A-LEVEL
MATHEMATICS

Pure Core 2 – MPC2
Mark scheme

6360
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Version/Stage: Final V1.0

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Mark	Total	Comment
1(a)	(Area =) $\frac{1}{2}\times 5\times 12\times \sin 47$	M1		$\frac{1}{2}\times 5\times 12\times \sin A$ stated or used
	$= 21.94\ldots = 22$ (cm^2)	A1	2	Correct area. If not 22 condone 21.9… NMS 22 or 'better' scores 2 marks
(b)	(BC^2 =) $5^2 + 12^2 - 2\times5\times12\times\cos 47$	M1		RHS of cosine rule used correctly
	$= 25 + 144 - 81.8(39..)$ (=87.16..)	m1		Correct evaluation of the three terms. PI by eg evaluation to a value 87 to 88 inclusive or correct final answer
	$BC = 9.3(359\ldots) = 9.3$ (cm)	A1	3	If not 9.3 accept 9.34 or 9.33 or 9.33…
	Total		**5**	
	Condone absent/incorrect units throughout this question.			
(a)	Candidates who find a perpendicular height do not score the M1 until 1/2base×height used ie the equivalent of $\frac{1}{2}\times 5\times 12\times \sin A$.			
(a)(b)	Cand who uses 47 rads can score a max of (a) M1A0 (b) M1m0A0			
(b)	Example: $169 - 120\cos 47$ (M1) $= 49\cos 47$ (m0) $= 33.4\ldots$			
(b)	$5^2 + 12^2 - 2\times5\times12\times\cos 47$ (M1); (BC=) 9.33 (m1A1)			

Q	Solution	Mark	Total	Comment
2(a)	$\int\left(1+3x^{\frac{1}{2}}+x^{\frac{3}{2}}\right)\mathrm{d}x = x + 2x^{1.5} + \frac{2}{5}x^{2.5}\ (+c)$	B1; B1 B1	3	ACF B1 for each correct term. Condone missing $+c$. (Can be left unsimplified)
(b)(i)	(n =) 3	B1	1	Correct value of n. Condone '$3y^2$'
(b)(ii)	$\left(1+\sqrt{x}\right)^3 = 1 + 3\sqrt{x} + 3\sqrt{x}^2 + \sqrt{x}^3$	B1F	1	Correct four term expansion ft c's n. Allow 'correct' alternatives eg $1 + 3x^{1/2} + "3"x + x^{3/2}$
(c)	$\int\left(1+\sqrt{x}\right)^3\mathrm{d}x = \int\left(1+3x^{\frac{1}{2}}+"3"x+x^{\frac{3}{2}}\right)\mathrm{d}x$			
	$= x + 2x^{1.5} + \frac{"3"x^2}{2} + \frac{2}{5}x^{2.5}\ (+c)$	B1F		Correct integration. If not correct, ft on c's answer to (a) + $\frac{nx^2}{2}$ for c's value of n in (b)(ii).
	$\int_0^1\left(1+\sqrt{x}\right)^3\mathrm{d}x =$			
	$1 + 2(1)^{1.5} + \frac{3(1)^2}{2} + \frac{2}{5}(1)^{2.5} - (0)$	M1		PI Attempt to find F(1)−F(0) following 'attempt' at integration. Condone the '−(0)' missing if cand's F(x) leads to F(0)=0.
	$= \frac{49}{10}$ (= 4.9)	A1	3	OE correct value. NMS ie 4.9 without any other work in (c) scores 0/3
	Total		**8**	
	Apply ISW after a correct answer but do not award the B1F in (c) if, for example, an incorrect simplification in (a) has been used in (c) and marked as 3/3 ISW in (a).			
(c)	Allow M1 PI if cand. has evaluated F(1) − F(0) correctly for their F(x), following integration.			
(c)	If 4.9 follows from incorrect working then A0 FIW			

