# Pure Core 2 Past Paper Questions

## Taken from MAME, MAP1, MAP2, MAP3

## Methods November 2003

**5** *In this question, no credit will be given for using approximate decimal values.*

It is given that $p = 8^{\frac{1}{2}}$ and $q = 4^{\frac{3}{4}}$.

(a) Show that $p = 2^{\frac{3}{2}}$. *(1 mark)*

(b) Similarly, express $q$ as a power of 2. *(1 mark)*

(c) Hence express $pq$ as a power of 2. *(2 marks)*

## Pure 1 January 2001

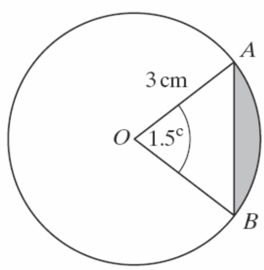
**1** Given that

$$y = x^2 - x^{-2},$$

(a) find the value of $\frac{dy}{dx}$ at the point where $x = 2$, *(3 marks)*

(b) find $\int y \, dx$. *(2 marks)*

**3**




The diagram shows a circle with centre $O$ and radius 3 cm. The points $A$ and $B$ on the circle are such that the angle $AOB$ is 1.5 radians.

(a) Find the length of the minor arc $AB$. *(1 mark)*

(b) Find the area of the minor sector $OAB$. *(2 marks)*

(c) Show that the area of the shaded segment is approximately 2.3 cm$^2$. *(4 marks)*

5 An electrician cuts a piece of wire of length 27 metres from a long roll of wire. He then cuts off more pieces, each piece being two-thirds as long as the preceding piece.

(a) Calculate the length of the fourth piece cut off. *(2 marks)*

(b) Show that, no matter how many pieces he cuts off, their total length cannot exceed 81 metres. *(2 marks)*

(c) Show that the total length of the first eleven pieces cut off is more than 80 metres. *(3 marks)*

(d) The length of the eleventh piece of wire is $u$ metres. Write down an expression for $u$. Hence show that

$$\ln u = a \ln 2 - b \ln 3 ,$$

where $a$ and $b$ are positive integers to be determined. *(4 marks)*

## Pure 1 June 2001

1 (a) Find the sum of the three hundred integers from 101 to 400 inclusive. *(3 marks)*

(b) Find the sum of the geometric series

$$2 + 6 + 18 + \ldots + 2 \times 3^{n-1},$$

giving your answer in the form

$$p^n - q,$$

where $p$ and $q$ are integers. *(3 marks)*

4 (a) Given that

$$2\cos^2 \theta - \sin \theta = 1,$$

show that

$$2\sin^2 \theta + \sin \theta - 1 = 0. \quad \textit{(2 marks)}$$

(b) *In this part of the question, no credit will be given for an approximate numerical method.*

Hence find all the values of $\theta$ in the interval $0 < \theta < 2\pi$ for which

$$2\cos^2 \theta - \sin \theta = 1,$$

giving each answer in terms of $\pi$. *(4 marks)*

(c) **Write down** all the values of $x$ in the interval $0 < x < \pi$ for which

$$2\cos^2 2x - \sin 2x = 1. \quad \textit{(2 marks)}$$

5 (a) Express $x^2 \sqrt{x}$ in the form $x^p$. *(1 mark)*

(b) Given that

$$y = x^2 \sqrt{x},$$

find the value of $\frac{dy}{dx}$ at the point where $x = 9$. *(3 marks)*

# Pure 1 January 2002

**1** It is given that $y = x^{\frac{1}{3}}$.

(a) Find $\dfrac{dy}{dx}$. *(2 marks)*

(b) (i) Find $\int y \, dx$. *(2 marks)*

(ii) Hence evaluate $\int_0^8 y \, dx$. *(2 marks)*

**2** (a) Show that $\log_2 8 = 3$. *(1 mark)*

(b) Find the value of

(i) $\log_2 (8^4)$, *(1 mark)*

(ii) $\log_2 \left(\dfrac{1}{\sqrt{8}}\right)$. *(2 marks)*

**3** A pipeline is to be constructed under a lake. It is calculated that the first mile will take 15 days to construct. Each further mile will take 3 days longer than the one before, so the 1st, 2nd and 3rd miles will take 15, 18 and 21 days, respectively, and so on.

