CORE MATHEMATICS (C) UNIT 2 TEST PAPER 9

1. Given that $ax^2 + bx + 4$ and $x^3 - 2x^2 + 2ax + b$ are both divisible by $(x - 2)$, find the values of the constants a and b. [4]

2. Find, and simplify fully, the complete binomial expansion of $(3x - 2y)^4$. [5]

3. Given that $\cos \frac{\pi}{12} = \frac{\sqrt{2}}{4}(1 + \sqrt{3})$, find expressions in surd form for

   (i) $\cos \frac{11\pi}{12}$, [2]

   (ii) $\sin^2 \frac{\pi}{12}$, simplifying your answer. [3]

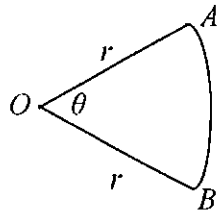
4. Integrate $(x + 1)(\sqrt{x} + 1)^2$ with respect to x, simplifying each term in the answer. [7]

5. (i) If a is a positive constant, state the exact values of

   (a) $\log_a a^7$, (b) $\log_a \sqrt{a}$, (c) $\log_a \frac{1}{a^2}$. [3]

   (ii) Solve for x the equation $5^{2x-1} = 7^{3-4x}$, giving the solution to 2 decimal places. [5]

6. A and B are two points on the circumference of a circle of radius r cm. The angle AOB is θ radians. The perimeter of the sector OAB is p cm.



   (i) Show that $\theta = \frac{p}{r} - 2$. [3]

   (ii) If the area of the sector OAB is $\frac{r^2}{3}$ cm^2, express p in terms of r. [3]

   (iii) If also the area of triangle OAB is kr^2 cm^2, find the value of k to 2 decimal places. [3]

7. When a new product is launched on the market, 9000 items are sold in the first week. In the second week, 8000 items are sold. The manufacturer knows that demand will continue to fall and considers two possible models: the weekly sales form either an arithmetic or a geometric sequence, with first two terms 9000 and 8000.

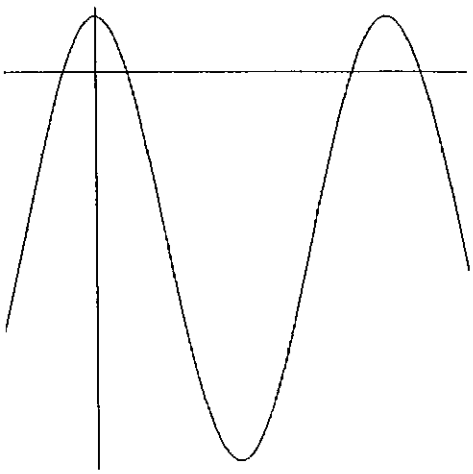
   (i) Using the arithmetic model, find the total number of items that will be sold. [3]

   (ii) Using the geometric model, find to the nearest integer

   (a) the number of items that will be sold in the tenth week, [3]

   (b) the total number of items that will be sold in the first twenty weeks. [3]

8. The diagram shows part of the curve with equation $y = p\cos qx - r$, where p, q, r are positive integers and x is measured in radians.
   The three turning points shown are at (0, 1), $\left(\frac{\pi}{2}, -7\right)$ and $(\pi, 1)$.



   (i) Find the values of p, q and r. [6]

   (ii) Find, in radians to 1 decimal place, the coordinates of the four points where the graph cuts the x-axis in the diagram. [6]

9. The equation of a curve is $y = \sqrt{x} + \frac{8}{x^2}$, where $x > 0$.

   (i) Find the values of x and y at the turning point of the curve. [6]

   (ii) Show that this turning point is a minimum. [2]

   (iii) Calculate the area of the region between the curve, the x-axis and the lines $x = 1$ and $x = 9$, giving your answer as an exact fraction. [5]