Q	Solution	Mark	Total	Comment
3(a)	$\{S_\infty =\}\ \dfrac{a}{1-r} = \dfrac{54}{1-\frac{8}{9}}$	M1		$\dfrac{a}{1-r}$ used with a=54 and r= 8/9 OE
	$\{S_\infty =\}$ 486	A1	2	Correct exact value for S_∞. 486 scores 2 marks unless rounding of a value to 486 seen in which case M1A0.
(b)	{2nd term =} $ar = 48$	B1	1	Correct value for 2nd term
(c)	{12th term =} ar^{12-1}	M1		ar^{12-1} stated or used. PI by 14.7(8…)
	$= 54\times\left(\dfrac{8}{9}\right)^{11} = 2\times3\times3\times3\times\left(\dfrac{2\times2\times2}{3\times3}\right)^{11}$	m1		Changing at least two of 54 and 8 and 9 in correct expression to correct products/powers of 2 and 3
	$= \dfrac{2\times3^3\times(2^3)^{11}}{(3^2)^{11}}$			
	$= \dfrac{3^3\times2^{34}}{3^{22}} = \dfrac{2^{34}}{3^{19}}\quad (p=34,\ q=19)$	A1	3	Showing 12th term $=\dfrac{2^{34}}{3^{19}}$ in a convincing manner
	Total		**6**	
(a)	Accept 0.8 or 0.9 or better as an OE to 8/9 or 0.2 or 0.1 or better as OE to 1−8/9 but a must be 54 (No MR)			
(c)	$54\times\dfrac{8}{9}^{11}$ (M1)			
(c)	$54\times\left(\dfrac{8}{9}\right)^{11}$ **(M1)** $= 54\times\left(\dfrac{8}{3^3}\right)^{11} = 2\times3\times3\times3\times\left(\dfrac{2\times2\times2}{3^3}\right)^{11}$ **(m1)** since 54 and 8 have been written as correct products of 2 and 3 starting with a correct expression, $54\times\left(\dfrac{8}{9}\right)^{11}$.			

Q	Solution	Mark	Total	Comment
4(a)	$\frac{1}{x^2}=x^{-2}$	B1		$\frac{1}{x^2}=x^{-2}$. PI by its **correct** derivative
	$(y=\frac{1}{x^2}+4x)$ $(\frac{dy}{dx}=)$ $-2x^{-3}+4$	M1		Correct differentiation of either $\frac{1}{x^2}$ or $4x$
		A1	3	Correct $\frac{dy}{dx}$ ACF
(b)	When $x=-1$, $\frac{dy}{dx}=-2(-1)^{-3}+4$ $(=6)$	M1		Attempt to find the value of $\frac{dy}{dx}$ when $x=-1$
	Gradient of normal = $-\frac{1}{6}$	m1		Correct use of $m\times m'=-1$, with c's value of $\frac{dy}{dx}$ when $x=-1$
	(Eqn of normal) $y+3=-\frac{1}{6}(x+1)$	A1F	3	A correct ft equation for normal with signs simplified; ft on c's $\frac{dy}{dx}$ expression in **(a)** **SC** $\frac{dy}{dx}$= const in **(a)**, mark **(b)** as M1A1F eg for $\frac{dy}{dx}$=4 in (a); grad of normal=$-\frac{1}{4}$ (M1), eqn $y+3=-\frac{1}{4}(x+1)$ (A1F)
(c)	$-2x^{-3}+4=-12$	M1		C's answer to (**a**) equated to −12 (or to 12) seen or used.
	$x^{-3}=8$	A1F		PI Correct rearrangement of $ax^{-n}+b=\pm 12$ or $\frac{a}{x^n}+b=\pm 12$ OE to form $x^{-n}=q$ or to form $x^n=p$, but only ft in case of n positive
	$x=0.5$	A1		$x=0.5$ OE
	When $x=0.5$, $y=6$	A1F		Correct ft y coordinate from $y_c=x_c^{-2}+4x_c$. Only ft if values are exact.
	(Eqn of tangent) $y-6=-12(x-0.5)$ (or eg $y=-12x+12$)	A1	5	Correct tangent equation ACF Apply ISW after ACF
	Total		**11**	

(a) Rearrange to $\frac{1+4x^3}{x^2}$ and then use quotient rule ($\frac{\pm vu'\pm uv'}{v^2}$)M1; A1(for correct v^2 and a correct term in the numerator); A1 (Correct $\frac{dy}{dx}$ ACF)

(b) Final answer as $y-(-3)=-\frac{1}{6}(x-(-1))$ is M1m1A0 as signs not simplified.

(c) Apply the PI only for the correct value of x with a correct M1 equation seen ie $-2x^{-3}+4=-12$, $x=½$ (M1A1FA1)

