezia in the explanation or diagram).</p>	B1 dB1 (2) [8]
(b)	<p>B1 for $\frac{1}{2} \times \frac{1}{2}$ or equivalent.</p> <p>For the M mark, the first bracket must contain the 'first and last' values, and the second bracket (which must be multiplied by 2) must have no additional values. If the only mistake is to <u>omit</u> one of the values from the second bracket, this can be considered as a slip and the M mark can be allowed.</p> <p>Bracketing mistake: i.e. $\left(\frac{1}{2} \times \frac{1}{2}\right)(1.414 + 3) + 2(1.554 + 1.732 + 1.957 + 2.236 + 2.580)$</p> <p>scores B1 M1 A0 A0 <u>unless</u> the final answer implies that the calculation has been done correctly (then full marks can be given).</p> <p><u>Alternative:</u> Separate trapezia may be used, and this can be marked equivalently.</p> $\left[\frac{1}{4}(1.414 + 1.554) + \frac{1}{4}(1.554 + 1.732) + \dots + \frac{1}{4}(2.580 + 3) \right]$ <p>1st A1ft for correct expression, ft their 2.236 and their 2.580</p>	
(c)	<p>1st B1 for 'overestimate', ignoring earlier mistakes and ignoring any reasons given. 2nd B1 is dependent upon the 1st B1 (overestimate).</p>	

Question Number	Scheme	Marks
Q5 (a)	$324r^3 = 96 \quad \text{or} \quad r^3 = \frac{96}{324} \quad \text{or} \quad r^3 = \frac{8}{27}$ $r = \frac{2}{3} \quad (*)$	M1 A1cso (2)
(b)	$a\left(\frac{2}{3}\right)^2 = 324 \quad \text{or} \quad a\left(\frac{2}{3}\right)^5 = 96 \quad a = \dots, \quad 729$	M1, A1 (2)
(c)	$S_{15} = \frac{729\left(1 - \left[\frac{2}{3}\right]^{15}\right)}{1 - \frac{2}{3}}, = 2182.00\dots \quad (\text{AWRT } 2180)$	M1A1ft, (3)
(d)	$S_{\infty} = \frac{729}{1 - \frac{2}{3}}, = 2187$	M1, A1 (2)
[9]		
(a)	<p>M1 for forming an equation for r^3 based on 96 and 324 (e.g. $96r^3 = 324$ scores M1). The equation must involve multiplication/division rather than addition/subtraction.</p> <p>A1 Do not penalise solutions with working in decimals, providing these are correctly rounded or truncated to at least 2dp <u>and</u> the final answer $2/3$ is seen.</p> <p><u>Alternative:</u> (verification)</p> <p>M1 Using $r^3 = \frac{8}{27}$ and multiplying 324 by this (or multiplying by $r = \frac{2}{3}$ three times).</p> <p>A1 Obtaining 96 (cso). (A conclusion is not required).</p> <p>$324 \times \left(\frac{2}{3}\right)^3 = 96$ (no real evidence of calculation) is not quite enough and scores M1 A0.</p>	
(b)	<p>M1 for the use of a correct formula or for 'working back' by dividing by $\frac{2}{3}$ (or by their r) twice from 324 (or 5 times from 96).</p> <p>Exceptionally, allow M1 also for using $ar^3 = 324$ or $ar^6 = 96$ instead of $ar^2 = 324$ or $ar^5 = 96$, or for dividing by r three times from 324 (or 6 times from 96)... but no other exceptions are allowed.</p>	
(c)	<p>M1 for use of sum to 15 terms formula with values of a and r. If the wrong power is used, e.g. 14, the M mark is scored only if the correct sum formula is stated.</p> <p>1st A1ft for a correct expression or correct ft their a with $r = \frac{2}{3}$.</p> <p>2nd A1 for awrt 2180, even following 'minor inaccuracies'.</p> <p>Condone missing brackets round the $\frac{2}{3}$ for the marks in part (c).</p> <p><u>Alternative:</u></p>	
(d)	<p>M1 for adding 15 terms and 1st A1ft for adding the 15 terms that ft from their a and $r = \frac{2}{3}$.</p> <p>M1 for use of correct sum to infinity formula with their a. For this mark, if a value of r different from the given value is being used, M1 can still be allowed providing $r < 1$.</p>	

Question Number	Scheme	Marks
Q6 (a)	$(x-3)^2 - 9 + (y+2)^2 - 4 = 12$ Centre is (3, -2)	M1 A1, A1
(b)	$(x-3)^2 + (y+2)^2 = 12 + "9" + "4"$ $r = \sqrt{12 + "9" + "4"} = 5$ (or $\sqrt{25}$) $PQ = \sqrt{(7-(-1))^2 + (-5-1)^2}$ or $\sqrt{8^2 + 6^2}$ $= 10 = 2 \times \text{radius}, \therefore \text{diam.}$ (N.B. For A1, need a comment or conclusion) [ALT: midpt. of PQ $\left(\frac{7+(-1)}{2}, \frac{1+(-5)}{2}\right)$: M1, $= (3, -2) = \text{centre: A1}$] [ALT: eqn. of PQ $3x + 4y - 1 = 0$: M1, verify (3, -2) lies on this: A1] [ALT: find two grads, e.g. PQ and P to centre: M1, equal \therefore diameter: A1] [ALT: show that point $S(-1, -5)$ or (7, 1) lies on circle: M1 because $\angle PSQ = 90^\circ$, semicircle \therefore diameter: A1]	M1 A1 (5) M1 A1 (2)
(c)	R must lie on the circle (angle in a semicircle theorem)... often <u>implied</u> by <u>a diagram with R on the circle</u> or by subsequent working) $x = 0 \Rightarrow y^2 + 4y - 12 = 0$ $(y - 2)(y + 6) = 0$ $y = \dots$ (M is dependent on previous M) $y = -6$ or 2 (Ignore $y = -6$ if seen, and 'coordinates' are not required))	B1 M1 dM1 A1 (4) [11]
(a)	1 st M1 for attempt to complete square. Allow $(x \pm 3)^2 \pm k$, or $(y \pm 2)^2 \pm k$, $k \neq 0$. 1 st A1 x -coordinate 3, 2 nd A1 y -coordinate -2 2 nd M1 for a full method leading to $r = \dots$, with their 9 and their 4, 3 rd A1 5 or $\sqrt{25}$ The 1 st M can be <u>implied</u> by $(\pm 3, \pm 2)$ but a full method must be seen for the 2 nd M. Where the 'diameter' in part (b) has <u>clearly</u> been used to answer part (a), no marks in (a), but in this case the M1 (<u>not</u> the A1) for part (b) can be given for work seen in (a). <u>Alternative</u> 1 st M1 for comparing with $x^2 + y^2 + 2gx + 2fy + c = 0$ to write down centre $(-g, -f)$ directly. Condone sign errors for this M mark. 2 nd M1 for using $r = \sqrt{g^2 + f^2 - c}$. Condone sign errors for this M mark.	
(c)	1 st M1 for setting $x = 0$ and getting a 3TQ in y by using eqn. of circle. 2 nd M1 (dep.) for attempt to solve a 3TQ leading to <u>at least one</u> solution for y . <u>Alternative 1:</u> (Requires the B mark as in the main scheme) 1 st M for using (3, 4, 5) triangle with vertices (3, -2), (0, -2), (0, y) to get a linear or quadratic equation in y (e.g. $3^2 + (y+2)^2 = 25$). 2 nd M (dep.) as in main scheme, but may be scored by simply solving a linear equation. <u>Alternative 2:</u> (Not requiring realisation that R is on the circle) B1 for attempt at $m_{PR} \times m_{QR} = -1$, (<u>NOT</u> m_{PQ}) or for attempt at Pythag. in triangle PQR . 1 st M1 for setting $x = 0$, i.e. (0, y), and proceeding to get a 3TQ in y . Then main scheme. <u>Alternative 2 by 'verification':</u> B1 for attempt at $m_{PR} \times m_{QR} = -1$, (<u>NOT</u> m_{PQ}) or for attempt at Pythag. in triangle PQR . 1 st M1 for trying (0, 2). 2 nd M1 (dep.) for performing all required calculations. A1 for fully correct working and conclusion.	

Question Number	Scheme	Marks
Q7 (i)	$\tan \theta = -1 \Rightarrow \theta = -45, 135$ $\sin \theta = \frac{2}{5} \Rightarrow \theta = 23.6, 156.4$ (AWRT: 24, 156)	B1, B1ft B1, B1ft (4)
(ii)	$4 \sin x = \frac{3 \sin x}{\cos x}$ $4 \sin x \cos x = 3 \sin x \Rightarrow \sin x(4 \cos x - 3) = 0$ Other possibilities (after squaring): $\sin^2 x(16 \sin^2 x - 7) = 0$, $(16 \cos^2 x - 9)(\cos^2 x - 1) = 0$ $x = 0, 180$ <u>seen</u> $x = 41.4, 318.6$ (AWRT: 41, 319)	M1 M1 B1, B1 B1, B1ft (6)
		[10]
(i)	1 st B1 for -45 seen (α , where $ \alpha < 90$) 2 nd B1 for 135 seen, <u>or ft</u> $(180 + \alpha)$ if α is negative, or $(\alpha - 180)$ if α is positive. If $\tan \theta = k$ is obtained from <u>wrong working</u> , 2 nd B1ft is still available. 3 rd B1 for awrt 24 (β , where $ \beta < 90$) 4 th B1 for awrt 156, <u>or ft</u> $(180 - \beta)$ if β is positive, or $-(180 + \beta)$ if β is negative. If $\sin \theta = k$ is obtained from <u>wrong working</u> , 4 th B1ft is still available.	
(ii)	1 st M1 for use of $\tan x = \frac{\sin x}{\cos x}$. Condone $\frac{3 \sin x}{3 \cos x}$. 2 nd M1 for correct work leading to 2 factors (may be implied). 1 st B1 for 0, 2 nd B1 for 180. 3 rd B1 for awrt 41 (γ , where $ \gamma < 180$) 4 th B1 for awrt 319, <u>or ft</u> $(360 - \gamma)$. If $\cos \theta = k$ is obtained from <u>wrong working</u> , 4 th B1ft is still available. N.B. Losing $\sin x = 0$ usually gives a maximum of 3 marks M1M0B0B0B1B1 <u>Alternative:</u> (squaring both sides) 1 st M1 for squaring both sides and using a 'quadratic' identity. e.g. $16 \sin^2 \theta = 9(\sec^2 \theta - 1)$ 2 nd M1 for reaching a factorised form. e.g. $(16 \cos^2 \theta - 9)(\cos^2 \theta - 1) = 0$ Then marks are equivalent to the main scheme. Extra solutions, if not rejected, are penalised as in the main scheme. <u>For both parts of the question:</u> <u>Extra solutions outside required range:</u> Ignore <u>Extra solutions inside required range:</u> For each <u>pair</u> of B marks, the 2 nd B mark is lost if there are two correct values and one or more extra solution(s), e.g. $\tan \theta = -1 \Rightarrow \theta = 45, -45, 135$ is B1 B0 <u>Answers in radians:</u> Loses a maximum of 2 B marks in the whole question (to be deducted at the first and second occurrence).	

Question Number	Scheme	Marks
Q8 (a)	$\log_2 y = -3 \Rightarrow y = 2^{-3}$ $y = \frac{1}{8} \text{ or } 0.125$	M1 A1 (2)
(b)	$32 = 2^5 \text{ or } 16 = 2^4 \text{ or } 512 = 2^9$ [or $\log_2 32 = 5 \log_2 2$ or $\log_2 16 = 4 \log_2 2$ or $\log_2 512 = 9 \log_2 2$] [or $\log_2 32 = \frac{\log_{10} 32}{\log_{10} 2}$ or $\log_2 16 = \frac{\log_{10} 16}{\log_{10} 2}$ or $\log_2 512 = \frac{\log_{10} 512}{\log_{10} 2}$] $\log_2 32 + \log_2 16 = 9$ $(\log x)^2 = \dots$ or $(\log x)(\log x) = \dots$ (May not be seen explicitly, so M1 may be implied by later work, and the base may be 10 rather than 2) $\log_2 x = 3 \Rightarrow x = 2^3 = 8$ $\log_2 x = -3 \Rightarrow x = 2^{-3} = \frac{1}{8}$	M1 A1 M1 A1 A1ft (5) [7]
(a)	M1 for <u>getting out of logs</u> correctly. If done by change of base, $\log_{10} y = -0.903\dots$ is insufficient for the M1, but $y = 10^{-0.903}$ scores M1. A1 for the <u>exact</u> answer, e.g. $\log_{10} y = -0.903 \Rightarrow y = 0.12502\dots$ scores M1 (implied) A0. <u>Correct answer</u> with no working scores both marks. <u>Allow</u> both marks for implicit statements such as $\log_2 0.125 = -3$.	
(b)	1 st M1 for expressing 32 or 16 or 512 as a power of 2, or for a change of base enabling evaluation of $\log_2 32$, $\log_2 16$ or $\log_2 512$ by calculator. (Can be implied by 5, 4 or 9 respectively). 1 st A1 for 9 (exact). 2 nd M1 for getting $(\log_2 x)^2 = \text{constant}$. The constant can be a log or a sum of logs. If written as $\log_2 x^2$ instead of $(\log_2 x)^2$, allow the M mark <u>only</u> if subsequent work implies correct interpretation. 2 nd A1 for 8 (exact). Change of base methods leading to a non-exact answer score A0. 3 rd A1ft for an answer of $\frac{1}{\text{their } 8}$. An ft answer may be non-exact. <u>Possible mistakes:</u> $\log_2(2^9) = \log_2(x^2) \Rightarrow x^2 = 2^9 \Rightarrow x = \dots$ scores M1A1(implied by 9)M0A0A0 $\log_2 512 = \log_2 x \times \log_2 x \Rightarrow x^2 = 512 \Rightarrow x = \dots$ scores M0A0(9 never seen)M1A0A0 $\log_2 48 = (\log_2 x)^2 \Rightarrow (\log_2 x)^2 = 5.585 \Rightarrow x = 5.145, x = 0.194$ scores M0A0M1A0A1ft <u>No working</u> (or 'trial and improvement'): $x = 8$ scores M0 A0 M1 A1 A0	

Question Number	Scheme	Marks
Q9 (a)	<p>(Arc length $\Rightarrow r\theta = r \times 1 = r$. Can be awarded by implication from later work, e.g. $3rh$ or $(2rh + rh)$ in the S formula. (Requires use of $\theta = 1$).</p> <p>(Sector area $\Rightarrow \frac{1}{2}r^2\theta = \frac{1}{2}r^2 \times 1 = \frac{r^2}{2}$. Can be awarded by implication from later work, e.g. the correct volume formula. (Requires use of $\theta = 1$).</p> <p>Surface area = 2 sectors + 2 rectangles + curved face $(= r^2 + 3rh)$ (See notes below for what is allowed here)</p> <p>Volume = $300 = \frac{1}{2}r^2h$</p> <p>Sub for h: $S = r^2 + 3 \times \frac{600}{r} = r^2 + \frac{1800}{r}$ (*)</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>B1</p> <p>A1cso (5)</p>
(b)	<p>$\frac{dS}{dr} = 2r - \frac{1800}{r^2}$ or $2r - 1800r^{-2}$ or $2r + -1800r^{-2}$</p> <p>$\frac{dS}{dr} = 0 \Rightarrow r^3 = \dots$, $r = \sqrt[3]{900}$, or AWRT 9.7 (NOT -9.7 or ± 9.7)</p>	<p>M1A1</p> <p>M1, A1 (4)</p>
(c)	<p>$\frac{d^2S}{dr^2} = \dots$ and consider sign, $\frac{d^2S}{dr^2} = 2 + \frac{3600}{r^3} > 0$ so point is a minimum</p>	<p>M1, A1ft (2)</p>
(d)	<p>$S_{\min} = (9.65\dots)^2 + \frac{1800}{9.65\dots}$</p> <p>(Using their value of r, however found, in the <u>given</u> S formula) $= 279.65\dots$ (AWRT: 280) (Dependent on full marks in part (b))</p>	<p>M1</p> <p>A1 (2)</p> <p>[13]</p>
(a)	<p>M1 for attempting a formula (with terms added) for surface area. May be incomplete or wrong and may have extra term(s), but must have an r^2 (or $r^2\theta$) term and an rh (or $rh\theta$) term.</p>	
(b)	<p><u>In parts (b), (c) and (d), ignore labelling of parts</u></p> <p>1st M1 for attempt at differentiation (one term is sufficient) $r^n \rightarrow kr^{n-1}$</p> <p>2nd M1 for setting their derivative (a 'changed function') = 0 and solving as far as $r^3 = \dots$ (depending upon their 'changed function', this could be $r = \dots$ or $r^2 = \dots$, etc., but the algebra <u>must deal with a negative power</u> of r and should be sound apart from possible <u>sign</u> errors, so that $r^n = \dots$ is consistent with their derivative).</p>	
(c)	<p>M1 for attempting second derivative (one term is sufficient) $r^n \rightarrow kr^{n-1}$, <u>and considering its sign</u>. Substitution of a value of r is not required. (<u>Equating it to zero is M0</u>).</p> <p>A1ft for a correct second derivative (or correct ft from their first derivative) <u>and</u> a valid reason (e.g. > 0), <u>and</u> conclusion. The actual <u>value</u> of the second derivative, if found, can be ignored. To score this mark as ft, their second derivative must indicate a minimum.</p> <p><u>Alternative:</u></p> <p>M1: Find <u>value</u> of $\frac{dS}{dr}$ on each side of their value of r and consider sign.</p> <p>A1ft: Indicate sign change of negative to positive for $\frac{dS}{dr}$, and conclude minimum.</p> <p><u>Alternative:</u></p> <p>M1: Find <u>value</u> of S on each side of their value of r and compare with their 279.65.</p> <p>A1ft: Indicate that both values are more than 279.65, and conclude minimum.</p>	

Mark Scheme (Results) January 2010

GCE

Core Mathematics C2 (6664)

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January 2010
Core Mathematics C2 6664
Mark Scheme

Question Number	Scheme	Marks
Q1	$\left[(3-x)^6 = 3^6 + 3^5 \times 6 \times (-x) + 3^4 \times \binom{6}{2} \times (-x)^2 \right.$ $= 729, -1458x, +1215x^2$	M1 B1, A1, A1 [4]
Notes	<p>M1 for <u>either</u> the x term <u>or</u> the x^2 term. Requires <u>correct</u> binomial coefficient in any form with the correct power of x – condone lack of negative sign and wrong power of 3. This mark may be given if no working is shown, but one of the terms including x is correct. Allow $\frac{6}{1}$, or $\frac{6}{2}$ (must have a power of 3, even if only power 1)</p> <p>First term must be 729 for B1, (writing just 3^6 is B0) can isw if numbers added to this constant later. Can allow 729(1...</p> <p>Term must be simplified to $-1458x$ for A1cao. The x is required for this mark.</p> <p>Final A1 is c.a.o and needs to be $+1215x^2$ (can follow omission of negative sign in working)</p> <p>Descending powers of x would be $x^6 + 3 \times 6 \times (-x)^5 + 3^2 \times \binom{6}{4} \times (-x)^4 + ..$</p> <p>i.e. $x^6 - 18x^5 + 135x^4 + ..$ This is M1B1A0A0 if completely “correct” or M1 B0A0A0 for <u>correct</u> binomial coefficient in any form with the correct power of x as before</p>	
Alternative	<p>NB Alternative method: $(3-x)^6 = 3^6(1 + 6 \times (-\frac{x}{3}) + \binom{6}{2} \times (-\frac{x}{3})^2 + ..)$ is M1B0A0A0</p> <p>– answers must be simplified to 729, $-1458x$, $+1215x^2$ for full marks (awarded as before)</p> <p>The mistake $(3-x)^6 = 3(1 - \frac{x}{3})^6 = 3(1 + 6 \times (-\frac{x}{3}) + \binom{6}{2} \times (-\frac{x}{3})^2 + ..)$ may also be awarded M1B0A0A0</p> <p>Another mistake $3^6(1 - 6x + 15x^2 ...) = 729...$ would be M1B1A0A0</p>	

Question Number	Scheme	Marks
Q2 (a)	$5 \sin x = 1 + 2(1 - \sin^2 x)$ $2 \sin^2 x + 5 \sin x - 3 = 0 \quad (*)$	M1 A1cso (2)
(b)	$(2s - 1)(s + 3) = 0 \text{ giving } s =$ $[\sin x = -3 \text{ has no solution}] \text{ so } \sin x = \frac{1}{2}$ $\therefore x = 30, 150$	M1 A1 B1, B1ft (4) [6]
(a)	<p>M1 for a correct method to change $\cos^2 x$ into $\sin^2 x$ (must use $\cos^2 x = 1 - \sin^2 x$)</p> <p>A1 need 3 term quadratic printed in any order with =0 included</p>	
(b)	<p>M1 for attempt to solve given quadratic (usual rules for solving quadratics) (can use any variable here, s, y, x, or $\sin x$)</p> <p>A1 requires no incorrect work seen and is for $\sin x = \frac{1}{2}$ or $x = \sin^{-1} \frac{1}{2}$</p> <p>$y = \frac{1}{2}$ is A0 (unless followed by $x = 30$)</p> <p>B1 for 30 (α) not depend on method</p> <p>2nd B1 for 180 - α provided in required range (otherwise 540 - α)</p> <p><u>Extra solutions outside required range:</u> Ignore</p> <p><u>Extra solutions inside required range:</u> Lose final B1</p> <p><u>Answers in radians:</u> Lose final B1</p> <p>S.C. Merely writes down two correct answers is M0A0B1B1</p> <p>Or $\sin x = \frac{1}{2} \therefore x = 30, 150$ is M1A1B1B1</p> <p>Just gives one answer : 30 only is M0A0B1B0 or 150 only is M0A0B0B1</p> <p>NB Common error is to factorise wrongly giving $(2 \sin x + 1)(\sin x - 3) = 0$</p> <p>$[\sin x = 3 \text{ gives no solution}] \sin x = -\frac{1}{2} \Rightarrow x = 210, 330$</p> <p>This earns M1 A0 B0 B1ft</p> <p>Another common error is to factorise correctly $(2 \sin x - 1)(\sin x + 3) = 0$ and follow this with $\sin x = \frac{1}{2}$, $\sin x = 3$ then $x = 30^\circ, 150^\circ$</p> <p>This would be M1 A0 B1 B1</p>	

Question Number	Scheme	Marks
Q3 (a)	$f\left(\frac{1}{2}\right) = 2 \times \frac{1}{8} + a \times \frac{1}{4} + b \times \frac{1}{2} - 6$ $f\left(\frac{1}{2}\right) = -5 \Rightarrow \frac{1}{4}a + \frac{1}{2}b = \frac{3}{4} \text{ or } a + 2b = 3$ $f(-2) = -16 + 4a - 2b - 6$ $f(-2) = 0 \Rightarrow 4a - 2b = 22$ <p>Eliminating one variable from 2 linear simultaneous equations in a and b</p> $a = 5 \text{ and } b = -1$	M1 A1 M1 A1 M1 A1 (6)
(b)	$2x^3 + 5x^2 - x - 6 = (x+2)(2x^2 + x - 3)$ $= (x+2)(2x+3)(x-1)$ <p>NB $(x+2)\left(x+\frac{3}{2}\right)(2x-2)$ is A0 But $2(x+2)\left(x+\frac{3}{2}\right)(x-1)$ is A1</p>	M1 M1A1 (3) [9]
(a)	<p>1st M1 for attempting $f(\pm\frac{1}{2})$ Treat the omission of the -5 here as a slip and allow the M mark.</p> <p>1st A1 for first correct equation in a and b simplified to three non zero terms (needs -5 used)</p> <p>s.c. If it is not simplified to three terms but is correct and is then used correctly with second equation to give correct answers- this mark can be awarded later.</p> <p>2nd M1 for attempting $f(\mp 2)$</p> <p>2nd A1 for the second correct equation in a and b. simplified to three terms (needs 0 used) s.c. If it is not simplified to three terms but is correct and is then used correctly with first equation to give correct answers - this mark can be awarded later.</p> <p>3rd M1 for an attempt to eliminate one variable from 2 linear simultaneous equations in a and b</p> <p>3rd A1 for both $a = 5$ and $b = -1$ (Correct answers here imply previous two A marks)</p>	
(b)	<p>1st M1 for attempt to divide by $(x+2)$ leading to a 3TQ beginning with correct term usually $2x^2$</p> <p>2nd M1 for attempt to factorize their quadratic provided no remainder</p> <p>A1 is cao and needs all three factors</p> <p>Ignore following work (such as a solution to a quadratic equation).</p>	
(a)	<p><u>Alternative;</u></p> <p>M1 for dividing by $(2x-1)$, to get $x^2 + (\frac{a+1}{2})x + \text{constant}$ with remainder as a function of a and b, and A1 as before for equations stated in scheme.</p> <p>M1 for dividing by $(x+2)$, to get $2x^2 + (a-4)x...$ (No need to see remainder as it is zero and comparison of coefficients may be used) with A1 as before</p>	
(b)	<p><u>Alternative;</u></p> <p>M1 for finding second factor correctly by factor theorem, usually $(x-1)$</p> <p>M1 for using two known factors to find third factor, usually $(2x \pm 3)$</p> <p>Then A1 for correct factorisation written as product $(x+2)(2x+3)(x-1)$</p>	

Question Number	Scheme	Marks
Q4		
(a)	<div> <p>Either $\frac{\sin(\hat{ACB})}{5} = \frac{\sin 0.6}{4}$ $\therefore \hat{ACB} = \arcsin(0.7058\dots)$ $= [0.7835\dots \text{ or } 2.358]$ Use angles of triangle $\hat{ABC} = \pi - 0.6 - \hat{ACB}$ (But as AC is the longest side so) $\hat{ABC} = 1.76 \text{ (*) (3sf) [Allow } 100.7^\circ \rightarrow 1.76]$ In degrees $0.6 = 34.377^\circ$, $\hat{ACB} = 44.9^\circ$</p> </div> <div> <p>or $4^2 = b^2 + 5^2 - 2 \times b \times 5 \cos 0.6$ $\therefore b = \frac{10 \cos 0.6 \pm \sqrt{(100 \cos^2 0.6 - 36)}}{2}$ $= [6.96 \text{ or } 1.29]$ Use sine / cosine rule with value for b $\sin B = \frac{\sin 0.6}{4} \times b$ or $\cos B = \frac{25 + 16 - b^2}{40}$ (But as AC is the longest side so) $\hat{ABC} = 1.76 \text{ (*) (3sf)}$</p> </div>	M1 M1 M1, A1 (4)
(b)	<p>$[\hat{CBD} = \pi - 1.76 = 1.38]$ Sector area $= \frac{1}{2} \times 4^2 \times (\pi - 1.76) = [11.0 \sim 11.1]$ $\frac{1}{2} \times 4^2 \times 79.3$ is M0</p> <p>Area of $\triangle ABC = \frac{1}{2} \times 5 \times 4 \times \sin(1.76) = [9.8]$ or $\frac{1}{2} \times 5 \times 4 \times \sin 101$</p> <p>Required area = awrt 20.8 or 20.9 or 21.0 or gives 21 (2sf) after correct work.</p>	M1 M1 A1 (3) [7]
(a)	<p>1st M1 for correct use of sine rule to find \hat{ACB} or cosine rule to find b (M0 for ABC here or for use of $\sin x$ where x could be \hat{ABC}) 2nd M1 for a correct expression for angle \hat{ACB} (This mark may be implied by .7835 or by $\arcsin(.7058)$) and needs accuracy. In second method this M1 is for correct expression for b – may be implied by 6.96. [Note $10 \cos 0.6 \approx 8.3$] (do not need two answers) 3rd M1 for a correct method to get angle \hat{ABC} in method (i) or $\sin \hat{ABC}$ or $\cos \hat{ABC}$, in method (ii) (If $\sin B > 1$, can have M1A0) A1cso for correct work leading to 1.76 3sf. Do not need to see angle 0.1835 considered and rejected.</p>	
(b)	<p>1st M1 for a correct expression for sector area or a value in the range 11.0 – 11.1 2nd M1 for a correct expression for the area of the triangle or a value of 9.8 Ignore 0.31 (working in degrees) as subsequent work. A1 for answers which round to 20.8 or 20.9 or 21.0. No need to see units.</p>	
(a)	<p>Special case If answer 1.76 is assumed then usual mark is M0 M0 M0 A0. A Fully checked method may be worth M1 M1 M0 A0. A maximum of 2 marks. The mark is either 2 or 0. <u>Either</u> M1 for \hat{ACB} is found to be 0.7816 (angles of triangle) then M1 for checking $\frac{\sin(\hat{ACB})}{5} = \frac{\sin 0.6}{4}$ with conclusion giving numerical answers This gives a maximum mark of 2/4 OR M1 for b is found to be 6.97 (cosine rule) M1 for checking $\frac{\sin(\hat{ABC})}{b} = \frac{\sin 0.6}{4}$ with conclusion giving numerical answers This gives a maximum mark of 2/4 Candidates making this assumption need a complete method. They cannot earn M1M0. So the score will be 0 or 2 for part (a). Circular arguments earn 0/4.</p>	

Question Number	Scheme	Marks
Q5 (a)	$\log_x 64 = 2 \Rightarrow 64 = x^2$ $\text{So } x = 8$	M1 A1 (2)
(b)	$\log_2(11-6x) = \log_2(x-1)^2 + 3$ $\log_2 \left[\frac{11-6x}{(x-1)^2} \right] = 3$ $\frac{11-6x}{(x-1)^2} = 2^3$ $\{11-6x = 8(x^2 - 2x + 1)\} \text{ and so } 0 = 8x^2 - 10x - 3$ $0 = (4x+1)(2x-3) \Rightarrow x = \dots$ $x = \frac{3}{2}, \left[-\frac{1}{4} \right]$	M1 M1 M1 A1 dM1 A1 (6) [8]
(a)	M1 for getting out of logs A1 Do not need to see $x = -8$ appear and get rejected. Ignore $x = -8$ as extra solution. $x = 8$ with no working is M1 A1	
(b)	1 st M1 for using the $n\log x$ rule 2 nd M1 for using the $\log x - \log y$ rule or the $\log x + \log y$ rule as appropriate 3 rd M1 for using 2 to the power – need to see 2^3 or 8 (May see $3 = \log_2 8$ used) If all three M marks have been earned and logs are still present in equation do not give final M1. So solution stopping at $\log_2 \left[\frac{11-6x}{(x-1)^2} \right] = \log_2 8$ would earn M1M1M0 1 st A1 for a correct 3TQ 4 th dependent M1 for attempt to solve or factorize their 3TQ to obtain $x = \dots$ (mark depends on three previous M marks) 2 nd A1 for 1.5 (ignore -0.25) s.c 1.5 only – no working – is 0 marks	
(a)	<u>Alternatives</u> Change base : (i) $\frac{\log_2 64}{\log_2 x} = 2$, so $\log_2 x = 3$ and $x = 2^3$, is M1 or (ii) $\frac{\log_{10} 64}{\log_{10} x} = 2$, $\log x = \frac{1}{2} \log 64$ so $x = 64^{\frac{1}{2}}$ is M1 then $x = 8$ is A1 BUT $\log x = 0.903$ so $x = 8$ is M1A0 (loses accuracy mark) (iii) $\log_{64} x = \frac{1}{2}$ so $x = 64^{\frac{1}{2}}$ is M1 then $x = 8$ is A1	

Question Number	Scheme	Marks
Q6	<p>(a) $18000 \times (0.8)^3 = £9216 *$ [may see $\frac{4}{5}$ or 80% or equivalent].</p> <p>(b) $18000 \times (0.8)^n < 1000$ $n \log(0.8) < \log\left(\frac{1}{18}\right)$ $n > \frac{\log\left(\frac{1}{18}\right)}{\log(0.8)} = 12.952....$ so $n = 13$.</p> <p>(c) $u_5 = 200 \times (1.12)^4, \quad = £314.70$ or $£314.71$</p> <p>(d) $S_{15} = \frac{200(1.12^{15} - 1)}{1.12 - 1}$ or $\frac{200(1 - 1.12^{15})}{1 - 1.12}, = 7455.94.....$ awrt $£7460$</p>	<p>B1cso (1)</p> <p>M1</p> <p>M1</p> <p>A1 cso (3)</p> <p>M1, A1 (2)</p> <p>M1A1, A1 (3)</p> <p>[9]</p>
	<p>(a) B1 NB Answer is printed so need working. May see as above or $\times 0.8$ in three steps giving 14400, 11520, 9216. Do not need to see £ sign but should see 9216 .</p> <p>(b) 1st M1 for an attempt to use nth term and 1000. Allow n or $n - 1$ and allow $>$ or $=$ 2nd M1 for use of logs to find n Allow n or $n - 1$ and allow $>$ or $=$ A1 Need $n = 13$ This is an accuracy mark and must follow award of both M marks but should not follow incorrect work using $n - 1$ for example. Condone slips in inequality signs here.</p> <p>(c) M1 for use of their a and r in formula for 5th term of GP A1 cao need one of these answers – answer can imply method here NB 314.7 – A0</p> <p>(d) M1 for use of sum to 15 terms of GP using their a and their r (allow if formula stated correctly and one error in substitution, but must use n not $n - 1$) 1st A1 for a fully correct expression (not evaluated)</p>	
	<p>(b) Alternative Methods Trial and Improvement See 989.56 (or 989 or 990) identified with 12, 13 or 14 years for first M1 See 1236.95 (or 1236 or 1237) identified with 11, 12 or 13 years for second M1 Then $n = 13$ is A1 (needs both Ms) Special case $18000 \times (0.8)^n < 1000$ so $n = 13$ as $989.56 < 1000$ is M1M0A0 (not discounted $n = 12$)</p> <p>(c) May see the terms 224, 250.88, 280.99, 314.71 with a small slip for M1 A0, or done accurately for M1A1</p> <p>(d) Adds 15 terms $200 + 224 + 250.88 + \dots + (977.42)$ M1 Seeing 977... is A1 Obtains answer 7455.94 A1 or awrt $£7460$ NOT 7450</p>	