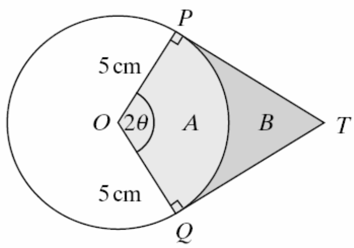
(a) Find the $n$th term of the arithmetic sequence $15, 18, 21, \ldots$ . *(2 marks)*

(b) Show that the total time taken to construct the first $n$ miles of the pipeline is $\frac{3}{2}n(n+9)$ days. *(3 marks)*

(c) Calculate the total length of pipeline that can be constructed in 600 days. *(3 marks)*

(b) The diagram shows a sector $A$ of a circle with centre $O$ and radius 5 cm. $OP$ and $OQ$ are the radii forming part of the boundary of $A$, and the angle $POQ$ is $2\theta$ radians.

The tangents to the circle at $P$ and $Q$ intersect at $T$, and the shaded region outside the circle is $B$.




(i) Write down the area of the sector $A$ in terms of $\theta$. *(2 marks)*

(ii) Find the area of the triangle $OPT$ in terms of $\theta$. *(2 marks)*

(iii) Deduce that the area of the region $B$ is $25(\tan\theta - \theta)$ cm$^2$. *(2 marks)*

(iv) Given that the regions $A$ and $B$ have equal areas, show that

$$\tan\theta - 2\theta = 0.$$ *(2 marks)*

(v) Use your result from part (a)(ii) to give the value of $\theta$ correct to one decimal place. *(1 mark)*

## Pure 1 June 2002

**2** (a) Find the sum of the 16 terms of the arithmetic series

$$2 + 5 + 8 + \ldots + 47.$$ *(3 marks)*

(b) An arithmetic sequence $u_1, u_2, u_3, \ldots$ has $r$th term $u_r$, where

$$u_r = 50 - 3r.$$

(i) Write down the values of $u_1, u_2, u_3$ and $u_4$. *(2 marks)*

(ii) Show that the sequence has exactly 16 positive terms. *(2 marks)*

**5** (a) Write down the exact values of:

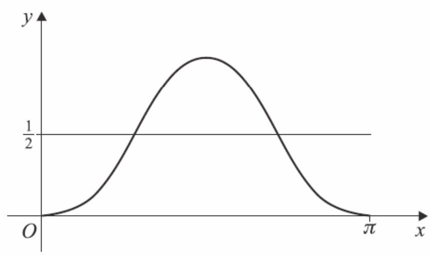
(i) $\sin \frac{\pi}{4}$;

(ii) $\cos \frac{\pi}{6}$;

(iii) $\tan \frac{\pi}{3}$. *(3 marks)*

The diagram shows the graphs of

$$y = \sin^2 x \ \text{ and } \ y = \tfrac{1}{2} \ \text{ for } \ 0 \leqslant x \leqslant \pi .$$



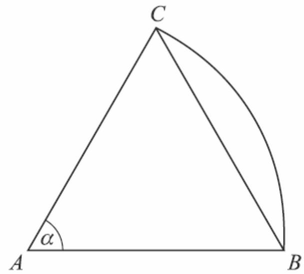

(b) Solve $\sin^2 x = \frac{1}{2}$ for $0 \leqslant x \leqslant \pi$. *(3 marks)*

(c) Hence solve $\sin^2 x > \frac{1}{2}$ for $0 \leqslant x \leqslant \pi$. *(2 marks)*

(d) Prove that

$$\sin^2 x > \tfrac{1}{2} \Rightarrow \cos^2 x < \tfrac{1}{2}.$$ *(2 marks)*

**6** (a)




The diagram shows an equilateral triangle $ABC$ with sides of length 6 cm and an arc $BC$ of a circle with centre $A$.

