CORE MATHEMATICS (C) UNIT 2 TEST PAPER 3

1. Given that $4 \sin x = 3 \cos x$,

   (i) find the value of $\tan x$. [1]

   (ii) Find all the values of x between 0 and 360 for which $4 \sin x° = 3 \cos x°$, giving your answers to 1 decimal place. [3]

2. In the binomial expansion of $(k - 3x)^8$ in ascending powers of x, the first two terms are

   $$a - 3072x.$$

   Find the values of the constants k and a. [6]

3. For the geometric series $a + ar + ar^2 + \dots$,

   (i) give a formula for the nth term. [1]

   The terms of this series are all different and positive. The sum of the first eight terms is 17 times the sum of the first four terms.

   (ii) Find the value of r. [5]

4. Find the values of x for which $\log_3 (x^2 - 2x - 1) - \log_3 (x + 3) = -1$. [6]

5. At the point (x, y) on a curve C, the gradient is equal to $x^{\frac{1}{3}} + x^{-\frac{2}{3}}$.

   The curve passes through the points (8, 8) and (27, a)

   (i) Find the equation of the curve. [5]

   (ii) Find the value of a. [2]

6. (i) Find an expression, in terms of p, for the nth term of the sequence

   $$\frac{1}{2},\ p + \frac{1}{2},\ 2p + \frac{1}{2},\ 3p + \frac{1}{2},\ \dots$$ [3]

   (ii) If the sum of the first ten terms of this sequence is 35, find the value of p. [4]

7. Given that $f(x) \equiv x^3 - 6x^2 + kx + 10$ and that $(x + 1)$ is a factor of $f(x)$,

   (i) find the value of k. [2]

   (ii) Write $f(x)$ as the product of three linear factors. [3]

   (iii) Solve the equation $2^{3y} - 6(2^{2y}) + k(2^y) + 10 = 0$, giving solutions correct to 1 decimal place where necessary. [3]

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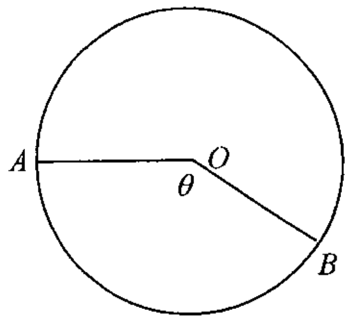
8. (i) Use the trapezium rule, with four intervals of equal length, to estimate the area bounded by the curve $y = \frac{3}{2\sqrt{x}}$, the lines $x = 1$, $x = 5$ and the x-axis. Give your answer to 2 decimal places. [4]

(ii) Use integration to find the true value of this area, also to 2 decimal places. [4]

(iii) Show that the estimate lies within 1.5 % of the true value. [1]

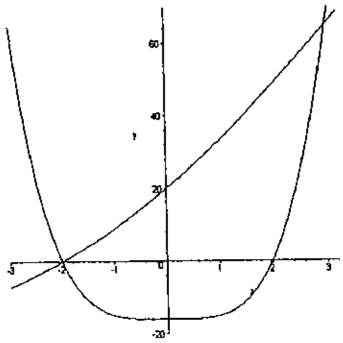
9. In the diagram, a circle is divided into two sectors by the radii OA and OB. The areas of the major and minor sectors are in the ratio 3 : 2. The obtuse angle AOB is θ radians.



(i) Show that $\theta = \frac{4\pi}{5}$. [4]

(ii) Express, in its simplest form in terms of π, the ratio of the perimeters of the two sectors. [5]

10. The diagram shows part of the curves with equations $y = x^4 - 16$ and $y = x^2 + 12x + 20$.



(i) Verify that the curves intersect at the points (–2, 0) and (3, 65). [2]

(ii) Show that the area of the finite region between the curves is given by $\int_{-2}^{3} (-x^4 + x^2 + 12x + 36)\,dx$. [2]

(iii) Hence evaluate this area. [6]