Question Number	Scheme	Marks
Q7	<p>(a) Puts $y = 0$ and attempts to solve quadratic e.g. $(x-4)(x-1) = 0$ Points are (1,0) and (4, 0)</p> <p>(b) $x = 5$ gives $y = 25 - 25 + 4$ and so (5, 4) lies on the curve</p> <p>(c) $\int (x^2 - 5x + 4) dx = \frac{1}{3}x^3 - \frac{5}{2}x^2 + 4x \quad (+ c)$</p> <p>(d) Area of triangle = $\frac{1}{2} \times 4 \times 4 = 8$ or $\int (x-1) dx = \frac{1}{2}x^2 - x$ with limits 1 and 5 to give 8 Area under the curve = $\int_4^5 \left(\frac{1}{3} \times 5^3 - \frac{5}{2} \times 5^2 + 4 \times 5 \right) \left[= -\frac{5}{6} \right]$ $\frac{1}{3} \times 4^3 - \frac{5}{2} \times 4^2 + 4 \times 4 \left[= -\frac{8}{3} \right]$ $\int_4^5 = -\frac{5}{6} - \left(-\frac{8}{3}\right) = \frac{11}{6}$ or equivalent (allow 1.83 or 1.8 here) Area of $R = 8 - \frac{11}{6} = 6\frac{1}{6}$ or $\frac{37}{6}$ or $6.16\bar{r}$ (not 6.17)</p>	<p>M1 A1 (2)</p> <p>B1cso (1)</p> <p>M1A1 (2)</p> <p>B1 M1 M1 A1 cao A1 cao (5)</p> <p>[10]</p>
	<p>(a) M1 for attempt to find L and M A1 Accept $x = 1$ and $x = 4$, then isw or accept $L = (1,0)$, $M = (4,0)$ Do not accept $L = 1$, $M = 4$ nor $(0, 1)$, $(0, 4)$ (unless subsequent work) Do not need to distinguish L and M. Answers imply M1A1.</p> <p>(b) See substitution, working should be shown, need conclusion which could be just $y = 4$ or a tick. Allow $y = 25 - 25 + 4 = 4$ But not $25 - 25 + 4 = 4$. ($y = 4$ may appear at start) Usually $0 = 0$ or $4 = 4$ is B0</p> <p>(c) M1 for attempt to integrate $x^2 \rightarrow kx^3$, $x \rightarrow kx^2$ or $4 \rightarrow 4x$ A1 for correct integration of all three terms (do not need constant) isw. Mark correct work when seen. So e.g. $\frac{1}{3}x^3 - \frac{5}{2}x^2 + 4x$ is A1 then $2x^3 - 15x^2 + 24x$ would be ignored as subsequent work.</p> <p>(d) B1 for this triangle only (not triangle LMN) 1st M1 for substituting 5 into their changed function 2nd M1 for substituting 4 into their changed function</p>	
	<p>(d) Alternative method: $\int_1^5 (x-1) - (x^2 - 5x + 4) dx + \int_1^4 x^2 - 5x + 4 dx$ can lead to correct answer Constructs $\int_1^5 (x-1) - (x^2 - 5x + 4) dx$ is B1 M1 for substituting 5 and 1 and subtracting in first integral M1 for substituting 4 and 1 and subtracting in second integral A1 for answer to first integral i.e. $\frac{32}{3}$ (allow 10.7) and A1 for final answer as before..</p>	

(d)	<p>Another alternative</p> $\int_4^5 (x-1) - (x^2 - 5x + 4) dx + \text{area of triangle } LMP$ <p>Constructs $\int_4^5 (x-1) - (x^2 - 5x + 4) dx$ is B1</p> <p>M1 for substituting 5 and 4 and subtracting in first integral</p> <p>M1 for complete method to find area of triangle (4.5)</p> <p>A1 for answer to first integral i.e. $\frac{5}{3}$ and A1 for final answer as before.</p>
(d)	<p>Could also use</p> $\int_4^5 (4x-16) - (x^2 - 5x + 4) dx + \text{area of triangle } LMN$ <p>Similar scheme to previous one. Triangle has area 6</p> <p>A1 for finding Integral has value $\frac{1}{6}$ and A1 for final answer as before.</p>

Question Number	Scheme	Marks
Q8	<p>(a) $N(2, -1)$</p> <p>(b) $r = \sqrt{\frac{169}{4}} = \frac{13}{2} = 6.5$</p> <p>(c) Complete Method to find x coordinates, $x_2 - x_1 = 12$ and $\frac{x_1 + x_2}{2} = 2$ then solve To obtain $x_1 = -4$, $x_2 = 8$ Complete Method to find y coordinates, using equation of circle or Pythagoras i.e. let d be the distance below N of A then $d^2 = 6.5^2 - 6^2 \Rightarrow d = 2.5 \Rightarrow y = ..$ So $y_2 = y_1 = -3.5$</p> <p>(d) Let $\hat{ANB} = 2\theta \Rightarrow \sin \theta = \frac{6}{"6.5"} \Rightarrow \theta = (67.38)...$ So angle ANB is 134.8°</p> <p>(e) AP is perpendicular to AN so using triangle ANP $\tan \theta = \frac{AP}{"6.5"}$ Therefore $AP = 15.6$</p>	<p>B1, B1 (2)</p> <p>B1 (1)</p> <p>M1 A1ft A1ft M1 A1 (5)</p> <p>M1 A1 (2)</p> <p>M1 A1cao (2)</p> <p>[12]</p>
	<p>(a) B1 for 2 (α), B1 for -1</p> <p>(b) B1 for 6.5 o.e.</p> <p>(c) 1st M1 for finding x coordinates – may be awarded if either x co-ord is correct A1ft, A1ft are for $\alpha - 6$ and $\alpha + 6$ if x coordinate of N is α 2nd M1 for a method to find y coordinates – may be given if y co-ordinate is correct A marks is for -3.5 only.</p> <p>(d) M1 for a full method to find θ or angle ANB (eg sine rule or cosine rule directly or finding another angle and using angles of triangle.) ft their 6.5 from radius or wrong y. $(\cos ANB = \frac{"6.5"{}^2 + "6.5"{}^2 - 12^2}{2 \times "6.5" \times "6.5"} = -0.704)$ A1 is a printed answer and must be 134.8 – do not accept 134.76.</p> <p>(e) M1 for a full method to find AP <u>Alternative Methods</u> N.B. May use triangle AXP where X is the mid point of AB. Or may use triangle ABP. From circle theorems may use angle $BAP = 67.38$ or some variation. Eg $\frac{AP}{\sin 67.4} = \frac{12}{\sin 45.2}$, $AP = \frac{6}{\sin 22.6}$ or $AP = \frac{6}{\cos 67.4}$ are each worth M1</p>	

Question Number	Scheme	Marks
Q9 (a)	$\left[y = 12x^{\frac{1}{2}} - x^{\frac{3}{2}} - 10 \right]$ $[y' =] \quad 6x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}}$ <p>Puts their $\frac{6}{x^{\frac{1}{2}}} - \frac{3}{2}x^{\frac{1}{2}} = 0$</p> <p>So $x = \frac{12}{3} = 4$ (If $x = 0$ appears also as solution then lose A1)</p> <p>$x = 4, \Rightarrow y = 12 \times 2 - 4^{\frac{3}{2}} - 10, \quad \text{so } y = 6$</p>	<p>M1 A1</p> <p>M1</p> <p>M1, A1</p> <p>dM1, A1 (7)</p>
(b)	$y'' = -3x^{-\frac{3}{2}} - \frac{3}{4}x^{-\frac{1}{2}}$	M1A1 (2)
(c)	[Since $x > 0$] It is a maximum	B1 (1) [10]
(a)	<p>1st M1 for an attempt to differentiate a fractional power $x^n \rightarrow x^{n-1}$ A1 a.e.f – can be unsimplified 2nd M1 for forming a suitable equation using their $y' = 0$ 3rd M1 for correct processing of fractional powers leading to $x = \dots$ (Can be implied by $x = 4$) A1 is for $x = 4$ only. If $x = 0$ also seen and not discarded they lose this mark only. 4th M1 for substituting their value of x back into y to find y value. Dependent on three previous M marks. Must see evidence of the substitution with attempt at fractional powers to give M1A0, but $y = 6$ can imply M1A1</p>	
(b)	<p>M1 for differentiating their y' again A1 should be simplified</p>	
(c)	<p>B1 . Clear conclusion needed and must follow correct y'' It is dependent on previous A mark (Do not need to have found x earlier).</p> <p>(Treat parts (a),(b) and (c) together for award of marks)</p>	

Mark Scheme (Results) Summer 2010

GCE

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Summer 2010

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SOME GENERAL PRINCIPLES FOR C2 MARKING

(But the particular mark scheme always takes precedence)

Method mark for solving 3 term quadratic:**1. Factorisation**

$$(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c|, \text{ leading to } x = \dots$$

$$(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where } |pq| = |c| \text{ and } |mn| = |a|, \text{ leading to } x = \dots$$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

$$\text{Solving } x^2 + bx + c = 0 : \left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0, \quad \text{leading to } x = \dots$$

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values (but refer to the mark scheme first... the application of this principle may vary). Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but will be lost if there is any mistake in the working.

Equation of a straight line

Apply the following conditions to the M mark for the equation of a line through (a, b) :

If the a and b are the wrong way round the M mark can still be given if a correct formula is seen, (e.g. $y - y_1 = m(x - x_1)$) otherwise M0.

If (a, b) is substituted into $y = mx + c$ to find c , the M mark is for attempting this.

Answers without working

The rubric says that these may gain no credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Misreads

A misread must be consistent for the whole question to be interpreted as such.

These are not common. In clear cases, please deduct the first 2 A (or B) marks which would have been lost by following the scheme. (Note that 2 marks is the maximum misread penalty, but that misreads which alter the nature or difficulty of the question cannot be treated so generously and it will usually be necessary here to follow the scheme as written).

Sometimes following the scheme as written is more generous to the candidate than applying the misread rule, so in this case use the scheme as written.

If in doubt, send the response to Review.

June 2010
Core Mathematics C2 6664
Mark Scheme

Question Number	Scheme	Marks
1.	<p>(a) 2.35, 3.13, 4.01 (One or two correct B1 B0, all correct B1 B1) <u>Important:</u> If part (a) is blank, or if answers have been crossed out and no replacement answers are visible, please send to Review as 'out of clip'.</p>	<p>B1 B1 (2)</p>
	<p>(b) $\frac{1}{2} \times 0.2 \dots\dots$ (or equivalent numerical value) $k\{(1+5)+2(1.65+p+q+r)\}$, k constant, $k \neq 0$ (See notes below) $= 2.828$ (awrt 2.83, allowed even after minor slips in values) The fractional answer $\frac{707}{250}$ (or other fraction wrt 2.83) is also acceptable. Answers with no working score no marks.</p>	<p>B1 M1 A1 A1 (4) 6</p>
	<p>(a) Answers must be given to 2 decimal places. <u>No marks</u> for answers given to only 1 decimal place.</p> <p>(b) The p, q and r below are positive numbers, none of which is equal to any of: 1, 5, 1.65, 0.2, 0.4, 0.6 or 0.8 M1 A1: $k\{(1+5)+2(1.65+p+q+r)\}$ M1 A0: $k\{(1+5)+2(1.65+p+q)\}$ or $k\{(1+5)+2(p+q+r)\}$ M0 A0: $k\{(1+5)+2(1.65+p+q+r+other\ value(s))\}$ Note that if the only mistake is to <u>omit</u> a value from the second bracket, this is considered as a slip and the M mark is allowed. <u>Bracketing mistake:</u> i.e. $\frac{1}{2} \times 0.2(1+5)+2(1.65+2.35+3.13+4.01)$ instead of $\frac{1}{2} \times 0.2\{(1+5)+2(1.65+2.35+3.13+4.01)\}$, so that only the $(1+5)$ is multiplied by 0.1 scores B1 M1 A0 A0 <u>unless</u> the final answer implies that the calculation has been done correctly (then full marks can be given). <u>Alternative:</u> Separate trapezia may be used, and this can be marked equivalently.</p>	

Question Number	Scheme	Marks
2	(a) Attempting to find $f(3)$ or $f(-3)$ $f(3) = 3(3)^3 - 5(3)^2 - (58 \times 3) + 40 = 81 - 45 - 174 + 40 = -98$	M1 A1 (2)
	(b) $\{3x^3 - 5x^2 - 58x + 40 = (x - 5)\} (3x^2 + 10x - 8)$ Attempt to <u>factorise</u> 3-term quadratic, or to use the quadratic formula (see general principles at beginning of scheme). This mark may be implied by the correct solutions to the quadratic. $(3x - 2)(x + 4) = 0 \quad x = \dots \quad \underline{\text{or}} \quad x = \frac{-10 \pm \sqrt{100 + 96}}{6}$ $\frac{2}{3}$ (or exact equiv.), -4 , 5 (Allow 'implicit' solns, e.g. $f(5) = 0$, etc.) Completely correct solutions without working: full marks.	M1 A1 M1 A1 ft A1 (5) 7
	(a) <u>Alternative (long division):</u> Divide by $(x - 3)$ to get $(3x^2 + ax + b)$, $a \neq 0$, $b \neq 0$. [M1] $(3x^2 + 4x - 46)$, and -98 seen. [A1] (If continues to say 'remainder = 98', isw)	
	(b) 1st M requires use of $(x - 5)$ to obtain $(3x^2 + ax + b)$, $a \neq 0$, $b \neq 0$. (Working need not be seen... this could be done 'by inspection'.) $(3x^2 + 10x - 8) \longleftarrow$ 2 nd M for the attempt to <u>factorise</u> their 3-term quadratic, or to solve it using the quadratic formula. Factorisation: $(3x^2 + ax + b) = (3x + c)(x + d)$, where $ cd = b $. A1ft: Correct factors for their 3-term quadratic <u>followed by a solution</u> (at least one value, which might be incorrect), <u>or</u> numerically correct expression from the quadratic formula for their 3-term quadratic. <u>Note</u> therefore that if the quadratic is correctly factorised but no solutions are given, the last 2 marks will be lost. <u>Alternative (first 2 marks):</u> $(x - 5)(3x^2 + ax + b) = 3x^3 + (a - 15)x^2 + (b - 5a)x - 5b = 0$, then compare coefficients to find <u>values</u> of a and b . [M1] $a = 10$, $b = -8$ [A1] <u>Alternative 1:</u> (factor theorem) M1: Finding that $f(-4) = 0$ A1: Stating that $(x + 4)$ is a factor. M1: Finding third factor $(x - 5)(x + 4)(3x \pm 2)$. A1: Fully correct factors (no ft available here) <u>followed by a solution</u> , (which might be incorrect). A1: All solutions correct. <u>Alternative 2:</u> (direct factorisation) M1: Factors $(x - 5)(3x + p)(x + q)$ A1: $pq = -8$ M1: $(x - 5)(3x \pm 2)(x \pm 4)$ Final A marks as in Alternative 1.	
Throughout this scheme, allow $\left(x \pm \frac{2}{3}\right)$ as an alternative to $(3x \pm 2)$.		

Question Number	Scheme	Marks
3	(a) $\left(\frac{dy}{dx} = \right) 2x - \frac{1}{2}kx^{\frac{1}{2}}$ (Having an extra term, e.g. +C, is A0)	M1 A1 (2)
	(b) Substituting $x = 4$ into their $\frac{dy}{dx}$ and ‘compare with zero’ (The mark is allowed for : $<, >, =, \leq, \geq$) $8 - \frac{k}{4} < 0 \quad k > 32 \quad (\text{or } 32 < k) \quad \underline{\text{Correct inequality needed}}$	M1 A1 (2) 4
	(a) M: $x^2 \rightarrow cx$ or $k\sqrt{x} \rightarrow cx^{\frac{1}{2}}$ (c constant, $c \neq 0$) (b) Substitution of $x = 4$ into y scores M0. However, $\frac{dy}{dx}$ is sometimes <u>called</u> y , and in this case the M mark can be given. $\frac{dy}{dx} = 0$ may be ‘implied’ for M1, when, for example, a value of k or an inequality solution for k is found. <u>Working</u> must be seen to justify marks in (b), i.e. $k > 32$ alone is M0 A0.	

Question Number	Scheme	Marks
4	<p>(a) $(1+ax)^7 = 1 + 7ax \dots$ or $1 + 7(ax) \dots$ (<u>Not</u> unsimplified versions)</p> <p>$+ \frac{7 \times 6}{2}(ax)^2 + \frac{7 \times 6 \times 5}{6}(ax)^3$ Evidence from <u>one</u> of these terms is enough</p> <p>$+ 21a^2x^2$ or $+ 21(ax)^2$ or $+ 21(a^2x^2)$</p> <p>$+ 35a^3x^3$ or $+ 35(ax)^3$ or $+ 35(a^3x^3)$</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>(4)</p>
	<p>(b) $21a^2 = 525$</p> <p>$a = \pm 5$ (Both values are required)</p> <p>(The answer $a = 5$ with no working scores M1 A0)</p>	<p>M1</p> <p>A1</p> <p>(2)</p> <p>6</p>
	<p>(a) The terms can be ‘listed’ rather than added.</p> <p>M1: Requires correct structure: a correct binomial coefficient in any form (perhaps from Pascal’s triangle) with the correct power of x. Allow missing a’s and wrong powers of a, e.g.</p> $\frac{7 \times 6}{2}ax^2, \quad \frac{7 \times 6 \times 5}{3 \times 2}x^3$ <p>However, $21 + a^2x^2 + 35 + a^3x^3$ or similar is M0.</p> <p>$1 + 7ax + 21 + a^2x^2 + 35 + a^3x^3 = 57 + \dots$ scores the B1 (isw).</p> <p>$\binom{7}{2}$ and $\binom{7}{3}$ or equivalent such as 7C_2 and 7C_3 are acceptable,</p> <p>but <u>not</u> $\left(\frac{7}{2}\right)$ or $\left(\frac{7}{3}\right)$ (unless subsequently corrected).</p> <p>1st A1: Correct x^2 term. 2nd A1: Correct x^3 term (The binomial coefficients <u>must</u> be simplified).</p> <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p><u>Special case:</u> If $(ax)^2$ and $(ax)^3$ are seen within the working, but then lost... ... A1 A0 can be given if $21ax^2$ and $35ax^3$ are <u>both</u> achieved.</p> </div> <p><u>a’s omitted throughout:</u> Note that only the M mark is available in this case.</p> <p>(b) M: Equating their coefficient of x^2 to 525.</p> <p>An equation in a or a^2 alone is required for this M mark, but allow ‘recovery’ that shows <u>the required coefficient</u>, e.g.</p> <p>$21a^2x^2 = 525 \Rightarrow 21a^2 = 525$ is acceptable, but $21a^2x^2 = 525 \Rightarrow a^2 = 25$ is not acceptable.</p> <p>After $21ax^2$ in the answer for (a), allow ‘recovery’ of a^2 in (b) so that full marks are available for (b) (but not retrospectively for (a)).</p>	

Question Number	Scheme	Marks
5	(a) $\tan \theta = \frac{2}{5}$ (or 0.4) (i.s.w. if a value of θ is subsequently found) Requires the correct value with no incorrect working seen.	B1 (1)
	(b) awrt 21.8 (α) (Also allow awrt 68.2, ft from $\tan \theta = \frac{5}{2}$ in (a), but no other ft) (This value must be seen in part (b). It may be implied by a correct solution, e.g. 10.9)	B1
	180 + α (= 201.8), or 90 + ($\alpha/2$) (if division by 2 has already occurred) (α found from $\tan 2x = \dots$ or $\tan x = \dots$ or $\sin 2x = \pm \dots$ or $\cos 2x = \pm \dots$)	M1
	360 + α (= 381.8), or 180 + ($\alpha/2$) (α found from $\tan 2x = \dots$ or $\sin 2x = \dots$ or $\cos 2x = \dots$) OR 540 + α (= 561.8), or 270 + ($\alpha/2$) (α found from $\tan 2x = \dots$)	M1
	Dividing at least one of the angles by 2 (α found from $\tan 2x = \dots$ or $\sin 2x = \dots$ or $\cos 2x = \dots$) $x = 10.9, 100.9, 190.9, 280.9$ (Allow awrt)	M1 A1 (5) 6

(b) Extra solution(s) in range: Loses the final A mark.

Extra solutions outside range: Ignore (whether correct or not).

Common answers:

10.9 and 100.9 would score B1 M1 M0 M1 A0 (Ensure that these M marks are awarded)

10.9 and 190.9 would score B1 M0 M1 M1 A0 (Ensure that these M marks are awarded)

Alternatives:

$$\begin{aligned}
 \text{(i)} \quad 2 \cos 2x - 5 \sin 2x = 0 \quad R \cos(2x + \lambda) = 0 \quad \lambda = 68.2 \Rightarrow 2x + 68.2 = 90 & \quad \text{B1} \\
 2x + \lambda = 270 & \quad \text{M1} \\
 2x + \lambda = 450 \quad \text{or} \quad 2x + \lambda = 630 & \quad \text{M1} \\
 \text{Subtracting } \lambda \text{ and dividing by 2 (at least once)} & \quad \text{M1}
 \end{aligned}$$

$$\text{(ii)} \quad 25 \sin^2 2x = 4 \cos^2 2x = 4(1 - \sin^2 2x)$$

$$29 \sin^2 2x = 4 \quad 2x = 21.8 \quad \text{B1}$$

The M marks are scored as in the main scheme, but extra solutions will be likely, losing the A mark.

Using radians:

B1: Can be given for awrt 0.38 (β)

M1: For $\pi + \beta$ or $180 + \beta$

M1: For $2\pi + \beta$ or $3\pi + \beta$ (Must now be consistently radians)

M1: For dividing at least one of the angles by 2

A1: For this mark, the answers must be in degrees.

(Correct) answers only (or by graphical methods):

B and M marks can be awarded by implication, e.g.

10.9 scores B1 M0 M0 M1 A0

10.9, 100.9 scores B1 M1 M0 M1 A0

10.9, 100.9, 190.9, 280.9 scores full marks.

Using 11, etc. instead of 10.9 can still score the M marks by implication.

Question Number	Scheme	Marks
6	(a) $r\theta = 9 \times 0.7 = 6.3$ (Also allow 6.30, or awrt 6.30)	M1 A1 (2)
	(b) $\frac{1}{2}r^2\theta = \frac{1}{2} \times 81 \times 0.7 = 28.35$ (Also allow 28.3 or 28.4, or awrt 28.3 or 28.4) (Condone 28.35^2 written instead of 28.35 cm^2)	M1 A1 (2)
	(c) $\tan 0.7 = \frac{AC}{9}$ $AC = 7.58$ (Allow awrt) <u>NOT</u> 7.59 (see below)	M1 A1 (2)
	(d) Area of triangle $AOC = \frac{1}{2}(9 \times \text{their } AC)$ (or other complete method) Area of $R = "34.11" - "28.35"$ (triangle – sector) or (sector – triangle) (needs a <u>value</u> for each) $= 5.76$ (Allow awrt)	M1 M1 A1 (3) 9
	(a) M: Use of $r\theta$ (with θ in radians), or equivalent (could be working in degrees with a correct degrees formula). (b) M: Use of $\frac{1}{2}r^2\theta$ (with θ in radians), or equivalent (could be working in degrees with a correct degrees formula). (c) M: Other methods must be fully correct, e.g. $\frac{AC}{\sin 0.7} = \frac{9}{\sin\left(\frac{\pi}{2} - 0.7\right)}$ $(\pi - 0.7)$ instead of $\left(\frac{\pi}{2} - 0.7\right)$ here is <u>not</u> a fully correct method. <u>Premature approximation (e.g. taking angle C as 0.87 radians):</u> This will often result in loss of A marks, e.g. $AC = 7.59$ in (c) is A0.	

Question Number	Scheme	Marks
7	(a) $2\log_3(x-5) = \log_3(x-5)^2$ $\log_3(x-5)^2 - \log_3(2x-13) = \log_3 \frac{(x-5)^2}{2x-13}$ $\log_3 3 = 1$ seen or used correctly $\log_3\left(\frac{P}{Q}\right) = 1 \Rightarrow P = 3Q \quad \left\{ \frac{(x-5)^2}{2x-13} = 3 \Rightarrow (x-5)^2 = 3(2x-13) \right\}$ $x^2 - 16x + 64 = 0$ (*)	B1 M1 B1 M1 A1 cso (5)
	(b) $(x-8)(x-8) = 0 \Rightarrow x = 8$ <u>Must</u> be seen in part (b). Or: Substitute $x = 8$ into original equation and verify. Having additional solution(s) such as $x = -8$ loses the A mark. $x = 8$ with no working scores both marks.	M1 A1 (2) 7

(a) Marks may be awarded if equivalent work is seen in part (b).

1st M: $\log_3(x-5)^2 - \log_3(2x-13) = \frac{\log_3(x-5)^2}{\log_3(2x-13)}$ is M0

$2\log_3(x-5) - \log_3(2x-13) = 2\log \frac{x-5}{2x-13}$ is M0

2nd M: After the first mistake above, this mark is available only if there is ‘recovery’ to the required

$\log_3\left(\frac{P}{Q}\right) = 1 \Rightarrow P = 3Q$. Even then the final mark (cso) is lost.

‘Cancelling logs’, e.g. $\frac{\log_3(x-5)^2}{\log_3(2x-13)} = \frac{(x-5)^2}{2x-13}$ will also lose the 2nd M.

A typical wrong solution:

$\log_3 \frac{(x-5)^2}{2x-13} = 1 \Rightarrow \log_3 \frac{(x-5)^2}{2x-13} = 3 \Rightarrow \frac{(x-5)^2}{2x-13} = 3 \Rightarrow (x-5)^2 = 3(2x-13)$



(Wrong step here)

This, with no evidence elsewhere of $\log_3 3 = 1$, scores B1 M1 B0 M0 A0

However, $\log_3 \frac{(x-5)^2}{2x-13} = 1 \Rightarrow \frac{(x-5)^2}{2x-13} = 3$ is correct and could lead to full marks.

(Here $\log_3 3 = 1$ is implied).

No log methods shown:

It is not acceptable to jump immediately to $\frac{(x-5)^2}{2x-13} = 3$. The only mark this scores is the 1st B1 (by generous implication).

(b) M1: Attempt to solve the given quadratic equation (usual rules), so the factors $(x-8)(x-8)$ with no solution is M0.

Question Number	Scheme	Marks
8	<p>(a) $\frac{dy}{dx} = 3x^2 - 20x + k$ (Differentiation is required)</p> <p>At $x = 2$, $\frac{dy}{dx} = 0$, so $12 - 40 + k = 0$ $k = 28$ (*)</p> <p><u>N.B. The '= 0' must be seen at some stage to score the final mark.</u></p> <p><u>Alternatively:</u> (using $k = 28$)</p> <p>$\frac{dy}{dx} = 3x^2 - 20x + 28$ (M1 A1)</p> <p>'Assuming' $k = 28$ only scores the final cso mark if there is justification that $\frac{dy}{dx} = 0$ at $x = 2$ represents the <u>maximum</u> turning point.</p>	<p>M1 A1</p> <p>A1 cso</p> <p>(3)</p>
	<p>(b) $\int (x^3 - 10x^2 + 28x) dx = \frac{x^4}{4} - \frac{10x^3}{3} + \frac{28x^2}{2}$ Allow $\frac{kx^2}{2}$ for $\frac{28x^2}{2}$</p> <p>$\left[\frac{x^4}{4} - \frac{10x^3}{3} + 14x^2 \right]_0^2 = \dots$ $\left(= 4 - \frac{80}{3} + 56 = \frac{100}{3} \right)$</p> <p>(With limits 0 to 2, substitute the limit 2 into a 'changed function')</p> <p>y-coordinate of $P = 8 - 40 + 56 = 24$ <u>Allow if seen in part (a)</u></p> <p>(The B1 for 24 may be scored by implication from later working)</p> <p>Area of rectangle = $2 \times$ (their y - coordinate of P)</p> <p>Area of $R =$ (their 48) $-$ $\left(\text{their } \frac{100}{3} \right) = \frac{44}{3} \left(14\frac{2}{3} \text{ or } 14.\dot{6} \right)$</p> <p>If the subtraction is the 'wrong way round', the final A mark is lost.</p>	<p>M1 A1</p> <p>M1</p> <p>B1</p> <p>M1 A1</p> <p>(6) 9</p>
	<p>(a) M: $x^n \rightarrow cx^{n-1}$ (c constant, $c \neq 0$) for one term, seen <u>in part (a)</u>.</p> <p>(b) 1st M: $x^n \rightarrow cx^{n+1}$ (c constant, $c \neq 0$) for one term.</p> <p>Integrating the <u>gradient function</u> loses this M mark.</p> <p>2ndM: Requires use of limits 0 and 2, with 2 substituted into a 'changed function'. (It may, for example, have been differentiated).</p> <p>Final M: Subtract their values either way round. This mark is dependent on the use of calculus and a correct method attempt for the area of the rectangle.</p> <p>A1: Must be <u>exact</u>, not 14.67 or similar, but isw after seeing, say, $\frac{44}{3}$.</p> <p><u>Alternative:</u> (effectively finding area of rectangle by integration)</p> <p>$\int \{24 - (x^3 - 10x^2 + 28x)\} dx = 24x - \left(\frac{x^4}{4} - \frac{10x^3}{3} + \frac{28x^2}{2} \right)$, etc.</p> <p>This can be marked equivalently, with the 1st A being for integrating the same 3 terms correctly. The 3rd M (for subtraction) will be scored at the same stage as the 2nd M. If the subtraction is the 'wrong way round', the final A mark is lost.</p>	

Question Number	Scheme	Marks
9	(a) $25\,000 \times 1.03 = 25750$ $\left\{ 25000 + 750 = 25750, \text{ or } 25000 \frac{(1 - 0.03^2)}{1 - 0.03} = 25750 \right\}$ (*)	B1 (1)
	(b) $r = 1.03$ Allow $\frac{103}{100}$ or $1\frac{3}{100}$ but no other alternatives	B1 (1)
	(c) $25000r^{N-1} > 40000$ (Either letter r or their r value) Allow '=' or '<' $r^M > 1.6 \Rightarrow \log r^M > \log 1.6$ Allow '=' or '<' (See below) OR (by change of base), $\log_{1.03} 1.6 < M \Rightarrow \frac{\log 1.6}{\log 1.03} < M$ $(N-1)\log 1.03 > \log 1.6$ (Correct bracketing required) (*) Accept work for part (c) seen in part (d)	M1 M1 A1 cso (3)
	(d) Attempt to evaluate $\frac{\log 1.6}{\log 1.03} + 1$ {or $25000(1.03)^{15}$ and $25000(1.03)^{16}$ } $N = 17$ (not 16.9 and not e.g. $N \geq 17$) Allow '17 th year' Accept work for part (d) seen in part (c)	M1 A1 (2)
	(e) Using formula $\frac{a(1-r^n)}{1-r}$ with values of a and r , and $n = 9, 10$ or 11 $\frac{25000(1-1.03^{10})}{1-1.03}$ 287 000 (must be rounded to the nearest 1 000) Allow 287000.00	M1 A1 A1 (3)

(c) 2nd M: Requires $\frac{40000}{25000}$ to be dealt with, and 'two' logs introduced.

With, say, N instead of $N-1$, this mark is still available.

Jumping straight from $1.03^{N-1} > 1.6$ to $(N-1)\log 1.03 > \log 1.6$ can score only M1 M0 A0.

(The intermediate step $\log 1.03^{N-1} > \log 1.6$ must be seen).

Longer methods require correct log work throughout for 2nd M, e.g.:

$$\log(25000r^{N-1}) > \log 40000 \Rightarrow \log 25000 + \log r^{N-1} > \log 40000 \Rightarrow$$

$$\log r^{N-1} > \log 40000 - \log 25000 \Rightarrow \log r^{N-1} > \log 1.6$$

(d) Correct answer with no working scores both marks.

Evaluating $\log\left(\frac{1.6}{1.03}\right) + 1$ does not score the M mark.

(e) M1 can also be scored by a "year by year" method, with terms added.

(Allow the M mark if there is evidence of adding 9, 10 or 11 terms).

1st A1 is scored if the 10 correct terms have been added (allow terms to be to the nearest 100).

To the nearest 100, these terms are:

25000, 25800, 26500, 27300, 28100, 29000, 29900, 30700, 31700, 32600

No working shown: Special case: 287 000 scores 1 mark, scored on ePEN as 1, 0, 0.

(Other answers with no working score no marks).

Question Number	Scheme	Marks
10	<p>(a) $(10-2)^2 + (7-1)^2$ or $\sqrt{(10-2)^2 + (7-1)^2}$ $(x \pm 2)^2 + (y \pm 1)^2 = k$ (k a positive <u>value</u>) $(x-2)^2 + (y-1)^2 = 100$ (Accept 10^2 for 100) (Answer only scores full marks)</p>	<p>M1 A1 M1 A1 (4)</p>
	<p>(b) (Gradient of radius) $= \frac{7-1}{10-2} = \frac{6}{8}$ (or equiv.) Must be seen in part (b) Gradient of tangent $= \frac{-4}{3}$ (Using perpendicular gradient method) $y-7 = m(x-10)$ Eqn., in any form, of a line through (10, 7) with any numerical gradient (except 0 or ∞) $y-7 = \frac{-4}{3}(x-10)$ or equiv (ft gradient of <u>radius</u>, dep. on <u>both</u> M marks) $\{3y = -4x + 61\}$ (N.B. The A1 is only available as <u>ft</u> after B0) The unsimplified version scores the A mark (isw if necessary... subsequent mistakes in simplification are not penalised here. The equation must at some stage be <u>exact</u>, not, e.g. $y = -1.3x + 20.3$</p>	<p>B1 M1 M1 A1ft (4)</p>
	<p>(c) $\sqrt{r^2 - \left(\frac{r}{2}\right)^2}$ Condone sign slip if there is evidence of correct use of Pythag. $= \sqrt{10^2 - 5^2}$ or numerically exact equivalent $PQ (= 2\sqrt{75}) = 10\sqrt{3}$ Simplest surd form $10\sqrt{3}$ required for final mark</p>	<p>M1 A1 A1 (3) 11</p>
	<p>(b) 2nd M: Using (10, 7) to find the equation, in any form, of a straight line through (10, 7), with any numerical gradient (except 0 or ∞). <u>Alternative</u>: 2nd M: Using (10, 7) and an m value in $y = mx + c$ to find a value of c. (b) <u>Alternative</u> for first 2 marks (differentiation): $2(x-2) + 2(y-1)\frac{dy}{dx} = 0$ or equiv. B1 Substitute $x = 10$ and $y = 7$ to find a value for $\frac{dy}{dx}$ M1 (This M mark can be awarded generously, even if the attempted 'differentiation' is not 'implicit'). (c) <u>Alternatives</u>: To score M1, must be a <u>fully</u> correct method to obtain $\frac{1}{2}PQ$ or PQ. 1st A1: For alternative methods that find PQ directly, this mark is for an <u>exact numerically correct version</u> of PQ.</p>	

Mark Scheme (Results)

January 2011

GCE

GCE Core Mathematics C2 (6664) Paper 1

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General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M marks:** method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A marks:** Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B marks** are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod - benefit of doubt
- ft - follow through
- the symbol \checkmark will be used for correct ft
- cao - correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw - ignore subsequent working
- awrt - answers which round to
- SC: special case
- oe - or equivalent (and appropriate)
- dep - dependent
- indep - independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- \square The second mark is dependent on gaining the first mark

January 2011
Core Mathematics C2 6664
Mark Scheme

Question Number	Scheme	Marks
1.		
(a)	$f(x) = x^4 + x^3 + 2x^2 + ax + b$ Attempting $f(1)$ or $f(-1)$. $f(1) = 1 + 1 + 2 + a + b = 7$ or $4 + a + b = 7 \Rightarrow a + b = 3$ (as required) AG	M1 A1 * cso (2)
(b)	Attempting $f(-2)$ or $f(2)$. $f(-2) = 16 - 8 + 8 - 2a + b = -8 \quad \{\Rightarrow -2a + b = -24\}$ Solving both equations simultaneously to get as far as $a = \dots$ or $b = \dots$ Any one of $a = 9$ or $b = -6$ Both $a = 9$ and $b = -6$	M1 A1 dM1 A1 A1 cso (5) [7]
Notes		
(a)	M1 for attempting either $f(1)$ or $f(-1)$. A1 for applying $f(1)$, setting the result equal to 7, and manipulating this correctly to give the result given on the paper as $a + b = 3$. Note that the answer is given in part (a).	
(b)	M1: attempting either $f(-2)$ or $f(2)$. A1: <u>correct underlined equation</u> in a and b ; eg <u>$16 - 8 + 8 - 2a + b = -8$</u> or equivalent, eg $-2a + b = -24$. dM1: an attempt to eliminate one variable from 2 linear simultaneous equations in a and b . Note that this mark is dependent upon the award of the first method mark. A1: any one of $a = 9$ or $b = -6$. A1: both $a = 9$ and $b = -6$ and a correct solution only.	
	<u>Alternative Method of Long Division:</u> (a) M1 for long division by $(x - 1)$ to give a remainder in a and b which is independent of x . A1 for $\{\text{Remainder} =\} b + a + 4 = 7$ leading to the correct result of $a + b = 3$ (answer given.) (b) M1 for long division by $(x + 2)$ to give a remainder in a and b which is independent of x . A1 for $\{\text{Remainder} =\} b - 2(a - 8) = -8 \quad \{\Rightarrow -2a + b = -24\}$. Then dM1A1A1 are applied in the same way as before.	