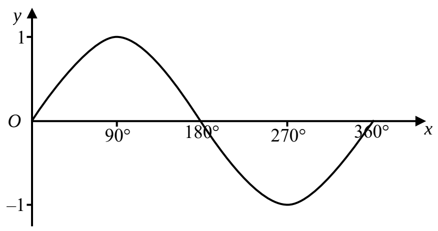
Q	Solution	Mark	Total	Comment
5	(Area of sector) = $\frac{1}{2}r^2\theta$	M1		$\frac{1}{2}r^2\theta$ seen, or used, for the sector area
	$\frac{1}{2}r^2\theta = 12$	A1		$\frac{1}{2}r^2\theta = 12$ OE
	(Arc length) = $r\theta$	M1		$r\theta$ seen, or used, for the arc length
	$r + r + r\theta = 4r\theta$	m1		$r + r + r\theta = 4r\theta$ OE in terms of r and θ or used with their value of $r\theta$.
	$3r\theta = 2r \Rightarrow \theta = \frac{2}{3}$	A1		$\theta = \frac{2}{3}$. Condone 0.66 or 0.67 or better PI by eg $\frac{1}{3}r^2 = 12$ OE
	$\frac{1}{3}r^2 = 12 \Rightarrow r = 6$	A1	6	$r = 6$ only with no evidence of a value seen being rounded to 6 .
	Total		**6**	

Example: $\frac{1}{2}r^2\theta = 12$ (M1A1) $r\theta = 4r\theta$ (M1m0)

Example: $r + r + r\theta = 4r\theta$ (M1m1) θ=0.67 (A1) $r^2\theta = 12$ (M0A0)

Example: $\frac{1}{2}r^2\theta = 12$ (M1A1) $2r + r\theta = 4r\theta$ (M1 m1) $2r^2 + r^2\theta = 4r^2\theta$, $2r^2 + 24 = 96$,

$2r^2 = 72$ (A1) $\Rightarrow$ $r = \pm 6$ (A0, since -6 still present)

Q	Solution	Mark	Total	Comment
6(a)		B2,1,0	2	Ignore parts of graph outside $0^\circ \le x \le 360^\circ$. B2: Correct graph including correct intersections and stationary points at/close to 90º and 270º with correct *y* values, 1 and −1 stated. If not B2 then award B1 for correct shape graph with either (i) at least 4 of the 5 critical points (intersections and stationary points) having *x*-coords. drawn within tolerance or (ii) at least 3 of the 5 critical points (intersections and stationary points) having *x*-coords. drawn within tolerance and *y* values, 1 and −1 stated for max and min respectively
(b)	Stretch (**I**) in *x*-direction (**II**) scale factor $\frac{1}{5}$ (**III**)	M1 A1	2	Need (**I**) **and** either (**II**) or (**III**) Need (**I**) and (**II**) and (**III**) More than one transformation scores 0/2.
(c)	Translation $\begin{bmatrix} -2^\circ \\ 0 \end{bmatrix}$	E2,1,0	2	E2: 'translat…' and $\begin{bmatrix} -2 \\ 0 \end{bmatrix}$ OE. If not E2 award E1for either 'translat… 2 in *x*-dir' OE. or 'translat…' and $\begin{bmatrix} -10 \\ 0 \end{bmatrix}$ OE. More than one transformation scores 0/2.
	Total		**6**	

(a)	For guidance, 'close to' means max pt is vertically above any part of the printed '90º' and min pt is vertically below any part of the printed '270º'. As a guideline, generally accept graph through 180 and 360 if graph goes through the printed *x*-axis markers at these points.
(b)	<u>**Stretch**</u> by <u>**0.2**</u> in <u>**x**</u> (direction) is sufficient for M1A1. Accept 'horizontal…' in place of '*x*'
(c)	<u>Lots of "correct" answers:</u> eg translate 70º in *x*-direction {in fact any translation of −2(mod72)º in *x*-direction would be correct} eg reflect in $x=17^\circ$ {in fact any reflection in $x=17(\text{mod}36)^\circ$ would be correct}
(c)	Examples: 'translate horizontally 2' scores E1; 'translating horizontally −2' scores E2; 'translated 2 in negative *x* ' scores E2 If using 'reflection' if not E2 then award E1 for eg 'reflection in $x=19$' OE (ie correct 17 replaced by 19)