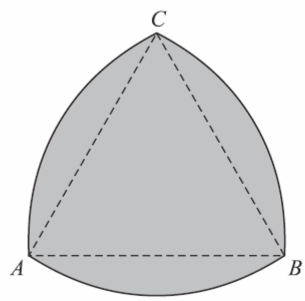
(i) Write down, in radians, the value of the angle $\alpha$. *(1 mark)*

(ii) Find the length of the arc $BC$. *(2 marks)*

(iii) Show that the area of the triangle $ABC$ is $9\sqrt{3}$ cm$^2$. *(3 marks)*

(iv) Show that the area of the sector $ABC$ is $6\pi$ cm$^2$. *(3 marks)*

(b)




The diagram shows an ornament made from a flat sheet of metal. Its boundary consists of three arcs of circles. The straight lines $AB$, $AC$ and $BC$ are each of length 6 cm. The arcs $BC$, $AC$ and $AB$ have centres $A$, $B$ and $C$ respectively.

(i) The boundary of the ornament is decorated with gilt edging. Find the total length of the boundary, giving your answer to the nearest centimetre. *(2 marks)*

(ii) Find the area of one side of the ornament, giving your answer to the nearest square centimetre. *(3 marks)*

# Pure 1 November 2002

**2** The $n$th term of an arithmetic sequence is $u_n$, where

$$u_n = 10 + 0.5n.$$

(a) Find the values of $u_1$ and $u_2$. *(2 marks)*

(b) Write down the common difference of the arithmetic sequence. *(1 mark)*

(c) Find the value of $n$ for which $u_n = 25$. *(2 marks)*

(d) Evaluate $\sum_{n=1}^{30} u_n$. *(3 marks)*

**3** (a) Show that $\int_1^4 x^{\frac{3}{2}}\,dx = \frac{62}{5}$. *(4 marks)*

**4** (a) Write down the value of $\log_2 8$. *(1 mark)*

(b) Express $\log_2 9$ in the form $n \log_2 3$. *(1 mark)*

(c) Hence show that

$$\log_2 72 = m + n\log_2 3,$$

where $m$ and $n$ are integers. *(1 mark)*

**5** The angle $\theta$ radians, where $0 \leqslant \theta \leqslant 2\pi$, satisfies the equation

$$3\tan\theta = 2\cos\theta.$$

(a) Show that

$$3\sin\theta = 2\cos^2\theta.$$ *(1 mark)*

(b) Hence use an appropriate identity to show that

$$2\sin^2\theta + 3\sin\theta - 2 = 0.$$ *(3 marks)*

(c) (i) Solve the quadratic equation in part (b). Hence explain why the only possible value of $\sin\theta$ which will satisfy it is $\frac{1}{2}$. *(3 marks)*

(ii) Write down the values of $\theta$ for which $\sin\theta = \frac{1}{2}$ and $0 \leqslant \theta \leqslant 2\pi$. *(2 marks)*

(iii) For the smaller of these values of $\theta$, write down the exact values, in surd form, of $\tan\theta$ and $\cos\theta$. *(2 marks)*

(iv) Verify that these exact values satisfy the original equation. *(1 mark)*

# Pure 1 January 2003

**1** The first four terms of a geometric sequence are

$$10,\ 9,\ 8.1,\ 7.29.$$

(a) Show that the common ratio of the sequence is 0.9. *(1 mark)*

(b) Find the $n$th term. *(2 marks)*

(c) Show that the sum of the first 25 terms is approximately 92.8. *(2 marks)*

(d) Find the sum to infinity. *(2 marks)*

**4** The acute angle $\theta$ radians is such that

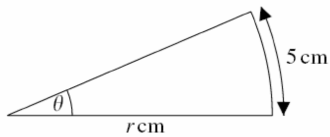
$$\sin\theta = \frac{5}{13}.$$

(a) (i) Show that $\cos\theta = \frac{12}{13}$. *(2 marks)*

(ii) Find the value of $\tan\theta$, giving your answer as a fraction. *(2 marks)*

(b) Use your calculator to find the value of $\theta$, giving your answer to three decimal places. *(1 mark)*

(c) The diagram shows a sector of a circle of radius $r$ cm and angle $\theta$ radians. The length of the arc which forms part of the boundary of the sector is 5 cm.




