



1.  $f(x) \equiv ax^3 + bx^2 - 7x + 14$ , where  $a$  and  $b$  are constants.

Given that when  $f(x)$  is divided by  $(x - 1)$  the remainder is 9,

- (a) write down an equation connecting  $a$  and  $b$ . **(2 marks)**

Given also that  $(x + 2)$  is a factor of  $f(x)$ ,

- (b) find the values of  $a$  and  $b$ . **(4 marks)**
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2. (i) Differentiate with respect to  $x$

$$2x^3 + \sqrt{x} + \frac{x^2 + 2x}{x^2}. \quad \text{(5 marks)}$$

- (ii) Evaluate

$$\int_1^4 \left( \frac{x}{2} + \frac{1}{x^2} \right) dx. \quad \text{(5 marks)}$$


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3. (a) An arithmetic series has first term  $a$  and common difference  $d$ . Prove that the sum of the first  $n$  terms of the series is

$$\frac{1}{2} n[2a + (n - 1)d]. \quad \text{(4 marks)}$$

A company made a profit of £54 000 in the year 2001. A model for future performance assumes that yearly profits will increase in an arithmetic sequence with common difference £ $d$ . This model predicts total profits of £619 200 for the 9 years 2001 to 2009 inclusive.

- (b) Find the value of  $d$ . **(4 marks)**

Using your value of  $d$ ,

- (c) find the predicted profit for the year 2011. **(2 marks)**

An alternative model assumes that the company's yearly profits will increase in a geometric sequence with common ratio 1.06. Using this alternative model and again taking the profit in 2001 to be £54 000,

- (d) find the predicted profit for the year 2011. **(3 marks)**
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4. (a) Write down formulae for  $\sin(A + B)$  and  $\sin(A - B)$ .

Using  $X = A + B$  and  $Y = A - B$ , prove that

$$\sin X + \sin Y = 2 \sin \frac{X+Y}{2} \cos \frac{X-Y}{2}.$$

**(4 marks)**

- (b) Hence, or otherwise, solve, for  $0 \leq \theta < 360$ ,

$$\sin 40^\circ + \sin 20^\circ = 0.$$

**(5 marks)**

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5.

Figure 1

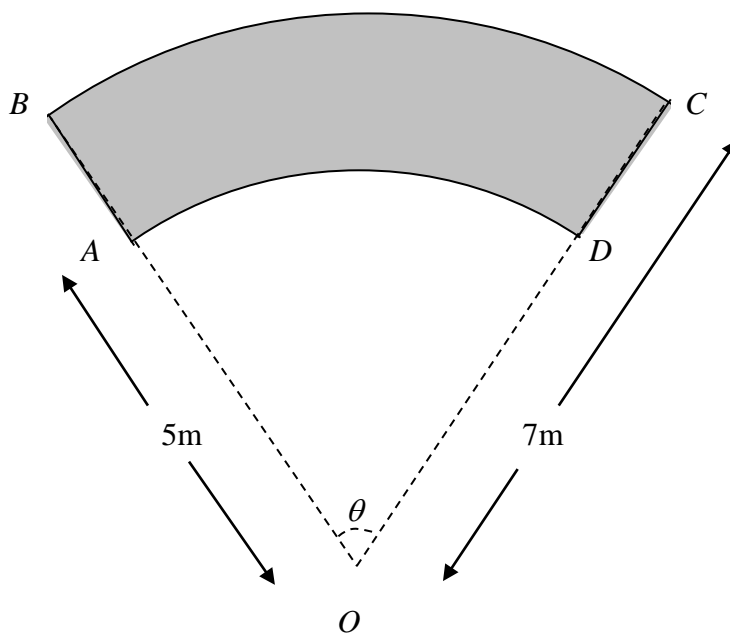


Figure 1 shows a gardener’s design for the shape of a flower bed with perimeter  $ABCD$ .

$AD$  is an arc of a circle with centre  $O$  and radius 5 m.

$BC$  is an arc of a circle with centre  $O$  and radius 7 m.

$OAB$  and  $ODC$  are straight lines and the size of  $\angle AOD$  is  $\theta$  radians.

(a) Find, in terms of  $\theta$ , an expression for the area of the flower bed. **(3 marks)**

Given that the area of the flower bed is  $15 \text{ m}^2$ ,

(b) show that  $\theta = 1.25$ , **(2 marks)**

(c) calculate, in m, the perimeter of the flower bed. **(3 marks)**

The gardener now decides to replace arc  $AD$  with the straight line  $AD$ .

(d) Find, to the nearest cm, the reduction in the perimeter of the flower bed. **(2 marks)**

6. (a) Given that

$$(2 + x)^5 + (2 - x)^5 \equiv A + Bx^2 + Cx^4,$$

Find the values of the constants  $A$ ,  $B$  and  $C$ .

(6 marks)

(b) Using the substitution  $y = x^2$  and your answers to part (a), solve,

$$(2 + x)^5 + (2 - x)^5 = 349.$$

(5 marks)

7.

Figure 2

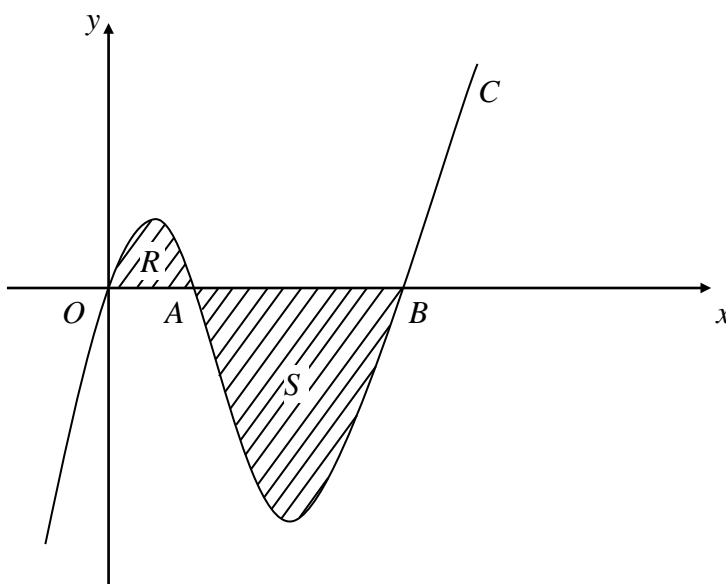


Figure 2 shows part of the curve  $C$  with equation  $y = f(x)$ , where

$$f(x) = x^3 - 6x^2 + 5x.$$

The curve crosses the  $x$ -axis at the origin  $O$  and at the points  $A$  and  $B$ .

(a) Factorise  $f(x)$  completely

(3 marks)

(b) Write down the  $x$ -coordinates of the points  $A$  and  $B$ .

(1 marks)

(c) Find the gradient of  $C$  at  $A$ .

(3 marks)

The region  $R$  is bounded by  $C$  and the line  $OA$ , and the region  $S$  is bounded by  $C$  and the line  $AB$ .

(d) Use integration to find the area of the combined regions  $R$  and  $S$ , shown shaded in Fig. 2.

(7 marks)

END