Question Number	Scheme	Marks
2.		
(a)	$11^2 = 8^2 + 7^2 - (2 \times 8 \times 7 \cos C)$ $\cos C = \frac{8^2 + 7^2 - 11^2}{2 \times 8 \times 7}$ (or equivalent) $\{\hat{C} = 1.64228...\} \Rightarrow \hat{C} = \text{awrt } 1.64$	M1 A1 A1 cso (3)
(b)	Use of Area $\Delta ABC = \frac{1}{2}ab\sin(\text{their } C)$, where a, b are any of 7, 8 or 11. $= \frac{1}{2}(7 \times 8)\sin C$ using the value of their C from part (a). $\{= 27.92848... \text{ or } 27.93297...\} = \text{awrt } 27.9$ (from angle of either 1.64° or 94.1°)	M1 A1 ft A1 cso (3) [6]
Notes		
(a)	M1 is also scored for $8^2 = 7^2 + 11^2 - (2 \times 7 \times 11 \cos C)$ or $7^2 = 8^2 + 11^2 - (2 \times 8 \times 11 \cos C)$ or $\cos C = \frac{7^2 + 11^2 - 8^2}{2 \times 7 \times 11}$ or $\cos C = \frac{8^2 + 11^2 - 7^2}{2 \times 8 \times 11}$ 1 st A1: Rearranged correctly to make $\cos C = \dots$ and numerically correct (possibly unsimplified). Award A1 for any of $\cos C = \frac{8^2 + 7^2 - 11^2}{2 \times 8 \times 7}$ or $\cos C = \frac{-8}{112}$ or $\cos C = -\frac{1}{14}$ or $\cos C = \text{awrt } -0.071$. SC: Also allow 1 st A1 for $112\cos C = -8$ or equivalent. Also note that the 1 st A1 can be implied for $\hat{C} = \text{awrt } 1.64$ or $\hat{C} = \text{awrt } 94.1^\circ$. Special Case: $\cos C = \frac{1}{14}$ or $\cos C = \frac{11^2 - 8^2 - 7^2}{2 \times 8 \times 7}$ scores a SC: M1A0A0. 2 nd A1: for awrt 1.64 cao Note that $A = 0.6876...^\circ$ (or $39.401...^\circ$), $B = 0.8116...^\circ$ (or $46.503...^\circ$)	
(b)	M1: alternative methods must be fully correct to score the M1. For any (or both) of the M1 or the 1 st A1; their C can either be in degrees or radians. Candidates who use $\cos C = \frac{1}{14}$ to give $C = 1.499...$, can achieve the correct answer of awrt 27.9 in part (b). These candidates will score M1A1A0cso, in part (b). Finding $C = 1.499...$ in part (a) and achieving awrt 27.9 with no working scores M1A1A0. Otherwise with no working in part (b), awrt 27.9 scores M1A1A1. Special Case: If the candidate gives awrt 27.9 from any of the below then award M1A1A1. $\frac{1}{2}(7 \times 11)\sin(0.8116^\circ \text{ or } 46.503^\circ) = \text{awrt } 27.9$, $\frac{1}{2}(8 \times 11)\sin(0.6876...^\circ \text{ or } 39.401...^\circ) = \text{awrt } 27.9$. Alternative: Hero's Formula: $A = \sqrt{13(13-11)(13-8)(13-7)} = \text{awrt } 27.9$, where M1 is attempt to apply $A = \sqrt{s(s-11)(s-8)(s-7)}$ and the first A1 is for the correct application of the formula.	

Question Number	Scheme	Marks
3.		
(a)	$ar = 750$ and $ar^4 = -6$ (could be implied from later working in either (a) or (b)). $r^3 = \frac{-6}{750}$ $r = -\frac{1}{5}$	B1 M1 Correct answer from no working, except for special case below gains all three marks. A1 (3)
(b)	$a(-0.2) = 750$ $a \left\{ = \frac{750}{-0.2} \right\} = -3750$	M1 A1 ft (2)
(c)	Applies $\frac{a}{1-r}$ correctly using both their a and their $ r < 1$. Eg. $\frac{-3750}{1-0.2}$ So, $S_{\infty} = -3125$	M1 A1 (2) [7]
Notes		
(a)	B1: for both $ar = 750$ and $ar^4 = -6$ (may be implied from later working in either (a) or (b)). M1: for eliminating a by either dividing $ar^4 = -6$ by $ar = 750$ or dividing $ar = 750$ by $ar^4 = -6$, to achieve an equation in r^3 or $\frac{1}{r^3}$. Note that $r^4 - r = -\frac{6}{750}$ is M0. Note also that any of $r^3 = \frac{-6}{750}$ or $r^3 = \frac{750}{-6} \{ = -125 \}$ or $\frac{1}{r^3} = \frac{-6}{750}$ or $\frac{1}{r^3} = \frac{750}{-6} \{ = -125 \}$ are fine for the award of M1. SC: $ar^{\alpha} = 750$ and $ar^{\beta} = -6$ leading to $r^{\delta} = \frac{-6}{750}$ or $r^{\delta} = \frac{750}{-6} \{ = -125 \}$ or $\frac{1}{r^{\delta}} = \frac{-6}{750}$ or $\frac{1}{r^{\delta}} = \frac{750}{-6} \{ = -125 \}$ where $\delta = \beta - \alpha$ and $\delta \geq 2$ are fine for the award of M1. SC: $ar^2 = 750$ and $ar^5 = -6$ leading to $r = -\frac{1}{5}$ scores B0M1A1.	
(b)	M1 for inserting their r into either of their original correct equations of either $ar = 750$ or $\{a = \} \frac{750}{r}$ or $ar^4 = -6$ or $\{a = \} \frac{-6}{r^4}$ – in both a and r . No slips allowed here for M1. A1 for either $a = -3750$ or a equal to the correct follow through result expressed either as an exact integer, or a fraction in the form $\frac{c}{d}$ where both c and d are integers, or correct to awrt 1 dp.	
(c)	M1 for applying $\frac{a}{1-r}$ correctly (only a slip in substituting r is allowed) using both their a and their $ r < 1$. Eg. $\frac{-3750}{1-0.2}$. A1 for -3125 In parts (a) or (b) or (c), the correct answer with no working scores full marks.	

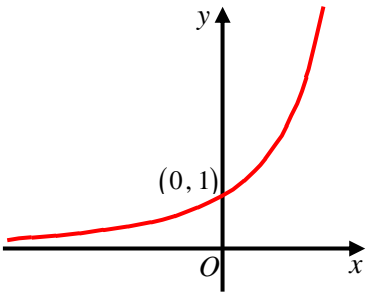
Question Number	Scheme	Marks
4.		
(a)	Seeing -1 and 5 . (See note below.)	B1 (1)
(b)	$(x+1)(x-5) = x^2 - 4x - 5$ or $x^2 - 5x + x - 5$ $\int (x^2 - 4x - 5) dx = \frac{x^3}{3} - \frac{4x^2}{2} - 5x \{+ c\}$ $\left[\frac{x^3}{3} - \frac{4x^2}{2} - 5x \right]_{-1}^5 = (\dots) - (\dots)$ $\left\{ \left(\frac{125}{3} - \frac{100}{2} - 25 \right) - \left(-\frac{1}{3} - 2 + 5 \right) \right\}$ $\left\{ = \left(-\frac{100}{3} \right) - \left(\frac{8}{3} \right) = -36 \right\}$ Hence, Area = 36	B1 M1A1ft A1 dM1 A1 (6) [7]
Notes		
(a)	B1: for -1 and 5 . Note that $(-1, 0)$ and $(5, 0)$ are acceptable for B1. Also allow $(0, -1)$ and $(0, 5)$ generously for B1. Note that if a candidate writes down that $A: (5, 0)$, $B: (-1, 0)$, (ie A and B interchanged,) then B0. Also allow values inserted in the correct position on the x -axis of the graph.	
(b)	B1 for $x^2 - 4x - 5$ or $x^2 - 5x + x - 5$. If you believe that the candidate is applying the Way 2 method then $-x^2 + 4x + 5$ or $-x^2 + 5x - x + 5$ would then be fine for B1. 1 st M1 for an attempt to integrate meaning that $x^n \rightarrow x^{n+1}$ for at least one of the terms. Note that $-5 \rightarrow 5x$ is sufficient for M1. 1 st A1 at least two out of three terms correctly ft from their multiplied out brackets. 2 nd A1 for correct integration only and no follow through. Ignore the use of a '+ c'. Allow 2 nd A1 also for $\frac{x^3}{3} - \frac{5x^2}{2} + \frac{x^2}{2} - 5x$. Note that $-\frac{5x^2}{2} + \frac{x^2}{2}$ only counts as one integrated term for the 1 st A1 mark. Do not allow any extra terms for the 2 nd A1 mark. 2 nd M1: Note that this method mark is dependent upon the award of the first M1 mark in part (b). Substitutes 5 and -1 (and not 1 if the candidate has stated $x = -1$ in part (a).) (or the limits the candidate has found from part(a)) into an "integrated function" and subtracts, either way round. 3 rd A1: For a final answer of 36 , not -36 . Note: An alternative method exists where the candidate states from the outset that Area (R) = $-\int_{-1}^5 (x^2 - 4x + 5) dx$ is detailed in the Appendix.	

Question Number	Scheme	Marks
5.		
(a)	$\binom{40}{4} = \frac{40!}{4!b!}$; $(1+x)^n$ coefficients of x^4 and x^5 are p and q respectively. $b = 36$ Candidates should usually “identify” two terms as their p and q respectively.	B1 (1)
(b)	<p>Term 1: $\binom{40}{4}$ or ${}^{40}C_4$ or $\frac{40!}{4!36!}$ or $\frac{40(39)(38)(37)}{4!}$ or 91390</p> <p>Term 2: $\binom{40}{5}$ or ${}^{40}C_5$ or $\frac{40!}{5!35!}$ or $\frac{40(39)(38)(37)(36)}{5!}$ or 658008</p> <p>Hence, $\frac{q}{p} = \frac{658008}{91390} \left\{ = \frac{36}{5} = 7.2 \right\}$</p>	<p>Any one of Term 1 or Term 2 correct. (Ignore the label of p and/or q.) M1</p> <p>Both of them correct. (Ignore the label of p and/or q.) A1</p> <p>for $\frac{658008}{91390}$ oe A1 oe cso (3) [4]</p>
Notes		
(a)	B1: for only $b = 36$.	
(b)	<p>The candidate may expand out their binomial series. At this stage no marks should be awarded until they start to identify either one or both of the terms that they want to focus on. Once they identify their terms then if one out of two of them (ignoring which one is p and which one is q) is correct then award M1. If both of the terms are identified correctly (ignoring which one is p and which one is q) then award the first A1.</p> <p>Term 1 = $\binom{40}{4}x^4$ or ${}^{40}C_4(x^4)$ or $\frac{40!}{4!36!}x^4$ or $\frac{40(39)(38)(37)}{4!}x^4$ or $91390x^4$,</p> <p>Term 2 = $\binom{40}{5}x^5$ or ${}^{40}C_5(x^5)$ or $\frac{40!}{5!35!}x^5$ or $\frac{40(39)(38)(37)(36)}{5!}x^5$ or $658008x^5$</p> <p>are fine for any (or both) of the first two marks in part (b).</p> <p>2nd A1 for stating $\frac{q}{p}$ as $\frac{658008}{91390}$ or equivalent. Note that $\frac{q}{p}$ must be independent of x.</p> <p>Also note that $\frac{36}{5}$ or 7.2 or any equivalent fraction is fine for the 2nd A1 mark.</p> <p>SC: If candidate states $\frac{p}{q} = \frac{5}{36}$, then award M1A1A0.</p> <p>Note that either $\frac{4!36!}{5!35!}$ or $\frac{5!35!}{4!36!}$ would be awarded M1A1.</p>	

Question Number	Scheme	Marks
6.		
(a)	$\begin{array}{c c c c c c} x & 2 & 2.25 & 2.5 & 2.75 & 3 \\ \hline y & 0.5 & 0.38 & 0.298507... & 0.241691... & 0.2 \end{array}$ <p>At $\{x = 2.5,\}$ $y = 0.30$ (only)</p> <p>At $\{x = 2.75,\}$ $y = 0.24$ (only)</p>	<p>At least one y-ordinate correct. B1</p> <p>Both y-ordinates correct. B1</p> <p>(2)</p>
(b)	$\frac{1}{2} \times 0.25 ; \times \{ 0.5 + 0.2 + 2(0.38 + \text{their } 0.30 + \text{their } 0.24) \}$ $\{ = \frac{1}{8}(2.54) \} = \text{awrt } 0.32$	<p>Outside brackets $\frac{1}{2} \times 0.25$ or $\frac{1}{8}$ B1 aef</p> <p>For structure of $\{ \dots \}$; M1</p> <p>Correct expression <u>inside brackets</u> which all must be multiplied by their “outside constant”. $\text{A1} \sqrt{}$</p> <p>awrt 0.32 A1</p> <p>(4)</p>
(c)	<p>Area of triangle $= \frac{1}{2} \times 1 \times 0.2 = 0.1$</p> <p>Area(S) = "0.3175" – 0.1</p> <p>= 0.2175</p>	<p>B1</p> <p>M1</p> <p>A1 ft</p> <p>(3)</p> <p>[9]</p>

Question Number	Scheme	Marks
	<u>Notes</u>	
(b)	<p>B1 for using $\frac{1}{2} \times 0.25$ or $\frac{1}{8}$ or equivalent.</p> <p>M1 requires the correct {.....} bracket structure. This is for the first bracket to contain first y-ordinate plus last y-ordinate and the second bracket to be the summation of the remaining y-ordinates in the table.</p> <p>No errors (eg. an omission of a y-ordinate or an extra y-ordinate or a repeated y-ordinate) are allowed in the second bracket and the second bracket must be multiplied by 2. Only one copying error is allowed here in the $2(0.38 + \text{their } 0.30 + \text{their } 0.24)$ bracket.</p> <p>A1ft for the correct bracket {.....} following through candidate's y-ordinates found in part (a).</p> <p>A1 for answer of awrt 0.32 .</p> <p><u>Bracketing mistake:</u> Unless the final answer implies that the calculation has been done correctly</p> <p>then award M1A0A0 for either $\frac{1}{2} \times 0.25 \times 0.5 + 2(0.38 + \text{their } 0.30 + \text{their } 0.24) + 0.2$</p> <p>(nb: yielding final answer of 2.1025) so that the 0.5 is only multiplied by $\frac{1}{2} \times 0.25$</p> <p>or $\frac{1}{2} \times 0.25 \times (0.5 + 0.2) + 2(0.38 + \text{their } 0.30 + \text{their } 0.24)$</p> <p>(nb: yielding final answer of 1.9275) so that the $(0.5 + 0.2)$ is multiplied by $\frac{1}{2} \times 0.25$.</p> <p>Need to see trapezium rule – answer only (with no working) gains no marks.</p> <p><u>Alternative:</u> Separate trapezia may be used, and this can be marked equivalently. (See appendix.)</p>	
(c)	<p>B1 for the area of the triangle identified as either $\frac{1}{2} \times 1 \times 0.2$ or 0.1. May be identified on the diagram.</p> <p>M1 for “part (b) answer” – “0.1 only” or “part (b) answer – their attempt at 0.1 only”. (Strict attempt!)</p> <p>A1ft for correctly following through “part (b) answer” – 0.1. This is also dependent on the answer to (b) being greater than 0.1. Note: candidates may round answers here, so allow A1ft if they round their answer correct to 2 dp.</p>	

Question Number	Scheme	Marks
7.		
(a)	$3\sin^2 x + 7\sin x = \cos^2 x - 4; \quad 0 \leq x < 360^\circ$ $3\sin^2 x + 7\sin x = (1 - \sin^2 x) - 4$ $4\sin^2 x + 7\sin x + 3 = 0$ AG	M1 A1 * cso (2)
(b)	<div style="display: flex; justify-content: space-between;"> <div> $(4\sin x + 3)(\sin x + 1) \{= 0\}$ $\sin x = -\frac{3}{4}, \quad \sin x = -1$ $(\alpha = 48.59\dots)$ $x = 180 + 48.59 \quad \text{or} \quad x = 360 - 48.59$ $x = 228.59\dots, \quad x = 311.41\dots$ $\{\sin x = -1\} \Rightarrow x = 270$ </div> <div> Valid attempt at factorisation and $\sin x = \dots$ Both $\sin x = -\frac{3}{4}$ and $\sin x = -1$. Either $(180 + \alpha)$ or $(360 - \alpha)$ Both awrt 228.6 and awrt 311.4 270 </div> </div>	M1 A1 dM1 A1 B1 (5) [7]
	Notes	
(a)	M1 for a correct method to change $\cos^2 x$ into $\sin^2 x$ (must use $\cos^2 x = 1 - \sin^2 x$). Note that applying $\cos^2 x = \sin^2 x - 1$, scores M0. A1 for obtaining the printed answer without error (except for implied use of zero.), although the equation at the end of the proof must be = 0 . Solution just written only as above would score M1A1.	
(b)	1 st M1 for a valid attempt at factorisation, can use any variable here, s , y , x or $\sin x$, and an attempt to find at least one of the solutions. <i>Alternatively</i> , using a correct formula for solving the quadratic. Either the formula must be stated correctly or the correct form must be implied by the substitution. 1 st A1 for the two correct values of $\sin x$. If they have used a substitution, a correct value of their s or their y or their x . 2 nd M1 for solving $\sin x = -k$, $0 < k < 1$ and realising a solution is either of the form $(180 + \alpha)$ or $(360 - \alpha)$ where $\alpha = \sin^{-1}(k)$. Note that you cannot access this mark from $\sin x = -1 \Rightarrow x = 270$. Note that this mark is dependent upon the 1 st M1 mark awarded. 2 nd A1 for both awrt 228.6 and awrt 311.4 B1 for 270. If there are any EXTRA solutions inside the range $0 \leq x < 360^\circ$ and the candidate would otherwise score FULL MARKS then withhold the final bA2 mark (the fourth mark in this part of the question). Also ignore EXTRA solutions outside the range $0 \leq x < 360^\circ$. Working in Radians: Note the answers in radians are $x = 3.9896\dots, 5.4351\dots, 4.7123\dots$ If a candidate works in radians then mark part (b) as above awarding the 2 nd A1 for both awrt 4.0 and awrt 5.4 and the B1 for awrt 4.7 or $\frac{3\pi}{2}$. If the candidate would then score FULL MARKS then withhold the final bA2 mark (the fourth mark in this part of the question.) No working: Award B1 for 270 seen without any working. Award M0A0M1A1 for awrt 228.6 and awrt 311.4 seen without any working. Award M0A0M1A0 for any one of awrt 228.6 or awrt 311.4 seen without any working.	

Question Number	Scheme	Marks
8.	<p>(a) Graph of $y = 7^x$, $x \in \mathbb{R}$ and solving $7^{2x} - 4(7^x) + 3 = 0$</p>  <p>At least two of the three criteria correct. (See notes below.)</p> <p>All three criteria correct. (See notes below.)</p>	<p>B1</p> <p>B1</p> <p>(2)</p>
(b)	<p>$y^2 - 4y + 3 = 0$</p> <p>$\{ (y-3)(y-1) = 0 \text{ or } (7^x-3)(7^x-1) = 0 \}$</p> <p>$y = 3, y = 1 \text{ or } 7^x = 3, 7^x = 1$</p> <p>$\{7^x = 3 \Rightarrow\} x \log 7 = \log 3$</p> <p>or $x = \frac{\log 3}{\log 7}$ or $x = \log_7 3$</p> <p>$x = 0.5645\dots$</p> <p>$x = 0$</p> <p>Forming a quadratic {using "y" = 7^x}.</p> <p>$y^2 - 4y + 3 = 0$</p> <p>Both $y = 3$ and $y = 1$.</p> <p>A valid method for solving $7^x = k$ where $k > 0, k \neq 1$</p> <p>0.565 or awrt 0.56</p> <p>$x = 0$ stated as a solution.</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>dM1</p> <p>A1</p> <p>B1</p> <p>(6)</p> <p>[8]</p>
	Notes	
(a)	<p>B1B0: Any two of the following three criteria below correct.</p> <p>B1B1: All three criteria correct.</p> <p>Criteria number 1: Correct shape of curve for $x \geq 0$.</p> <p>Criteria number 2: Correct shape of curve for $x < 0$.</p> <p>Criteria number 3: (0, 1) stated or 1 marked on the y-axis. Allow (1, 0) rather than (0, 1) if marked in the "correct" place on the y-axis.</p>	

Question Number	Scheme	Marks
(b)	<p>1st M1 is an attempt to form a quadratic equation {using "$y = 7^x$".}</p> <p>1st A1 mark is for the correct quadratic equation of $y^2 - 4y + 3 = 0$.</p> <p>Can use any variable here, eg: y, x or 7^x. Allow M1A1 for $x^2 - 4x + 3 = 0$.</p> <p>Writing $(7^x)^2 - 4(7^x) + 3 = 0$ is also sufficient for M1A1.</p> <p>Award M0A0 for seeing $7^{x^2} - 4(7^x) + 3 = 0$ by itself without seeing $y^2 - 4y + 3 = 0$ or $(7^x)^2 - 4(7^x) + 3 = 0$.</p> <p>1st A1 mark for both $y = 3$ and $y = 1$ or both $7^x = 3$ and $7^x = 1$. Do not give this accuracy mark for both $x = 3$ and $x = 1$, unless these are recovered in later working by candidate applying logarithms on these.</p> <p>Award M1A1A1 for $7^x = 3$ and $7^x = 1$ written down with no earlier working.</p> <p>3rd dM1 for solving $7^x = k, k > 0, k \neq 1$ to give either $x \ln 7 = \ln k$ or $x = \frac{\ln k}{\ln 7}$ or $x = \log_7 k$.</p> <p>dM1 is dependent upon the award of M1.</p> <p>2nd A1 for 0.565 or awrt 0.56. B1 is for the solution of $x = 0$, from any working.</p>	

Question Number	Scheme	Marks
9.		
(a)	$C\left(\frac{-2+8}{2}, \frac{11+1}{2}\right) = C(3, 6)$ AG	Correct method (no errors) for finding the mid-point of AB giving $(3, 6)$ B1* (1)
(b)	$(8-3)^2 + (1-6)^2$ or $\sqrt{(8-3)^2 + (1-6)^2}$ or $(-2-3)^2 + (11-6)^2$ or $\sqrt{(-2-3)^2 + (11-6)^2}$ $(x-3)^2 + (y-6)^2 = 50$ (or $(\sqrt{50})^2$ or $(5\sqrt{2})^2$)	Applies distance formula in order to find the radius. Correct application of formula. $(x \pm 3)^2 + (y \pm 6)^2 = k$, k is a positive <u>value</u> . $(x-3)^2 + (y-6)^2 = 50$ (Not 7.07^2) M1 A1 M1 A1 (4)
(c)	{For $(10, 7)$, } $\underline{(10-3)^2 + (7-6)^2 = 50}$, {so the point lies on C .}	<u>B1</u> (1)
(d)	{Gradient of radius} = $\frac{7-6}{10-3}$ or $\frac{1}{7}$ Gradient of tangent = $\frac{-7}{1}$ $y-7 = -7(x-10)$ $y = -7x + 77$	This must be seen in part (d). Using a perpendicular gradient method. $y-7 = (\text{their gradient})(x-10)$ $y = -7x + 77$ or $y = 77 - 7x$ B1 M1 M1 A1 cao (4) [10]
Notes		
(a)	Alternative method: $C\left(-2 + \frac{8-2}{2}, 11 + \frac{1-11}{2}\right)$ or $C\left(8 + \frac{-2-8}{2}, 1 + \frac{11-1}{2}\right)$	
(b)	You need to be convinced that the candidate is attempting to work out the radius and not the diameter of the circle to award the first M1. Therefore allow 1 st M1 generously for $\frac{(-2-8)^2 + (11-1)^2}{2}$ Award 1 st M1A1 for $\frac{(-2-8)^2 + (11-1)^2}{4}$ or $\frac{\sqrt{(-2-8)^2 + (11-1)^2}}{2}$. Correct answer in (b) with no working scores full marks.	
(c)	B1 awarded for correct verification of $\underline{(10-3)^2 + (7-6)^2 = 50}$ with no errors. Also to gain this mark candidates need to have the correct equation of the circle either from part (b) or re-attempted in part (c). They cannot verify $(10, 7)$ lies on C without a correct C . Also a candidate could either substitute $x = 10$ in C to find $y = 7$ or substitute $y = 7$ in C to find $x = 10$.	

Question Number	Scheme	Marks
(d)	<p>2nd M1 mark also for the complete method of applying $7 = (\text{their gradient})(10) + c$, finding c. Note: Award 2nd M0 in (d) if their numerical gradient is either 0 or ∞.</p> <p>Alternative: For first two marks (differentiation): $2(x - 3) + 2(y - 6)\frac{dy}{dx} = 0$ (or equivalent) scores B1.</p> <p>1st M1 for substituting both $x = 10$ and $y = 7$ to find a value for $\frac{dy}{dx}$, which must contain both x and y. (This M mark can be awarded generously, even if the attempted “differentiation” is not “implicit”.) Alternative: $(10 - 3)(x - 3) + (7 - 6)(y - 6) = 50$ scores B1M1M1 which leads to $y = -7x + 77$.</p>	

Question Number	Scheme		Marks
10.			
(a)	$V = 4x(5 - x)^2 = 4x(25 - 10x + x^2)$ So, $V = 100x - 40x^2 + 4x^3$ $\frac{dV}{dx} = 100 - 80x + 12x^2$	$\pm \alpha x \pm \beta x^2 \pm \gamma x^3$, where $\alpha, \beta, \gamma \neq 0$ $V = 100x - 40x^2 + 4x^3$ At least two of their expanded terms differentiated correctly. $100 - 80x + 12x^2$	M1 A1 M1 A1 cao (4)
(b)	$100 - 80x + 12x^2 = 0$ $\left\{ \Rightarrow 4(3x^2 - 20x + 25) = 0 \Rightarrow 4(3x - 5)(x - 5) = 0 \right\}$ $\left\{ \text{As } 0 < x < 5 \right\} \quad x = \frac{5}{3}$ $x = \frac{5}{3}, \quad V = 4\left(\frac{5}{3}\right)\left(5 - \frac{5}{3}\right)^2$ So, $V = \frac{2000}{27} = 74\frac{2}{27} = 74.074\dots$	Sets their $\frac{dV}{dx}$ from part (a) = 0 $x = \frac{5}{3}$ or $x = \text{awrt } 1.67$ Substitute candidate's value of x where $0 < x < 5$ into a formula for V . Either $\frac{2000}{27}$ or $74\frac{2}{27}$ or awrt 74.1	M1 A1 dM1 A1 (4)
(c)	$\frac{d^2V}{dx^2} = -80 + 24x$ When $x = \frac{5}{3}, \quad \frac{d^2V}{dx^2} = -80 + 24\left(\frac{5}{3}\right)$ $\frac{d^2V}{dx^2} = -40 < 0 \Rightarrow V$ is a maximum	Differentiates their $\frac{dV}{dx}$ correctly to give $\frac{d^2V}{dx^2}$. $\frac{d^2V}{dx^2} = -40$ and <u>< 0 or negative</u> and <u>maximum</u> .	M1 A1 cso (2) [10]
Notes			
(a)	1 st M1 for a three term cubic in the form $\pm \alpha x \pm \beta x^2 \pm \gamma x^3$. Note that an un-combined $\pm \alpha x \pm \lambda x^2 \pm \mu x^2 \pm \gamma x^3$, $\alpha, \lambda, \mu, \gamma \neq 0$ is fine for the 1 st M1. 1 st A1 for either $100x - 40x^2 + 4x^3$ or $100x - 20x^2 - 20x^2 + 4x^3$. 2 nd M1 for any two of their expanded terms differentiated correctly. NB: If expanded expression is divided by a constant, then the 2 nd M1 can be awarded for at least two terms are correct. Note for un-combined $\pm \lambda x^2 \pm \mu x^2, \pm 2\lambda x \pm 2\mu x$ counts as one term differentiated correctly. 2 nd A1 for $100 - 80x + 12x^2$, cao . Note: See appendix for those candidates who apply the product rule of differentiation.		

Question Number	Scheme	Marks
(b)	<p>Note you can mark parts (b) and (c) together.</p> <p>Ignore the extra solution of $x = 5$ (and $V = 0$). Any extra solutions for V inside found for values inside the range of x, then award the final A0.</p>	
(c)	<p>M1 is for their $\frac{dV}{dx}$ differentiated correctly (follow through) to give $\frac{d^2V}{dx^2}$.</p> <p>A1 for all three of $\frac{d^2V}{dx^2} = -40$ and <u>< 0 or negative</u> and <u>maximum</u>.</p> <p>Ignore any second derivative testing on $x = 5$ for the final accuracy mark.</p> <p><u>Alternative Method: Gradient Test:</u> M1 for finding the gradient either side of their x-value from part (b) where $0 < x < 5$. A1 for <u>both gradients calculated correctly to the near integer, using > 0 and < 0 respectively or a correct sketch and maximum</u>. (See appendix for gradient values.)</p>	

Question Number	Scheme	Marks
<p><i>Aliter</i></p> <p>4 (b) Way 2</p>	$(x+1)(x-5) = \underline{x^2 - 4x - 5} \text{ or } \underline{x^2 - 5x + x - 5}$ $- \int (x^2 - 4x - 5) dx = -\frac{x^3}{3} + \frac{4x^2}{2} + 5x \{+c\}$ $\left[-\frac{x^3}{3} + \frac{4x^2}{2} + 5x \right]_{-1}^5 = (\dots) - (\dots)$ $\left\{ \left(-\frac{125}{3} + \frac{100}{2} + 25 \right) - \left(\frac{1}{3} + 2 - 5 \right) \right\}$ $= \left(\frac{100}{3} \right) - \left(-\frac{8}{3} \right)$ <p>Hence, Area = 36</p>	<p>Can be implied by later working.</p> <p>M: $x^n \rightarrow x^{n+1}$ for any one term.</p> <p>1st A1 any two out of three terms correctly ft.</p> <p>Substitutes 5 and -1 (or limits from part(a)) into an “integrated function” and subtracts, either way round.</p> <p>B1</p> <p>M1A1ft A1</p> <p>dM1</p> <p>A1</p> <p>(6)</p>

Question Number	Scheme	Marks
<i>Aliter</i> 6 (b) Way 2	$0.25 \times \left\{ \frac{0.5 + 0.38}{2} + \frac{0.38 + 0.30}{2} + \frac{0.30 + 0.24}{2} + \frac{0.24 + 0.2}{2} \right\}$ <p>which is equivalent to:</p> $\frac{1}{2} \times 0.25 ; \times \left\{ (0.5 + 0.2) + 2(0.38 + \text{their } 0.30 + \text{their } 0.24) \right\}$ $\left\{ = \frac{1}{8}(2.54) \right\} = \text{awrt } 0.32$	<p>0.25 and a divisor of 2 on all terms inside brackets.</p> <p>One of first and last ordinates, two of the middle ordinates inside brackets ignoring the denominator of 2.</p> <p>Correct expression inside brackets if $\frac{1}{2}$ was to be factorised out.</p> <p>awrt 0.32</p> <p>B1</p> <p>M1</p> <p>A1 $\sqrt{\quad}$</p> <p>A1</p> <p>(4)</p>

Question Number	Scheme	Marks
<i>Aliter</i> 10 (a) Way2	Product Rule Method: $\left\{ \begin{array}{ll} u = 4x & v = (5 - x)^2 \\ \frac{du}{dx} = 4 & \frac{dv}{dx} = 2(5 - x)^1(-1) \end{array} \right\}$ $\frac{dy}{dx} = 4(5 - x)^2 + 4x(2)(5 - x)^1(-1)$ $\frac{dy}{dx} = 4(5 - x)^2 - 8x(5 - x)$	$\pm(\text{their } u')(5 - x)^2 \pm (4x)(\text{their } v')$ M1 A correct attempt at differentiating any one of either u or v correctly. dM1 Both $\frac{du}{dx}$ and $\frac{dv}{dx}$ correct A1 $4(5 - x)^2 - 8x(5 - x)$ A1 (4)
<i>Aliter</i> 10 (a) Way3	$\left\{ \begin{array}{ll} u = 4x & v = 25 - 10x + x^2 \\ \frac{du}{dx} = 4 & \frac{dv}{dx} = -10 + 2x \end{array} \right\}$ $\frac{dy}{dx} = 4(25 - 10x + x^2) + 4x(-10 + 2x)$ $\frac{dV}{dx} = 100 - 80x + 12x^2$	$\pm(\text{their } u')(\text{their } (5 - x)^2) \pm (4x)(\text{their } v')$ M1 A correct attempt at differentiating any one of either u or their v correctly. dM1 Both $\frac{du}{dx}$ and $\frac{dv}{dx}$ correct A1 $100 - 80x + 12x^2$ A1 (4)
	Note: The candidate needs to use a complete product rule method in order for you to award the the first M1 mark here. The second method mark is dependent on the first method mark awarded.	

