CORE MATHEMATICS (C) UNIT 2 TEST PAPER 1

1. Use the Remainder Theorem to decide whether $2x^3 + 3x^2 - 4x + 1$ is divisible by $2x + 1$. [3]

2. The first three terms of a geometric series are $4, -1, \frac{1}{4}$. Find

   (i) the seventh term, [2]

   (ii) the sum to infinity of the series. [2]

3. The gradient of a curve at the point (x, y) is given by

$$\frac{dy}{dx} = 1 - \frac{4}{x^2}.$$

   The curve passes through the point P with coordinates $(1, 7)$.

   Find the equation of the curve, in the form $y = f(x)$. [6]

4. (i) Show that the equation $2 \sin^2 x = 2 - \cos x$ can be written as a quadratic equation in $\cos x$. [2]

   (ii) Hence find, in degrees, the values of x between 0 and 360° for which $2 \sin^2 x = 2 - \cos x$. [5]

5. (i) A sequence of numbers is given by $x_{n+1} = x_n^2 - 3$. Given that $x_1 = 2$,

   (a) find the value of x_5, [2]

   (b) deduce the value of x_{55}. [2]

   (ii) Evaluate $\sum_{r=1}^{21} (2r + 3)$. [4]

6. (i) Show that $x = 1$ is a solution of the equation $x^3 - 7x^2 + 15x - 9 = 0$, and hence solve this equation completely. [5]

   (ii) Hence find all solutions of the equation $3^{3y} - 7(3^{2y}) + 15(3^y) - 9 = 0$. [4]

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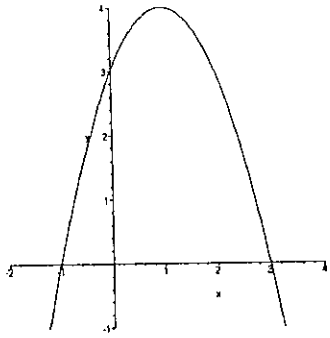
7. The diagram shows the curve with equation $y = 3 + 2x - x^2$.



The area of the finite region between the curve and the x-axis is to be estimated using the trapezium rule.

x	-1	0	1	2	3
y	0	3			0

(i) Calculate the values of y for $x = 1$ and $x = 2$ and hence find an estimate of the area. [5]

(ii) Use integration to find the true value of the area. [4]

8. In the binomial expansion of $(a + x)^n$, where $n > 4$, the coefficient of x^3 is twice that of x^4.

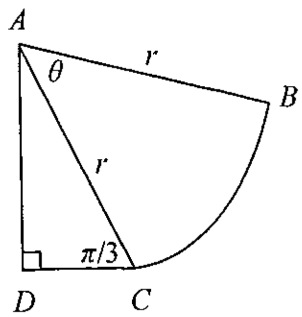
(i) Show that $n = 2a + 3$. [4]

In the same expansion, the coefficient of x^2 is $\frac{3}{2}$ times the coefficient of x.

(ii) Obtain another relation between n and a. Hence find the values of n and a. [4]

(iii) State the constant term in the expansion. [2]

9.



In the diagram, BC is an arc of a circle with centre A and radius r cm.

Angle $BAC = \theta$ radians and angle $ACD = \frac{\pi}{3}$ radians.

AD is perpendicular to DC.

(i) Show that $AD = r\frac{\sqrt{3}}{2}$ cm and find the length of CD. [3]

(ii) Show that the perimeter of the figure $ABCD$ is $r\left(\frac{3+\sqrt{3}}{2} + \theta\right)$ cm. [3]

(iii) Show that the area of the figure $ABCD$ is $\frac{r^2}{2}\left(\frac{\sqrt{3}}{4} + \theta\right)$ cm^2. [3]

The perimeter of $ABCD$ has a fixed length of 8 cm.

