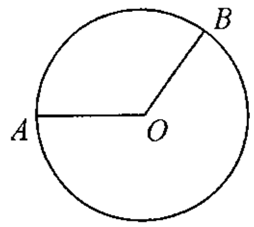
CORE MATHEMATICS (C) UNIT 2 TEST PAPER 6

1. Use the trapezium rule with three intervals to estimate $\int_2^5 \sqrt{2x^2 - 1}\, dx$ to 1 decimal place. [4]

2. Geri's uncle gave her £3 on her first birthday. Subsequently he gave her £$(2n + 1)$ on her nth birthday, until her 18th birthday. How much did he give her altogether? [4]

3. The diagram shows a circle with center O and radius 8 cm.
The minor sector AOB has perimeter 36 cm. Calculate
(i) the angle AOB, in radians, [2]
(ii) the area of the sector AOB, in cm^2. [2]



4. The third term of a geometric series is 160 and the eighth term is 5. Find
(i) the common ratio, [2]
(ii) the first term, [2]
(iii) the sum to infinity of the series. [2]

5. $f(x) \equiv \log_2 (4 - x^2)$, $-2 < x < 2$.
(i) Find f(1) to 2 decimal places. [2]
(ii) Express $f(x) - \log_2 (2 - x)$ in its simplest form. [3]
(iii) Solve for x the equation $f(x) = -1$. [2]

6. (i) Expand $(1 - 2x)^9$ in ascending powers of x as far as the term in x^3. [4]
(ii) By substituting a suitable value for x in your expansion, obtain an estimate of 0.98^9. [3]

7. The gradient at the point (x, y) on a curve C is equal to $x^{\frac{3}{2}}$. The curve passes through the point P (4, 13). Find
(i) the equation of C, in the form $y = f(x)$, [4]
(ii) an equation of the normal to C at P, in the form $ax + by + c = 0$. [5]

8. (i) Divide $(4x^3 - 2x^2 - 8x + 5)$ by $(2x - 3)$. [5]
(ii) Given that the remainder when $(x^2 - 6x - 27)$ is divided by $(x - k)$ is positive, find the set of possible values of k. [4]

9. The equation of a curve is $y = 2\sin\left(x + \frac{2\pi}{3}\right)$.

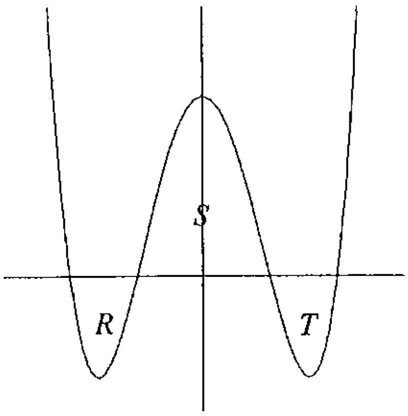
   (i) Find the values of x in the interval $0 \leq x \leq 2\pi$ at which the curve cuts the x-axis. [4]

   (ii) Sketch the curve for $0 \leq x \leq 2\pi$, clearly showing the values found in part (i) and giving the coordinates of any maximum and minimum points in terms of π. [5]

10. $f(x) \equiv x^4 - 5x^2 + 4$.

   (i) By using the substitution $t = x^2$, or otherwise, factorise f(x) completely. [4]

   The diagram shows the graph of $y = f(x)$.



   (ii) Write down the coordinates of all the points where the curve meets the x-axis. [2]

   (iii) Calculate the total area of the regions R, S and T between the curve and the x-axis. [7]

CORE MATHS 2 (C) TEST PAPER 6 : ANSWERS AND MARK SCHEME

1. (2, 2.646), (3, 4.123), (4, 5.568), (5, 7) ½ (9.646 + 2(9.691)) = 14.5 B2 M1 A1 4

2. Sequence is 3, 5, 7, ..., 37 Sum = 9(40) = £360 M1 A1 M1 A1 4

3. (i) $16 + 8\theta = 36$ $\theta = 2.5$ radians (ii) Area = ½ (64)(2.5) = 80 cm^2 M1 A1 M1 A1 4

4. (i) $r^5 = 1/32$ $r = 1/2$ (ii) $a(1/2^2) = 160$ $a = 640$ M1 A1 M1 A1
   (iii) $a/(1 - r) = 1280$ M1 A1 6

5. (i) f(1) = $\log_2 3 = \log 3 / \log 2 = 1.59$ M1 A1
   (ii) $\log_2 ((4 - x^2)/(2 - x)) = \log_2 (2 + x)$ M1 A1 A1
   (iii) $4 - x^2 = ½$ $x = \sqrt{(7/2)}$ (= ±1.87 to 2 decimal places) M1 A1 7

6. (i) $(1 - 2x)^9 = 1 + 9(-2x) + 36(-2x)^2 + 84(-2x)^3 + \ldots$ M1 M1
   $= 1 - 18x + 144x^2 - 672x^3 + \ldots$ A1 A1
   (ii) Put $x = 0.01$ to get $1 - 0.18 + 0.0144 - 0.000672 = 0.833728$ M1 A1 A1 7

7. (i) $y = \frac{2}{5}x^{5/2} + c$ $13 = \frac{64}{5} + c$ $y = \frac{2}{5}x^{5/2} + \frac{1}{5}$ M1 A1 M1 A1
   (ii) Gradient at $P = 8$, so normal is $y - 13 = -1/8\ (x - 4)$ B1 M1 A1
   $8y - 104 = 4 - x$ $x + 8y - 108 = 0$ M1 A1 9

8. (i) Long division gives $2x^2 + 2x - 1 + \frac{2}{2x - 3}$ M1 A1 A1 A1 A1
   (ii) $k^2 - 6k - 27 > 0$ $(k + 3)(k - 9) > 0$ $k < -3$ or $k > 9$ M1 A1 M1 A1 9

9. (i) When $y = 0$, $x + 2\pi/3 = \pi, 2\pi$ $x = \pi/3$, $x = 4\pi/3$ M1 A1 M1 A1
   (ii) Graph thro' ($\pi/3$, 0), ($4\pi/3$, 0); min. at ($5\pi/6$, −2), max. at ($11\pi/6$, 2) B1 B2 B2 9

10. (i) $f(x) = (t - 1)(t - 4) = (x^2 - 1)(x^2 - 4) = (x + 1)(x - 1)(x + 2)(x - 2)$ M1 A1 M1 A1
   (ii) (−2, 0), (−1, 0), (1, 0), (2, 0) B1 B1
   (iii) $R = T = \int_1^2 (x^4 - 5x^2 + 4)\ dx = \left[\frac{x^5}{5} - \frac{5x^3}{3} + 4x\right]_1^2 = \frac{31}{5} - \frac{35}{3} + 4 = -\frac{22}{15}$ M1 A1 A1
   $S = 2\int_0^1 (x^4 - 5x^2 + 4)\ dx = 2\left[\frac{x^5}{5} - \frac{5x^3}{3} + 4x\right]_0^1 = \frac{76}{15}$ Total area = 8 M1 A1 A1 A1 13