Question Number	Scheme	Marks																																								
<i>Aliter</i> 10 (c) <i>Way 2</i>	Gradient Test Method: $\frac{dV}{dx} = 100 - 80x + 12x^2$ Helpful table! <table><tr><th>x</th><th>$\frac{dV}{dx}$</th></tr><tr><td>0.8</td><td>43.68</td></tr><tr><td>0.9</td><td>37.72</td></tr><tr><td>1</td><td>32</td></tr><tr><td>1.1</td><td>26.52</td></tr><tr><td>1.2</td><td>21.28</td></tr><tr><td>1.3</td><td>16.28</td></tr><tr><td>1.4</td><td>11.52</td></tr><tr><td>1.429</td><td>10.204</td></tr><tr><td>1.5</td><td>7</td></tr><tr><td>1.6</td><td>2.72</td></tr><tr><td>1.7</td><td>-1.32</td></tr><tr><td>1.8</td><td>-5.12</td></tr><tr><td>1.9</td><td>-8.68</td></tr><tr><td>2</td><td>-12</td></tr><tr><td>2.1</td><td>-15.08</td></tr><tr><td>2.2</td><td>-17.92</td></tr><tr><td>2.3</td><td>-20.52</td></tr><tr><td>2.4</td><td>-22.88</td></tr><tr><td>2.5</td><td>-25</td></tr></table>	x	$\frac{dV}{dx}$	0.8	43.68	0.9	37.72	1	32	1.1	26.52	1.2	21.28	1.3	16.28	1.4	11.52	1.429	10.204	1.5	7	1.6	2.72	1.7	-1.32	1.8	-5.12	1.9	-8.68	2	-12	2.1	-15.08	2.2	-17.92	2.3	-20.52	2.4	-22.88	2.5	-25	
x	$\frac{dV}{dx}$																																									
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2.5	-25																																									

Question Number	Scheme	Marks																																																				
8 (b)	<p>Method of trial and improvement Helpful table:</p> <table><tr><td>x</td><td>$y = 7^{2x} - 4(7^x) + 3$</td></tr><tr><td>0</td><td>0</td></tr><tr><td>0.1</td><td>-0.38348</td></tr><tr><td>0.2</td><td>-0.72519</td></tr><tr><td>0.3</td><td>-0.95706</td></tr><tr><td>0.4</td><td>-0.96835</td></tr><tr><td>0.5</td><td>-0.58301</td></tr><tr><td>0.51</td><td>-0.51316</td></tr><tr><td>0.52</td><td>-0.43638</td></tr><tr><td>0.53</td><td>-0.3523</td></tr><tr><td>0.54</td><td>-0.26055</td></tr><tr><td>0.55</td><td>-0.16074</td></tr><tr><td>0.56</td><td>-0.05247</td></tr><tr><td>0.561</td><td>-0.04116</td></tr><tr><td>0.562</td><td>-0.02976</td></tr><tr><td>0.563</td><td>-0.01828</td></tr><tr><td>0.564</td><td>-0.0067</td></tr><tr><td>0.565</td><td>0.00497</td></tr><tr><td>0.57</td><td>0.064688</td></tr><tr><td>0.58</td><td>0.19118</td></tr><tr><td>0.59</td><td>0.327466</td></tr><tr><td>0.6</td><td>0.474029</td></tr><tr><td>0.7</td><td>2.62723</td></tr><tr><td>0.8</td><td>6.525565</td></tr><tr><td>0.9</td><td>13.15414</td></tr><tr><td>1</td><td>24</td></tr></table> <p>For a full method of trial and improvement by trialing f(value between 0.1 and 0.5645) = value and f(value between 0.5645 and 1) = value Any one of these values correct to 1sf or truncated 1sf. Both of these values correct to 1sf or truncated 1sf. A method to confirm root to 2 dp by finding by trialing f(value between 0.56and 0.5645) = value and f(value between 0.5645 and 0.565) = value Both values correct to 1sf or truncated 1sf and the confirmation that the root is $x = 0.56$ (only) $x = 0$</p>	x	$y = 7^{2x} - 4(7^x) + 3$	0	0	0.1	-0.38348	0.2	-0.72519	0.3	-0.95706	0.4	-0.96835	0.5	-0.58301	0.51	-0.51316	0.52	-0.43638	0.53	-0.3523	0.54	-0.26055	0.55	-0.16074	0.56	-0.05247	0.561	-0.04116	0.562	-0.02976	0.563	-0.01828	0.564	-0.0067	0.565	0.00497	0.57	0.064688	0.58	0.19118	0.59	0.327466	0.6	0.474029	0.7	2.62723	0.8	6.525565	0.9	13.15414	1	24	<p>M1 A1 A1 M1 A1 B1 (6)</p>
x	$y = 7^{2x} - 4(7^x) + 3$																																																					
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	<p>Note: If a candidate goes from $7^x = 3$ with no working to $x = 0.5645...$ then give M1A1 implied.</p>																																																					

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Mark Scheme (Results)

June 2011

GCE Core Mathematics C2 (6664) Paper 1

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June 2011

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EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod – benefit of doubt
- ft – follow through
- the symbol $\hat{=}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- \square The second mark is dependent on gaining the first mark

June 2011
Core Mathematics C2 6664
Mark Scheme

Question Number	Scheme	Marks
1.	$f(x) = 2x^3 - 7x^2 - 5x + 4$	
(a)	Remainder = $f(1) = 2 - 7 - 5 + 4 = -6$ $= -6$	Attempts $f(1)$ or $f(-1)$. -6 M1 A1 [2]
(b)	$f(-1) = 2(-1)^3 - 7(-1)^2 - 5(-1) + 4$ and so $(x + 1)$ is a factor.	Attempts $f(-1)$. $f(-1) = 0$ with no sign or substitution errors and for conclusion. M1 A1 [2]
(c)	$f(x) = \{(x + 1)\}(2x^2 - 9x + 4)$ $= (x + 1)(2x - 1)(x - 4)$ (Note: Ignore the ePEN notation of (b) (should be (c)) for the final three marks in this part).	M1 A1 dM1 A1 [4] 8
(a)	M1 for attempting either $f(1)$ or $f(-1)$. Can be implied. Only one slip permitted. M1 can also be given for an attempt (at least two “subtracting” processes) at long division to give a remainder which is independent of x . A1 can be given also for -6 seen at the bottom of long division working. Award A0 for a candidate who finds -6 but then states that the remainder is 6. Award M1A1 for -6 without any working.	
(b)	M1: attempting only $f(-1)$. A1: must correctly show $f(-1) = 0$ and give a conclusion in part (b) only . Note: Stating “hence factor” or “it is a factor” or a “tick” or “QED” is fine for the conclusion. Note also that a conclusion can be implied from a <u>preamble</u> , eg: “If $f(-1) = 0$, $(x + 1)$ is a factor...” Note: Long division scores no marks in part (b). The factor theorem is required.	
(c)	1 st M1: Attempts long division or other method, to obtain $(2x^2 \pm ax \pm b)$, $a \neq 0$, even with a remainder. Working need not be seen as this could be done “by inspection.” $(2x^2 \pm ax \pm b)$ must be seen in part (c) only . Award 1 st M0 if the quadratic factor is clearly found from dividing $f(x)$ by $(x - 1)$. Eg. Some candidates use their $(2x^2 - 5x - 10)$ in part (c) found from applying a long division method in part (a). 1 st A1: For seeing $(2x^2 - 9x + 4)$. 2 nd dM1: Factorises a 3 term quadratic. (see rule for factorising a quadratic). This is dependent on the previous method mark being awarded. This mark can also be awarded if the candidate applies the quadratic formula correctly. 2 nd A1: is cao and needs all three factors on one line. Ignore following work (such as a solution to a quadratic equation.) Note: Some candidates will go from $\{(x + 1)\}(2x^2 - 9x + 4)$ to $\{x = -1\}$, $x = \frac{1}{2}$, 4 , and not list all three factors. Award these responses M1A1M1A0. Alternative: 1 st M1: For finding either $f(4) = 0$ or $f(\frac{1}{2}) = 0$. 1 st A1: A second correct factor of usually $(x - 4)$ or $(2x - 1)$ found. Note that any one of the other correct factors found would imply the 1 st M1 mark. 2 nd dM1: For using two known factors to find the third factor, usually $(2x \pm 1)$. 2 nd A1 for correct answer of $(x + 1)(2x - 1)(x - 4)$. Alternative: (for the first two marks) 1 st M1: Expands $(x + 1)(2x^2 + ax + b)$ {giving $2x^3 + (a + 2)x^2 + (b + a)x + b$ } then compare coefficients to find <u>values</u> for a and b . 1 st A1: $a = -9$, $b = 4$ Not dealing with a factor of 2: $(x + 1)(x - \frac{1}{2})(x - 4)$ or $(x + 1)(x - \frac{1}{2})(2x - 8)$ scores M1A1M1A0. Answer only, with one sign error: eg. $(x + 1)(2x + 1)(x - 4)$ or $(x + 1)(2x - 1)(x + 4)$ scores M1A1M1A0. (c) Award M1A1M1A1 for Listing all three correct factors with no working.	

Question Number	Scheme	Marks
2.	243 as a constant term seen. $\{(3 + bx)^5\} = (3)^5 + {}^5C_1(3)^4(bx) + {}^5C_2(3)^3(bx)^2 + \dots$ $= 243 + 405bx + 270b^2x^2 + \dots$ $({}^5C_1 \times \dots \times x) \text{ or } ({}^5C_2 \times \dots \times x^2)$ $270b^2x^2 \text{ or } 270(bx)^2$	B1 B1 M1 A1 [4]
(a)	Establishes an equation from their coefficients. Condone 2 on the wrong side of the equation. $\{2(\text{coeff } x) = \text{coeff } x^2\} \Rightarrow 2(405b) = 270b^2$ $b = 3 \text{ (Ignore } b = 0, \text{ if seen.)}$	M1 A1
(b)	So, $\left\{b = \frac{810}{270} \Rightarrow\right\} b = 3$	[2] 6
(a)	<p>The terms can be “listed” rather than added. Ignore any extra terms. 1st B1: A constant term of 243 seen. Just writing $(3)^5$ is B0. 2nd B1: Term must be simplified to $405bx$ for B1. The x is required for this mark. Note $405 + bx$ is B0. M1: For <u>either</u> the x term <u>or</u> the x^2 term. Requires <u>correct</u> binomial coefficient in any form <u>with the correct power of x</u>, but the other part of the coefficient (perhaps including powers of 3 and/or b) may be wrong or missing. <u>Allow</u> binomial coefficients such as $\binom{5}{2}, \left(\frac{5}{2}\right), \binom{5}{1}, \left(\frac{5}{1}\right), {}^5C_2, {}^5C_1$. A1: For either $270b^2x^2$ or $270(bx)^2$. (If $270bx^2$ follows $270(bx)^2$, isw and allow A1.) <u>Alternative:</u> Note that a factor of 3^5 can be taken out first: $3^5\left(1 + \frac{bx}{3}\right)^5$, but the mark scheme still applies. <u>Ignore subsequent working (isw):</u> Isw if necessary after correct working: e.g. $243 + 405bx + 270b^2x^2 + \dots$ leading to $9 + 15bx + 10b^2x^2 + \dots$ scores B1B1M1A1 isw. Also note that full marks could also be available in part (b), here. <u>Special Case:</u> Candidate writing down the first three terms in <i>descending</i> powers of x usually get $(bx)^5 + {}^5C_4(3)^1(bx)^4 + {}^5C_3(3)^2(bx)^3 + \dots = b^5x^5 + 15b^4x^4 + 90b^3x^3 + \dots$ So award SC: B0B0M1A0 for either $({}^5C_4 \times \dots \times x^4)$ or $({}^5C_3 \times \dots \times x^3)$</p>	
(b)	<p>M1 for equating 2 times their coefficient of x to the coefficient of x^2 to get an equation in b, <u>or</u> equating their coefficient of x to 2 times that of x^2, to get an equation in b. Allow this M mark even if the equation is trivial, providing their coefficients from part (a) have been used, eg: $2(405b) = 270b$, but beware $b = 3$ from this, which is A0. <u>An equation in b alone is required:</u> e.g. $2(405b)x = 270b^2x^2 \Rightarrow b = 3$ or similar will be Special Case SC: M1A0 (as equation in coefficients only is not seen here). e.g. $2(405b)x = 270b^2x^2 \Rightarrow 2(405b) = 270b^2 \Rightarrow b = 3$ will get M1A1 (as coefficients rather than terms have now been considered). Note: Answer of 3 from no working scores M1A0. Note: The mistake $k\left(1 + \frac{bx}{3}\right)^5$, $k \neq 243$ would give a maximum of 3 marks: B0B0M1A0, M1A1 Note: For $270bx^2$ in part (a), followed by $2(405b) = 270b^2 \Rightarrow b = 3$, in part (b), allow recovery M1A1.</p>	

Question Number	Scheme	Marks
3.	(a) $5^x = 10$ and (b) $\log_3(x - 2) = -1$	
(a)	$x = \frac{\log 10}{\log 5}$ or $x = \log_5 10$ $x \{= 1.430676558...\} = 1.43$ (3 sf)	M1 1.43 A1 cao [2]
(b)	$(x - 2) = 3^{-1}$ $x \{= \frac{1}{3} + 2\} = 2\frac{1}{3}$	$(x - 2) = 3^{-1}$ or $\frac{1}{3}$ $2\frac{1}{3}$ or $\frac{7}{3}$ or $2.\dot{3}$ or awrt 2.33 M1 oe A1 [2] 4
(a)	M1: for $x = \frac{\log 10}{\log 5}$ or $x = \log_5 10$. Also allow M1 for $x = \frac{1}{\log 5}$ 1.43 with no working (or any working) scores M1A1 (even if left as $5^{1.43}$). Other answers which round to 1.4 with no working score M1A0. <u>Trial & Improvement Method:</u> M1: For a method of trial and improvement by trialing f (value between 1.4 and 1.43) = Value below 10 and f (value between 1.431 and 1.5) = Value over 10. A1 for 1.43 cao. <u>Note:</u> $x = \log_{10} 5$ by itself is M0; but $x = \log_{10} 5$ followed by $x = 1.430676558...$ is M1.	
(b)	M1: Is for correctly eliminating log out of the equation. Eq 1: $\log_3(x - 2) = \log_3(\frac{1}{3}) \Rightarrow x - 2 = \frac{1}{3}$ only gets M1 when the logs are correctly removed. Eq 2: $\log_3(x - 2) = -\log_3(3) \Rightarrow \log_3(x - 2) + \log_3(3) = 0 \Rightarrow \log_3(3(x - 2)) = 0$ $\Rightarrow 3(x - 2) = 3^0$ only gets M1 when the logs are correctly removed, but $3(x - 2) = 0$ would score M0. Note: $\log_3(x - 2) = -1 \Rightarrow \log_3\left(\frac{x}{2}\right) = -1 \Rightarrow \frac{x}{2} = 3^{-1}$ would score M0 for incorrect use of logs. <u>Alternative: changing base</u> $\frac{\log_{10}(x - 2)}{\log_{10} 3} = -1 \Rightarrow \log_{10}(x - 2) = -\log_{10} 3 \Rightarrow \log_{10}(x - 2) + \log_{10} 3 = 0$ $\Rightarrow \log_{10} 3(x - 2) = 0 \Rightarrow 3(x - 2) = 10^0$. At this point M1 is scored. A correct answer in (b) without any working scores M1A1.	

Question Number	Scheme	Marks
4.	$x^2 + y^2 + 4x - 2y - 11 = 0$	
(a)	$\{(x+2)^2 - 4 + (y-1)^2 - 1 - 11 = 0\}$ ($\pm 2, \pm 1$), see notes. Centre is $(-2, 1)$. ($-2, 1$).	M1 A1 cao [2]
(b)	$(x+2)^2 + (y-1)^2 = 11 + 1 + 4$ $r = \sqrt{11 + "1" + "4"}$ So $r = \sqrt{11 + 1 + 4} \Rightarrow r = 4$ 4 or $\sqrt{16}$ (Award A0 for ± 4).	M1 A1 [2]
(c)	When $x = 0$, $y^2 - 2y - 11 = 0$ Putting $x = 0$ in C or their C. $y = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-11)}}{2(1)} \left\{ = \frac{2 \pm \sqrt{48}}{2} \right\}$ $y^2 - 2y - 11 = 0$ or $(y-1)^2 = 12$, etc So, $y = 1 \pm 2\sqrt{3}$ Attempt to use formula or a method of completing the square in order to find $y = \dots$ $1 \pm 2\sqrt{3}$	M1 A1 aef M1 A1 cao cso [4]
8		
(a)	<p>Note: Please mark parts (a) and (b) together. Answers only in (a) and/or (b) get full marks. Note in part (a) the marks are now M1A1 and not B1B1 as on ePEN.</p> <p>M1: for $(\pm 2, \pm 1)$. Otherwise, M1 for an attempt to complete the square eg. $(x \pm 2)^2 \pm \alpha$, $\alpha \neq 0$ or $(y \pm 1)^2 \pm \beta$, $\beta \neq 0$. M1A1: Correct answer of $(-2, 1)$ stated from any working gets M1A1.</p>	
(b)	<p>M1: to find the radius using 11, "1" and "4", ie. $r = \sqrt{11 + "1" + "4"}$. By applying this method candidates will usually achieve $\sqrt{16}$, $\sqrt{6}$, $\sqrt{8}$ or $\sqrt{14}$ and not 16, 6, 8 or 14.</p> <p>Note: $(x+2)^2 + (y-1)^2 = -11 - 5 = -16 \Rightarrow r = \sqrt{16} = 4$ should be awarded M0A0.</p> <p>Alternative: M1 in part (a): For comparing with $x^2 + y^2 + 2gx + 2fy + c = 0$ to write down centre $(-g, -f)$ directly. Condone sign errors for this M mark. M1 in part (b): For using $r = \sqrt{g^2 + f^2 - c}$. Condone sign errors for this method mark.</p> <p>$(x+2)^2 + (y-1)^2 = 16 \Rightarrow r = 8$ scores M0A0, but $r = \sqrt{16} = 8$ scores M1A1 isw.</p>	
(c)	<p>1st M1: Putting $x = 0$ in either $x^2 + y^2 + 4x - 2y - 11 = 0$ or their circle equation usually given in part (a) or part (b). 1st A1 for a correct equation in y in any form which can be implied by later working.</p> <p>2nd M1: See rules for using the formula. Or completing the square on a 3TQ to give $y = a \pm \sqrt{b}$, where \sqrt{b} is a surd, $b \neq$ their 11 and $b > 0$. This mark should not be given for an attempt to factorise.</p> <p>2nd A1: Need exact pair in simplified surd form of $\{y = \} 1 \pm 2\sqrt{3}$. This mark is also cso.</p> <p>Do not need to see $(0, 1 + 2\sqrt{3})$ and $(0, 1 - 2\sqrt{3})$. Allow 2nd A1 for bod $(1 + 2\sqrt{3}, 0)$ and $(1 - 2\sqrt{3}, 0)$. Any incorrect working in (c) gets penalised the final accuracy mark. So, beware: incorrect $(x-2)^2 + (y-1)^2 = 16$ leading to $y^2 - 2y - 11 = 0$ and then $y = 1 \pm 2\sqrt{3}$ scores M1A1M1A0.</p> <p>Special Case for setting $y = 0$: Award SC: M0A0M1A0 for an attempt at applying the formula</p> <div style="display: flex; align-items: center;"> <div style="flex: 1;"> $x = \frac{-4 \pm \sqrt{(-4)^2 - 4(1)(-11)}}{2(1)} \left\{ = \frac{-4 \pm \sqrt{60}}{2} = -2 \pm \sqrt{15} \right\}$ </div> <div style="flex: 1; border-left: 1px solid black; padding-left: 10px;"> <p>Award SC: M0A0M1A0 for completing the square to their equation in x which will usually be $x^2 + 4x - 11 = 0$ to give $a \pm \sqrt{b}$, where \sqrt{b} is a surd, $b \neq$ their 11 and $b > 0$.</p> </div> </div> <p>Special Case: For a candidate not using \pm but achieving one of the correct answers then award SC: M1A1 M1A0 for one of either $y = 1 + 2\sqrt{3}$ or $y = 1 - 2\sqrt{3}$ or $y = 1 + \sqrt{12}$ or $y = 1 - \sqrt{12}$.</p>	

Question Number	Scheme	Marks
5. (a)	$\frac{1}{2}r^2\theta = \frac{1}{2}(6)^2\left(\frac{\pi}{3}\right) = 6\pi$ or 18.85 or awrt 18.8 (cm) ² Using $\frac{1}{2}r^2\theta$ (See notes) 6π or 18.85 or awrt 18.8	M1 A1 [2]
(b)	$\sin\left(\frac{\pi}{6}\right) = \frac{r}{6-r}$ $\frac{1}{2} = \frac{r}{6-r}$ $6-r = 2r \Rightarrow r = 2$ $\sin\left(\frac{\pi}{6}\right)$ or $\sin 30^\circ = \frac{r}{6-r}$ Replaces sin by numeric value $r = 2$	M1 dM1 A1 cso [3]
(c)	Area = $6\pi - \pi(2)^2 = 2\pi$ or awrt 6.3 (cm) ² their area of sector – πr^2 2π or awrt 6.3	M1 A1 cao [2] 7
(a)	M1: Needs θ in radians for this formula. Candidate could convert to degrees and use the degrees formula. A1: Does not need units. Answer should be either 6π or 18.85 or awrt 18.8 Correct answer with no working is M1A1. This M1A1 can only be awarded in part (a).	
(b)	M1: Also allow $\cos\left(\frac{\pi}{3}\right)$ or $\cos 60^\circ = \frac{r}{6-r}$. 1 st M1: Needs correct trigonometry method. Candidates could state $\sin\left(\frac{\pi}{6}\right) = \frac{r}{x}$ and $x + r = 6$ or equivalent in their working to gain this method mark. dM1: Replaces sin by numerical value. $0.009... = \frac{r}{6-r}$ from working “incorrectly” in degrees is fine here for dM1. A1: For $r = 2$ from correct solution only. Alternative: 1 st M1 for $\frac{r}{OC} = \sin 30$ or $\frac{r}{OC} = \cos 60$. 2 nd M1 for $OC = 2r$ and then A1 for $r = 2$. Note seeing $OC = 2r$ is M1M1. Special Case: If a candidate states an answer of $r = 2$ (must be in part (b)) as a guess or from an incorrect method then award SC: M0M0B1. Such a candidate could then go on to score M1A1 in part (c).	
(c)	M1: For “their area of sector – their area of circle”, where $r > 0$ is ft from their answer to part (b). Allow the method mark if “their area of sector” < “their area of circle”. The candidate must show somewhere in their working that they are subtracting the correct way round, even if their answer is negative. Some candidates in part (c) will either use their value of r from part (b) or even introduce a value of r in part (c). You can apply the scheme to award either M0A0 or M1A0 or M1A1 to these candidates. Note: Candidates can get M1 by writing “their part (a) answer – πr^2 ”, where the radius of the circle is not substituted. A1: cao – accept exact answer or awrt 6.3 Correct answer only with no working in (c) gets M1A1 Beware: The answer in (c) is the same as the arc length of the pendant	

Question Number	Scheme	Marks
6.		
(a)	$\{ ar = 192 \text{ and } ar^2 = 144 \}$ $r = \frac{144}{192}$ $r = \frac{3}{4} \text{ or } 0.75$	<p>Attempt to eliminate a. (See notes.)</p> <p>M1</p> <p>A1</p> <p>[2]</p>
(b)	$a(0.75) = 192$ $a \left\{ = \frac{192}{0.75} \right\} = 256$	<p>256</p> <p>M1</p> <p>A1</p> <p>[2]</p>
(c)	$S_{\infty} = \frac{256}{1-0.75}$ So, $\{S_{\infty}\} = 1024$	<p>Applies $\frac{a}{1-r}$ correctly using both their a and their $r < 1$.</p> <p>1024</p> <p>M1</p> <p>A1 cao</p> <p>[2]</p>
(d)	$\frac{256(1 - (0.75)^n)}{1 - 0.75} > 1000$ $(0.75)^n < 1 - \frac{1000(0.25)}{256} \left\{ = \frac{6}{256} \right\}$ $n \log(0.75) < \log\left(\frac{6}{256}\right)$ $n > \frac{\log\left(\frac{6}{256}\right)}{\log(0.75)} = 13.0471042... \Rightarrow n = 14$	<p>Applies S_n with their a and r and “uses” 1000 at any point in their working. (Allow with = or <).</p> <p>Attempt to isolate $(r)^n$ from S_n formula. (Allow with = or >).</p> <p>Uses the power law of logarithms correctly. (Allow with = or >). (See notes.)</p> <p>See notes and $n = 14$</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>A1 cso</p> <p>[4]</p> <p>10</p>
(a)	<p>M1: for eliminating a by eg. $192r = 144$ or by either dividing $ar^2 = 144$ by $ar = 192$ or dividing $ar = 192$ by $ar^2 = 144$, to achieve an equation in r or $\frac{1}{r}$. Note that $r^2 - r = \frac{144}{192}$ is M0.</p> <p>Note also that any of $r = \frac{144}{192}$ or $r = \frac{192}{144} \left\{ = \frac{4}{3} \right\}$ or $\frac{1}{r} = \frac{192}{144}$ or $\frac{1}{r} = \frac{144}{192}$ are fine for the award of</p> <p>M1. Note: A candidate just writing $r = \frac{144}{192}$ with no reference to a can also get the method mark.</p> <p>Note: $ar^2 = 192$ and $ar^3 = 144$ leading to $r = \frac{3}{4}$ scores M1A1. This is because r is the ratio between any two consecutive terms. These candidates, however, will usually be penalised in part (b).</p>	
(b)	<p>M1 for inserting their r into either of the correct equations of either $ar = 192$ or $\{a = \} \frac{192}{r}$ or $ar^2 = 144$ or $\{a = \} \frac{144}{r^2}$. No slips allowed here for M1.</p> <p>M1: can also be awarded for writing down $144 = a \left(\frac{192}{a} \right)^2$</p> <p>A1 for $a = 256$ only. Note 256 from any working scores M1A1.</p> <p>Note: Some candidates incorrectly confuse notation to give $r = \frac{4}{3}$ or 1.33 in part (a) (getting M1A0). In part (b), they recover to write $a = 192 \times \frac{4}{3}$ for M1 and then 256 for A1.</p>	

Question Number	Scheme	Marks						
(c)	M1: for applying $\frac{a}{1-r}$ correctly (no slips allowed!) using both their a and their r , where $ r < 1$. A1: for 1024, cao.							
(d)	In parts (a) or (b) or (c), the correct answer with no working scores full marks. 1 st M1: For applying S_n with their a and either “the letter r ” or their r and “uses” 1000. 2 nd M1: For isolating $+(r)^n$ and not $(ar)^n$, (eg. $(192)^n$) as the subject of an equation or inequality. $+(r)^n$ must be derived from the S_n formula. 3 rd M1: For applying the power law to $\lambda^k = \mu$ to give $k \log \lambda = \log \mu$ oe. where $\lambda, \mu > 0$. or 3 rd M1: For solving $\lambda^k = \mu$ to give $k = \log_{\lambda} \mu$, where $\lambda, \mu > 0$. A1: cso If a candidate uses inequalities, a fully correct method with inequalities is required here. So, an <u>incorrect</u> inequality statement at any stage in a candidate’s working for this part loses this mark. Note: Some candidates do not realise that the direction of the inequality is reversed in the final line of their solution. Or A1: cso Note a candidate can achieve full marks here if they do not use inequalities. So, if a candidate uses equations rather than inequalities in their working then they need to state in the final line of their working that $n = 13.04$ (truncated) or $n = \text{awrt } 13.05 \Rightarrow n = 14$ for A1. $n = 14$ from no working gets SC: M0M0M1A1. A method of $T_n > 1000 \Rightarrow 256(0.75)^{n-1} > 1000$ can score M0M0M1A0 for a correct application of the power law of logarithms. Trial & Improvement Method: For $a = 256$ and $r = 0.75$, apply the following scheme: <table border="0" style="width: 100%;"> <tr> <td style="width: 40%;">$S_{13} = \frac{256(1 - (0.75)^{13})}{1 - 0.75} = 999.6725616\dots$</td> <td style="width: 40%;">Attempt to find either S_{13} or S_{14}. EITHER (1) $S_{13} = \text{awrt } 999.7$ or truncated 999 OR (2) $S_{14} = \text{awrt } 1005.8$ or truncated 1005.</td> <td style="width: 20%; text-align: right;">M1</td> </tr> <tr> <td>$S_{14} = \frac{256(1 - (0.75)^{14})}{1 - 0.75} = 1005.754421\dots$</td> <td>Attempt to find both S_{13} and S_{14}. BOTH (1) $S_{13} = \text{awrt } 999.7$ or truncated 999 AND (2) $S_{14} = \text{awrt } 1005.8$ or truncated 1005 AND $n = 14$.</td> <td style="text-align: right;">M1 A1</td> </tr> </table> So, $n = 14$.	$S_{13} = \frac{256(1 - (0.75)^{13})}{1 - 0.75} = 999.6725616\dots$	Attempt to find either S_{13} or S_{14} . EITHER (1) $S_{13} = \text{awrt } 999.7$ or truncated 999 OR (2) $S_{14} = \text{awrt } 1005.8$ or truncated 1005.	M1	$S_{14} = \frac{256(1 - (0.75)^{14})}{1 - 0.75} = 1005.754421\dots$	Attempt to find both S_{13} and S_{14} . BOTH (1) $S_{13} = \text{awrt } 999.7$ or truncated 999 AND (2) $S_{14} = \text{awrt } 1005.8$ or truncated 1005 AND $n = 14$.	M1 A1	
$S_{13} = \frac{256(1 - (0.75)^{13})}{1 - 0.75} = 999.6725616\dots$	Attempt to find either S_{13} or S_{14} . EITHER (1) $S_{13} = \text{awrt } 999.7$ or truncated 999 OR (2) $S_{14} = \text{awrt } 1005.8$ or truncated 1005.	M1						
$S_{14} = \frac{256(1 - (0.75)^{14})}{1 - 0.75} = 1005.754421\dots$	Attempt to find both S_{13} and S_{14} . BOTH (1) $S_{13} = \text{awrt } 999.7$ or truncated 999 AND (2) $S_{14} = \text{awrt } 1005.8$ or truncated 1005 AND $n = 14$.	M1 A1						

Question Number	Scheme	Marks
	<p>Note: A similar scheme would apply for T&I for candidates using their a and their r. So,...</p> <p>1st M1: For attempting to find one of the correct S_n's either side (but next to) 1000.</p> <p>2nd M1: For one of these S_n's correct for their a and their r. (You may need to get your calculators out!)</p> <p>3rd M1: For attempting to find both of the correct S_n's either side (but next to) 1000.</p> <p>A1: Cannot be gained for wrong a and/or r.</p> <p>Trial & Improvement Cumulative Approach: A similar scheme to T&I will be applied here:</p> <p>1st M1: For getting as far as the cumulative sum of 13 terms. 2nd M1: (1)S_{13} = awrt 999.7 or truncated 999. 3rd M1: For getting as far as the cumulative sum to 14 terms. Also at this stage $S_{13} < 1000$ and $S_{14} > 1000$. A1: BOTH (1)S_{13} = awrt 999.7 or truncated 999 AND (2) S_{14} = awrt 1005.8 or truncated 1005 AND $n = 14$.</p> <p>Trial & Improvement Method: for $(0.75)^n < \frac{6}{256} = 0.0234375$</p> <p>3rd M1: For evidence of examining both $n = 13$ and $n = 14$.</p> <p>Eg: $(0.75)^{13} \{ = 0.023757... \}$ and $(0.75)^{14} \{ = 0.0178179... \}$</p> <p>A1: $n = 14$</p> <p>Any misreads, $S_n > 10000$ etc, please escalate up to your Team Leader.</p>	
7.	<p>(a) $3\sin(x + 45^\circ) = 2$; $0 \leq x < 360^\circ$ (b) $2\sin^2 x + 2 = 7\cos x$; $0 \leq x < 2\pi$</p> <p>(a) $\sin(x + 45^\circ) = \frac{2}{3}$, so $(x + 45^\circ) = 41.8103...$ ($\alpha = 41.8103...$) $\sin^{-1}\left(\frac{2}{3}\right)$ or awrt 41.8 or awrt 0.73°</p> <p>So, $x + 45^\circ = \{138.1897..., 401.8103...\}$ $x + 45^\circ = \text{either "180 - their } \alpha \text{" or "360}^\circ + \text{their } \alpha \text{" (} \alpha \text{ could be in radians).}$</p> <p>and $x = \{93.1897..., 356.8103...\}$ $\text{Either awrt } 93.2^\circ \text{ or awrt } 356.8^\circ$ $\text{Both awrt } 93.2^\circ \text{ and awrt } 356.8^\circ$</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[4]</p>
(b)	<p>$2(1 - \cos^2 x) + 2 = 7\cos x$ Applies $\sin^2 x = 1 - \cos^2 x$</p> <p>$2\cos^2 x + 7\cos x - 4 = 0$ Correct 3 term, $2\cos^2 x + 7\cos x - 4 \{ = 0 \}$</p> <p>$(2\cos x - 1)(\cos x + 4) \{ = 0 \}$, $\cos x = ...$ Valid attempt at solving and $\cos x = ...$</p> <p>$\cos x = \frac{1}{2}$, $\{ \cos x = -4 \}$ $\cos x = \frac{1}{2}$ (See notes.)</p> <p>$\left(\beta = \frac{\pi}{3} \right)$</p> <p>$x = \frac{\pi}{3}$ or $1.04719...^\circ$ Either $\frac{\pi}{3}$ or awrt 1.05°</p> <p>$x = \frac{5\pi}{3}$ or $5.23598...^\circ$ Either $\frac{5\pi}{3}$ or awrt 5.24° or $2\pi - \text{their } \beta$ (See notes.)</p>	<p>M1</p> <p>A1 oe</p> <p>M1</p> <p>A1 cso</p> <p>B1</p> <p>B1 ft</p> <p>[6] 10</p>