(iv) Express the area of $ABCD$ in terms of r only. [3]

(v) Show that as r varies, the area is maximum when $r = \frac{16}{6+\sqrt{3}}$. [4]

CORE MATHS 2 (C) TEST PAPER 1 : ANSWERS AND MARK SCHEME

1.	$f(-1/2) = -1/4 + 3/4 + 2 + 1 = 7/2 \neq 0$, so not divisible	M1 A1 A1	3
2.	(i) $r = -1/4$ $T_7 = ar^6 = 1/4^5 = 1/1024$ (ii) $a/(1-r) = 4/(5/4) = 3.2$	M1 A1 M1 A1	4
3.	Integrating, $y = x + 4/x + c$ $7 = 5 + c$ $y = x + 4/x + 2$	M2 A1 A1 M1 A1	6
4.	(i) $2 - 2\cos^2 x + \cos x - 2 = 0$ (ii) $\cos x\,(2\cos x - 1) = 0$	M1 A1 M1	
	$\cos x = 0$ or $\cos x = ½$ $x = 60, 90, 270, 300$	A1 M1 A1 A1	7
5.	(i) (a) Sequence is 2, 1, −2, 1, −2 $x_5 = -2$	M1 A1	
	(b) All odd-numbered terms from x_3 are equal, so $x_{55} = -2$	M1 A1	
	(ii) Series is 5, 7, 9, …, 45 Sum = 21(5 + 45)/2 = 525	B1 M1 A1 A1	8
6.	(i) $f(1) = 1 - 7 + 15 - 9 = 0$ $(x-1)(x^2 - 6x + 9) = 0$	B1 M1 A1	
	$(x-1)(x-3)^2 = 0$ $x = 1, x = 3$	M1 A1	
	(ii) $3^y = 1$ or $3^y = 3$ $y = 0$ or $y = 1$	M1 A1 M1 A1	9
7.	(i) $y(1) = 4, y(2) = 3$ Area $\approx ½(0 + 0 + 2(3 + 4 + 3)) = 10$	B1 B1 M1 A1 A1	
	(ii) $\left[3x + x^2 - \frac{x^3}{3}\right]_{-1}^{3} = 9 - \left[-\frac{5}{3}\right] = 10\frac{2}{3}$	M1 A1 M1 A1	9
8.	(i) $\frac{n(n-1)(n-2)}{6}a^{n-3} = 2\frac{n(n-1)(n-2)(n-3)}{24}a^{n-4}$ $a = ½\,(n-3)$, etc	M1 A1 M1 A1	
	(ii) $\frac{n(n-1)}{2}a^{n-2} = \frac{3}{2}na^{n-1}$ $3a = n - 1$ $n = 3a + 1$	M1 A1	
	$2a + 3 = 3a + 1$ $a = 2, n = 7$ (iii) Constant term $= 2^7 = 128$	M1 A1 M1 A1	10
9.	(i) $AD = r\sin\frac{\pi}{3} = r\frac{\sqrt{3}}{2}$ $CD = r\cos\frac{\pi}{3} = \frac{r}{2}$	M1 A1 A1	
	(ii) Perimeter $= r + \frac{r}{2} + r\frac{\sqrt{3}}{2} + r\theta = r\left(\frac{3+\sqrt{3}}{2} + \theta\right)$	M1 A1 A1	
	(iii) Area $= \frac{1}{2}.\frac{r}{2}.\frac{r\sqrt{3}}{2} + \frac{1}{2}r^2\theta = \frac{r^2}{2}\left(\frac{\sqrt{3}}{4} + \theta\right)$	M1 A1 A1	
	(iv) $\theta = \frac{8}{r} - \frac{3+\sqrt{3}}{2}$ $A = \frac{r^2}{2}\left(\frac{8}{r} - \frac{6+\sqrt{3}}{4}\right) = 4r - \left(\frac{6+\sqrt{3}}{8}\right)r^2$	B1 M1 A1	
	(v) $\frac{dA}{dr} = 4 - \left(\frac{6+\sqrt{3}}{4}\right)r = 0$ when $r = \frac{16}{6+\sqrt{3}}$ $\frac{d^2A}{dr^2} < 0$ so max.	M1 A1 M1 A1	16