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Q	Solution	Mark	Total	Comment
7(a)	$\dfrac{\cos^2 x+4\sin^2 x}{1-\sin^2 x}=\dfrac{\cos^2 x+4\sin^2 x}{\cos^2 x}$ (=7)	M1		A correct use of identity $\sin^2 x+\cos^2 x=1$
	$(1+\dfrac{4\sin^2 x}{\cos^2 x}=7)$; $\Rightarrow 1+4\tan^2 x=7$	m1		Correct use of identity $\tan^2 x=\dfrac{\sin^2 x}{\cos^2 x}$ to obtain a correct equation in $\tan^2 x$ only.
	$\Rightarrow 4\tan^2 x=6 \Rightarrow \tan^2 x=\dfrac{3}{2}$	A1	3	AG $\tan^2 x=\dfrac{3}{2}$ obtained convincingly
(b)	$\tan^2 2\theta=\dfrac{3}{2}$	M1		Using printed answer to part **(a).** PI by either $\tan 2\theta=\sqrt{\dfrac{3}{2}}$ or $\tan 2\theta=-\sqrt{\dfrac{3}{2}}$ or later equivalent work
	$\tan 2\theta=\pm\sqrt{\dfrac{3}{2}}=\pm1.22(47..)$,	A1		$\tan 2\theta=\pm\sqrt{\dfrac{3}{2}}$ OE Must see the ±
	$(\theta=)$ 25°, 65°, 115°, 155°	B2,1,0	4	B2: All 4 integer values correct. If not B2 award B1 for 2 AWRT correct integer values. If more than 4 solutions inside given interval deduct 1 mark (to min of B0) for each extra solution. Ignore values outside given interval
	Total		**7**	
(a)	Altn. Finding value for $\cos^2 x$ and value for $\sin^2 x$ then using to find $\tan^2 x$ Example: $\cos^2 x+4\sin^2 x=7(1-\sin^2 x)$; $1+3\sin^2 x=7(1-\sin^2 x)$ (M1); $10\sin^2 x=6$; $\sin^2 x=3/5$; So $\cos^2 x=1-3/5=2/5$; $\tan^2 x=\dfrac{\sin^2 x}{\cos^2 x}=\dfrac{3/5}{2/5}$ (m1) $=\dfrac{3}{2}$ (A1)			
(b)	Eg. $\tan 2\theta=\sqrt{\dfrac{3}{2}}$ (M1); $\theta=25.4,\ 115.4$ (B1) Example showing the M1 PI $\tan x=\sqrt{\dfrac{3}{2}}$; x=50.76… , 230.76.. ; (no marks yet) $\theta=25.4,\ 115.4$ (M1B1) Cand solving tan$x$ = 3/2 and dividing answers for x by 2 will score 0/4 since not taken sq root. Candidate who solves $\tan^2 x=\dfrac{3}{2}$ without ever linking it with 2θ (eg by dividing answers for x by 2) will score 0/4.			

Q	Solution	Mark	Total	Comment
8(a)	$[S_5=]\ \frac{5}{2}[2a+(5-1)d]$	M1		$\frac{5}{2}[2a+(5-1)d]$ OE
	$\frac{5}{2}[2a+(5-1)d]=575$; $5(2a+4d)=575\times2$	m1		Forming correct eqn and attempt to remove fraction or expand brackets or better
	$2a+4d=115\times2 \Rightarrow a+2d=115$	A1	3	AG $a+2d=115$ convincingly obtained
(b)	$a+(10-1)d=87$ $a+2d=115,\ a+9d=87 \Rightarrow 7d=87-115$	M1 m1		$87=a+(10-1)d$ OE Solving $a+2d=115$ simultaneously with $a+9d=87$ as far as eliminating either a or d.
	$7d=-28,\quad d=-4$	A1	3	$d=-4$
(c)	When $d=-4,\ a=123$	B1F		Correct value of a or correct ft value for a. Ft only on $a=115-2\times$ cand's d
	$u_k=123+(k-1)(-4)>0$ $u_{k+1}=123+(k)(-4)<0$	M1		Either inequality, ft c's values for a and d. Condone equality and also n written for k.
	$k<31.75,\ k>30.75 \Rightarrow k=31$	E1		Justification of k=31 with no errors seen in relevant working and k=31 stated or used.
	$\sum_{n=1}^{31}u_n=\frac{31}{2}[2a+(31-1)d]$	M1		$\sum_{n=1}^{31}u_n=\frac{31}{2}[2a+(31-1)d]$ OE Must be using 31 for n.
	$=1953$	A1	5	$\sum_{n=1}^{k}u_n=1953$ dep. on previous B1FM1M1 being awarded
	Total		**11**	

(b)	Cand who recognises **(a)** answer as 3rd term = 115: $115+7d=87$ (M1m1) $d=-4$ (A1)
(c)	Can award the B1F for the value of a if seen in (b) with no contradiction in (c).
(c)	Examples sufficient for the E1: $123+(k-1)(-4)>0$, $k<31.75$, $\Rightarrow k=31$ (E1); $123+(k)(-4)<0$, $k>30.75 \Rightarrow k=31$ (E1); (T&I approach) M1 for either $u_{31}=3$ **or** $u_{32}=-1$ $u_{31}=3$ **and** $u_{32}=-1 \Rightarrow k=31$ (E1);
(c)	Example $123+(n-1)(-4)=0$ (M1), $n=31.75$ (no E yet) **and d<0** (OE) so n=31 (E1)
(c)	An OE for 2nd M1 is $\sum_{n=1}^{31}u_n=\frac{31}{2}[a+3]$