(i) Show that $r \approx 12.7$. *(2 marks)*

(ii) Find the area of the sector, giving your answer to the nearest square centimetre. *(3 marks)*

**6** The function f is defined for $x \geqslant 0$ by

$$f(x) = x^{\frac{1}{2}} + 2.$$

(a) (i) Find $f'(x)$. *(2 marks)*

(ii) Hence find the gradient of the curve $y = f(x)$ at the point for which $x = 4$. *(1 mark)*

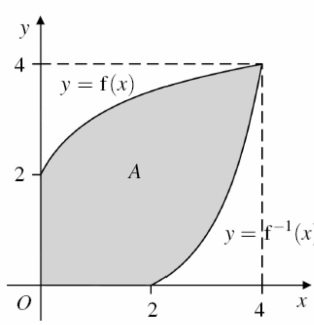
(b) (i) Find $\int f(x)\,dx$. *(3 marks)*

(ii) Hence show that $\int_0^4 f(x)\,dx = \frac{40}{3}$. *(2 marks)*

(c) Show that $f^{-1}(x) = (x-2)^2$. *(2 marks)*

(d) The diagram shows a symmetrical shaded region $A$ bounded by:

parts of the coordinate axes;
the curve $y = f(x)$ for $0 \leqslant x \leqslant 4$; and
the curve $y = f^{-1}(x)$ for $2 \leqslant x \leqslant 4$.




(i) Write down the equation of the line of symmetry of $A$. *(1 mark)*

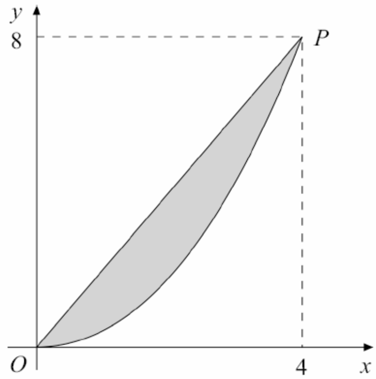
(ii) Calculate the area of $A$. *(4 marks)*

# Pure 1 June 2003

**1** The diagram shows the graph of

$$y = x^{\frac{3}{2}}, \quad 0 \leqslant x \leqslant 4,$$

and a straight line joining the origin to the point $P$ which has coordinates $(4, 8)$.




(a) (i) Find $\int x^{\frac{3}{2}}\,\mathrm{d}x$. *(2 marks)*

(ii) Hence find the value of $\int_0^4 x^{\frac{3}{2}}\,\mathrm{d}x$. *(2 marks)*

(b) Calculate the area of the shaded region. *(2 marks)*

**2** The graph of

$$y = x + 4x^{-2}$$

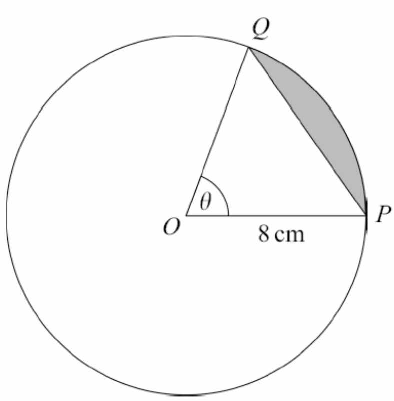
has one stationary point.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. *(2 marks)*

(b) Find the coordinates of the stationary point. *(3 marks)*

(c) Find the value of $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ at the stationary point, and hence determine whether the stationary point is a maximum or a minimum. *(4 marks)*

**3** The diagram shows a circle with centre $O$ and radius 8 cm. The angle between the radii $OP$ and $OQ$ is $\theta$ radians.




