CORE MATHEMATICS (C) UNIT 2 TEST PAPER 5

1. Find, in radians, all the values of θ between 0 and 2π for which $\sin(2\pi - \theta) = \frac{1}{2}$. [4]

2. Given that $2^x = y$,
   (i) find, to 2 decimal places, the value of x when $y = 7$. [3]
   (ii) Express $\log_4 y$ in terms of x. [2]

3. Use the trapezium rule, with six equal intervals, to estimate the value of

$$\int_2^8 \log_{10}\left(\sin\frac{\pi}{x}\right) dx.$$

   Give your answer to two decimal places. [6]

4. $f(x) \equiv x^3 + ax^2 + bx + 8$. When $f(x)$ is divided by $(x - 1)$, the remainder is 5.
   When $f(x)$ is divided by $(x + 2)$, the remainder is 20.
   (i) Find the values of the constants a and b. [4]
   (ii) Show that $(x + 4)$ is a factor of $f(x)$. [2]
   (iii) Find the number of real roots of the equation $f(x) = 0$. [2]

5. In the binomial expansion of $(1 + kx)^n$ in ascending powers of x, where n is a positive integer, the first three terms are $1 + 28x + 98kx^2$.
   Find the values of the constants k and n. [8]

6. The first three terms of an arithmetic series are $a(1 + b)$, $a(1 + 3b)$, $a(1 + 5b)$.
   (i) State the common difference of the series. [2]
   (ii) Find an expression for the nth term of the series. [2]
   (iii) If the fifth term of the series is 25 and the tenth term is 55, find the values of a and b. [5]

7. The function f is such that $f'(x) = (3x - 1)^2 - \frac{2}{x^2}$.
   (i) Given that $f(1) = 0$, find $f(x)$. [5]
   (ii) Find an equation of the tangent to the curve $y = f(x)$ at the point $(1, 0)$. [3]
   (iii) Find the second derivative of $f(x)$ with respect to x. [2]

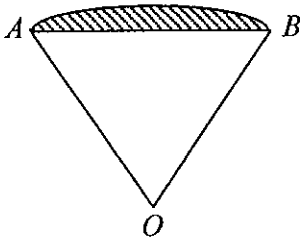
8. The curve C has equation $y = \tan\left(x - \frac{\pi}{3}\right)$, where $-\pi \le x \le \pi$.

   (i) Sketch C, showing the exact coordinates of all points of intersection with the coordinate axes and the equations of any asymptotes. [5]

   (ii) Explain how the graph shows that for any real constant k there are exactly two values of x in the interval $-\pi < x \le \pi$ for which $\tan\left(x - \frac{\pi}{3}\right) = k$. [2]

   (iii) Find the solutions of $\tan\left(x - \frac{\pi}{3}\right) = 1$ in this interval, giving your answers in terms of π. [4]

9. The shaded segment is bounded by the arc AB and the chord AB of a circle with centre O and radius 4 cm. The angle AOB is 2θ radians. The arc AB is x cm longer than the chord AB.



   (i) Show that $x = 8(\theta - \sin\theta)$. [4]

   (ii) Find an expression for the area of the shaded segment in terms of θ. [3]

   Given further that the shaded area is $2x + 8(\sqrt{2} - 1)$ cm^2,

   (iii) show that $\sin 2\theta = 2\sin\theta + 1 - \sqrt{2}$ [2]

   (iv) Verify that this equation is satisfied when $\theta = \frac{\pi}{4}$. [2]

CORE MATHS 2 (C) TEST PAPER 5 : ANSWERS AND MARK SCHEME

1. $2\pi - \theta = \frac{\pi}{6}, \frac{5\pi}{6}$ $\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$ M1 M1 A1 A1 4

2. (i) $x = \log 7 / \log 2 = 2.81$ (ii) $y = 2^x = 4^{x/2}$ so $\log_4 y = x/2$ M1 A1 A1 M1 A1 5

3. (2, 0), (3, –0.062), (4, –0.151), (5, –0.231), (6, –0.301), (7, –0.363), (8, –0.417) B1 B1 B1
   ½ (1)(–0.417 + 2(–1.108)) = –1.32 M1 A1 A1 6

4. (i) $f(1) = a + b + 9 = 5$ $f(-2) = 4a - 2b = 20$ B1
   $a + b = -4,\ 2a - b = 10$ $a = 2, b = -6$ M1 A1 A1
   (ii) $f(-4) = -64 + 32 + 24 + 8 = 0$ so $(x + 4)$ is a factor M1 A1
   (iii) $f(x) = (x + 4)(x^2 - 2x + 2)$ so one real root M1 A1 8

5. $(1 + kx)^n = 1 + nkx + \frac{n(n-1)}{2}k^2x^2 + \ldots$ $nk = 28$, ½ $n(n-1)k^2 = 98k$ M1 M1 A1 A1
   $n(n-1)k = 196$ $28(n-1) = 196$ $n = 8, k = 7/2$ M1 M1 A1 A1 8

6. (i) Common difference $d = 2ab$ M1 A1
   (ii) $T_n = a + ab + (n-1)(2ab)$ or $a(1 + (2n-1)b)$ M1 A1
   (iii) $5d = 30$ so $ab = 3$ $a(1 + 9b) = 25$ so $a + 27 = 25$ B1 M1 A1
   $a = -2, b = -3/2$ A1 A1 9

7. (i) $f'(x) = 9x^2 - 6x + 1 - 2x^{-2}$ $f(x) = 3x^3 - 3x^2 + x + 2x^{-1} + c$ B1 M1 A1
   $3 - 3 + 1 + 2 + c = 0$, so $c = -3$ $f(x) = 3x^3 - 3x^2 + x - 3 + 2x^{-1}$ M1 A1
   (ii) At (1, 0), $f'(x) = 2$ Tangent is $y = 2x - 2$ B1 M1 A1
   (iii) $f''(x) = 18x - 6 + 4x^{-3}$ B1 B1 10

8. (i) Cuts axes at $(-2\pi/3, 0)$, $(0, -\sqrt{3})$, $(\pi/3, 0)$ Asymp. $x = -\pi/6$, $x = 5\pi/6$ B1 B2 B2
   (ii) Any horizontal line cuts the graph twice in this interval B2
   (iii) $x - \pi/3 = \pi/4, -3\pi/4$ $x = -5\pi/12, x = 7\pi/12$ M1 A1 A1 A1 11

9. (i) Arc $AB = 8\theta$ Chord $AB = 2(4 \sin \theta)$ $8\theta - 8 \sin \theta = x$ B1 M1 A1 A1
   (ii) Area = ½ $r^2 (2\theta)$ – ½ $r^2 \sin 2\theta = 16\theta - 8 \sin 2\theta$ M1 A1 A1
   (iii) $16\theta - 8 \sin 2\theta = 16\theta - 16 \sin \theta + 8(\sqrt{2} - 1)$ $\sin 2\theta = 2 \sin \theta + 1 - \sqrt{2}$ M1 A1
   (iv) When $\theta = \pi/4$, $\sin 2\theta = 1$ $2 \sin \theta + 1 - \sqrt{2} = 2/\sqrt{2} + 1 - \sqrt{2} = 1$ M1 A1 11