CORE MATHS 2 (C) TEST PAPER 3 : ANSWERS AND MARK SCHEME

	Working	Marks	Total
1.	(i) $\tan x = ¾$ (ii) $x = 36.9, x = 216.9$	B1 M1 A1 A1	4
2.	$k^8 + 8k^7(-3x) = a - 3072x$ $k^8 = a, 24k^7 = 3072$	M1 A1 A1	
	$k = 128^{1/7} = 2$ $a = 2^8 = 256$	M1 A1 A1	6
3.	(i) nth term $= ar^{n-1}$	B1	
	(ii) $a(r^8 - 1)/(r - 1) = 17a(r^4 - 1)/(r - 1)$ $r^8 - 1 = 17(r^4 - 1)$	M1 A1	
	$r^8 - 17r^4 + 16 = 0$ $(r^4 - 1)(r^4 - 16) = 0$ $r > 0$ and $r \neq 1$ so $r = 2$	M1 A1 A1	6
4.	$\log_3 [(x^2 - 2x - 1)/(x + 3)] = -1.$ $(x^2 - 2x - 1)/(x + 3) = 1/3$	B1 M1	
	$3x^2 - 7x - 6 = 0$ $(3x + 2)(x - 3) = 0$ $x = -2/3$ or $x = 3$	A1 M1 A1 A1	6
5.	(i) Integrating, $y = \frac{3}{4}x^{4/3} + 3x^{1/3} + c$ $8 = 18 + c$, so $c = -10$	M1 A1 A1 M1 A1	
	(ii) $y = \frac{3}{4}x^{4/3} + 3x^{1/3} - 10$ $a = 60.75 - 1 = 59.75$	M1 A1	7
6.	(i) Common difference $= p$, so $T_n = (n-1)p + \frac{1}{2}$	B1 M1 A1	
	(ii) $S_n = \frac{n}{2}\left(1 + (n-1)p\right) = 5(9p + 1) = 35$, so $p = \frac{2}{3}$	M1 A1 M1 A1	7
7.	(i) $f(-1) = 0$ $-1 - 6 - k + 10 = 0$ $k = 3$	M1 A1	
	(ii) $f(x) = (x + 1)(x^2 - 7x + 10) = (x + 1)(x - 2)(x - 5)$	M1 A1 A1	
	(iii) $2^y = x = -1, 2, 5$ $y = 1$ or $y = \log_2 5 = 2.3$	M1 A1 A1	8
8.	(i) (1, 1.5), (2, 1.061), (3, 0.866), (4, 0.75), (5, 0.671)	B2	
	Area $\approx ½ (2.171 + 2(2.677)) = 3.76$	M1 A1	
	(ii) $\int_1^5 \frac{3}{2}x^{-1/2}\,dx = \left[3x^{1/2}\right]_1^5 \approx 3.71$	M1 A1 M1 A1	
	(iii) $0.054/3.708 \times 100\% \approx 1.45\%$	B1	9
9.	(i) $½ r^2(2\pi - \theta) : ½ r^2\theta = 3 : 2$ $2(2\pi - \theta) = 3\theta$ $5\theta = 4\pi$	M1 A1 M1 A1	
	(ii) $r(2\pi - \theta) + 2r : \theta + 2r = (6\pi/5 + 2) : (4\pi/5 + 2) = (3\pi + 5) : (2\pi + 5)$	M1 A1 M1 A1 A1	9
10.	(i) $16 - 16 = 4 - 24 + 20 = 0$, $81 - 16 = 9 + 36 + 20 = 65$	B1 B1	
	(ii) Area $= \int_{-2}^{3} (x^2 + 12x + 20) - (x^4 - 16)\ dx = \int_{-2}^{3} (-x^4 + x^2 + 12x + 36)\,dx$	M1 A1	
	(iii) $\left[-\frac{x^5}{5} + \frac{x^3}{3} + 6x^2 + 36x\right]_{-2}^{3} = \left[-\frac{243}{5} + 9 + 54 + 108\right] - \left[\frac{32}{5} - \frac{8}{3} + 24 - 72\right]$	M1 A1 A1 M1 A1	
	$= 500/3$	A1	10