Question Number	Scheme	Marks
(a)	<p>1st M1: can also be implied for $x = \text{awrt } -3.2$</p> <p>2nd M1: for $x + 45^\circ =$ either "$180 - \text{their } \alpha$" or "$360^\circ + \text{their } \alpha$". This can be implied by later working. The candidate's α could also be in radians.</p> <p>Note that this mark is not for $x =$ either "$180 - \text{their } \alpha$" or "$360^\circ + \text{their } \alpha$".</p> <p>Note: Imply the first two method marks or award M1M1A1 for either awrt 93.2° or awrt 356.8°.</p> <p>Note: Candidates who apply the following incorrect working of $3\sin(x + 45^\circ) = 2$ $\Rightarrow 3(\sin x + \sin 45) = 2$, etc will usually score M0M0A0A0.</p> <p>If there are any EXTRA solutions inside the range $0 \leq x < 360$ and the candidate would otherwise score FULL MARKS then withhold the final aA2 mark (the final mark in this part of the question). Also ignore EXTRA solutions outside the range $0 \leq x < 360$.</p> <p>Working in Radians: Note the answers in radians are $x = \text{awrt } 1.6$, awrt 6.2</p> <p>If a candidate works in radians then mark part (a) as above awarding the A marks in the same way. If the candidate would then score FULL MARKS then withhold the final aA2 mark (the final mark in this part of the question.)</p> <p>No working: Award M1M1A1A0 for one of awrt 93.2° or awrt 356.8° seen without any working. Award M1M1A1A1 for both awrt 93.2° and awrt 356.8° seen without any working.</p> <p>Allow benefit of the doubt (FULL MARKS) for final answer of $\sin x \{ \text{and not } x \} = \{ \text{awrt } 93.2, \text{ awrt } 356.8 \}$</p>	

Question Number	Scheme	Marks
(b)	<p>1st M1: for a correct method to use $\sin^2 x = 1 - \cos^2 x$ on the given equation. Give bod if the candidate omits the bracket when substituting for $\sin^2 x$, but $2 - \cos^2 x + 2 = 7 \cos x$, without supporting working, (eg. seeing “$\sin^2 x = 1 - \cos^2 x$”) would score 1st M0. Note that applying $\sin^2 x = \cos^2 x - 1$, scores M0. 1st A1: for obtaining either $2\cos^2 x + 7\cos x - 4$ or $-2\cos^2 x - 7\cos x + 4$. 1st A1: can also awarded for a correct three term equation eg. $2\cos^2 x + 7\cos x = 4$ or $2\cos^2 x = 4 - 7\cos x$ etc. 2nd M1: for a valid attempt at factorisation of a quadratic (either 2TQ or 3TQ) in \cos, can use any variable here, c, y, x or $\cos x$, and an attempt to find at least one of the solutions. See introduction to the Mark Scheme. <i>Alternatively</i>, using a correct formula for solving the quadratic. Either the formula must be stated correctly or the correct form must be implied by the substitution. 2nd A1: for $\cos x = \frac{1}{2}$, BY A CORRECT SOLUTION ONLY UP TO THIS POINT. Ignore extra answer of $\cos x = -4$, but penalise if candidate states an incorrect result e.g. $\cos x = 4$. If they have used a substitution, a correct value of their c or their y or their x. Note: 2nd A1 for $\cos x = \frac{1}{2}$ can be implied by later working. 1st B1: for either $\frac{\pi}{3}$ or awrt 1.05° 2nd B1: for either $\frac{5\pi}{3}$ or awrt 5.24° or can be ft from 2π – their β or 360° – their β where $\beta = \cos^{-1}(k)$, such that $0 < k < 1$ or $-1 < k < 0$, but $k \neq 0$, $k \neq 1$ or $k \neq -1$. If there are any EXTRA solutions inside the range $0 \leq x < 2\pi$ and the candidate would otherwise score FULL MARKS then withhold the final bB2 mark (the final mark in this part of the question). Also ignore EXTRA solutions outside the range $0 \leq x < 2\pi$. Working in Degrees: Note the answers in degrees are $x = 60, 300$ If a candidate works in degrees then mark part (b) as above awarding the B marks in the same way. If the candidate would then score FULL MARKS then withhold the final bB2 mark (the final mark in this part of the question). Answers from no working: $x = \frac{\pi}{3}$ and $x = \frac{5\pi}{3}$ scores M0A0M0A0B1B1, $x = 60$ and $x = 300$ scores M0A0M0A0B1B0, $x = \frac{\pi}{3}$ ONLY or $x = 60$ ONLY scores M0A0M0A0B1B0, $x = \frac{5\pi}{3}$ ONLY or $x = 120$ ONLY scores M0A0M0A0B0B1. No working: You cannot apply the ft in the B1ft if the answers are given with NO working. Eg: $x = \frac{\pi}{5}$ and $x = \frac{9\pi}{3}$ FROM NO WORKING scores M0A0M0A0B0B0. For candidates using trial & improvement, please forward these to your Team Leader.</p>	

Question Number	Scheme	Marks
8. (a)	$\{V = \} \quad 2x^2y = 81$ $\{L = 2(2x + x + 2x + x) + 4y \Rightarrow L = 12x + 4y\}$ $y = \frac{81}{2x^2} \Rightarrow L = 12x + 4\left(\frac{81}{2x^2}\right)$ So, $L = 12x + \frac{162}{x^2}$ AG	$2x^2y = 81$ B1 oe Making y the subject of their expression and substitute this into the correct L formula. M1 Correct solution only. AG . A1 cso [3]
(b)	$\frac{dL}{dx} = 12 - \frac{324}{x^3} \quad \{ = 12 - 324x^{-3} \}$ $\left\{ \frac{dL}{dx} = \right\} 12 - \frac{324}{x^3} = 0 \Rightarrow x^3 = \frac{324}{12}; = 27 \Rightarrow x = 3$ $\{x = 3, \} \quad L = 12(3) + \frac{162}{3^2} = 54 \text{ (cm)}$	Either $12x \rightarrow 12$ or $\frac{162}{x^2} \rightarrow \frac{\pm \lambda}{x^3}$ Correct differentiation (need not be simplified). $L' = 0$ and “their $x^3 = \pm$ value” or “their $x^{-3} = \pm$ value” $x = \sqrt[3]{27}$ or $x = 3$ Substitute candidate's value of $x (\neq 0)$ into a formula for L . 54 M1 A1 aef M1; A1 cso ddM1 A1 cao [6]
(c)	$\{ \text{For } x = 3 \}, \quad \frac{d^2L}{dx^2} = \frac{972}{x^4} > 0 \Rightarrow \text{Minimum}$	Correct ft L'' and considering sign. $\frac{972}{x^4}$ and > 0 and conclusion. M1 A1 [2] 11
(a) (b) (c)	B1: For any correct form of $2x^2y = 81$. (may be unsimplified). Note that $2x^3 = 81$ is B0. Otherwise, candidates can use any symbol or letter in place of y . M1: Making y the subject of their formula and substituting this into a correct expression for L . A1: Correct solution only. Note that the answer is given. Note you can mark parts (b) and (c) together. 2 nd M1: Setting their $\frac{dL}{dx} = 0$ and “candidate's ft correct power of $x =$ a value”. The power of x must be consistent with their differentiation. If inequalities are used this mark cannot be gained until candidate states value of x or L from their x without inequalities. $L' = 0$ can be implied by $12 = \frac{324}{x^3}$. 2 nd A1: $x^3 = 27 \Rightarrow x = \pm 3$ scores A0. 2 nd A1: can be given for no value of x given but followed through by correct working leading to $L = 54$. 3 rd M1: Note that this method mark is dependent upon the two previous method marks being awarded. M1: for attempting correct ft second derivative and <u>considering its sign</u> . A1: Correct second derivative of $\frac{972}{x^4}$ (need not be simplified) <u>and</u> a valid reason (e.g. > 0), <u>and</u> conclusion. Need to conclude minimum (allow x and not L is a minimum) or indicate by a tick that it is a minimum. The actual value of the second derivative, if found, can be ignored, although substituting their L and not x into L'' is A0. Note: 2 marks can be scored from a wrong value of x , no value of x found or from not substituting in the value of their x into L'' . Gradient test or testing values either side of their x scores M0A0 in part (c). Throughout this question allow confused notation such as $\frac{dy}{dx}$ for $\frac{dL}{dx}$.	

Question Number	Scheme	Marks
9. (a)	<p>Curve: $y = -x^2 + 2x + 24$, Line: $y = x + 4$</p> <p>{Curve = Line} $\Rightarrow -x^2 + 2x + 24 = x + 4$</p> <p>$x^2 - x - 20 \{= 0\} \Rightarrow (x - 5)(x + 4) \{= 0\} \Rightarrow x = \dots$</p> <p>So, $x = 5, -4$</p> <p>So corresponding y-values are $y = 9$ and $y = 0$.</p>	<p>Eliminating y correctly. B1</p> <p>Attempt to solve a resulting quadratic to give x = their values. M1</p> <p>Both $x = 5$ and $x = -4$. A1</p> <p>See notes below. B1ft [4]</p>
(b)	<p>$\left\{ \int (-x^2 + 2x + 24) dx \right\} = -\frac{x^3}{3} + \frac{2x^2}{2} + 24x \{+ c$</p> <p>$\left[-\frac{x^3}{3} + \frac{2x^2}{2} + 24x \right]_{-4}^5 = (\dots) - (\dots)$</p> <p>$\left\{ \left(-\frac{125}{3} + 25 + 120 \right) - \left(\frac{64}{3} + 16 - 96 \right) = \left(103\frac{1}{3} \right) - \left(-58\frac{2}{3} \right) = 162 \right\}$</p> <p>Area of $\Delta = \frac{1}{2}(9)(9) = 40.5$</p> <p>So area of R is $162 - 40.5 = 121.5$</p>	<p>M1: $x^n \rightarrow x^{n+1}$ for any one term. M1A1A1</p> <p>1st A1 at least two out of three terms.</p> <p>2nd A1 for <u>correct answer</u>.</p> <p>Substitutes 5 and -4 (or their limits from part(a)) into an “integrated function” and subtracts, either way round. dM1</p> <p>Uses correct method for finding area of triangle. M1</p> <p>Area under curve – Area of triangle. M1</p> <p>121.5 A1 oe cao [7]</p> <p>11</p>

Question Number	Scheme	Marks
(a)	<p>1st B1: For correctly eliminating either x or y. Candidates will usually write $-x^2 + 2x + 24 = x + 4$. This mark can be implied by the resulting quadratic.</p> <p>M1: For solving their quadratic (which must be different to $-x^2 + 2x + 24$) to give $x = \dots$ See introduction for Method mark for solving a 3TQ. It must result from some attempt to eliminate one of the variables.</p> <p>A1: For both $x = 5$ and $x = -4$.</p> <p>2nd B1ft: For correctly substituting their values of x in equation of line or parabola to give both correct ft y-values. (You may have to get your calculators out if they substitute their x into $y = -x^2 + 2x + 24$).</p> <p>Note: For $x = 5, -4 \Rightarrow y = 9$ and $y = 0 \Rightarrow$ eg. $(-4, 9)$ and $(5, 0)$, award B1 isw.</p> <p>If the candidate gives additional answers to $(-4, 0)$ and $(5, 9)$, then withhold the final B1 mark.</p> <p>Special Case: Award SC: B0M0A0B1 for $\{A\}(-4, 0)$. You may see this point marked on the diagram.</p> <p>Note: SC: B0M0A0B1 for solving $0 = -x^2 + 2x + 24$ to give $\{A\}(-4, 0)$ and/or $(6, 10)$.</p> <p>Note: Do not give marks for working in part (b) which would be creditable in part (a).</p>	
(b)	<p>1st M1 for an attempt to integrate meaning that $x^n \rightarrow x^{n+1}$ for at least one of the terms.</p> <p>Note that $24 \rightarrow 24x$ is sufficient for M1.</p> <p>1st A1 at least two out of three terms correctly integrated.</p> <p>2nd A1 for correct integration only and no follow through. Ignore the use of a '+ c'.</p> <p>2nd M1: Note that this method mark is dependent upon the award of the first M1 mark in part (b). Substitutes 5 and -4 (and not 4 if the candidate has stated $x = -4$ in part (a).) (or the limits the candidate has found from part(a)) into an "integrated function" and subtracts, either way round. Allow one slip!</p> <p>3rd M1: Area of triangle = $\frac{1}{2}(\text{their } x_2 - \text{their } x_1)(\text{their } y_2)$ or Area of triangle = $\int_{x_1}^{x_2} x + 4 \{dx\}$.</p> <p>Where $x_1 = \text{their } -4$, $x_2 = \text{their } 5$ and $y_2 = \text{their } y$ usually found in part (a).</p> <p>4th M1: Area under curve – Area under triangle, where both Area under curve > 0 and Area under triangle > 0 and Area under curve $>$ Area under triangle.</p> <p>3rd A1: 121.5 or $\frac{243}{2}$ oe cao.</p>	

Question Number	Scheme	Marks
Aliter 9.(b) Way 2	<p>Curve: $y = -x^2 + 2x + 24$, Line: $y = x + 4$</p> <p>Area of $R = \int_{-4}^5 (-x^2 + 2x + 24) - (x + 4) dx$</p> $= -\frac{x^3}{3} + \frac{x^2}{2} + 20x \{+c\}$ $\left[-\frac{x^3}{3} + \frac{x^2}{2} + 20x \right]_{-4}^5 = (\dots) - (\dots)$ $\left\{ \left(-\frac{125}{3} + \frac{25}{2} + 100 \right) - \left(\frac{64}{3} + 8 - 80 \right) \right\} = \left(70\frac{5}{6} \right) - \left(-50\frac{2}{3} \right)$ <p><i>See above working to decide to award 3rd M1 mark here:</i> <i>See above working to decide to award 4th M1 mark here:</i></p> <p>So area of R is = 121.5</p>	<p>3rd M1: Uses integral of $(x + 4)$ with correct ft limits. 4th M1: Uses “curve” – “line” function with correct ft limits. M: $x^n \rightarrow x^{n+1}$ for any one term. A1 at least two out of three terms Correct answer (Ignore + c). Substitutes 5 and -4 (or <i>their limits</i> from part(a)) into an “integrated function” and subtracts, either way round.</p> <p>M1 A1ft A1 dM1</p> <p>M1 M1 A1 oe cao</p> <p>[7] 11</p>
(b)	<p>1st M1 for an attempt to integrate meaning that $x^n \rightarrow x^{n+1}$ for at least one of the terms. Note that $20 \rightarrow 20x$ is sufficient for M1. 1st A1 at least two out of three terms correctly ft. Note this accuracy mark is ft in Way 2. 2nd A1 for correct integration only and no follow through. Ignore the use of a '+ c'.</p> <p>Allow 2nd A1 also for $-\frac{x^3}{3} + \frac{2x^2}{2} + 24x - \left(\frac{x^2}{2} + 4x \right)$. Note that $\frac{2x^2}{2} - \frac{x^2}{2}$ or $24x - 4x$ only counts as one integrated term for the 1st A1 mark. Do not allow any extra terms for the 2nd A1 mark.</p> <p>2nd M1: Note that this method mark is dependent upon the award of the first M1 mark in part (b). Substitutes 5 and -4 (and not 4 if the candidate has stated $x = -4$ in part (a).) (or the limits the candidate has found from part(a)) into an “integrated function” and subtracts, either way round. Allow one slip!</p> <p>3rd M1: Uses the integral of $(x + 4)$ with correct ft limits of their x_1 and their x_2 (usually found in part (a)) {where $(x_1, y_1) = (-4, 0)$ and $(x_2, y_2) = (5, 9)$.} This mark is usually found in the first line of the candidate’s working in part (b).</p> <p>4th M1: Uses “curve” – “line” function with correct ft (usually found in part (a)) limits. Subtraction must be correct way round. This mark is usually found in the first line of the candidate’s working in part (b).</p> <p>Allow $\int_{-4}^5 (-x^2 + 2x + 24) - x + 4 \{dx\}$ for this method mark.</p> <p>3rd A1: 121.5 oe cao. Note: SPECIAL CASE for this alternative method</p> $\text{Area of } R = \int_{-4}^5 (x^2 - x - 20) dx = \left[\frac{x^3}{3} - \frac{x^2}{2} - 20x \right]_{-4}^5 = \left(\frac{125}{3} - \frac{25}{2} - 100 \right) - \left(-\frac{64}{3} - 8 + 80 \right)$ <p>The working so far would score SPEICAL CASE M1A1A1M1M1M0A0.</p> <p>The candidate may then go on to state that $= \left(-70\frac{5}{6} \right) - \left(50\frac{2}{3} \right) = -\frac{243}{2}$</p> <p>If the candidate then multiplies their answer by -1 then they would gain the 4th M1 and 121.5 would gain the final A1 mark.</p>	

Question Number	Scheme	Marks
Aliter 9. (a) Way 2	<p>Curve: $y = -x^2 + 2x + 24$, Line: $y = x + 4$</p> <p>{Curve = Line} $\Rightarrow y = -(y-4)^2 + 2(y-4) + 24$</p> <p>$y^2 - 9y \{= 0\} \Rightarrow y(y-9) \{= 0\} \Rightarrow y = \dots$</p> <p>So, $y = 0, 9$</p> <p>So corresponding y-values are $x = -4$ and $x = 5$.</p>	<p>Eliminating x correctly. B1</p> <p>Attempt to solve a resulting quadratic to give $y =$ their values. M1</p> <p>Both $y = 0$ and $y = 9$. A1</p> <p>See notes below. B1ft</p> <p style="text-align: right;">[4]</p>
	2 nd B1ft: For correctly substituting their values of y in equation of line or parabola to give both correct ft x -values.	
9. (b)	<p>Alternative Methods for obtaining the M1 mark for use of limits:</p> <p>There are two alternative methods candidates can apply for finding "162".</p> <p>Alternative 1:</p> $\int_{-4}^0 (-x^2 + 2x + 24)dx + \int_0^5 (-x^2 + 2x + 24)dx$ $= \left[-\frac{x^3}{3} + \frac{2x^2}{2} + 24x \right]_{-4}^0 + \left[-\frac{x^3}{3} + \frac{2x^2}{2} + 24x \right]_0^5$ $= (0) - \left(\frac{64}{3} + 16 - 96 \right) + \left(-\frac{125}{3} + 25 + 120 \right) - (0)$ $= \left(103\frac{1}{3} \right) - \left(-58\frac{2}{3} \right) = 162$ <p>Alternative 2:</p> $\int_{-4}^6 (-x^2 + 2x + 24)dx - \int_5^6 (-x^2 + 2x + 24)dx$ $= \left[-\frac{x^3}{3} + \frac{2x^2}{2} + 24x \right]_{-4}^6 - \left[-\frac{x^3}{3} + \frac{2x^2}{2} + 24x \right]_5^6$ $= \left\{ \left(-\frac{216}{3} + 36 + 144 \right) - \left(\frac{64}{3} + 16 - 96 \right) \right\} - \left\{ \left(-\frac{216}{3} + 36 + 144 \right) - \left(-\frac{125}{3} + 25 + 120 \right) \right\}$ $= \left\{ (108) - \left(-58\frac{2}{3} \right) \right\} - \left\{ (108) - \left(103\frac{1}{3} \right) \right\}$ $= \left(166\frac{2}{3} \right) - \left(4\frac{2}{3} \right) = 162$	

Appendix

List of Abbreviations

- dM1 denotes a method mark which is dependent upon the award of the previous method mark.
- ft or $\sqrt{\quad}$ denotes “follow through”
- cao denotes “correct answer only”
- aef denotes “any equivalent form”
- cso denotes “correct solution only”
- AG or * denotes “answer given” (in the question paper.)
- awrt denotes “anything that rounds to”
- aliter denotes “alternative methods”

Extra Solutions

If a candidate makes more than one attempt at any question:

- If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
- If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

Question Number	Scheme		Marks
Aliter 4. (c) Way 2	$(x + 2)^2 + (y - 1)^2 = 16$, centre $(x_1, y_1) = (-2, 1)$ and radius $r = 4$. $d_1 = \sqrt{4^2 - 2^2} = \sqrt{12}$ Hence, $y = 1 \pm \sqrt{12}$ So, $y = 1 \pm 2\sqrt{3}$	Applying $\sqrt{\text{their } r^2 - \text{their } x_1 ^2}$ $\sqrt{12}$ Applies $y = \text{their } y_1 \pm \text{their } d$ $1 \pm 2\sqrt{3}$	M1 A1 aef M1 A1 cao cso [4]
	Special Case: Award Final SC: M1A1 M1A0 if candidate achieves any one of either $y = 1 + 2\sqrt{3}$ or $y = 1 - 2\sqrt{3}$ or $y = 1 + \sqrt{12}$ or $y = 1 - \sqrt{12}$.		
Aliter 8. (a) Way 2	$2x^2 \left(\frac{L - 12x}{4} \right) = 81$ $\Rightarrow x^2(L - 12x) = 162 \Rightarrow L = 12x + \frac{162}{x^2}$	$2x^2 \left(\frac{L - 12x}{4} \right) = 81$ Rearranges their equation to make y the subject. Correct solution only. AG.	B1 oe M1 A1 cso [3]

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Mark Scheme (Results)

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GCE Mathematics
Core Mathematics 2 (6664)

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SOME GENERAL PRINCIPLES FOR C2 MARKING

(But the particular mark scheme always takes precedence)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values (but refer to the mark scheme first... the application of this principle may vary).

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but will be lost if there is any mistake in the working.

Equation of a straight line

Apply the following conditions to the M mark for the equation of a line through (a, b) :

If the a and b are the wrong way round the M mark can still be given if a correct formula is seen, (e.g. $y - y_1 = m(x - x_1)$) otherwise M0.

If (a, b) is substituted into $y = mx + c$ to find c , the M mark is for attempting this.

Answers without working

The rubric says that these may gain no credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Misreads

A misread must be consistent for the whole question to be interpreted as such.

These are not common. In clear cases, please deduct the first 2 A (or B) marks which would have been lost by following the scheme. (Note that 2 marks is the maximum misread penalty, but that misreads which alter the nature or difficulty of the question cannot be treated so generously and it will usually be necessary here to follow the scheme as written).

Sometimes following the scheme as written is more generous to the candidate than applying the misread rule, so in this case use the scheme as written.

If in doubt, send the response to Review.

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod - benefit of doubt
- ft - follow through
- the symbol \surd will be used for correct ft
- cao - correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw - ignore subsequent working
- awrt - answers which round to
- SC: special case
- oe - or equivalent (and appropriate)
- dep - dependent
- indep - independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- \square The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.
6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
7. Ignore wrong working or incorrect statements following a correct answer.
8. Marks for each question are scored by clicking in the marking grids that appear below each student response on ePEN. The maximum mark allocation for each question/part question(item) is set out in the marking grid and you should allocate a score of '0' or '1' for each mark, or "trait", as shown:

	0	1
aM		•
aA	•	
bM1		•
bA1	•	
bB	•	
bM2		•
bA2		•

9. Be careful when scoring a response that is either all correct or all incorrect. It is very easy to click down the '0' column when it was meant to be '1' and all correct.

January 2012
C2 6664
Mark Scheme

Question number	Scheme	Marks
1 (a) (b) (c)	Uses $360 \times \left(\frac{7}{8}\right)^{19}$, to obtain 28.5 Uses $S = \frac{360(1 - (\frac{7}{8})^{20})}{1 - \frac{7}{8}}$, or $S = \frac{360((\frac{7}{8})^{20} - 1)}{\frac{7}{8} - 1}$ to obtain 2680 Uses $S = \frac{360}{1 - \frac{7}{8}}$, to obtain 2880	M1, A1 (2) M1, A1 (2) M1, A1cao (2) 6
Notes	<p>(a) M1: Correct use of formula with power = 19 A1: Accept 28.47, or 28.474 or indeed 28.47446075</p> <p>(b) M1: Correct use of formula with $n = 20$ A1: Accept 2681, 2680.7, 2680.68 or 2680.679 or indeed 2680.678775 (N.B. 2680.67 or 2680.0 is A0)</p> <p>(c) M1: Correct use of formula A1: Accept 2880 only</p>	
Alternative method	<p>Alternative to (a) Gives all 20 terms 315, 275.6(25), 241.17(1875), ... (1st 3 accurate)</p> <p>All correct and last term as above A1: Accept 28.5, 28.47, or 28.474 or indeed 28.47446075</p> <p>Alternative to (b) Gives all 20 terms 315, 275.6(25), 241.17(1875), ... (1st 3 accurate) and adds</p> <p>Sum correct A1: Accept 2680, 2681, 2680.7, 2680.68 or 2680.679 or indeed 2680.678775</p>	M1 A1 M1 A1

Question number	Scheme	Marks
2	<p>The equation of the circle is $(x+1)^2 + (y-7)^2 = (r^2)$</p> <p>The radius of the circle is $\sqrt{(-1)^2 + 7^2} = \sqrt{50}$ or $5\sqrt{2}$ or $r^2 = 50$</p> <p>So $(x+1)^2 + (y-7)^2 = 50$ or equivalent</p>	<p>M1 A1</p> <p>M1</p> <p>A1</p> <p>(4)</p> <p>4</p>
Notes	<p>M1 is for this expression on left hand side– allow <i>errors in sign</i> of 1 and 7. A1 correct signs (just LHS)</p> <p>M1 is for Pythagoras or substitution into equation of circle to give r or r^2 Giving this value as diameter is M0</p> <p>A1, cao for cartesian equation with numerical values but allow $(\sqrt{50})^2$ or $(5\sqrt{2})^2$ or any exact equivalent</p> <p>A correct answer implies a correct method – so answer given with no working earns all four marks for this question.</p>	
Alternative method	<p>Equation of circle is $x^2 + y^2 \pm 2x \pm 14y + c = 0$</p> <p>Equation of circle is $x^2 + y^2 + 2x - 14y + c = 0$</p> <p>Uses (0,0) to give $c = 0$, or finds $r = \sqrt{(-1)^2 + 7^2} = \sqrt{50}$ or $5\sqrt{2}$ or $r^2 = 50$</p> <p>So $x^2 + y^2 + 2x - 14y = 0$ or equivalent</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>

Question number	Scheme	Marks
3 (a).	$(1 + \frac{x}{4})^8 = 1 + 2x + \dots$ $+ \frac{8 \times 7}{2} (\frac{x}{4})^2 + \frac{8 \times 7 \times 6}{2 \times 3} (\frac{x}{4})^3,$ $= \quad + \frac{7}{4}x^2 + \frac{7}{8}x^3 \quad \text{or} \quad = \quad + 1.75x^2 + 0.875x^3$	B1 M1 A1 A1 (4)
(b)	States or implies that $x = 0.1$ Substitutes their value of x (provided it is <1) into series obtained in (a) i.e. $1 + 0.2 + 0.0175 + 0.000875, = 1.2184$	B1 M1 A1 cao (3)
Alternative for (b) Special case	Starts again and expands $(1 + 0.025)^8$ to $1 + 8 \times 0.025 + \frac{8 \times 7}{2} (0.025)^2 + \frac{8 \times 7 \times 6}{2 \times 3} (0.025)^3, = 1.2184$ (Or $1 + 1/5 + 7/400 + 7/8000 = 1.2184$)	B1,M1,A1
Notes	<p>(a) B1 must be simplified</p> <p>The method mark (M1) is awarded for an attempt at Binomial to get the third and/or fourth term – need correct binomial coefficient combined with correct power of x. Ignore bracket errors or errors in powers of 4. Accept any notation for 8C_2 and 8C_3, e.g. $\binom{8}{2}$ and $\binom{8}{3}$ (unsimplified) or 28 and 56 from Pascal's triangle. (The terms may be listed without + signs)</p> <p>First A1 is for two completely correct unsimplified terms</p> <p>A1 needs the fully simplified $\frac{7}{4}x^2$ and $\frac{7}{8}x^3$.</p> <p>(b) B1 – states or uses $x=0.1$ or $\frac{x}{4} = \frac{1}{40}$</p> <p>M1 for substituting their value of x ($0 < x < 1$) into expansion (e.g. 0.1 (correct) or 0.01, 0.00625 or even 0.025 but not 1 nor 1.025 which would earn M0)</p> <p>A1 Should be answer printed cao (not answers which round to) and should follow correct work.</p> <p>Answer with no working at all is B0, M0, A0</p> <p>States 0.1 then just writes down answer is B1 M0A0</p>	

Question number	Scheme	Marks
4. (a)	$\log_3 3x^2 = \log_3 3 + \log_3 x^2$ or $\log y - \log x^2 = \log 3$ or $\log y - \log 3 = \log x^2$ $\log_3 x^2 = 2 \log_3 x$ Using $\log_3 3 = 1$	B1 B1 B1 (3)
(b)	$3x^2 = 28x - 9$ Solves $3x^2 - 28x + 9 = 0$ to give $x = \frac{1}{3}$ or $x = 9$	M1 M1 A1 (3) 6
Notes (a)	B1 for correct use of addition rule (or correct use of subtraction rule) B1: replacing $\log x^2$ by $2 \log x$ – not $\log 3x^2$ by $2 \log 3 x$ this is B0 These first two B marks are often earned in the first line of working B1. for replacing $\log 3$ by 1 (or use of $3^1 = 3$) If candidate has been awarded 3 marks and their proof includes an error or omission of reference to logy withhold the last mark. So just B1 B1 B0 These marks must be awarded for work in part (a) only	
(b)	M1 for removing logs to get an equation in x – statement in scheme is sufficient. This needs to be accurate without any errors seen in part (b). M1 for attempting to solve three term quadratic to give $x =$ (see notes on marking quadratics) A1 for the two correct answers – this depends on second M mark only. Candidates often begin again in part (b) and do not use part (a). If such candidates make errors in log work in part (b) they score first M0 . The second M and the A are earned as before. It is possible to get M0M1A1 or M0M1A0.	
Alternative to (b) using y	Eliminates x to give $3y^2 - 730y + 243 = 0$ with no errors is M1 Solves quadratic to find y, then uses values to find x M1 A1 as before See extra sheet with examples illustrating the scheme.	