Q	Solution	Mark	Total	Comment
9(a)	$6 = 3\times12^k$; $12^k = 2$ $k\log 12 = \log 2$ ($k =$) $0.27894\ldots = 0.279$ (to 3sf)	B1 M1 A1	 3	$6 = 3\times12^k$ OE Condone x for k throughout. From $12^k = c$, correct application of 3rd law of logs OE eg $k = \log_{12} c$ Must see logs being used. Condone >3sf.
(b)	$h = 0.5$ $F(x) = 3\times12^x$ $I \approx \frac{h}{2}\{F(0)+F(1.5)+2[F(0.5)+F(1)]\}$ $\frac{h}{2}$ with $\{\ldots\}=3+36\sqrt{12}+2\left(3\sqrt{12}+36\right)$ $= 3+124.7...+2(10.39\ldots + 36)$ $= 127.7\ldots + 2\times46.39\ldots$ ($I \approx 0.25[220.492..]$ $(= 55.1..)$) $= 55$ (to 2sf)	B1 M1 A1 A1	 4	$h = 0.5$ stated or used. (PI by x-values 0, 0.5, 1, 1.5 provided no contradiction) $h/2\{F(0)+F(1.5)+2[F(0.5)+F(1)]\}$ OE summing of areas of the 'trapezia'.. OE Accept 2sf or better evidence for surds. Can be implied by later correct work provided >1 term or a single term which rounds to 55 or is 55 CAO Must be 55 SC 4 strips used: **max** B0M1A0; 52 A1
(c)	$f(x) = 3\times12^{x-1} + p$ $f(0) = 0 \Rightarrow 3\times12^{-1} + p = 0 \Rightarrow p = -0.25$ **Altn** (0, ..) on y=f(x) from translating $(-1, 3\times12^{-1})$ (.., 0) on y=f(x) from translating $(\ldots, -p)$ $-p = 3\times12^{-1} \Rightarrow p = -0.25$	M2,1,0 A1 (M1) (M1) (A1)	 3 (3)	M2 for $3\times12^{x-1} + p$; M1 if one sign error $p = -1/4$ OE identified PI by seeing 3×12^{-1} equated to p or $-p$ PI
(d)	$2^{2-x} = 3\times12^x$ $(2-x)\log_2 2 = \log_2\left(3\times12^x\right)$ $(2-x)\log_2 2 = \log_2 3 + \log_2 12^x$ $= \log_2 3 + x\log_2 12$ $= \log_2 3 + x(\log_2 3 + \log_2 4)$ $2 - x = \log_2 3 + x\log_2 3 + 2x$ $2 - \log_2 3 = x\log_2 3 + 3x$ $x = \dfrac{2-\log_2 3}{3+\log_2 3}$ $(q = 3)$	B1 M1 m1 A1 A1	 5	$2^{2-x} = 3\times12^x$ OE Elimination of y Attempting to takes logs of both sides of a correct eqn and applies a law of logs correctly to either side; condone missing base Using log laws correctly to reach a correct eqn where any log terms other than log3 are of the form logN where N = 2,4 or 8. condone missing base. $\log_2 2 = 1$ used to reach a correct eqn involving no log terms other than $\log_2 3$ $x = \dfrac{2-\log_2 3}{3+\log_2 3}$ obtained convincingly
	Total		15	
	TOTAL		75	
(a)	$6 = 3\times12^x$ (B1); $\log 6 = \log 3 + \log 12^x$ (M not scored yet); $\log 6 = \log 3 + x\log 12$ (M1)			
(b)	For guidance sep. trap. 3.34…+11.59..+40.17.. . (b) MR of F(x) **max** B1M1A0A0			
(d)	NB $(2-x)\log_2 2 = 2 - x\log_2 2 = 2-x$			
(d)	$4 = 3\times24^x$ (B1); $\log 4 = \log 3 + \log 24^x$ (M1); $\log 4 = \log 3 + x\log 24$; $\log 4 = \log 3 + x(\log 3 + \log 8)$ (m1) $2 = \log_2 3 + x(\log_2 3 + 3)$ (A1); $x = \dfrac{2-\log_2 3}{3+\log_2 3}$ (A1).			
(d)	Example: $2^{2-x} = 3\times12^x$ (B1) $\log 2^{2-x} = \log 3\times x\log 12 = x\log 36$, $2 - x\log 2 = x\log 36$ **(M1m0)**			