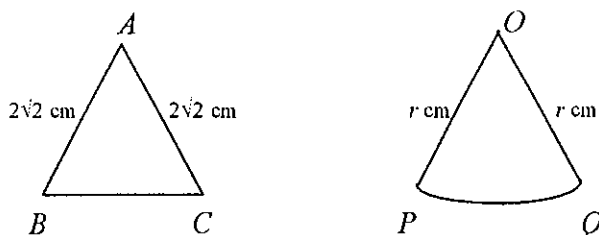


CORE MATHEMATICS (C) UNIT 2 TEST PAPER 7

1. Find the remainder when $(x^4 - 3x^3 + 2x - 1)$ is divided by $(x + 1)$. [3]
2. The diagram shows an isosceles triangle ABC whose equal sides are $2\sqrt{2}$ cm long and a sector OPQ of a circle of radius r cm. Angle $BAC = \text{angle } POQ = \frac{\pi}{4}$ radians.



Given that the triangle and the sector have equal areas, find r correct to 2 decimal places. [4]

3. (i) Find $\int (3x - 1)^2 dx$. [4]
- (ii) Hence write down *two* possible functions $f(x)$ such that $f'(x) = (3x - 1)^2$. [2]

4. In the binomial expansion of $\left(2x + \frac{1}{2x}\right)^6$ in descending powers of x , find in their simplest form
- (i) the first two terms, [4]
- (ii) the term which is independent of x . [3]

5. In the triangle XYZ , $XY = (x + 1)$ cm, $YZ = x$ cm and $XZ = y$ cm. Angle $XYZ = 120^\circ$.
- (i) Show that $y^2 = 3x^2 + 3x + 1$. [3]
- (ii) When $x = 7$, find the size of angle XZY , in degrees to 1 decimal place. [4]

6. (i) A car is bought for £12,000. Each year, its value decreases by 15%.
Show that the values of the car in successive years form a geometric sequence, and
find the value of the car after 12 years, to the nearest pound. [4]
- (ii) Find the exact value of $\sum_{r=1}^{15} (7 - 3r)$. [3]

CORE MATHEMATICS 2 (C) TEST PAPER 7 Page 2

7. (i) Solve for x the equation $2 \log_{10} 5 + \log_{10} (10 - x) = 3 + \log_{10} x$, giving your answer as an exact fraction. [5]
- (ii) By substituting $y = 2^x$, or otherwise, find to 1 decimal place the value of x for which $2^{2x+1} - 2^{x+2} = 6$. [5]
8. (i) Find, to 1 decimal place, the values of x between -180 and 180 for which $\cos 2x^\circ = 0.35$. [3]
- (ii) Given that $\sin \theta = \frac{15}{17}$ and $-180^\circ < \theta < 180^\circ$, find the possible values of
 (a) $\cos \theta$, (b) $\tan \theta$. [4]
- (iii) Show that the equation $3 \cos^2 x = 2(\sin x + 1)$ can be written as a quadratic equation in $\sin x$. Hence solve this equation for $0 \leq x \leq 2\pi$, giving your answers in radians to two decimal places. [5]
9. The equation of a curve C is $y = x^3 - 4x^2 + 5x - 2$.
- (i) Show that $y = 0$ when $x = 1$. Hence express y as the product of three linear factors. [4]
- (ii) Calculate the coordinates of the turning points of C , identifying each as a maximum or a minimum. [5]
- (iii) Sketch the curve C , clearly showing the turning points and any points of intersection with the axes. [3]
- (iv) Find the area of the finite region contained between C and the x -axis. [4]

CORE MATHS 2 (C) TEST PAPER 7 : ANSWERS AND MARK SCHEME

1. Remainder = $f(-1) = 1 + 3 - 2 - 1 = 1$ M1 A1 A1 3
2. $4 \sin \pi/4 = \frac{1}{2} r^2 (\pi/4)$ $r^2 = 16\sqrt{2}/\pi$ $r = 2.68$ M1 A1 M1 A1 4
3. (i) $\int (9x^2 - 6x + 1) dx = 3x^3 - 3x^2 + x + c$ M1 A1 A1 A1
 (ii) e.g. $3x^3 - 3x^2 + x$, $3x^3 - 3x^2 + x + 1$ B1 B1 6
4. (i) $\left(2x + \frac{1}{2x}\right)^6 = (2x)^6 + 6(2x)^5\left(\frac{1}{2x}\right) + \dots = 64x^6 + 96x^4 + \dots$ M1 A1 M1 A1
 (ii) Term is $\binom{6}{3}(2x)^3\left(\frac{1}{2x}\right)^3 = 20$ M1 A1 A1 7
5. (i) By cosine rule, $y^2 = x^2 + (x + 1)^2 - 2x(x + 1)(-1/2) = 3x^2 + 3x + 1$ M1 M1 A1
 (ii) When $x = 7$, $y^2 = 169$ so $y = 13$ $\sin Z / 8 = \sin 120^\circ / 13$ B1 M1
 $\sin Z = 0.533$ Angle $XZY = 32.2^\circ$ M1 A1 7
6. (i) Each year's value is 0.85 times the previous years, hence geometric B2
 $\pounds 12,000 \times 0.85^{12} = \pounds 1707$ M1 A1
 (ii) $4 + 1 + \dots + (-38) = 15(4 - 38)/2 = 15(-17) = -255$ M1 A1 A1 7
7. (i) $\log_{10} 25(10 - x) = \log_{10} 1000 + \log_{10} x$ $250 - 25x = 1000x$ M1 A1 A1
 $1025x = 250$ $x = 10/41$ M1 A1
 (ii) $2y^2 - 4y - 6 = 0$ $2(y + 1)(y - 3) = 0$ M1 A1 A1
 $2^x = -1$ (impossible) or $2^x = 3$ $x = \log_2 3 = 1.6$ M1 A1 10
8. (i) $2x = \pm 69.51, \pm 290.49$ $x = \pm 34.8, \pm 145.2$ M1 A1 A1
 (ii) Using 8, 15, 17 triangle, (i) $\cos \theta = \pm 8/17$, (ii) $\tan \theta = \pm 15/8$ M1 A1 M1 A1
 (iii) $3(1 - \sin^2 x) = 2(\sin x + 1)$ $3 \sin^2 x + 2 \sin x - 1 = 0$ M1 A1
 $(3 \sin x - 1)(\sin x + 1) = 0$ $\sin x = 1/3$ or -1 $x = 0.34, 2.80, 4.71$ M1 A1 A1 12
9. (i) When $x = 1$, $y = 1 - 4 + 5 - 2 = 0$ B1
 $y = (x - 1)(x^2 - 3x + 2) = (x - 1)(x - 1)(x - 2)$ M1 A1 A1
 (ii) $dy/dx = 3x^2 - 8x + 5 = (3x - 5)(x - 1) = 0$ when $x = 1, x = 5/3$ M1 M1 A1
 T.P.s are (1, 0) maximum, (5/3, -4/27) minimum A1 A1
 (iii) Curve sketched, cutting y-axis at (0, -2) and touching x-axis at (1, 0) B1 B1 B1
 (iv) $\int_1^2 y dx = \left[\frac{x^4}{4} - \frac{4x^3}{3} + \frac{5x^2}{2} - 2x \right]_1^2 = \frac{15}{4} - \frac{28}{3} + \frac{15}{2} - 2 = -\frac{1}{12}$ Area = $\frac{1}{12}$ M1 A1 M1 A1 16