Question number	Scheme	Marks
5 (a)	$f(-2) = -8 + 4a - 2b + 3 = 7$ so $2a - b = 6$ *	M1 A1 (2)
(b)	$f(1) = 1 + a + b + 3 = 4$ Solve two linear equations to give $a = 2$ and $b = -2$	M1 A1 M1 A1 (4)
		6
Notes	<p>(a) M1 : Attempts $f(\pm 2) = 7$ or attempts long division as far as putting remainder equal to 7 (There may be sign slips) A1 is for correct equation with remainder = 7 and for the printed answer with no errors and no wrong working between the two</p> <p>(b) M1 : Attempts $f(\pm 1) = 4$ or attempts long division as far as putting remainder equal to 4 A1 is for correct equation with remainder = 4 and powers calculated correctly M1 : Solving simultaneous equations (may be implied by correct answers). This mark may be awarded for attempts at elimination or substitution leading to values for both a and b. Errors are penalised in the accuracy mark. A1 is cao for values of a and b and explicit values are needed. Special case: Misreads and puts remainder as 7 again in (b). This may earn M1A0M1A0 in part (b) and will result in a maximum mark of 4/6</p>	
Long Divisions	$(x+2) \overline{) \begin{array}{r} x^2 + (a-2)x + (b-2a+4) \\ x^3 + ax^2 + bx + 3 \\ \hline x^3 + 2x^2 \end{array}}$ <p>.....</p> $(x-1) \overline{) \begin{array}{r} x^2 + (a+1)x + (b+a+1) \\ x^3 + ax^2 + bx + 3 \\ \hline x^3 - x^2 \end{array}}$ <p>.....</p> <p>A marks as before</p> <p>and reach their "$3 - 2b + 4a - 8 = 7$" M1</p> <p>and reach their "$3 + b + a + 1 = 4$" M1</p>	

Question number	Scheme	Marks																
6: (a)	<table><tr><td>x</td><td>1</td><td>1.5</td><td>2</td><td>2.5</td><td>3</td><td>3.5</td><td>4</td></tr><tr><td>y</td><td>16.5</td><td>7.361</td><td>4</td><td>2.31</td><td>1.278</td><td>0.556</td><td>0</td></tr></table>	x	1	1.5	2	2.5	3	3.5	4	y	16.5	7.361	4	2.31	1.278	0.556	0	B1, B1
x	1	1.5	2	2.5	3	3.5	4											
y	16.5	7.361	4	2.31	1.278	0.556	0											
(b)	$\frac{1}{2} \times 0.5, \{ (16.5 + 0) + 2(7.361 + 4 + 2.31 + 1.278 + 0.556) \}$ <p>= 11.88 (or answers listed below in note)</p>	(2) B1, M1A1ft A1 (4)																
(c)	$\int_1^4 \frac{16}{x^2} - \frac{x}{2} + 1 \, dx = \left[-\frac{16}{x} - \frac{x^2}{4} + x \right]_1^4$ $= [-4 - 4 + 4] - [-16 - \frac{1}{4} + 1]$ $= 11\frac{1}{4} \text{ or equivalent}$	M1 A1 A1 dM1 A1 (5) 11																
Notes	<p>(a) B1 for 4 or any correct equivalent e.g. 4.000 B1 for 2.31 or 2.310</p> <p>(b) B1: Need 0.25 or ½ of 0.5</p> <p>M1: requires first bracket to contain first y value plus last y value (0 may be omitted or be at end) and second bracket to include no additional y values from those in the scheme. They may however omit one value as a slip.</p> <p>N.B. Special Case - Bracketing mistake</p> <p>$\frac{1}{2} \times 0.5(16.5 + 0) + 2(7.361 + 4 + 2.31 + 1.278 + 0.556)$ scores B1 M1 A0 A0 unless the final answer implies that the calculation has been done correctly (then full marks)</p> <p>A1ft: This should be correct but ft their 4 and 2.31</p> <p>A1: Accept 11.8775 or 11.878 or 11.88 only</p> <p>(c) M1 Attempt to integrate ie power increased by 1 or 1 becomes x ,</p> <p>A1 two correct terms, next A1 all three correct unsimplified (ignore +c)</p> <p>(Allow $-16x^{-1} - 0.25x^2 + 1x$ or equivalent)</p> <p>dM1 (This cannot be earned if previous M mark has not been awarded) Uses limits 4 and 1 in their integrated expression and subtracts (either way round)</p> <p>A1 11.25 or $11\frac{1}{4}$ or $45/4$ or equivalent (penalise negative final answer here)</p>																	
Alternative Method for (b)	Separate trapezia may be used : B1 for 0.25, M1 for $\frac{1}{2}h(a+b)$ used 5 or 6 times (and A1ft all correct for their “4” and “2.31”) final A1 for 11.88 etc. as before																	
	In part (b) Need to use trapezium rule – answer only (with no working) is 0/4 -any doubts send to review In part (c) need to see integration																	

Question number	Scheme	Marks
7 (a)	$r\theta = 6 \times 0.95, = 5.7 \quad (\text{cm})$	M1, A1 (2)
(b)	$\frac{1}{2}r^2\theta = \frac{1}{2} \times 6^2 \times 0.95, = 17.1 \quad (\text{cm}^2)$	M1, A1 (2)
(c)	Let $AD = x$ then $\frac{x}{\sin 0.95} = \frac{6}{\sin 1.24}$ so $x = 5.16$ *	M1 A1 (2)
	OR $x = 3 / \cos 0.95$ OR so $x = 3 / \sin 0.62$ so $x = 5.16$ *	
(d)	OR $x^2 = 6^2 + x^2 - 12x \cos 0.95$ leading to $x =$, so $x = 5.16$ * Perimeter = '5.7'+5.16 +6 – 5.16= "11.7" or 6 + their 5.7	M1A1 ft (2)
(e)	Area of triangle $ABD = \frac{1}{2} \times 6 \times 5.16 \times \sin 0.95 = 12.6$ or $\frac{1}{2} \times 6 \times 3 \times \tan 0.95 = 12.6$ ($\frac{1}{2}$ base x height) or $\frac{1}{2} \times 5.16 \times 5.16 \times \sin 1.24 = 12.6$ So Area of $R = '17.1' - '12.6' = 4.5$	M1 A1 M1 A1 (4) 12
Notes	<p>(a) M1: Needs θ in radians for this formula. Could convert to degrees and use degrees formula. A1: Does not need units</p> <p>(b) M1: Needs θ in radians for this formula. Could convert to degrees and use degrees formula. A1: Does not need units</p> <p>(c) M1: Needs complete correct trig method to achieve $x =$ May have worked in degrees, using 54.4 degrees and 71.1 degrees Using angles of triangle sum to 360degrees is not correct method so is M0 A1: accept answers which round to 5.16 (NB This is given answer) If the answer 5.16 is assumed and verified award M1A0 for correct work</p> <p>(d) M1: Accept answer only as implying method, or just $6 + 5.7$ A1 : can be scored even following wrong answer to part (c)</p> <p>(e) M1: needs complete method for area of triangle ABD not ABC A1: Accept awrt 12.6 (If area of triangle is not evaluated or is given as 12.5 (truncated) this mark may be implied by 4.5 later) M1: Uses area of $R =$ area of sector – area of triangle ABD (not ABC) A1: Answers wrt 4.5</p>	
Alternative For part (e)	<p>Finds area of segment and area of triangle BDC by correct methods M1 Obtains 2.4585 and 2.0498 – accept answers wrt 2.5, 2.1 A1 Uses area of segment + area of triangle BDC ,to obtain 4.5 (not 4.6) M1, A1 NB Just finding area of segment is M0</p>	

Question number	Scheme	Marks
8 (a)	$kr^2 + cxy = 4$ or $kr^2 + c[(x+y)^2 - x^2 - y^2] = 4$ $\frac{1}{4}\pi x^2 + 2xy = 4$ $y = \frac{4 - \frac{1}{4}\pi x^2}{2x} = \frac{16 - \pi x^2}{8x}$ *	M1 A1 B1 cso (3)
(b)	$P = 2x + cy + k\pi r$ where $c = 2$ or 4 and $k = \frac{1}{4}$ or $\frac{1}{2}$ $P = \frac{\pi x}{2} + 2x + 4\left(\frac{4 - \frac{1}{4}\pi x^2}{2x}\right)$ or $P = \frac{\pi x}{2} + 2x + 4\left(\frac{16 - \pi x^2}{8x}\right)$ o.e. $P = \frac{\pi x}{2} + 2x + \frac{8}{x} - \frac{\pi x}{2}$ so $P = \frac{8}{x} + 2x$ *	M1 A1 A1 (3)
(c)	$\left(\frac{dP}{dx}\right) - \frac{8}{x^2} + 2$ $-\frac{8}{x^2} + 2 = 0 \Rightarrow x^2 = ..$ and so $x = 2$ o.e. (ignore extra answer $x = -2$) $P = 4 + 4 = 8$ (m)	M1 A1 M1 A1 B1 (5)
(d)	$y = \frac{4 - \pi}{4}$, (and so width) = 21 (cm)	M1, A1 (2)
13		
Notes	<p>(a) M1: Putting sum of one or two xy terms and one kr^2 term equal to 4 (k and c may be wrong) A1: For any correct form of this equation with x for radius (may be unsimplified) B1: Making y the subject of their formula to give this printed answer with no errors</p> <p>(b) M1: Uses Perimeter formula of the form $2x + cy + k\pi r$ where $c = 2$ or 4 and $k = \frac{1}{4}$ or $\frac{1}{2}$ A1: Correct unsimplified formula with y substituted as shown, i.e. $c = 4$, $k = \frac{1}{2}$, $r = x$ and $y = \frac{16 - \pi x^2}{8x}$ or $y = \left(\frac{4 - \frac{1}{4}\pi x^2}{2x}\right)$</p> <p>A1: obtains printed answer with at least one line of correct simplification or expansion before giving printed answer or stating result has been shown or equivalent</p> <p>(c) M1: At least one power of x decreased by 1 (Allow $2x$ becomes 2) A1: accept any equivalent correct answer</p> <p>M1: Setting $\frac{dP}{dx} = 0$ and finding a value for correct power of x for candidate A1: For $x = 2$. (This mark may be given for equivalent and may be implied by correct P) B1: 8 (cao) N.B. This may be awarded if seen in part (d)</p> <p>(d) M1: Substitute x value found in (c) into equation for y from (a) (or substitute x and P into equation for P from (b)) and evaluate (may see 0.2146 and correct answer implies M1 or need to see substitution if x value was wrong.) A1 is for 21 or 21cm or 0.21m as this is to nearest cm</p>	

Question number	Scheme	Marks
9 (i)	$\sin(3x-15) = \frac{1}{2}$ so $3x-15 = 30$ (α) and $x = 15$ Need $3x-15 = 180 - \alpha$ or $3x-15 = 540 - \alpha$ Need $3x-15 = 180 - \alpha$ and $3x-15 = 360 + \alpha$ and $3x-15 = 540 - \alpha$ $x = 55$ or 175 $x = 55, 135, 175$	M1 A1 M1 M1 A1 A1 (6)
Notes	<p>M1 Correct order of operation: inverse sine then linear algebra - not just $3x-15 = 30$ (slips in linear algebra lose Accuracy mark)</p> <p>A1 Obtains first solution 15</p> <p>M1 Uses either $180 - \alpha$ or $540 - \alpha$,</p> <p>M1 uses all three $180 - \alpha$ and $360 + \alpha$ and $540 - \alpha$</p> <p>A1, for one further correct solution 55 or 175, (depends only on second M1)</p> <p>A1 – all 3 further correct solutions</p> <p>If more than 4 solutions in range, lose last A1</p> <p>Common slips: Just obtains 15 and 55, or 15 and 175 – usually M1A1M1M0A1A0</p> <p>Just obtains 15 and 135 is usually M1A1M0M0A0A0 (It is easy to get this erroneously)</p> <p>Obtains 5, 45, 125 and 165 – usually M1A0M1M1A0A0</p> <p>Obtains 25, 65, 145, (185) usually M1A0M1M1A0A0</p> <p>Working in radians – lose last A1 earned for $\frac{\pi}{12}, \frac{11\pi}{36}, \frac{3\pi}{4}$ and $\frac{35\pi}{36}$ or numerical equivalents</p> <p>Mixed radians and degrees is usually Method marks only</p> <p>Methods involving no working should be sent to Review</p>	
9 (ii)	<p>At least one of $(\frac{a\pi}{10} - b) = 0$ (or $n\pi$)</p> <p>$(\frac{a3\pi}{5} - b) = \pi$ {or $(n+1)\pi$} or in degrees</p> <p>or $(\frac{a11\pi}{10} - b) = 2\pi$ {or $(n+2)\pi$}</p> <p>If two of above equations used eliminates a or b to find one or both of these or uses period property of curve to find a</p> <p>or uses other valid method to find either a or b (May see $\frac{5\pi}{10}a = \pi$ so $a =$)</p> <p>Obtains $a = 2$</p> <p>Obtains $b = \frac{\pi}{5}$ (must be in radians)</p>	M1 M1 A1 A1 (4)

Notes	<p>M1: Award for $(\frac{a\pi}{10} - b) = 0$ or $\frac{a\pi}{10} = b$ BUT $\sin(\frac{a\pi}{10} - b) = 0$ is M0</p> <p>M1: As described above but solving $(\frac{a\pi}{10} - b) = 0$ with $(\frac{a3\pi}{5} - b) = 0$ is M0 (It gives $a = b = 0$)</p> <p>Special cases: Can obtain full marks here for both correct answers with no working M1M1A1A1 For $a = 2$ only, with no working, award M0M1A1A0 For $b = \frac{\pi}{5}$ only with no working M1M0A0A1</p>
Alternative	<p>Some use translations and stretches to give answers. If they achieve $a=2$ they earn second method and first accuracy. If they achieve correct value for b they earn first method and second accuracy.</p> <p>Common error is $a = 2$ and $b = \frac{\pi}{10}$. This is usually M0M1A1A0 unless they have stated $(\frac{a\pi}{10} - b) = 0$ earlier in which case they earn first M1.</p>

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Mark Scheme (Results)

Summer 2012

GCE Core Mathematics C2
(6664) Paper 1

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Summer 2012
6664 Core Mathematics
C2 Mark Scheme

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod – benefit of doubt
 - ft – follow through
 - the symbol \checkmark will be used for correct ft
 - cao – correct answer only
 - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper
 - \square The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c), leading to $x = \dots$

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Summer 2012
6664 Core Mathematics 2
Mark Scheme

Question number	Scheme	Marks
1	$\begin{aligned} [(2-3x)^5] &= \dots + \binom{5}{1} 2^4 (-3x) + \binom{5}{2} 2^3 (-3x)^2 + \dots, \dots \\ &= 32, -240x, +720x^2 \end{aligned}$	M1 B1, A1, A1
Notes	<p>M1: The method mark is awarded for an attempt at Binomial to get the second and/or third term – need correct binomial coefficient combined with correct power of x. Ignore errors (or omissions) in powers of 2 or 3 or sign or bracket errors. Accept any notation for 5C_1 and 5C_2, e.g. $\binom{5}{1}$ and $\binom{5}{2}$ (unsimplified) or 5 and 10 from Pascal's triangle This mark may be given if no working is shown, but either or both of the terms including x is correct.</p> <p>B1: must be simplified to 32 (writing just 2^5 is B0). 32 must be the only constant term in the final answer- so $32 + 80 - 3x + 80 + 9x^2$ is B0 but may be eligible for M1A0A0 .</p> <p>A1: is cao and is for $-240x$. (not $+240x$) The x is required for this mark</p> <p>A1: is c.a.o and is for $720x^2$ (can follow omission of negative sign in working)</p> <p>A list of correct terms may be given credit i.e. series appearing on different lines</p> <p>Ignore extra terms in x^3 and/or x^4 (isw)</p>	Total 4
Special Case	<p>Special Case: <i>Descending powers</i> of x would be</p> $(-3x)^5 + 2 \times 5 \times (-3x)^4 + 2^2 \times \binom{5}{3} \times (-3x)^3 + \dots \text{ i.e. } -243x^5 + 810x^4 - 1080x^3 + \dots$ <p>This is a misread but award as s.c. M1B1A0A0 if completely “correct” or M1 B0A0A0 for <u>correct</u> binomial coefficient in any form with the correct power of x</p>	
Alternative Method	<p>Method 1: $[(2-3x)^5] = 2^5 (1 + \binom{5}{1}(-\frac{3x}{2}) + \binom{5}{2}(\frac{-3x}{2})^2 + \dots)$ is M1B0A0A0 { The M1 is for the expression in the bracket and as in first method– need correct binomial coefficient combined with correct power of x. Ignore bracket errors or errors (or omissions) in powers of 2 or 3 or sign or bracket errors }</p> <p>– answers must be simplified to $= 32, -240x, +720x^2$ for full marks (awarded as before)</p> $[(2-3x)^5] = 2(1 + \binom{5}{1}(-\frac{3x}{2}) + \binom{5}{2}(\frac{-3x}{2})^2 + \dots)$ <p>would also be awarded M1B0A0A0</p> <p>Method 2: Multiplying out : B1 for 32 and M1A1A1 for other terms with M1 awarded if x or x^2 term is correct. Completely correct is 4/4</p>	

Question number	Scheme	Marks
2	$2 \log x = \log x^2$ $\log_3 x^2 - \log_3 (x-2) = \log_3 \frac{x^2}{x-2}$ $\frac{x^2}{x-2} = 9$ <p>Solves $x^2 - 9x + 18 = 0$ to give $x = \dots$</p> <p>$x = 3, x = 6$</p>	<p>B1</p> <p>M1</p> <p>A1 o.e.</p> <p>M1</p> <p>A1</p>
		Total 5
Notes	<p>B1 for this correct use of power rule (may be implied)</p> <p>M1: for correct use of subtraction rule (or addition rule) for logs</p> <p>N.B. $2 \log_3 x - \log_3 (x-2) = 2 \log_3 \frac{x}{x-2}$ is M0</p> <p>A1. for correct equation without logs (Allow any correct equivalent including 3^2 instead of 9.)</p> <p>M1 for attempting to solve $x^2 - 9x + 18 = 0$ to give $x =$ (see notes on marking quadratics)</p> <p>A1 for these two correct answers</p>	
Alternative Method	<p>$\log_3 x^2 = 2 + \log_3 (x-2)$ is B1,</p> <p>so $x^2 = 3^{2+\log_3 (x-2)}$ needs to be followed by $(x^2) = 9(x-2)$ for M1 A1</p> <p>Here M1 is for complete method i.e. correct use of powers after logs are used correctly</p>	
Common Slips	<p>$2 \log x - \log x + \log 2 = 2$ may obtain B1 if $\log x^2$ appears but the statement is M0 and so leads to no further marks</p> <p>$2 \log_3 x - \log_3 (x-2) = 2$ so $\log_3 x - \log_3 (x-2) = 1$ and $\log_3 \frac{x}{x-2} = 1$ can earn M1 for <i>correct</i> subtraction rule following error, but no other marks</p>	
Special Case	<p>$\frac{\log x^2}{\log(x-2)} = 2$ leading to $\frac{x^2}{x-2} = 9$ and then to $x=3, x=6$, usually earns B1M0A0, but may then earn M1A1 (special case) so 3/5 [This <i>recovery</i> after uncorrected error is very common]</p> <p>Trial and error, Use of a table or just stating answer with both $x=3$ and $x=6$ should be awarded B0M0A0 then final M1A1 i.e. 2/5</p>	

Question number	Scheme	Marks
3	Obtain $\underline{(x \pm 10)^2}$ and $\underline{(y \pm 8)^2}$	M1
(a)	Obtain $\underline{(x - 10)^2}$ and $\underline{(y - 8)^2}$	A1
	Centre is (10, 8). N.B. This may be indicated on diagram only as (10, 8)	A1
(b)	See $\underline{(x \pm 10)^2} + \underline{(y \pm 8)^2} = 25 (= r^2)$ or $(r^2 =) "100" + "64" - 139$ $r = 5$ * (this is a printed answer so need one of the above two reasons)	M1 A1 (3)
(c)	Use $x = 13$ in either form of equation of circle and solve resulting quadratic to give $y =$ e.g. $x = 13 \Rightarrow (13 - 10)^2 + (y - 8)^2 = 25 \Rightarrow (y - 8)^2 = 16$ so $y =$ $or 13^2 + y^2 - 20 \times 13 - 16y + 139 = 0 \Rightarrow y^2 - 16y + 48 = 0$ so $y =$ $y = 4$ or 12 (on EPEN mark one correct value as A1A0 and both correct as A1 A1)	M1 A1 (2)
(d)	Use of $r\theta$ with $r = 5$ and $\theta = 1.855$ (may be implied by 9.275) Perimeter $PTQ = 2r +$ their arc PQ (Finding perimeter of triangle is M0 here) $= 19.275$ or 19.28 or 19.3	M1 M1 A1 (3)
		11 marks
Alternatives	<i>Method 2:</i> From $x^2 + y^2 + 2gx + 2fy + c = 0$ centre is $(\pm g, \pm f)$	M1
(a)	Centre is $(-g, -f)$, and so centre is (10, 8).	A1, A1
OR	<i>Method 3:</i> Use any value of y to give two points (L and M) on circle. x co-ordinate of mid point of LM is "10" and Use any value of x to give two points (P and Q) on circle. y co-ordinate of mid point of PQ is "8" (Centre – chord theorem) . (10,8) is M1A1A1	M1 A1 A1 (3)
(b)	<i>Method 2:</i> Using $\sqrt{g^2 + f^2 - c}$ or $(r^2 =) "100" + "64" - 139$ $r = 5$ *	M1 A1
OR	<i>Method 3:</i> Use point on circle with centre to find radius. Eg $\sqrt{(13 - 10)^2 + (12 - 8)^2}$ $r = 5$ *	M1 A1 cao (2)
(c)	Divide triangle PTQ and use Pythagoras with $r^2 - (13 - "10")^2 = h^2$, then evaluate $"8 \pm h"$ - (N.B. Could use 3,4,5 Triangle and 8 ± 4). Accuracy as before	M1
Notes	Mark (a) and (b) together M1 as in scheme and can be <u>implied</u> by $(\pm 10, \pm 8)$. Correct centre (10, 8) implies M1A1A1 M1 for a correct method leading to $r = \dots$, or $r^2 = "100" + "64" - 139$ (not $139 - "100" - "64"$) or for using equation of circle in $\underline{(x \pm 10)^2} + \underline{(y \pm 8)^2} = k^2$ form to identify $r =$ 3rd A1 $r = 5$ (NB This is a given answer so should follow $k^2 = 25$ or $r^2 = 100 + 64 - 139$) Special case: if centre is given as (-10, -8) or (10, -8) or (-10, 8) allow M1A1 for $r = 5$ worked correctly as $r^2 = 100 + 64 - 139$	
(d)	Full marks available for calculation using major sector so Use of $r\theta$ with $r = 5$ and $\theta = 4.428$ leading to perimeter of 32.14 for major sector	

Question number	Scheme	Marks
4 (a)	$f(-2) = 2.(-2)^3 - 7.(-2)^2 - 10.(-2) + 24$ $= 0 \text{ so } (x+2) \text{ is a factor}$	M1 A1 (2)
(b)	$f(x) = (x+2)(2x^2 - 11x + 12)$ $f(x) = (x+2)(2x-3)(x-4)$	M1 A1 dM1 A1 (4)
		6 marks
Notes (a)	<p>M1 : Attempts $f(\pm 2)$ (Long division is M0) A1 : is for $=0$ and conclusion Note: Stating “hence factor” or “it is a factor” or a “√” (tick) or “QED” is fine for the conclusion. Note also that a conclusion can be implied from a <u>preamble</u>, eg: “If $f(-2) = 0$, $(x+2)$ is a factor...” (Not just $f(-2)=0$)</p>	
(b)	<p>1st M1: Attempts long division by correct factor or other method leading to obtaining $(2x^2 \pm ax \pm b)$, $a \neq 0$, $b \neq 0$, even with a remainder. Working need not be seen as could be done “by inspection.” Or <i>Alternative Method</i> : 1st M1: Use $(x+2)(ax^2 + bx + c) = 2x^3 - 7x^2 - 10x + 24$ with expansion and comparison of coefficients to obtain $a = 2$ and to obtain values for b and c 1st A1: For seeing $(2x^2 - 11x + 12)$. [Can be seen here in (b) after work done in (a)] 2nd M1: Factorises quadratic. (see rule for factorising a quadratic). This is dependent on the previous method mark being awarded and needs factors 2nd A1: is cao and needs all three factors together. Ignore subsequent work (such as a solution to a quadratic equation.) Note: Some candidates will go from $\{(x+2)\}(2x^2 - 11x + 12)$ to $\{x = -2\}$, $x = \frac{3}{2}$, 4, and not list all three factors. Award these responses M1A1M0A0. Finds $x = 4$ and $x = 1.5$ by factor theorem, formula or calculator and produces factors M1 $f(x) = (x+2)(2x-3)(x-4)$ or $f(x) = 2(x+2)(x-1.5)(x-4)$ o.e. is full marks $f(x) = (x+2)(x-1.5)(x-4)$ loses last A1</p>	

Question number	Scheme		Marks
Method 1 5 (a)	<p>Puts $10 - x = 10x - x^2 - 8$ and rearranges to give three term quadratic</p> <p>Solves their "$x^2 - 11x + 18 = 0$" using acceptable method as in general principles to give $x =$</p> <p>Obtains $x = 2$, $x = 9$ (may be on diagram or in part (b) in limits)</p> <p>Substitutes their x into a given equation to give $y =$ (may be on diagram)</p> <p>$y = 8, y = 1$</p>	<p>Or puts $y = 10(10 - y) - (10 - y)^2 - 8$ and rearranges to give three term quadratic</p> <p>Solves their "$y^2 - 9y + 8 = 0$" using acceptable method as in general principles to give $y =$</p> <p>Obtains $y = 8, y = 1$ (may be on diagram)</p> <p>Substitutes their y into a given equation to give $x =$ (may be on diagram or in part (b))</p> <p>$x = 2, x = 9$</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 (5)</p>
(b)	$\int (10x - x^2 - 8) dx = \frac{10x^2}{2} - \frac{x^3}{3} - 8x \{ + c \}$ $\left[\frac{10x^2}{2} - \frac{x^3}{3} - 8x \right]_2^9 = (\dots) - (\dots)$ $= 90 - \frac{4}{3} = 88\frac{2}{3} \text{ or } \frac{266}{3}$ <p>Area of trapezium $= \frac{1}{2}(8+1)(9-2) = 31.5$</p> <p>So area of R is $88\frac{2}{3} - 31.5 = 57\frac{1}{6} \text{ or } \frac{343}{6}$</p>		<p>M1 A1 A1</p> <p>dM1</p> <p>B1</p> <p>M1A1 cao (7)</p>
Notes (a)	<p>First M1: See scheme Second M1: See notes relating to solving quadratics</p> <p>Third M1: This may be awarded if one substitution is made</p> <p>Two correct Answers following tables of values, or from Graphical calculator are 5/5</p> <p>Just one pair of correct coordinates – no working or from table is M0M0A0M1A0</p>		
(b)	<p>M1: $x^n \rightarrow x^{n+1}$ for any one term.</p> <p>1st A1: at least two out of three terms correct 2nd A1: All three correct</p> <p>dM1: Substitutes 9 and 2 (or limits from part(a)) into an "integrated function" and subtracts, either way round</p> <p>(NB: If candidate changes all signs to get $\int (-10x + x^2 + 8) dx = -\frac{10x^2}{2} + \frac{x^3}{3} + 8x \{ + c \}$ This is M1 A1 A1</p> <p>Then uses limits dM1 and trapezium is B1</p> <p>Needs to <i>change sign of value obtained</i> from integration for final M1A1 so $-88\frac{2}{3} - 31.5$ is M0A0)</p> <p>B1: Obtains 31.5 for area under line using any correct method (could be integration) or triangle minus triangle $\frac{1}{2} \times 8 \times 8 - \frac{1}{2}$ or rectangle plus triangle [may be implied by correct 57 1/6]</p> <p>M1: Their Area under curve – Their Area under line (if integrate both need same limits)</p> <p>A1: Accept 57.16recurring but not 57.16</p> <p>PTO for Alternative method</p>		12 marks

Method 2 for (b)	<div>Area of R</div> $= \int_2^9 (10x - x^2 - 8) - (10 - x) \, dx$ $\int_2^9 -x^2 + 11x - 18 \, dx$ $= -\frac{x^3}{3} + \frac{11x^2}{2} - 18x \{+c\}$ $\left[-\frac{x^3}{3} + \frac{11x^2}{2} - 18x\right]_2^9 = (\dots) - (\dots)$ <div>This mark is implied by final answer which rounds to 57.2</div> <div><i>See above working(allow bracketing errors) to decide to award 3rd M1 mark for (b) here:</i></div> $40.5 - (-16\frac{2}{3}) = 57\frac{1}{6} \text{ cao}$	<div>3rd M1 (in (b)): Uses difference between two functions in integral.</div> <div>M: $x^n \rightarrow x^{n+1}$ for any one term.</div> <div>A1 at least two out of these three simplified terms</div> <div>Correct integration. (Ignore + c).</div> <div>Substitutes 9 and 2 (or limits from part(a)) into an “integrated function” and subtracts, either way round.</div>	<div>M1</div> <div>A1</div> <div>A1</div> <div>dM1</div> <div>B1</div> <div>M1</div> <div>A1</div> <div>(7)</div>
Special case of above method	$\int_2^9 x^2 - 11x + 18 \, dx = \frac{x^3}{3} - \frac{11x^2}{2} + 18x \{+c\}$ $\left[\frac{x^3}{3} - \frac{11x^2}{2} + 18x\right]_2^9 = (\dots) - (\dots)$ <div>This mark is implied by final answer which rounds to 57.2 (not -57.2)</div> <div>Difference of functions implied (see above expression)</div> $40.5 - (-16\frac{2}{3}) = 57\frac{1}{6} \text{ cao}$	<div>M1A1A1</div> <div>DM1</div> <div>B1</div> <div>M1</div> <div>A1</div> <div>(7)</div>	
Special Case 2	Integrates expression in y e.g. " $y^2 - 9y + 8 = 0$ ": This can have first M1 in part (b) and no other marks. (It is not a method for finding this area)		
Notes	Take away trapezium again having used Method 2 loses last two marks Common Error: Integrates $-x^2 + 9x - 18$ is likely to be M1A1A0dM1B0M1A0 Integrates $2 - 11x - x^2$ is likely to be M1A0A0dM1B0M1A0 Writing $\int_2^9 (10x - x^2 - 8) - (10 - x) \, dx$ only earns final M mark		

Question number	Scheme		Marks
6(a)	States or uses $\tan 2x = \frac{\sin 2x}{\cos 2x}$ $\frac{\sin 2x}{\cos 2x} = 5 \sin 2x \Rightarrow \sin 2x - 5 \sin 2x \cos 2x = 0 \Rightarrow \sin 2x(1 - 5 \cos 2x) = 0$ *		M1 A1 (2)
(b)	$\sin 2x = 0$ gives $2x = 0, 180, 360$ so $x = 0, 90, 180$ $\cos 2x = \frac{1}{5}$ gives $2x = 78.46$ (or 78.5 or 78.4) or $2x = 281.54$ (or 281.6) $x = 39.2$ (or 39.3), 140.8 (or 141)	B1 for two correct answers, second B1 for all three correct. Excess in range – lose last B1 M1 A1, A1 (5)	B1, B1 M1 A1, A1 (5)
7 marks			
Notes	<p>(a) M1: Statement that $\tan \theta = \frac{\sin \theta}{\cos \theta}$ or Replacement of tan (wherever it appears). Must be a correct statement but may involve θ instead of $2x$. A1: the answer is given so all steps should be given.</p> <p>N.B. $\sin 2x - 5 \sin 2x \cos 2x = 0$ or $-5 \sin 2x \cos 2x + \sin 2x = 0$ or $\sin 2x(\frac{1}{\cos 2x} - 5) = 0$ o.e. must be seen and be followed by printed answer for A1 mark $\sin 2x = 5 \sin 2x \cos 2x$ is not sufficient.</p> <p>(b) Statement of 0 and 180 with no working gets B1 B0 (bod) as it is two solutions M1: This mark for one of the two statements given (must relate to $2x$ not just to x) A1, A1: first A1 for 39.2, second for 140.8 <i>Special case</i> solving $\cos 2x = -1/5$ giving $2x = 101.5$ or 258.5 is awarded M1A0A0 140.8 omitted would give M1A1A0 Allow answers which round to 39.2 or 39.3 and which round to 140.8 and allow 141 Answers in radians lose last A1 awarded (These are 0, 0.68, 1.57, 2.46 and 3.14) Excess answers in range lose last A1 Ignore excess answers outside range. All 5 correct answers with no extras and no working gets full marks in part (b). The answers imply the method here</p>		

Question number	Scheme	Marks												
7 (a)	<table><tr><td>x</td><td>0</td><td>0.25</td><td>0.5</td><td>0.75</td><td>1</td></tr><tr><td>y</td><td>1</td><td>1.251</td><td>1.494</td><td>1.741</td><td>2</td></tr></table>	x	0	0.25	0.5	0.75	1	y	1	1.251	1.494	1.741	2	B1, B1 (2)
x	0	0.25	0.5	0.75	1									
y	1	1.251	1.494	1.741	2									
(b)	$\frac{1}{2} \times 0.25, \{(1+2)+2(1.251+1.494+1.741)\} \text{ o.e.}$ $=1.4965$	B1, M1,A1 ft A1 (4)												
		6 marks												
Notes	<p>(a) first B1 for 1.494 and second B1 for 1.741 (1.740 is B0) Wrong accuracy e.g. 1.49, 1.74 is B1B0</p> <p>(b) B1: Need $\frac{1}{2}$ of 0.25 or 0.125 o.e. M1: requires first bracket to contain first plus last values and second bracket to include no additional values from the three in the table. If the only mistake is to omit one value from second bracket this may be regarded as a slip and M mark can be allowed (An extra repeated term forfeits the M mark however) x values: M0 if values used in brackets are x values instead of y values A1ft follows their answers to part (a) and is for {correct expression} Final A1: Accept 1.4965, 1.497. or 1.50 only after correct work. (No follow through except one special case below following 1.740 in table) Separate trapezia may be used : B1 for 0.125, M1 for $\frac{1}{2}h(a+b)$ used 3 or 4 times (and A1ft if it is all correct) e.g.. $0.125(1+ 1.251) + 0.125(1.251+1.494) + 0.125(1.741 + 2)$ is M1 A0 equivalent to missing one term in { } in main scheme</p> <p>Special Case: Bracketing mistake: i.e. $0.125(1+2) + 2(1.251+1.494+1.741)$ scores B1 M1 A0 A0 for 9.347 If the final answer implies that the calculation has been done correctly i.e. 1.4965 (then full marks can be given). Need to see trapezium rule – answer only (with no working) is 0/4 any doubts send to review</p> <p>Special Case; Uses 1.740 to give 1.49625 or 1.4963 or 1.496 or 1.50 gets, B1 B0 B1M1A1ft then A1 (lose 1 mark)</p> <p>NB Bracket is 11.972</p>													

Question number	Scheme	Marks
8		
(a)	$(h =) \frac{60}{\pi x^2}$ or equivalent exact (not decimal) expression e.g. $(h =) 60 \div \pi x^2$	B1 (1)
(b)	$(A =) 2\pi x^2 + 2\pi xh$ or $(A =) 2\pi r^2 + 2\pi rh$ or $(A =) 2\pi r^2 + \pi dh$ may not be simplified and may appear on separate lines Either $(A) = 2\pi x^2 + 2\pi x \left(\frac{60}{\pi x^2} \right)$ or As $\pi xh = \frac{60}{x}$ then $(A =) 2\pi x^2 + 2 \left(\frac{60}{x} \right)$	B1 M1
(c)	$A = 2\pi x^2 + \left(\frac{120}{x} \right)$ *	A1 cso (3)
	$\left(\frac{dA}{dx} \right) = 4\pi x - \frac{120}{x^2}$ or $= 4\pi x - 120x^{-2}$	M1 A1
	$4\pi x - \frac{120}{x^2} = 0$ implies $x^3 =$ (Use of > 0 or < 0 is M0 then M0A0)	M1
	$x = \sqrt[3]{\frac{120}{4\pi}}$ or answers which round to 2.12 (-2.12 is A0)	dM1 A1 (5)
(d)	$A = 2\pi(2.12)^2 + \frac{120}{2.12}, = 85$ (only ft $x = 2$ or 2.1 – both give 85)	M1, A1 (2)
(e)	Either $\frac{d^2 A}{dx^2} = 4\pi + \frac{240}{x^3}$ and sign considered (May appear in (c))	M1
	Or (method 2) considers gradient to left and right of their 2.12 (e.g at 2 and 2.5)	
	Or (method 3) considers value of A either side	
	Find numerical values for gradients and observes which is > 0 and therefore minimum (most substitute 2.12 but it is not essential to see a substitution) (may appear in (c))	A1 (2)
	concludes minimum	
	OR finds numerical values of A , observing greater than minimum value and draws conclusion	
		13 marks
Notes	<p>(a) B1: This expression must be correct and in part (a) $\frac{60}{\pi r^2}$ is B0</p> <p>(b) B1: Accept any equivalent correct form – may be on two or more lines.</p> <p>M1 : substitute their expression for h in terms of x into Area formula of the form $kx^2 + cxh$</p> <p>A1: There should have been no errors in part (b) in obtaining this printed answer</p> <p>(c) M1: At least one power of x decreased by 1 A1 accept any equivalent correct answer</p> <p>M1: Setting $\frac{dA}{dx} = 0$ and finding a value for x^3 ($x^3 =$ may be implied by answer). Allow $\frac{dy}{dx} = 0$</p> <p>dM1: Using cube root to find x</p> <p>A1 : For any equivalent correct answer (need 3sf or more) Correct answer implies previous M mark</p> <p>(d) M1 : Substitute the (+ve) x value found in (c) into equation for A and evaluate . A1 is for 85 only</p> <p>(e) M1: Complete method, usually one of the three listed in the scheme. For first method $A''(x)$ must be attempted and sign considered</p> <p>A1: Clear statements and conclusion. (numerical substitution of x is not necessary in first method shown, and x or calculation could be wrong but $A''(x)$ must be correct . Must not see 85 substituted)</p>	

Question	Scheme	Marks
9 (a)	$(S_n =) a + ar + (ar^2) + \dots + ar^{n-1}$ and $rS_n = ar + ar^2 + (ar^3) \dots + ar^n$ $S_n - rS_n = a - ar^n$ $S_n(1-r) = a(1-r^n)$ And so result $S_n = \frac{a(1-r^n)}{(1-r)}$ *	M1 M1 dM1 A1 (4)
(b)	Divides one term by other (either way) to give $r^2 = \dots$ then square roots to give $r =$ $r^2 = \frac{1.944}{5.4}$, $r = 0.6$ (ignore -0.6)	Or: (Method 2) Finds geometric mean i.e 3.24 and divides one term by 3.24 or 3.24 by one term M1 A1 (2)
(c)	Uses $5.4 \div r^2$ or $1.944 \div r^4$, to give $a =$ $a = 15$	M1, A1ft (2)
(d)	Uses $S = \frac{15}{1-0.6}$, to obtain 37.5	M1A1 ,A1 (3)
		11 marks
Notes	<p>(a) M1: Lists both of these sums ($S_n =$) may be omitted, rS_n (or rS) must be stated 1st two terms must be correct in each series. Last term must be ar^{n-1} or ar^n in first series and the corresponding ar^n or ar^{n+1} in second series. Must be n and not a number. Reference made to other terms e.g. space or dots to indicate missing terms M1: Subtracts series for rS from series for S (or other way round) to give $RHS = \pm(a - ar^n)$. This may have been obtained by following a pattern. If wrong power stated on line 1 M0 here. (Ignore LHS)M0M0M0A0 dM1: Factorises both sides correctly– must follow from a previous M1 (It is possible to obtain M0M1M1A0 or M1M0M1A0) A1: completes the proof with no errors seen No errors seen: First line absolutely correct, omission of second line, third and fourth lines correct: M1M0M1A1 See next sheet of common errors. Refer any attempts involving sigma notation, or any proofs by induction to team leader. Also attempts which begin with the answer and work backwards.</p>	
Special Case	<p>(b) M1: Deduces r^2 by dividing either term by other and attempts square root A1: any correct equivalent for r e.g. $3/5$ Answer only is $2/2$ (Method 2) Those who find fourth term must use \sqrt{ab} and not $\frac{1}{2}(a+b)$ then must use it in a division with given term to obtain $r =$ (c) M1: May be done in two steps or more e.g. $5.4 \div r$ then divided by r again A1ft: follow through their value of r. Just $a = 15$ with no wrong working implies M1A1 (d) M1: States sum to infinity formula with values of a and r found earlier, provided $r < 1$ A1 : uses 15 and 0.6 (or $3/5$) (This is not a ft mark) A1: 37.5 or exact equivalent</p>	
Common errors	<p>(i) Fraction inverted in (b) $r^2 = \frac{5.4}{1.944}$ and $r = 1\frac{2}{3}$, then correct ft gives M1A0 M1 A1ft M0A0A0 i.e. $3/7$ (ii) Uses $r = 0.36$: (b)M0A0 (c)M1A1ft (d) M1A0A0 i.e. $3/7$ (iii) Uses $ar^3 = 5.4$, $ar^5 = 1.944$ Likely to have (b)M1A1 (c)M0A0 (d) M1A0A0 i.e.$3/7$</p>	

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Mark Scheme (Results)

January 2013

GCE Core Mathematics C2 (6664/01)

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January 2013

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- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
- Unless indicated in the mark scheme a correct answer with no working should gain full marks for that part of the question.