(a) (i) Find the area of the sector $OPQ$ in terms of $\theta$. *(2 marks)*

(ii) Find the area of the triangle $OPQ$ in terms of $\sin\theta$. *(2 marks)*

(iii) Hence write down the area of the shaded segment. *(1 mark)*

**4** It is given that $x$ satisfies the equation

$$2\cos^2 x = 2 + \sin x.$$

(a) Use an appropriate trigonometrical identity to show that

$$2\sin^2 x + \sin x = 0. \qquad \textit{(2 marks)}$$

(b) Solve this quadratic equation and hence find all the possible values of $x$ in the interval $0 \leqslant x < 2\pi$. *(6 marks)*

**6** (a) The first three terms of a geometric sequence are $a$, $b$, $c$. Each term represents an increase of $p$ per cent on the preceding term.

(i) Show that the common ratio is $\left(1 + \frac{p}{100}\right)$. *(2 marks)*

(ii) It is given that $a = 2000$. Express $b$ and $c$ in terms of $p$. *(2 marks)*

(b) A deposit of £2000 is put into a bank account. After each year, the balance in the account is increased by $p$ per cent. There are no other deposits or withdrawals. After two years the balance is £2332.80.

(i) Show that $p = 8$. *(3 marks)*

(ii) Given that after $n$ years the balance is £$u_n$, write down an expression for $u_n$ in terms of $n$. *(2 marks)*

(iii) Use your answer to part (b)(ii) to find the balance after 10 years. *(2 marks)*

## Pure 1 November 2003

**1** The first four terms of a geometric sequence are

$$6, \quad 2, \quad \tfrac{2}{3}, \quad \tfrac{2}{9}.$$

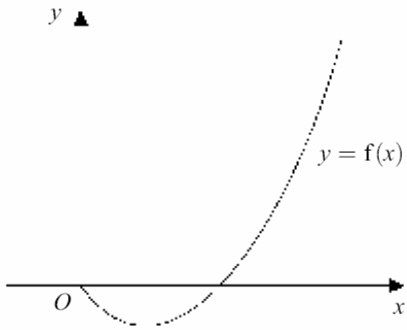
(a) Write down the common ratio. *(1 mark)*

(b) Find the tenth term, giving your answer to three significant figures. *(2 marks)*

(c) Find the sum to infinity. *(2 marks)*

**2** The diagram shows the graph of $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 5x^{\frac{3}{2}} - 3x\,.$$




(a) (i) Differentiate $\mathrm{f}(x)$ to find $\mathrm{f}'(x)$. *(2 marks)*

(ii) Show that, at the stationary point on the graph, $x = 0.16$. *(3 marks)*

**3** (a) Sketch the graph of

$$y = \tan x \qquad \text{for } 0 \leqslant x \leqslant 2\pi,$$

giving the equations of the asymptotes. *(4 marks)*

(b) Solve the equation

$$\tan x = \sqrt{3} \qquad \text{for } 0 \leqslant x \leqslant 2\pi.$$ *(3 marks)*

**5** (a) Explain briefly why $\log_5 125 = 3$. *(1 mark)*

(b) Find the value of:

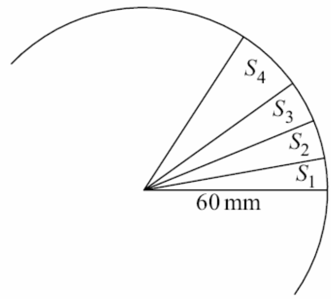
(i) $\log_5(125^2)$; *(1 mark)*

(ii) $\log_5 \sqrt{125}$; *(1 mark)*

(iii) $\log_5\left(\frac{1}{\sqrt{125}}\right)$. *(1 mark)*

(c) Solve the equation $\log_5(125x) = 4$. *(2 marks)*

**6** Ashley is cutting a circular disc of radius 60 mm into unequal sectors $S_1, S_2, S_3, \ldots$




(a) The first sector, $S_1$, has an angle of $10°$.

(i) Write the angle $10°$ in radians, in terms of $\pi$. *(2 marks)*

(ii) Show that the area of $S_1$ is $100\pi$ mm$^2$. *(3 marks)*

(b) The angles of the sectors $S_2, S_3, S_4, \ldots$ are $12°$, $14°$, $16°$ and so on in an arithmetic sequence.