CORE MATHS 2 (C) TEST PAPER 9 : ANSWERS AND MARK SCHEME

1. $4a + 2b + 4 = 0,\ 4a + b = 0$ $\quad a = 1, b = -4$ — M1 M1 A1 A1 — 4

2. $(3x - 2y)^4 = (3x)^4 + 4(3x)^3(-2y) + 6(3x)^2(-2y)^2 + 4(3x)(-2y)^3 + (-2y)^4$ — M1 A1 A1
$= 81x^4 - 216x^3y + 216x^2y^2 - 96xy^3 + 16y^4$ — M1 A1 — 5

3. (i) $\cos\dfrac{11\pi}{12} = -\cos\dfrac{\pi}{12} = -\dfrac{\sqrt{2}}{4}(1+\sqrt{3})$ — M1 A1

(ii) $\sin^2\dfrac{\pi}{12} = 1 - \cos^2\dfrac{\pi}{12} = 1 - \dfrac{1}{8}(4+2\sqrt{3}) = \dfrac{2-\sqrt{3}}{4}.$ — M1 A1 A1 — 5

4. $\int (x+1)(x+2\sqrt{x}+1)\,dx = \int x^2 + 2x^{3/2} + 2x + 2x^{1/2} + 1\,dx$ — M1 A1 A1
$= \dfrac{1}{3}x^3 + \dfrac{4}{5}x^{5/2} + x^2 + \dfrac{4}{3}x^{3/2} + x + c$ — M1 A1 A1 A1 — 7

5. (i) (a) 7 (b) ½ (c) –2 — B1 B1 B1
(ii) $(2x - 1)\log 5 = (3 - 4x)\log 7$ — M1 A1
$(2\log 5 + 4\log 7)x = 3\log 7 + \log 5$ $\quad x = 3.234/4.778 = 0.68$ — M1 A1 A1 — 8

6. (i) $p = r\theta + 2r$ $\quad \theta = (p - 2r)/r = p/r - 2$ — M1 A1 A1
(ii) $r^2\theta/2 = r^2/3$ so $\theta = 2/3$ $\quad p/r = 8/3$ $\quad p = 8r/3$ — M1 A1 A1
(iii) $r^2 \sin\theta/2 = kr^2$ so $k = \frac{1}{2}\sin(2/3) = 0.31$ — M1 A1 A1 — 9

7. (i) $9000 + 8000 + \ldots + 1000 = 9(10{,}000)/2 = 45{,}000$ — M1 A1 A1
(ii) (a) $r = 8/9$ $\quad T_{10} = 9000(8/9)^9 = 3118$ — B1 M1 A1
(b) $S_{20} = 9000(1 - (8/9)^{20})/(1 - 8/9) = 73{,}319$ — M1 A1 A1 — 9

8. (i) $1 = p - r,\ -7 = -p - r$ $\quad p = 4, r = 3$ — B2 M1 A1
Period $= \pi$, so $q = 2$ — M1 A1
(ii) Where $4\cos 2x - 3 = 0$, $\cos 2x = 3/4$ $\quad 2x = \pm 0.723, 5.560, 7.006$ — M1 A1 A1
$x = -0.36, 0.36, 2.78, 3.50$ — M1 A1 A1 — 12

9. (i) $y = x^{1/2} + 8x^{-2}$ $\quad \dfrac{dy}{dx} = \dfrac{1}{2}x^{-1/2} - 16x^{-3} = 0$ when $x^{5/2} = 32$ — B1 B1 M1 A1
$x = 4, y = 5/2$ $\quad$ (ii) $\dfrac{d^2y}{dx^2} = -\dfrac{1}{4}x^{-3/2} + 48x^{-4} = \dfrac{5}{32} > 0$ so min. — A1 A1 M1 A1
(iii) $\displaystyle\int_1^9 y\,dx = \left[\frac{2}{3}x^{3/2} - \frac{8}{x}\right]_1^9 = 18 - \frac{8}{9} - \frac{2}{3} + 8 = \frac{220}{9}$ — M1 A1 A1 M1 A1