EDEXCEL GCE MATHEMATICS

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 - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.

In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme.

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- bod – benefit of doubt
 - ft – follow through
 - the symbol $\frac{\Delta}{\Delta}$ will be used for correct ft
 - cao – correct answer only
 - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper
 - \square The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but incorrect answers should never be awarded A marks.
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 6. If a candidate makes more than one attempt at any question:

- If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
7. Ignore wrong working or incorrect statements following a correct answer.
8. The maximum mark allocation for each question/part question(item) is set out in the marking grid and you should allocate a score of '0' or '1' for each mark, or "trait", as shown:

	0	1
aM		•
aA	•	
bM1		•
bA1	•	
bB	•	
bM2		•
bA2		•

January 2013
6664 Core Mathematics C2
Mark Scheme

Question Number	Scheme		Marks
1.	$(2 - 5x)^6$		
	$(2^6 =) 64$	Award this when first seen (not $64x^0$)	B1
	$+ 6 \times (2)^5 (-5x) + \frac{6 \times 5}{2} (2)^4 (-5x)^2$	<p>Attempt binomial expansion with correct structure for at least one of these terms. E.g. a term of the form: $\binom{6}{p} \times (2)^{6-p} (-5x)^p$</p> <p>with $p = 1$ or $p = 2$ consistently. Condone sign errors. Condone missing brackets if later work implies correct structure and allow alternative forms for binomial coefficients</p> <p>e.g. 6C_1 or $\binom{6}{1}$ or even $\left(\frac{6}{1}\right)$</p>	M1
	$-960x$	Not $+ -960x$	A1 (first)
	$(+)6000x^2$		A1 (Second)
			(4)
Way 2	$64(1 \pm \dots\dots\dots)$	64 and $(1 \pm \dots\dots -$ Award when first seen.	B1
	$\left(1 - \frac{5x}{2}\right)^6 = 1 - 6 \times \frac{5x}{2} + \frac{6 \times 5}{2} \left(-\frac{5x}{2}\right)^2$	<p>Correct structure for at least one of the underlined terms. E.g. a term of the form: $\binom{6}{p} \times (kx)^p$ with $p = 1$ or $p = 2$</p> <p>consistently and $k \neq \pm 5$</p> <p>Condone sign errors. Condone missing brackets if later work implies correct structure but it must be an expansion of $(1 - kx)^6$ where $k \neq \pm 5$</p>	M1
	$-960x$	Not $+ -960x$	A1
	$(+)6000x^2$		A1
			(4)

[illegible]

3.			
(a)	$120000 \times (1.05)^3 = 138915 *$	Or $120000 \times 1.05 \times 1.05 \times 1.05 = 138915$ Or 120000, 126000, 132300, 138915 Or $a = 120000$ and $a \times (1.05)^3 = 138915$	B1
			(1)
(b)	$120000 \times (1.05)^{n-1} > 200000$	Allow n or $n - 1$ and “>”, “<”, or “=” etc.	M1
	$\log 1.05^{n-1} > \log\left(\frac{5}{3}\right)$	Takes logs correctly Allow n or $n - 1$ and “>”, “<”, or “=” etc.	M1
	$(n-1 >) \frac{\log\left(\frac{5}{3}\right)}{\log 1.05}$ or equivalent e.g. $(n >) \frac{\log\left(\frac{7}{4}\right)}{\log 1.05}$	Allow n or $n - 1$ and “>”, “<”, or “=” etc. Allow 1.6 or awrt 1.67 for 5/3.	A1
	2024	M1: Identifies a calendar year using their value of n or $n - 1$ A1: 2024	M1A1
			(5)
(c)	$\frac{a(1-r^n)}{1-r} = \frac{120000(1-1.05^{11})}{1-1.05}$	M1: Correct sum formula with $n = 10, 11$ or 12 A1: Correct numerical expression with $n = 11$	M1 A1
	1704814	Cao (Allow 1704814.00)	A1
			(3)
			[9]
	Listing or trial/improvement in (b)		
	$U_{10} = 186\,159.39, U_{11} = 195\,467.36, U_{12} = 205\,240.72$		
	Attempt to find at least the 10 th or 11 th or 12 th terms correctly using a common ratio of 1.05 (all the terms need not be listed)		M1
	Forms the geometric progression correctly to reach a term > 200 000		M1
	Obtains an “11 th ” term of awrt 195 500 and a “12 th ” term of awrt 205 200		A1
	Uses their number of terms to identify a calendar year		M1
	2024		A1
			(5)

[illegible]

[illegible]

7.			
(a)	$9^2 = 4^2 + 6^2 - 2 \times 4 \times 6 \cos \alpha \Rightarrow \cos \alpha = \dots$	Correct use of cosine rule leading to a value for $\cos \alpha$	M1
	$\cos \alpha = \frac{4^2 + 6^2 - 9^2}{2 \times 4 \times 6} \left(= -\frac{29}{48} = -0.604\ldots \right)$		
	$\alpha = 2.22 \quad *$	Cso (2.22 must be seen here)	A1
	(NB $\alpha = 2.219516005$)		(2)
(a) Way 2	$XY^2 = 4^2 + 6^2 - 2 \times 4 \times 6 \cos 2.22 \Rightarrow XY^2 = \dots$	Correct use of cosine rule leading to a value for XY^2	M1
	$XY^2 = 81.01\dots$		
	$XY = 9.00\dots$		A1
			(2)
(b)	$2\pi - 2.22 (= 4.06366\dots)$	$2\pi - 2.22$ or awrt 4.06	B1
	$\frac{1}{2} \times 4^2 \times "4.06"$	Correct method for major sector area.	M1
	32.5	Awrt 32.5	A1
			(3)
(b) Way2	Circle – Minor sector		
	$\pi \times 4^2$	Correct expression for circle area	B1
	$\pi \times 4^2 - \frac{1}{2} \times 4^2 \times 2.22 = 32.5$	Correct method for circle - minor sector area	M1
	$= 32.5$	Awrt 32.5	A1
			(3)
(c)	Area of triangle = $\frac{1}{2} \times 4 \times 6 \times \sin 2.22 (= 9.56)$	Correct expression for the area of triangle XYZ	B1
	So area required = "9.56" + "32.5"	Their Triangle XYZ + (part (b) answer or correct attempt at major sector)	M1
	$= 42.1 \text{ cm}^2$ or 42.0 cm^2	Awrt 42.1 or 42.0 (Or <u>just</u> 42)	A1
			(3)
(d)	Arc length = $4 \times 4.06 (= 16.24)$ Or $8\pi - 4 \times 2.22$	M1: $4 \times \text{their}(2\pi - 2.22)$ Or circumference – minor arc A1: Correct ft expression	M1A1ft
	Perimeter = $ZY + WY + \text{Arc Length}$	$9 + 2 + \text{Any Arc}$	M1
	Perimeter = 27.2 or 27.3	Awrt 27.2 or awrt 27.3	A1
			(4)
			[12]

8.	$y = 6 - 3x - \frac{4}{x^3}$		
(a)	$\frac{dy}{dx} = -3 + \frac{12}{x^4}$ or $-3 + 12x^{-4}$	M1: $x^n \rightarrow x^{n-1}$ ($x^1 \rightarrow x^0$ or $x^{-3} \rightarrow x^{-4}$ or $6 \rightarrow 0$)	M1 A1
		A1: Correct derivative	
	$\frac{dy}{dx} = 0 \Rightarrow -3 + \frac{12}{x^4} = 0 \Rightarrow x = \dots$ or $\frac{dy}{dx} = -3 + \frac{12}{\sqrt{2}^4}$	$y' = 0$ and attempt to solve for x May be implied by $\frac{dy}{dx} = -3 + \frac{12}{x^4} = 0 \Rightarrow \frac{12}{x^4} = 3 \Rightarrow x = \dots$ or Substitutes $x = \sqrt{2}$ into their y'	M1
	So $x^4 = 4$ and $x = \sqrt{2}$ or $\frac{dy}{dx} = -3 + \frac{12}{(\sqrt{2})^4}$ or $-3 + 12(\sqrt{2})^{-4} = 0$	Correct completion to answer with no errors by solving their $y' = 0$ or substituting $x = \sqrt{2}$ into their y'	A1
			(4)
(b)	$x = -\sqrt{2}$	Awrt -1.41	B1
			(1)
(c)	$\frac{d^2y}{dx^2} = \frac{-48}{x^5}$ or $-48x^{-5}$	Follow through their first derivative from part (a)	B1ft
			(1)
(d)	An appreciation that either $y'' > 0 \Rightarrow$ a minimum or $y'' < 0 \Rightarrow$ a maximum		B1
	Maximum at P as $y'' < 0$	Cso	B1
	Need a fully correct solution for this mark. y'' need not be evaluated but must be correct and there must be reference to P or to $\sqrt{2}$ and negative or < 0 and maximum. There must be no incorrect or contradictory statements (NB allow $y'' =$ awrt -8 or -9)		
	Minimum at Q as $y'' > 0$	Cso	B1
	Need a fully correct solution for this mark. y'' need not be evaluated but must be correct and part (b) must be correct and there must be reference to P or to $-\sqrt{2}$ and positive or > 0 and minimum. There must be no incorrect or contradictory statements (NB allow $y'' =$ awrt 8 or 9)		
			(3)
			[9]
	Other methods for identifying the nature of the turning points are acceptable. The first B1 is for finding values of y or dy/dx either side of $\sqrt{2}$ or their x at Q and the second and third B1's for fully correct solutions to identify the maximum/minimum.		

9.	$y = 27 - 2x - 9\sqrt{x} - \frac{16}{x^2}$		
(a)	6.272 , 3.634		B1, B1
			(2)
(b)	$\frac{1}{2} \times \frac{1}{2}$ or $\frac{1}{4}$		B1
 $\{(0+0) + 2(5.866 + "6.272" + 5.210 + "3.634" + 1.856)\}$	Need {} or implied later for A1ft	M1A1ft
	$\frac{1}{2} \times 0.5 \{(0+0) + 2(5.866 + "6.272" + 5.210 + "3.634" + 1.856)\}$ $= \frac{1}{4} \times 45.676$		
	= 11.42	cao	A1
			(4)
(c)	$\int y dx = 27x - x^2 - 6x^{\frac{3}{2}} + 16x^{-1} (+c)$	M1: $x^n \rightarrow x^{n+1}$ on any term	M1A1A1A1
		A1: $27x - x^2$	
		A1: $-6x^{\frac{3}{2}}$	
		A1: $+16x^{-1}$	
	$\left(27(4) - (4)^2 - 6(4)^{\frac{3}{2}} + 16(4)^{-1}\right)$ $- \left(27(1) - (1)^2 - 6(1)^{\frac{3}{2}} + 16(1)^{-1}\right)$	Attempt to subtract either way round using the limits 4 and 1. Dependent on the previous M1	dM1
	= (48 - 36)		
	12	Cao	A1
			(6)
			[12]

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Mark Scheme (Results)

Summer 2013

GCE Core Mathematics 2 (6664/01R)

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Summer 2013

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General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x =$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x =$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Question Number	Scheme	Marks
1.	$\frac{dy}{dx} = 2 - 16x^{-3}$ $2 - 16x^{-3} = 0 \text{ so } x^{-3} = \text{ or } x^3 = \text{ , or } 2 - 16x^{-3} = 0 \text{ so } x = 2$ $x = 2 \text{ only (after correct derivative)}$ $y = 2 \times "2" + 3 + \frac{8}{"2^2"}$ $= 9$	M1 A1 M1 A1 M1 A1 (6) Total 6
Notes for Question 1		
	<p>1st M1: At least one term differentiated (not integrated) correctly, so $2x \rightarrow 2$, or $\frac{8}{x^2} \rightarrow -16x^{-3}$, or $3 \rightarrow 0$</p> <p>A1: This answer or equivalent e.g. $2 - \frac{16}{x^3}$</p> <p>2nd M1: Sets $\frac{dy}{dx}$ to 0, and solves to give $x^3 = \text{value}$ or $x^{-3} = \text{value}$ (or states $x = 2$ with no working following correctly stated $2 - 16x^{-3} = 0$)</p> <p>A1: $x = 2$ cso (if $x = -2$ is included this is A0 here)</p> <p>3rd M1: Attempts to substitutes their positive x (found from attempt to differentiate) into $y = 2x + 3 + \frac{8}{x^2}$, $x > 0$</p> <p>Or may be implied by $y = 9$ or correct follow through from their positive x</p> <p>A1: 9 cao (Does not need to be written as coordinates) (ignore the extra (-2,1) here)</p>	

Question Number	Scheme	Marks
2.(a)	$\{x = 1.3\} \quad y = 0.8572 \text{ (only)}$	B1 cao (1)
(b)	$\frac{1}{2} \times 0.1 \dots\dots\dots$ $\{0.7071 + 0.9487 + 2(0.7591 + 0.8090 + "0.8572" + 0.9037)\}$ $\dots\{0.7071 + 0.9487 + 2(0.7591 + 0.8090 + "0.8572" + 0.9037)\}$ $\{0.05(8.3138)\} = 0.41569 = \text{awrt } 0.416$	B1 M1 A1ft A1 (4) Total 5
Notes for Question 2		
(a)	B1: 0.8572 cao	
(b)	B1 for using $\frac{1}{2} \times 0.1$ or 0.05 or equivalent. M1 It needs the first bracket to contain first y value plus last y value and the second bracket to be multiplied by 2 and to be the summation of the remaining y values in the table with no additional values. If the only mistake is a copying error or is to omit one value from 2nd bracket this may be regarded as a slip and the M mark can be allowed (An extra repeated term forfeits the M mark however). M0 if values used in brackets are x values instead of y values A1ft for the correct bracket $\{ \dots\dots \}$ following through candidate's y value found in part (a). NB: Separate trapezia may be used : B1 for 0.05, M1 for $\frac{1}{2} h(a + b)$ used 4 or 5 times (and A1ft if it is all correct) Then A1 as before. (Equivalent correct formulae may be used) Special case: Bracketing mistake $0.05 \times (0.7071 + 0.9487) + 2(0.7591 + 0.8090 + "0.8572" + 0.9037)$ scores B1 M1 A0 A0 (usually for 6.74079) unless the final answer implies that the calculation has been done correctly (then full marks can be given).	

Question Number	Scheme	Marks
3. Way 1	$\left(2 - \frac{1}{2}x\right)^8 = 2^8 + \binom{8}{1} \cdot 2^7 \left(-\frac{1}{2}x\right) + \binom{8}{2} 2^6 \left(-\frac{1}{2}x\right)^2 + \binom{8}{3} 2^5 \left(-\frac{1}{2}x\right)^3$ <p>First term of 256</p> $\left({}^8C_1 \times \dots \times x\right) + \left({}^8C_2 \times \dots \times x^2\right) + \left({}^8C_3 \times \dots \times x^3\right)$ $= (256) - 512x + 448x^2 - 224x^3$	B1 M1 A1, A1 (4) Total 4
Way 2	$\left(2 - \frac{1}{2}x\right)^8 = 2^8 \left(1 - \frac{1}{4}x\right)^8 = 2^8 \left(1 + \binom{8}{1} \cdot \left(-\frac{1}{4}x\right) + \binom{8}{2} \left(-\frac{1}{4}x\right)^2 + \binom{8}{3} \left(-\frac{1}{4}x\right)^3\right)$ <p>Scheme is applied exactly as before except in special case below*</p>	
Notes for Question 3		
<p>B1: The first term should be 256 in their expansion M1: Two binomial coefficients must be correct and must be with the correct power of x. Accept 8C_1 or $\binom{8}{1}$ or 8 as a coefficient, and 8C_2 or $\binom{8}{2}$ or 28 as another..... Pascal's triangle may be used to establish coefficients. A1: Any two of the final three terms correct (but allow +- instead of -) A1: All three of the final three terms correct and simplified. (Deduct last mark for $+512x$ and $+224x^3$ in the series). Also deduct last mark for the three terms correct but unsimplified. (Accept answers without + signs, can be listed with commas or appear on separate lines) The common error $\left(2 - \frac{1}{2}x\right)^8 = 256 + \binom{8}{1} \cdot 2^7 \left(-\frac{1}{2}x\right) + \binom{8}{2} 2^6 \left(-\frac{1}{2}x\right)^2 + \binom{8}{3} 2^5 \left(-\frac{1}{2}x\right)^3$ would earn B1, M1, A0, A0 Ignore extra terms involving higher powers. Condone terms in reverse order i.e. $= -224x^3 + 448x^2 - 512x + (256)$ *In Way 2 the error $= 2 \left(1 + \binom{8}{1} \cdot \left(-\frac{1}{4}x\right) + \binom{8}{2} \left(-\frac{1}{4}x\right)^2 + \binom{8}{3} \left(-\frac{1}{4}x\right)^3\right)$ giving $= 2 - 4x + \frac{7}{2}x^2 - \frac{7}{4}x^3$ is a special case B0, M1, A1, A0 i.e. 2/4</p>		

Question Number	Scheme		Marks
	Way 1	Way 2	
4.(a)	<p>Attempting $f(\pm 1)$ or $f(\pm 3)$</p> <p>Sets $f(3)=55$ i.e. $27a - 99 + 3b + 4 = 55$</p> <p>Sets $f(-1)=-9$ i.e. $-a - 11 - b + 4 = -9$ $a = \dots / b = \dots$</p> <p>$a = 6$ and $b = -4$</p>	<p>Divides by $(x-3)$ and reaches remainder or divides by $(x+1)$ and reaches remainder</p> <p>Sets remainder $= 55$ $27a - 99 + 3b + 4 = 55$</p> <p>Sets remainder $= -9$ $-a - 11 - b + 4 = -9$ $a = \dots / b = \dots$</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1cao</p> <p>(5)</p>
(b)	<p>$f(x) = (3x+2)(2x^2-5x+2)$ or $(x+\frac{2}{3})(6x^2-15x+6)$</p> <p>$= (3x+2)(x-2)(2x-1)$ or $= (3x+2)(2-x)(1-2x)$</p>		<p>M1 A1</p> <p>M1 A1</p> <p>(4)</p> <p>Total 9</p>
Notes for Question 4			
(a)	<p>Way 1 M1: Attempting $f(\pm 1)$ or $f(\pm 3)$ – with numbers substituted into expression A1: for applying $f(3)$ correctly and setting the result equal to 55 (see correct equation then isw) A1: for applying $f(-1)$ correctly and setting the result equal to -9 (see correct equation then isw) M1: Solve simultaneously to obtain a value for either a or b (may make slip) A1: Both values correct- this could imply M1 if no working Or Way 2 M1: Long division by $(x-3)$ or $(x+1)$ A1: A1: correct remainders put equal to 55 or -9 respectively M1A1: As in Way 1</p>		
(b)	<p>1st M1: attempting to divide a multiple of their cubic by $(3x+2)$ or $(x+\frac{2}{3})$ leading to a 3TQ beginning with the correct term, usually $\frac{a}{3}x^2$. A1: for either $(2x^2-5x+2)$ after division by $(3x+2)$ or $(6x^2-15x+6)$ after division by $(x+\frac{2}{3})$ 2nd M1: for a valid attempt to factorise their quadratic 2nd A1 is for any equivalent correct answer and needs all three factors together. As shown in scheme Apply isw if candidate continues to “solve” and give $x = \dots$</p>		

Question Number	Scheme	Marks
5.(a)	$a = 4p$, $ar = (3p+15)$ and $ar^2 = 5p+20$ (So $r =$) $\frac{5p+20}{3p+15} = \frac{3p+15}{4p}$ or $4p(5p+20) = (3p+15)^2$ or equivalent See $(3p+15)^2 = 9p^2 + 90p + 225$ $20p^2 + 80p = 9p^2 + 90p + 225 \rightarrow 11p^2 - 10p - 225 = 0$ *	B1 M1 M1 A1 * (4)
(b)	$(p-5)(11p+45)$ so $p =$ $p = 5$ only (after rejecting - 45/11) N.B. Special case $p = 5$ can be verified in (b) (1 mark only) $11 \times 5^2 - 10 \times 5 - 225 = 275 - 50 - 225 = 0$ M1A0	M1 A1 (2)
(c)	$\frac{3 \times 5 + 15}{4 \times 5}$ or $\frac{5 \times 5 + 20}{3 \times 5 + 15}$ $r = \frac{3}{2}$	M1 A1 (2)
(d)	$S_{10} = \frac{20 \left(1 - \left(\frac{3}{2} \right)^{10} \right)}{\left(1 - \frac{3}{2} \right)}$ $(= 2266.601568...) = 2267$	M1A1ft A1 (3) Total 11
Notes for Question 5		
(a)	B1: Correct statement (needs all three terms)– this may be omitted and implied by correct statement in p only as candidates may use geometric mean, or may use ratio of terms being equal and give a correct line 2 without line 1. (This would earn the B1M1 immediately) M1: Valid Attempt to eliminate a and r and to obtain equation in p only M1: Correct expansion of $(3p+15)^2 = 9p^2 + 90p + 225$ A1cso: No incorrect work seen. The printed answer is obtained. NB Those who show $p = 5$ in part (a) obtain no credit for this	
(b)	M1: Attempt to solve quadratic by usual methods (factorisation, completion of square or formula) Must appear in part (b) – not part (a) A1: 5 only and -45/11 should be seen and rejected or $(11p+45)$ seen and statement $p > 0$	
(c)	M1: Substitutes $p = 5$ completely and attempt ratio (correct way up) A1: 1.5 or any equivalent	
(d)	M1: Use of correct formula with $n = 10$ a and/or r may still be in terms of p A1ft: Correct expression ft on their r only – must have $a = 20$ and power = 10 here A1 2267 (accept awrt 2267)	

Question Number	Scheme		Marks
6.(a)	Way 1: $\log_3(9x) = \log_3 9 + \log_3 x$ $= 2 + a$	or Way 2: $\log_3(9x) = \log_3 3^{a+2}$ $= 2 + a$	M1 A1 (2)
(b)	Way 1: $\log_3\left(\frac{x^5}{81}\right) = \log_3 x^5 - \log_3 81$ $\log x^5 = 5 \log x$ or $\log 81 = 4 \log 3$ or $\log 81 = 4$ $= 5a - 4$	or Way 2 $= \log_3 \frac{3^{5a}}{3^4}$ $= \log_3 3^{5a-4}$	M1 M1 A1 cso (3)
(c)	$\log_3(9x) + \log_3\left(\frac{x^5}{81}\right) = 3$ Method 1 $\Rightarrow 2 + a + 5a - 4 = 3$ $\Rightarrow a = \frac{5}{6}$ $\Rightarrow x = 3^{\frac{5}{6}}$ or $\log_{10} x = a \log_{10} 3$ so $x =$ $x = 2.498$ or awrt	Method 2 $\log_3\left(9x \cdot \frac{x^5}{81}\right) = (3 \text{ or } \log 27)$ $\log_3\left(\frac{x^6}{9}\right) = 3 \text{ or } \log 27$ $\Rightarrow \frac{x^6}{9} = 3^3 \Rightarrow x^6 = 3^5 \Rightarrow x =$ $x = 2.498$ or awrt	M1 A1 M1 A1 (4)
If $x = -2.498$ appears as well or instead this is A0		Total 9	
Notes for Question 6			
(a)	Way 1: M1: Use of $\log(ab) = \log(a) + \log(b)$ A1: must be $a + 2$ or $2 + a$ Way 2: Uses $x = 3^a$ to give $\log_3(9x) = \log_3 3^{a+2}$, A1 for $a + 2$ or $2 + a$		
(b)	Way 1: M1: Use of $\log(a/b) = \log(a) - \log(b)$ M1: Use of $n \log(a) = \log(a)^n$ Way 2: M1 Use of correct powers of 3 in numerator and denominator M1: Subtracts powers A1: No errors seen		
(c)	Method 1: M1: Uses (a) and (b) results to form an equation in a (may not be linear) A1: $a = \text{awrt } 0.833$ M1: Finds x by use of 3 to a power, or change of base performed correctly A1: $x = 2.498$ (accept answer which round to this value from 2.498049533...) Method 2: M1: Use of $\log(ab) = \log(a) + \log(b)$ in an equation (RHS may be wrong) A1: Equation correct and simplified M1: Tries to undo log by 3 to power correctly, and uses root to obtain x A1: $x = 2.498$ (accept answer which round to this value from 2.498049533...) Lose this mark if negative answer is given as well as or instead of positive answer.		

Question Number	Scheme	Marks
7.(a)	$x^2 + 2x + 2 = 10 \Rightarrow x^2 + 2x - 8 = 0$ (so $(x+4)(x-2) = 0$) $\Rightarrow x = \dots\dots$ $x = -4, 2$	M1 A1 (2)
(b) Way 1	$\int (x^2 + 2x + 2) dx = \frac{x^3}{3} + \frac{2x^2}{2} + 2x (+C)$ $\left[\frac{x^3}{3} + \frac{2x^2}{2} + 2x \right]_{-4}^{2} = \left(\frac{8}{3} + \frac{8}{2} + 4 \right) - \left(-\frac{64}{3} + \frac{32}{2} - 8 \right) \quad (= 24)$ Rectangle: $10 \times (2 - (-4)) = 60$ $R = "60" - "24"$ $= 36$	M1A1A1 M1 B1 cao M1 A1 (7) Total 9
(b) Way 2	$\int (8 - x^2 - 2x) dx = 8x - \frac{x^3}{3} - \frac{2x^2}{2} (+C)$ $\left[8x - \frac{x^3}{3} - \frac{2x^2}{2} \right]_{-4}^{2} = \left(16 - \frac{8}{3} - 4 \right) - \left(-32 + \frac{64}{3} - 16 \right) = (9.3 - (-26.7))$ Implied by final answer of 36 after correct work $10 - (x^2 + 2x + 2) = 8 - x^2 - 2x, = 36$	M1 A1ft A1 M1 B1 M1, A1
Notes for Question 7		
(a)	M1 Set the curve equation equal to 10 and collect terms. Solves quadratic to $x = \dots$	
(b)	A1 cao : Both values correct – allow $A = -4, B = 2$ M1: One correct integration A1: Two correct integrations(ft slips subtracting in Way 2) A1: All 3 terms correct (penalise subtraction errors here in Way 2) M1: Substitute their limits from (a) into the integrated function and subtract (either way round) B1: Way 1: Find area under the line by integration or area of rectangle – should be 60 here (no follow through) Way 2: (implied by final correct answer in second method) M1: Subtract one area from the other (implied by subtraction of functions in second method)- award even after differentiation A1: Must be 36 not -36. Special case 1: Combines both methods. Uses Way 2 integration, but continues after reaching “36” to subtract “36” from rectangle giving answer as “24” This loses final M1 A1 Special case 2: Integrates $(x^2 + 2x - 8)$ between limits -4 and 2 to get -36 and then changes sign and obtains 36. Do not award final A mark – so M1A1A1M1B1M1A0 If the answer is left as -36, then M1A1A1M1B0M1A0 N.B. Allow full marks for modulus used earlier in working e.g. $\left \int_{-4}^2 x^2 + 2x - 2 dx - \int_{-4}^2 10 dx \right $	

Question Number	Scheme	Marks
8.(a)	<p>Way 1: $10^2 = 7^2 + 13^2 - 2 \times 7 \times 13 \cos \theta$ or $\cos \theta = \frac{7^2 + 13^2 - 10^2}{2 \times 7 \times 13}$</p> <p>$\cos \theta = \frac{59}{91}$ or $\cos \theta = \frac{7^2 + 13^2 - 10^2}{2 \times 7 \times 13}$ or $\cos \theta = 0.6483$ or 0.8644</p> <p>$(\theta = 0.8653789549...) = 0.865^*$ (to 3 dp)</p>	<p>M1</p> <p>A1 o.e</p> <p>A1* cso</p> <p>(3)</p>
(b)	<p>Way 2: Uses $\cos \theta = \frac{x}{7}$, where $7^2 - x^2 = 10^2 - (13 - x)^2$ and finds x ($= 59/13$)</p> <p>$\cos \theta = \frac{59}{91}$ and $(\theta = 0.8653789549...) = 0.865^*$ (to 3 dp) – as in Way 1</p> <p>Area triangle $ABC = \frac{1}{2} \times 13 \times 7 \sin 0.865$ or $\frac{1}{2} \times 13 \times 7 \sin 49.6$ or $20\sqrt{3}$</p> <p>Area sector $ABD = \frac{1}{2} \times 7^2 \times 0.865$ or $\frac{49.6}{360} \times \pi \times 7^2$</p> <p>$= 34.6$ (triangle) or 21.2 (Sector)</p> <p>Area of $S = \frac{1}{2} \times 13 \times 7 \sin 0.865 - \frac{1}{2} \times 7^2 \times 0.865$ ($= 13.4$)</p> <p>(Amount of seed =) $13.4 \times 50 = 670\text{g}$ or 680g (need one of these two answers)</p>	<p>M1</p> <p>A1, A1</p> <p>(3)</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1 A1</p> <p>M1 A1 (7)</p> <p>Total 10</p>
Notes for Question 8		
(a)	<p>M1: use correct cosine formula in any form A1: give a value for $\cos \theta$</p> <p>NB $\cos \theta = \frac{7^2 + 13^2 - 10^2}{2 \times 7 \times 13}$ earns M1A1</p>	
(b)	<p>A1: deduce and state the printed answer $\theta = 0.865$</p> <p>M1: Uses Correct method for area of the correct triangle i.e. ABC</p> <p>M1: Uses Correct method for the area of the sector</p> <p>A1: This is earned for one of the correct answers. May be implied if these answers are not calculated but the final answer is correct with no errors (or shaded area is 13.4 or 13.5)</p> <p>M1: Their area of Triangle ABC – Area of Sector (may have $kr^2\theta$ but not $k\theta$)</p> <p>A1: Correct expression or awrt 13.4 or 13.5 (may be implied by final answer)</p> <p>M1: Multiply their previous answer by 50</p> <p>A1: 670g or 680 g (There is an argument for rounding answer up to provide enough seed)</p>	
<p>N.B. $(\frac{1}{2} \times 13 \times 7 \sin 0.865 - \frac{1}{2} \times 7^2 \times 0.865) \times 50 = 670$ or 680 earns full marks</p> <p>$(\frac{1}{2} \times 13 \times 7 \sin 0.865 - \frac{1}{2} \times 7^2 \times 0.865) \times 50 = \text{awrt } 670$ or 680 just loses last mark</p> <p>$(\frac{1}{2} \times 13 \times 7 \sin 0.865 - \frac{1}{2} \times 7^2 \times 0.865) \times 50 = \text{wrong answer}$ M1M1A0M1A1M1A0</p>		

Question Number	Scheme	Marks
9.(a)	$\sin(2\theta - 30) = -0.6$ or $2\theta - 30 = -36.9$ or implied by 216.9 $2\theta - 30 = 216.87 = (180 + 36.9)$ $\theta = \frac{216.87 + 30}{2} = 123.4$ or 123.5 $2\theta - 30 = 360 - 36.9$ or 323.1 $\theta = \frac{323.1 + 30}{2} = 176.6$	B1 M1 A1 M1 A1 (5)
(b)	$9\cos^2 x - 11\cos x + 3(1 - \cos^2 x) = 0$ or $6\cos^2 x - 11\cos x + 3(\sin^2 x + \cos^2 x) = 0$ $6\cos^2 x - 11\cos x + 3 = 0$ { as $(\sin^2 x + \cos^2 x) = 1$ } $(3\cos x - 1)(2\cos x - 3) = 0$ implies $\cos x =$ $\cos x = \frac{1}{3}, \left(\frac{3}{2}\right)$ $x = 70.5$	M1 A1 M1 A1 B1 (5)
	$x = 360 - "70.5"$ $x = 289.5$	M1 A1cao (7) Total 12
Notes for Question 9		
(a)	B1: This statement seen and must contain no errors or may implied by -36.9 M1: Uses $180 - \arcsin(-0.6)$ i.e. $180 + 36.9$ (or $\pi + \arcsin(0.6)$ in radians) (in 3 rd quadrant) A1: allow answers which round to 123.4 or 123.5 must be in degrees M1: Uses $360 + \arcsin(-0.6)$ i.e. $360 - 36.9$ (or $2\pi + \arcsin(-0.6)$ in radians) (in 4th quadrant) A1: allow answers which round to 176.6 must be in degrees (A1 implies M1) Ignore extra answers outside range but lose final A1 for extra answers in the range if both B and A marks have been earned) Working in radians may earn B1M1A0M1A0	
(b)	M1: Use of $\sin^2 x = (1 - \cos^2 x)$ or $(\sin^2 x + \cos^2 x) = 1$ in the given equation A1: Correct three term quadratic in any equivalent form M1: Uses standard method to solve quadratic and obtains $\cos x =$ A1: A1 for $\frac{1}{3}$ with $\frac{3}{2}$ ignored but A0 if $\frac{3}{2}$ is incorrect B1: 70.5 or answers which round to this value M1: $360 - \arccos(\text{their } 1/3)$ (or $2\pi - \arccos(\text{their } 1/3)$ in radians) A1: Second answer Working in radians in (b) may earn M1A1M1A1B0M1A0 Extra values in the range coming from $\arccos(1/3)$ – deduct final A mark - so A0	