(i) Find the areas of $S_2$, $S_3$ and $S_4$ in terms of $\pi$. *(3 marks)*

(ii) Show that the sum of the areas of $S_1, S_2, S_3, \ldots, S_n$ is

$$10\pi n(n+9) \text{ mm}^2.$$ *(4 marks)*

(iii) Verify that Ashley will obtain exactly 15 sectors from the circular disc. *(3 marks)*

## Pure 1 January 2004

**1** (a) Find $\int x^{\frac{1}{2}}\,dx$. *(2 marks)*

(b) Hence find the value of $\int_0^2 x^{\frac{1}{2}}\,dx$,

giving your answer in the form $p\sqrt{2}$, where $p$ is a rational number. *(3 marks)*

**2** The $n$th term of a geometric sequence is $u_n$, where

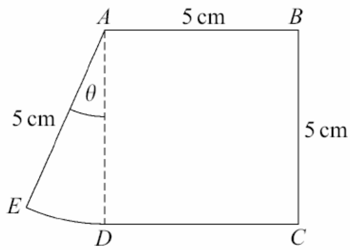
$$u_n = 2 \times 3^n.$$

(a) Find the values of $u_1$ and $u_2$. *(2 marks)*

(b) Write down the common ratio of the geometric sequence. *(1 mark)*

(c) Show that the sum of the first 10 terms is $3(3^{10} - 1)$. *(3 marks)*

**3** The diagram shows a shape $ABCDE$. The shape consists of a square $ABCD$, with sides of length 5 cm, and a sector $ADE$ of a circle with centre $A$ and radius 5 cm. The angle of the sector is $\theta$ radians.




(a) Find the area of the sector $ADE$ in terms of $\theta$. *(2 marks)*

(b) The area of the sector $ADE$ is a quarter of the area of the square $ABCD$.

(i) Find the value of $\theta$. *(2 marks)*

(ii) Find the perimeter of the shape $ABCDE$. *(2 marks)*

**4** (a) An arithmetic sequence has first term 100 and common difference 2.

(i) Write down the second and third terms. *(2 marks)*

(ii) Given that the last term is 200, find the number of terms. *(3 marks)*

(b) A tape dispenser has a length of tape wrapped round a circular cylinder. The length of tape in the first layer (nearest to the cylinder) is 100 mm. Each further layer is 2 mm longer than the one before. The outer layer has 200 mm of tape.

Calculate the total length of tape. *(3 marks)*

**7** (a) Write down the exact values of $\sin\frac{\pi}{6}$, $\cos\frac{\pi}{6}$ and $\tan\frac{\pi}{6}$. *(3 marks)*

(b) It is given that $x$ satisfies the equation

$$3\sin^2 x = \cos^2 x.$$

By first using an appropriate trigonometrical identity to simplify this equation, find all the solutions of the equation in the interval $0 \leqslant x \leqslant 2\pi$. *(6 marks)*

## Pure 1 June 2004

**1** (a) Find the sum of the 100 terms of the arithmetic series

$$3 + 7 + 11 + \ldots + 399.$$ *(3 marks)*

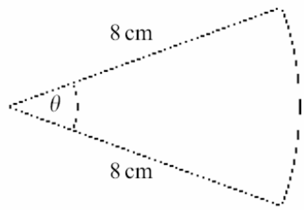
(b) An arithmetic sequence $u_1, u_2, u_3, \ldots$ has $r$th term $u_r$, where

$$u_r = 8r - 2.$$

(i) Write down the values of $u_1$, $u_2$, $u_3$ and $u_4$. *(2 marks)*

(ii) Using your answer to part (a), or otherwise, find the sum of the first 100 terms of this sequence. *(2 marks)*

**2** The diagram shows a sector of a circle of radius 8 cm. The sector has angle $\theta$ radians. The perimeter of the sector is $P$ cm and its area is $A\text{ cm}^2$.




