General Certificate of Education
Advanced Subsidiary Examination
June 2013

Mathematics MPC2

Unit Pure Core 2

Monday 13 May 2013 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 A geometric series has first term 80 and common ratio $\frac{1}{2}$.

(a) Find the third term of the series. *(1 mark)*

(b) Find the sum to infinity of the series. *(2 marks)*

(c) Find the sum of the first 12 terms of the series, giving your answer to two decimal places. *(2 marks)*

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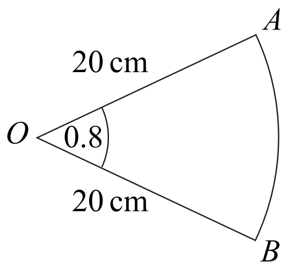
2 The diagram shows a sector *OAB* of a circle with centre *O*.

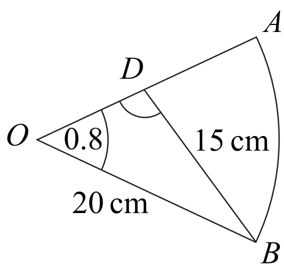


The radius of the circle is 20 cm and the angle $AOB = 0.8$ radians.

(a) Find the length of the arc *AB*. *(2 marks)*

(b) Find the area of the sector *OAB*. *(2 marks)*

(c) A line from *B* meets the radius *OA* at the point *D*, as shown in the diagram below.



The length of *BD* is 15 cm. Find the size of the **obtuse** angle *ODB*, in **radians**, giving your answer to three significant figures. *(4 marks)*

3 (a) (i) Using the binomial expansion, or otherwise, express $(2+y)^3$ in the form $a+by+cy^2+y^3$, where a, b and c are integers. *(2 marks)*

(ii) Hence show that $(2+x^{-2})^3+(2-x^{-2})^3$ can be expressed in the form $p+qx^{-4}$, where p and q are integers. *(3 marks)*

(b) (i) Hence find $\int\left[(2+x^{-2})^3+(2-x^{-2})^3\right]\mathrm{d}x$. *(2 marks)*

(ii) Hence find the value of $\int_1^2\left[(2+x^{-2})^3+(2-x^{-2})^3\right]\mathrm{d}x$. *(2 marks)*

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4 (a) Sketch the graph of $y=9^x$, indicating the value of the intercept on the y-axis. *(2 marks)*

(b) Use logarithms to solve the equation $9^x=15$, giving your value of x to three significant figures. *(2 marks)*

(c) The curve $y=9^x$ is reflected in the y-axis to give the curve with equation $y=\mathrm{f}(x)$. Write down an expression for $\mathrm{f}(x)$. *(1 mark)*

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5 (a) Use the trapezium rule with five ordinates (four strips) to find an approximate value for $\int_0^2\sqrt{8x^3+1}\,\mathrm{d}x$, giving your answer to three significant figures. *(4 marks)*

(b) Describe the single transformation that maps the graph of $y=\sqrt{8x^3+1}$ onto the graph of $y=\sqrt{x^3+1}$. *(2 marks)*

(c) The curve with equation $y=\sqrt{x^3+1}$ is translated by $\begin{bmatrix}2\\-0.7\end{bmatrix}$ to give the curve with equation $y=\mathrm{g}(x)$. Find the value of $\mathrm{g}(4)$. *(3 marks)*

6 A curve has the equation

$$y = \frac{12 + x^2\sqrt{x}}{x}, \quad x > 0$$

(a) Express $\dfrac{12 + x^2\sqrt{x}}{x}$ in the form $12x^p + x^q$. *(3 marks)*

(b) (i) Hence find $\dfrac{dy}{dx}$. *(2 marks)*

(ii) Find an equation of the normal to the curve at the point on the curve where $x = 4$. *(4 marks)*

(iii) The curve has a stationary point P. Show that the x-coordinate of P can be written in the form 2^k, where k is a rational number. *(3 marks)*

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7 The nth term of a sequence is u_n. The sequence is defined by

$$u_{n+1} = pu_n + q$$

where p and q are constants.

The first two terms of the sequence are given by $u_1 = 96$ and $u_2 = 72$.

The limit of u_n as n tends to infinity is 24.

(a) Show that $p = \frac{2}{3}$. *(4 marks)*

(b) Find the value of u_3. *(2 marks)*

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8 (a) Given that $\log_a b = c$, express b in terms of a and c. *(1 mark)*

(b) By forming a quadratic equation, show that there is only one value of x which satisfies the equation $2\log_2(x+7) - \log_2(x+5) = 3$. *(6 marks)*

9 (a) (i) On the axes given below, sketch the graph of $y = \tan x$ for $0° \leqslant x \leqslant 360°$. *(3 marks)*

(ii) Solve the equation $\tan x = -1$, giving all values of x in the interval $0° \leqslant x \leqslant 360°$. *(2 marks)*

(b) (i) Given that $6 \tan \theta \sin \theta = 5$, show that $6 \cos^2 \theta + 5 \cos \theta - 6 = 0$. *(3 marks)*

(ii) **Hence** solve the equation $6 \tan 3x \sin 3x = 5$, giving all values of x to the nearest degree in the interval $0° \leqslant x \leqslant 180°$. *(6 marks)*