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Mark Scheme (Results)

Summer 2013

GCE Core Mathematics 2 (6664/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod – benefit of doubt
 - ft – follow through
 - the symbol \surd will be used for correct ft
 - cao – correct answer only
 - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper
 - \square The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
 5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
 6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
 7. Ignore wrong working or incorrect statements following a correct answer.
 8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x =$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x =$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Question Number	Scheme	Marks
1. (a)	$\{r = \} \frac{2}{3}$	B1 (1)
(b)	$\{p = \} 8$	B1 cao (1)
(c)	$\{S_{15} = \} \frac{18(1 - (\frac{2}{3})^{15})}{1 - \frac{2}{3}}$ $\{S_{15} = 53.87668...\} \Rightarrow S_{15} = \text{awrt } 53.877$	M1 A1 (2) [4]
Notes for Question 1		
(a)	B1: Accept $\frac{12}{18}$, $0.\dot{6}$ or 0.6 recurring, or even 0.667 (3sf) but not 0.6 or 0.67	
(b)	B1: accept 8 only	
(c)	M1: Applies this formula $S_{15} = \frac{18(1 - (\text{their } r)^{15})}{1 - (\text{their } r)}$, can be implied by their answer. For this mark they may use any value for r except $r = 1$ or $r = 0$ (even $3/2$ or -6 may be used) A1: Answers which round to 53.877	
Alternative method for (c)	M1: (Adding terms is an unlikely method for this question) Need to see 15 terms listed as $18+12+\dots+0.06165877$ or can be implied by correct answer A1: awrt 53.877 Answer only : 53.9 is M0A0 with no working, but 53.877 with no working is M1A1	

Question Number	Scheme	Marks
2. (a)	$(2 + 3x)^4$ - Mark (a) and (b) together $2^4 + {}^4C_1 2^3(3x) + {}^4C_2 2^2(3x)^2 + {}^4C_3 2^1(3x)^3 + (3x)^4$ First term of 16 $({}^4C_1 \times \dots \times x) + ({}^4C_2 \times \dots \times x^2) + ({}^4C_3 \times \dots \times x^3) + ({}^4C_4 \times \dots \times x^4)$ $= (16 +) 96x + 216x^2 + 216x^3 + 81x^4$ Must use Binomial – otherwise A0, A0	B1 M1 A1 A1 (4)
(b)	$(2 - 3x)^4 = 16 - 96x + 216x^2 - 216x^3 + 81x^4$	B1ft (1) 5
Alternative method (a)	$(2 + 3x)^4 = 2^4(1 + \frac{3x}{2})^4$ $2^4(1 + {}^4C_1(\frac{3x}{2}) + {}^4C_2(\frac{3x}{2})^2 + {}^4C_3(\frac{3x}{2})^3 + (\frac{3x}{2})^4)$ Scheme is applied exactly as before	
Notes for Question 2		
(a)	B1: The constant term should be 16 in their expansion M1: Two binomial coefficients must be correct and must be with the correct power of x . Accept 4C_1 or $\binom{4}{1}$ or 4 as a coefficient, and 4C_2 or $\binom{4}{2}$ or 6 as another..... Pascal's triangle may be used to establish coefficients. A1: Any two of the final four terms correct (i.e. two of $96x + 216x^2 + 216x^3 + 81x^4$) in expansion following Binomial Method. A1: All four of the final four terms correct in expansion. (Accept answers without + signs, can be listed with commas or appear on separate lines)	
(b)	B1ft: Award for correct answer as printed above or ft their previous answer provided it has five terms ft and must be subtracting the x and x^3 terms Allow terms in (b) to be in descending order and allow $-96x$ and $-216x^3$ in the series. (Accept answers without + signs, can be listed with commas or appear on separate lines)	
	e.g. The common error $2^4 + {}^4C_1 2^3 3x + {}^4C_2 2^2 3x^2 + {}^4C_3 2^1 3x^3 + 3x^4 = (16) + 96x + 72x^2 + 24x^3 + 3x^4$ would earn B1, M1, A0, A0, and if followed by $= (16) - 96x + 72x^2 - 24x^3 + 3x^4$ gets B1ft so 3/5 Fully correct answer with no working can score B1 in part (a) and B1 in part (b). The question stated use the Binomial theorem and if there is no evidence of its use then M mark and hence A marks cannot be earned. Squaring the bracket and squaring again may also earn B1 M0 A0 A0 B1 if correct Omitting the final term but otherwise correct is B1 M1 A1 A0 B0ft so 3/5 If the series is divided through by 2 or a power of 2 at the final stage after an error or omission resulting in all even coefficients then apply scheme to series before this division and ignore subsequent work (isw)	

Question Number	Scheme		Marks
3. (a)	Either (Way 1) : Attempt $f(3)$ or $f(-3)$ $f(3) = 54 - 45 + 3a + 18 = 0 \Rightarrow 3a = -27 \Rightarrow a = -9$	Or (Way 2): Assume $a = -9$ and attempt $f(3)$ or $f(-3)$ $f(3) = 0$ so $(x - 3)$ is factor	M1 A1 * cso (2)
	Or (Way 3): $(2x^3 - 5x^2 + ax + 18) \div (x - 3) = 2x^2 + px + q$ where p is a number and q is an expression in terms of a Sets the remainder $18 + 3a + 9 = 0$ and solves to give $a = -9$		M1 A1* cso (2)
(b)	Either (Way 1): $f(x) = (x - 3)(2x^2 + x - 6)$ $= (x - 3)(2x - 3)(x + 2)$		M1A1 M1A1 (4)
	Or (Way 2) Uses trial or factor theorem to obtain $x = -2$ or $x = 3/2$ Uses trial or factor theorem to obtain both $x = -2$ and $x = 3/2$ Puts three factors together (see notes below) Correct factorisation : $(x - 3)(2x - 3)(x + 2)$ or $(3 - x)(3 - 2x)(x + 2)$ or $2(x - 3)(x - \frac{3}{2})(x + 2)$ oe		M1 A1 M1 A1 (4)
	Or (Way 3) No working three factors $(x - 3)(2x - 3)(x + 2)$ otherwise need working		M1A1M1A1
(c)	$\{3^y = 3 \Rightarrow\} \underline{y = 1}$ or $g(1) = 0$		B1
	$\{3^y = 1.5 \Rightarrow\} \log(3^y) = \log 1.5$ or $y = \log_3 1.5$		M1
	$\{y = 0.3690702...\} \Rightarrow y = \text{awrt } 0.37$		A1 (3) (9)
Notes for Question 3			
(a)	M1 for attempting either $f(3)$ or $f(-3)$ – with numbers substituted into expression A1 for applying $f(3)$ correctly , setting the result equal to 0 , and manipulating this correctly to give the result given on the paper i.e. $a = -9$. (Do not accept $x = -9$) Note that the answer is given in part (a). If they assume $a = -9$ and verify by factor theorem or division they must state $(x - 3)$ is a factor for A1 (or equivalent such as QED or a tick).		
(b)	1 st M1: attempting to divide by $(x - 3)$ leading to a 3TQ beginning with the correct term, usually $2x^2$. (Could divide by $(3 - x)$, in which case the quadratic would begin $-2x^2$.) This may be done by a variety of methods including long division, comparison of coefficients, inspection etc. 1 st A1: usually for $2x^2 + x - 6 \dots$ Credit when seen and use isw if miscopied 2 nd M1: for a valid * attempt to factorise their quadratic (* see notes on page 6 - General Principles for Core Mathematics Marking section 1) 2 nd A1 is cao and needs all three factors together. Ignore subsequent work (such as a solution to a quadratic equation.) NB: $(x - 3)(x - \frac{3}{2})(x + 2)$ is M1A1M0A0, $(x - 3)(x - \frac{3}{2})(2x + 4)$ is M1A1M1A0, but $2(x - 3)(x - \frac{3}{2})(x + 2)$ is M1A1M1A1.		
(c)	B1: $\underline{y = 1}$ seen as a solution – may be spotted as answer – no working needed. Allow also for $g(1) = 0$. M1: Attempt to take logs to solve $3^y = \alpha$ or even $3^{ky} = \alpha$, but not $6^y = \alpha$ where $\alpha > 0$ and $\alpha \neq 3$ & was a root of $f(x) = 0$ (ft their factorization) A1: for an answer that rounds to 0.37. If a third answer is included (and not “rejected”) such as $\ln(-2)$ lose final A mark		

Question Number	Scheme								Marks	
4.		x	0	0.5	1	1.5	2	2.5	3	
		y	5	4	2.5	1.538	1	0.690	0.5	
	(a)	{At $x = 1.5,$ } $y = 1.538$ (only)								B1 cao [1]
	(b)	$\frac{1}{2} \times 0.5 ;$ $\frac{\{ 5 + 0.5 + 2(4 + 2.5 + \text{their } 1.538 + 1 + 0.690) \}}{2}$ For structure of $\{ \dots \} ;$ $\frac{1}{2} \times 0.5 \times \{ (5 + 0.5) + 2(4 + 2.5 + \text{their } 1.538 + 1 + 0.690) \} \{ = \frac{1}{4}(24.956) = 6.239 \} = \text{awrt } 6.24$								B1 oe M1A1ft A1 [4]
(c)	Adds Area of Rectangle or first integral $= 3 \times 4$ or $[4x]_0^3$ to previous answer So required estimate $= \{ "6.239" + 12 = "18.239" \} = \text{"awrt } 18.24"$ (or $12 + \text{previous answer}$). N.B. $7 \times 4 + \text{previous answer}$ is M0A0 (added 4 seven times because 7 numbers in table)								M1 A1ft [2] 7	
	Notes for Question 4									
(a)	B1: 1.538									
(b)	B1: for using $\frac{1}{2} \times 0.5$ or $\frac{1}{4}$ or equivalent. M1: requires the correct $\{ \dots \}$ bracket structure. It needs the first bracket to contain first y value plus last y value and the second bracket to be multiplied by 2 and to be the summation of the remaining y values in the table with no additional values. If the only mistake is a copying error or is to omit one value from 2nd bracket this may be regarded as a slip and the M mark can be allowed (An extra repeated term forfeits the M mark however). M0 if values used in brackets are x values instead of y values A1ft: for the correct bracket $\{ \dots \}$ following through candidate's y value found in part (a). A1: for answer which rounds to 6.24. NB: Separate trapezia may be used : B1 for 0.25, M1 for $\frac{1}{2} h(a + b)$ used 5 or 6 times (and A1ft if it is all correct) Then A1 as before. Special case: Bracketing mistake $0.25 \times (5 + 0.5) + 2(4 + 2.5 + \text{their } 1.538 + 1 + 0.690)$ scores B1 M1 A0 A0 unless the final answer implies that the calculation has been done correctly (then full marks can be given). An answer of 20.831 usually indicates this error.									
(c)	M1: Relates previous answer (not integral of previous answer) to this question by integrating 4 between limits, and adding, or by using geometry to find rectangle and adding. A1ft: for $12 + \text{answer to (b)}$									
Alternative method (c)	Those who do a trapezium rule for part (b)- using the table from (a) with 4 added to each cell of the table Get: M1 for $\text{"their } \frac{1}{4} " \times \{ 9 + 4.5 + 2(8 + 6.5 + \text{their } 5.538 + 5 + 4.690) \} =$ (structure must be correct – allow one copying error only) And A1ft: for awrt 18.24 (or $12 + \text{previous answer}$).									

Question Number	Scheme	Marks
5. (a)	<p>Mark (a) and (b) together.</p> <p>Usually answered in radians: Uses either $\frac{1}{2}ab\sin(\text{angle})$ or $\frac{1}{2}(12)^2(\text{angle})$ or both</p> <p>Area = $\frac{1}{2}(23)(12)\sin 0.64$ or $\frac{1}{2}(12)^2(\pi - 0.64)$ { = 82.41297091... or 180.1146711... }</p> <p>Area = $\frac{1}{2}(23)(12)\sin 0.64 + \frac{1}{2}(12)^2(\pi - 0.64)$ { = 82.41297091... + 180.1146711... }</p> <p>{ Area = 262.527642... } = awrt 262.5 (m²) or 262.4(m²) or 262.6 (m²)</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>(4)</p>
(b)	<p>$CDE = 12 \times (\text{angle}), = 12(\pi - 0.64) \Rightarrow CDE = 30.01911...$</p> <p>$AE^2 = 23^2 + 12^2 - 2(23)(12)\cos(0.64) \Rightarrow AE^2 =$ or $AE =$ { $AE = 15.17376...$ }</p> <p>Perimeter = $23 + 12 + 15.17376... + 30.01911...$</p> <p>= 80.19287... = awrt 80.2 (m)</p>	<p>M1, A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>(5)</p> <p>[9]</p>
Notes for Question 5		
(a)	<p>M1: uses either area of triangle formula as given in mark scheme, or area of sector or both (may be implied by answer)</p> <p>A1: one correct area expression (with correct angle used) $\frac{1}{2}(23)(12)\sin 0.64$ or $\frac{1}{2}(12)^2(\pi - 0.64)$ or see awrt 82.4 or awrt 180 (180 may be split as 226.2(semicircle) minus 46.1(small sector))</p> <p>A1: two correct area expressions (with correct angles) added together (allow 2.5 as implying $\pi - 0.64$) or see awrt 82.4 + awrt 180 (or 226 - 46)</p>	
(b)	<p>A1: answers which round to 262.5 or 262.4 or 262.6</p> <p>1st M1 for attempt to use $s = r \theta$ (any angle)</p> <p>1st A1 for $\pi - 0.64$ in the formula (or 2.5)</p> <p>2nd M1: Uses correct cosine rule to obtain AE or AE^2 (this may appear in part (a))</p> <p>3rd M1(independent): Perimeter = $23 + 12 +$ their AE + their CDE</p> <p>2nd A1: awrt 80.2 (ignore units – even incorrect units)</p>	
Degrees		
(a)	<p>Uses either $\frac{1}{2}ab\sin(\text{angle})$ or $\frac{\text{angle in degrees}}{360} \times \pi(12)^2$ or both for M1</p> <p>Area = $\frac{1}{2}(23)(12)\sin 36.7$ or $\frac{(180-36.7)}{360} \times \pi(12)^2$ { = awrt 82.4... or 180 } A1</p> <p>Area = $\frac{1}{2}(23)(12)\sin 36.7 + \frac{(180-36.7)}{360} \times \pi(12)^2$ { = awrt 82.4... + 180 } A1</p> <p>Final mark as before</p>	
(b)	<p>$CDE = \frac{\text{Angle in degrees}}{360} \times 24\pi, = \frac{180-36.7}{360} \times 24\pi \Rightarrow CDE = 30.01268...$ M1, A1</p> <p>Final three marks as before</p>	

Question Number	Scheme	Marks
6. (a)	Seeing -4 and 2.	B1 (1)
(b)	$x(x+4)(x-2) = x^3 + 2x^2 - 8x \quad \text{or} \quad x^3 - 2x^2 + 4x^2 - 8x \text{ (without simplifying)}$ $\int (x^3 + 2x^2 - 8x) dx = \frac{x^4}{4} + \frac{2x^3}{3} - \frac{8x^2}{2} \{+c\} \quad \text{or} \quad \frac{x^4}{4} - \frac{2x^3}{3} + \frac{4x^3}{3} - \frac{8x^2}{2} \{+c\}$ $\left[\frac{x^4}{4} + \frac{2x^3}{3} - \frac{8x^2}{2} \right]_{-4}^0 = (0) - \left(64 - \frac{128}{3} - 64 \right) \quad \text{or} \quad \left[\frac{x^4}{4} + \frac{2x^3}{3} - \frac{8x^2}{2} \right]_0^2 = \left(4 + \frac{16}{3} - 16 \right) - (0)$ <p>One integral = $\pm 42\frac{2}{3}$ (42.6 or awrt 42.7) or other integral = $\pm 6\frac{2}{3}$ (6.6 or awrt 6.7)</p> <p>Hence Area = "their $42\frac{2}{3}$" + "their $6\frac{2}{3}$" or Area = "their $42\frac{2}{3}$" - "their $6\frac{2}{3}$"</p> <p>= $49\frac{1}{3}$ or 49.3 or $\frac{148}{3}$ (NOT $-\frac{148}{3}$)</p> <p>(An answer of $= 49\frac{1}{3}$ may not get the final two marks – check solution carefully)</p>	B1 B1 M1A1ft dM1 A1 dM1 A1 (7)
Notes for Question 6		
(a)	B1: Need both -4 and 2. May see (-4,0) and (2,0) (correct) but allow (0,-4) and (0, 2) or $A = -4$, $B = 2$ or indeed any indication of -4 and 2 – check graph also	
(b)	<p>B1: Multiplies out cubic correctly (terms may not be collected, but if they are, mark collected terms here)</p> <p>M1: Tries to integrate their expansion with $x^n \rightarrow x^{n+1}$ for at least one of the terms</p> <p>A1ft: completely correct integral following through from their CUBIC expansion (if only quadratic or quartic this is A0)</p> <p>dM1: (dependent on previous M) substituting EITHER -a and 0 and subtracting either way round OR similarly for 0 and b. If their limits -a and b are used in ONE integral, apply the Special Case below.</p> <p>A1: Obtain either $\pm 42\frac{2}{3}$ (or 42.6 or awrt 42.7) from the integral from -4 to 0 or $\pm 6\frac{2}{3}$ (6.6 or awrt 6.7) from the integral from 0 to 2; NO follow through on their cubic (allow decimal or improper equivalents $\frac{128}{3}$ or $\frac{20}{3}$) isw such as subtracting from rectangles. This will be penalized in the next two marks, which will be M0A0.</p> <p>dM1 (depends on first method mark) Correct method to obtain shaded area so adds two positive numbers (areas) together or uses their positive value minus their negative value, obtained from two separate definite integrals.</p> <p>A1: Allow 49.3, 49.33, 49.333 etc. Must follow correct logical work with no errors seen.</p> <p>For full marks on this question there must be two definite integrals, from -4 to 0 and from 0 to 2, though the evaluations for 0 may not be seen.</p> <p>(Trapezium rule gets no marks after first two B marks)</p>	
(b)	<p>Special Case: one integral only from -a to b: B1M1A1 available as before, then</p> $\left[\frac{x^4}{4} + \frac{2x^3}{3} - \frac{8x^2}{2} \right]_{-4}^2 = \left(4 + \frac{16}{3} - 16 \right) - \left(64 - \frac{128}{3} - 64 \right) = -6\frac{2}{3} + 42\frac{2}{3} = \dots$ <p>dM1 for correct use of their limits -a and b and subtracting either way round.</p> <p>A1 for 36: NO follow through. Final M and A marks not available. Max 5/7 for part (b)</p>	

[8]

Question Number	Scheme	Marks
7. (i) Method 1	$\log_2\left(\frac{2x}{5x+4}\right) = -3$ or $\log_2\left(\frac{5x+4}{2x}\right) = 3$, or $\log_2\left(\frac{5x+4}{x}\right) = 4$ (see special case 2) $\left(\frac{2x}{5x+4}\right) = 2^{-3}$ or $\left(\frac{5x+4}{2x}\right) = 2^3$ or $\left(\frac{5x+4}{x}\right) = 2^4$ or $\left(\log_2\left(\frac{2x}{5x+4}\right)\right) = \log_2\left(\frac{1}{8}\right)$ $16x = 5x + 4 \Rightarrow x =$ (depends on previous Ms and must be this equation or equivalent) $x = \frac{4}{11}$ or exact recurring decimal $0.\dot{3}\dot{6}$ after correct work	M1 M1 dM1 A1 cso (4)
7(i) Method 2	$\log_2(2x) + 3 = \log_2(5x + 4)$ So $\log_2(2x) + \log_2(8) = \log_2(5x + 4)$ (3 replaced by $\log_2 8$) Then $\log_2(16x) = \log_2(5x + 4)$ (addition law of logs) Then final M1 A1 as before	2 nd M1 1 st M1 dM1A1
(ii)	$\log_a y + \log_a 2^3 = 5$ $\log_a 8y = 5$ $y = \frac{1}{8}a^5$ <p style="text-align: right;">Applies product law of logarithms. $y = \frac{1}{8}a^5$</p>	M1 dM1 A1cao (3) [7]
Notes for Question 7		
(i)	1 st M1: Applying the subtraction or addition law of logarithms correctly to make two log terms in x into one log term in x 2 nd M1: For RHS of either 2^{-3} , 2^3 , 2^4 or $\log_2\left(\frac{1}{8}\right)$, $\log_2 8$ or $\log_2 16$ i.e. using connection between log base 2 and 2 to a power. This may follow an earlier error. Use of 3^2 is M0 3 rd dM1: Obtains correct linear equation in x. usually the one in the scheme and attempts $x =$ A1: cso Answer of $4/11$ with no suspect log work preceding this.	
(ii)	M1: Applies power law of logarithms to replace $3\log_a 2$ by $\log_a 2^3$ or $\log_a 8$ dM1: (should not be following M0) Uses addition law of logs to give $\log_a 2^3 y = 5$ or $\log_a 8y = 5$	
(i)	Special case 1: $\log_2(2x) = \log_2(5x + 4) - 3 \Rightarrow \frac{\log_2(2x)}{\log_2(5x + 4)} = -3 \Rightarrow \frac{2x}{5x + 4} = 2^{-3} \Rightarrow x = \frac{4}{11}$ or $\log_2(2x) = \log_2(5x + 4) - 3 \Rightarrow \frac{\log_2(2x)}{\log_2(5x + 4)} = -3 \Rightarrow \log_2 \frac{2x}{5x + 4} = -3 \Rightarrow \frac{2x}{5x + 4} = 2^{-3} \Rightarrow x = \frac{4}{11}$ each attempt scores M0M1M1A0 – special case Special case 2: $\log_2(2x) = \log_2(5x + 4) - 3 \Rightarrow \log_2 2 + \log_2 x = \log_2(5x + 4) - 3$, is M0 until the two log terms are combined to give $\log_2\left(\frac{5x+4}{x}\right) = 3 + \log_2 2$. This earns M1 Then $\left(\frac{5x+4}{x}\right) = 2^4$ or $\log_2\left(\frac{5x+4}{x}\right) = \log_2 2^4$ gets second M1. Then scheme as before.	

Question Number	Scheme	Marks
8. (i)	$(\alpha = 56.3099\dots)$ $x = \{\alpha + 40 = 96.309993\dots\} = \text{awrt } 96.3$ $x - 40^\circ = -180 + "56.3099"\dots$ or $x - 40^\circ = -\pi + "0.983"\dots$ $x = \{-180 + 56.3099\dots + 40 = -83.6901\dots\} = \text{awrt } -83.7$	B1 M1 A1 (3)
(ii)(a)	$\sin \theta \left(\frac{\sin \theta}{\cos \theta} \right) = 3 \cos \theta + 2$ $\left(\frac{1 - \cos^2 \theta}{\cos \theta} \right) = 3 \cos \theta + 2$ $1 - \cos^2 \theta = 3 \cos^2 \theta + 2 \cos \theta \Rightarrow 0 = 4 \cos^2 \theta + 2 \cos \theta - 1^*$	M1 dM1 A1 cso * (3)
(b)	$\cos \theta = \frac{-2 \pm \sqrt{4 - 4(4)(-1)}}{8}$ or $4(\cos \theta \pm \frac{1}{4})^2 \pm q \pm 1 = 0$, or $(2 \cos \theta \pm \frac{1}{2})^2 \pm q \pm 1 = 0$, $q \neq 0$ so $\cos \theta = \dots$ One solution is 72° or 144° , Two solutions are 72° and 144° $\theta = \{72, 144, 216, 288\}$	M1 A1, A1 M1 A1 (5) [11]
Notes for Question 8		
(i)	B1: 96.3 by any or no method M1: Takes 180 degrees from arctan (1.5) or from their "96.3" May be implied by A1. (Could be obtained by adding 180, then subtracting 360). A1: awrt -83.7 Extra answers: ignore extra answers outside range. Any extra answers in range lose final A mark (if earned) Working in radians – could earn M1 for $x - 40^\circ = -\pi + "0.983"\dots$ so B0M1A0	
(ii) (a)	M1: uses $\tan \theta = \frac{\sin \theta}{\cos \theta}$ or equivalent in equation (not just $\tan = \frac{\sin}{\cos}$, with no argument) dM1: uses $\sin^2 \theta = 1 - \cos^2 \theta$ (quoted correctly) in equation A1: completes proof correctly, with no errors to give printed answer*. Need at least three steps in proof and need to achieve the correct quadratic with all terms on one side and "=0"	
(b)	M1: Attempts to solve quadratic by correct quadratic formula, or completion of the square . Factorisation attempts score M0. 1 st A1: Either 72 or 144, 2 nd A1: both 72 and 144 (allow 72.0 etc.) M1: 360 – "a previous solution" (provided that cos was being used) (not dependent on previous M) A1: All four solutions correct (Extra solutions in range lose this A mark, but outside range - ignore) (Premature approximation: e.g. 71.9, 144.1, 288.1 and 215.9 – lose first A1 then fit other angles) Do not require degrees symbol for the marks Special case: Working in radians M1: as before, A1 for either $\theta = \frac{2}{5}\pi$ or $\theta = \frac{4}{5}\pi$ or decimal equivalents, and 2 nd A1: both M1: $2\pi - \alpha_1$ or $2\pi - \alpha_2$ then A0 so 4/5	

Question Number	Scheme	Marks
9. (a)	$\left\{ \frac{dy}{dx} = \right\} 2x - 16x^{-\frac{1}{2}}$ $2x - 16x^{-\frac{1}{2}} = 0 \Rightarrow x^{\frac{3}{2}} = , x^{-\frac{3}{2}} = , \text{or } 2x - 16x^{-\frac{1}{2}} \text{ then squared then obtain } x^3 =$ [or $2x - 16x^{-\frac{1}{2}} = 0 \Rightarrow x = 4$ (no wrong work seen)] $(x^{\frac{3}{2}} = 8 \Rightarrow) x = 4$ $x = 4, y = 4^2 - 32\sqrt{4} + 20 = -28$ (ignore $y = 100$ as second answer)	M1 A1 M1 A1 M1 A1 (6)
(b)	$\left\{ \frac{d^2y}{dx^2} = \right\} 2 + 8x^{-\frac{3}{2}}$ $(\frac{d^2y}{dx^2} > 0 \Rightarrow) y \text{ is a minimum (there should be no wrong reasoning)}$	M1 A1 A1 (3) [9]
(b)	Alternative Method: Gradient Test: M1 for finding the gradient either side of their x -value from part (a). A1 for <u>both</u> gradients calculated correctly to 1 significant figure, then using < 0 and > 0 respectively <u>maybe by use of sketch or table</u> . (See appendix for gradient values. This is not ft their x) A1 states minimum needs M1A1 to have been awarded.	
	Notes for Question 9	
(a)	1 st M1: At least one term differentiated correctly, so $x^2 \rightarrow 2x$, or $32\sqrt{x} \rightarrow 16x^{-\frac{1}{2}}$, or $20 \rightarrow 0$ A1: This answer or equivalent e.g. $2x - \frac{16}{\sqrt{x}}$ 2 nd M1: Sets their $\frac{dy}{dx}$ to 0, and solves to give $x^{\frac{3}{2}} = , x^{-\frac{3}{2}} = \text{or } x^3 =$ after correct squaring or spots $x = 4$ (NB $\left\{ \frac{d^2y}{dx^2} = 0 \right\}$ so $2 + 8x^{-\frac{3}{2}} = 0$ is M0) N.B. Common error: Putting derivative = 0 and merely obtaining $x = 0$ is M0A0, then M0A0 for next two marks. (The first two marks in (a) and marks for second derivative may be earned in part (b).) A1: $x = 4$ cao [$x = -4$ is A0 and $x = \pm 4$ is also A0] 3 rd M1: Substitutes their positive found x (NOT zero) into $y = x^2 - 32\sqrt{x} + 20, x > 0$. Should follow attempting to set $\frac{dy}{dx} = 0$ and not setting $\frac{d^2y}{dx^2} = 0$	
(b)	A1: -28 cao (Does not need to be written as coordinates) M1: Attempts differentiation of their first derivative with at least one term differentiated correctly. Should be seen or referred to (in part (b)) in determining the nature of the stationary point. A1: Answer in scheme or equivalent A1: States minimum (Second derivative should be correct- can follow incorrect positive x . Needs M1A1 to have been awarded- should not follow incorrect reasoning – (need not say $\frac{d^2y}{dx^2} > 0$ but should not have said $\frac{d^2y}{dx^2} = 0$ for example)	

Question Number	Scheme	Marks
10. (a)	Equation of form $(x \pm 5)^2 + (y \pm 9)^2 = k$, $k > 0$ Equation of form $(x - a)^2 + (y - b)^2 = 5^2$, with values for a and b $(x + 5)^2 + (y - 9)^2 = 25 = 5^2$	M1 M1 A1 (3)
(b)	$P(8, -7)$. Let centre of circle = $X(-5, 9)$ $PX^2 = (8 - (-5))^2 + (-7 - 9)^2$ or $PX = \sqrt{(8 - (-5))^2 + (-7 - 9)^2}$ ($PX = \sqrt{425}$ or $5\sqrt{17}$) $PT^2 = (PX)^2 - 5^2$ with numerical PX $PT = \{\sqrt{400}\} = 20$ (allow 20.0)	M1 dM1 A1 cso (3) [6]
Alternative 2 for (a)	Equation of the form $x^2 + y^2 \pm 10x \pm 18y + c = 0$ Uses $a^2 + b^2 - 5^2 = c$ with their a and b or substitutes $(0, 9)$ giving $+9^2 \pm 2b \times 9 + c = 0$ $x^2 + y^2 + 10x - 18y + 81 = 0$	M1 M1 A1 (3)
Alternative 2 for (b)	An attempt to find the point T may result in pages of algebra, but solution needs to reach $(-8, 5)$ or $\left(\frac{-8}{17}, 11\frac{2}{17}\right)$ to get first M1 (even if gradient is found first) M1: Use either of the correct points with $P(8, -7)$ and distance between two points formula A1: 20	M1 dM1 A1cso (3)
Alternative 3 for (b)	Substitutes $(8, -7)$ into circle equation so $PT^2 = 8^2 + (-7)^2 + 10 \times 8 - 18 \times (-7) + 81$ Square roots to give $PT = \{\sqrt{400}\} = 20$	M1 dM1A1 (3)
Notes for Question 10		
(a)	The three marks in (a) each require a circle equation – (see special cases which are not circles) M1: Uses coordinates of centre to obtain LHS of circle equation (RHS must be r^2 or $k > 0$ or a positive value) M1: Uses $r = 5$ to obtain RHS of circle equation as 25 or 5^2 A1: correct circle equation in any equivalent form Special cases $(x \pm 5)^2 + (x \pm 9)^2 = (5^2)$ is not a circle equation so M0M0A0 Also $(x \pm 5)^2 + (y - 9) = (5^2)$ And $(x \pm 5)^2 - (y \pm 9)^2 = (5^2)$ are not circles and gain M0M0A0 But $(x - 0)^2 + (y - 9)^2 = 5^2$ gains M0M1A0	
(b)	M1: Attempts to find distance from their centre of circle to P (or square of this value). If this is called PT and given as answer this is M0. Solution may use letter other than X , as centre was not labelled in the question. N.B. Distance from $(0, 9)$ to $(8, -7)$ is incorrect method and is M0, followed by M0A0. dM1: Applies the subtraction form of Pythagoras to find PT or PT^2 (depends on previous method mark for distance from centre to P) or uses appropriate complete method involving trigonometry A1: 20 cso	

Question Number	Scheme	Marks																																												
<i>Aliter</i> 9. (b) <i>Way 2</i>	Gradient Test Method: $\frac{dy}{dx} = 2x - 16x^{-\frac{1}{2}}$ <i>Helpful table!</i> <table><tr><th>x</th><th>$\frac{dy}{dx}$</th></tr><tr><td>3</td><td>-3.2376</td></tr><tr><td>3.1</td><td>-2.88739</td></tr><tr><td>3.2</td><td>-2.54427</td></tr><tr><td>3.3</td><td>-2.20771</td></tr><tr><td>3.4</td><td>-1.87722</td></tr><tr><td>3.5</td><td>-1.55236</td></tr><tr><td>3.6</td><td>-1.23274</td></tr><tr><td>3.7</td><td>-0.918</td></tr><tr><td>3.8</td><td>-0.60783</td></tr><tr><td>3.9</td><td>-0.30191</td></tr><tr><td>4</td><td>0</td></tr><tr><td>4.1</td><td>0.298163</td></tr><tr><td>4.2</td><td>0.592799</td></tr><tr><td>4.3</td><td>0.884115</td></tr><tr><td>4.4</td><td>1.172299</td></tr><tr><td>4.5</td><td>1.457528</td></tr><tr><td>4.6</td><td>1.739962</td></tr><tr><td>4.7</td><td>2.01975</td></tr><tr><td>4.8</td><td>2.297033</td></tr><tr><td>4.9</td><td>2.571937</td></tr><tr><td>5</td><td>2.844582</td></tr></table>	x	$\frac{dy}{dx}$	3	-3.2376	3.1	-2.88739	3.2	-2.54427	3.3	-2.20771	3.4	-1.87722	3.5	-1.55236	3.6	-1.23274	3.7	-0.918	3.8	-0.60783	3.9	-0.30191	4	0	4.1	0.298163	4.2	0.592799	4.3	0.884115	4.4	1.172299	4.5	1.457528	4.6	1.739962	4.7	2.01975	4.8	2.297033	4.9	2.571937	5	2.844582	
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