(a) Show that $P = 8(\theta + 2)$. *(2 marks)*

(b) Find $A$ in terms of $\theta$. *(2 marks)*

(c) Given that $A = P$, find the value of $\theta$. *(3 marks)*

**3** (a) Show that the equation

$$2x^{\frac{3}{2}} - 9x + 6 = 0$$

has a root between 0 and 1. *(3 marks)*

(b) A curve has equation

$$y = 2x^{\frac{3}{2}} - 9x.$$

(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. *(5 marks)*

(ii) Calculate the coordinates of the stationary point on the curve. *(3 marks)*

(iii) Find the value of $\frac{d^2y}{dx^2}$ at the stationary point and hence determine whether this point is a maximum or a minimum. *(2 marks)*

**4** Write each of the following in the form $a \ln p + b \ln q$:

(a) $\ln(pq)$; *(1 mark)*

(b) $\ln(p^2q^3)$; *(1 mark)*

(c) $\ln\left(\frac{p}{q}\right)$; *(1 mark)*

(d) $\ln\sqrt{\frac{p}{q}}$. *(1 mark)*

**5** (a) A geometric sequence has first term 230 and second term 345.

(i) Show that the common ratio is 1.5. *(1 mark)*

(ii) Calculate the third and fourth terms. *(2 marks)*

(b) In 1501 the population of a country was 2 300 000.

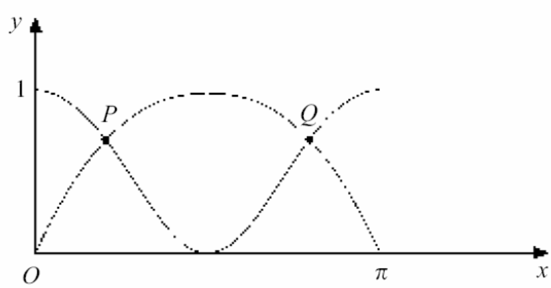
In 1601 the population was 3 450 000.

Assuming that the population in the years 1501, 1601, 1701 and 1801 can be modelled as a geometric sequence, write down an estimate for the population in 1801. Give your answer to three significant figures. *(2 marks)*

**6** The diagram shows the graphs of

$$y = \cos^2 x \text{ and } y = \sin x \text{ for } 0 \leqslant x \leqslant \pi.$$

The graphs intersect each other at two points $P$ and $Q$.




(a) Use a trigonometric identity to show that the $x$-coordinates of $P$ and $Q$ satisfy the equation

$$\sin^2 x + \sin x - 1 = 0.$$ *(2 marks)*

(b) (i) Solve this quadratic equation. *(2 marks)*

(ii) Show that the only possible value for $\sin x$ is approximately 0.618. *(2 marks)*

(c) Find the $x$-coordinates of $P$ and $Q$, giving each answer to two decimal places. *(3 marks)*

# Pure 2 June 2002

**2** In a clinical trial, the concentration of a drug in the blood at time $t$ hours from the start of the trial is denoted by $p_t$.

It is given that

$$p_{t+1} = a + bp_t,$$

where $a$ and $b$ are constants.

Measurements give $p_0 = 5.0$, $p_1 = 13.0$ and $p_2 = 14.6$.

(a) Find:

(i) $a$ and $b$; *(3 marks)*

(ii) the concentration of the drug in the blood after 3 hours. *(1 mark)*

(b) The concentration converges to a limiting value $w$. Write down and solve an equation for $w$. *(2 marks)*

# Pure 2 June 2004

**6** (a) The circle $(x-4)^2 + (y-3)^2 = 4$ has centre $C$ and radius $r$.

Write down:

(i) the coordinates of $C$;

(ii) the value of $r$. *(2 marks)*

(b) The line $y = x + 1$ intersects this circle at two points $A$ and $B$.

(i) Find the coordinates of $A$ and $B$. *(5 marks)*

(ii) Show that the area of the minor segment bounded by the circle and the chord $AB$ is $\pi - 2$. *(3 marks)*

# Pure 3 January 2002

**1** Find the coefficient of the term in $x^4$ in the binomial expansion of

$$(3 + 2x)^7.$$ *(3 marks)*

# Pure 3 June 2003

**1** Find the coefficient of $x^3$ in the binomial expansion of $(2 + 3x)^9$. Give your answer as an integer. *(3 marks)*