General Certificate of Education
Advanced Subsidiary Examination
June 2012

Mathematics MPC2

Unit Pure Core 2

Wednesday 16 May 2012 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 The arithmetic series

$$23 + 32 + 41 + 50 + \ldots + 2534$$

has 280 terms.

(a) Write down the common difference of the series. *(1 mark)*

(b) Find the 100th term of the series. *(2 marks)*

(c) Find the sum of the 280 terms of the series. *(2 marks)*

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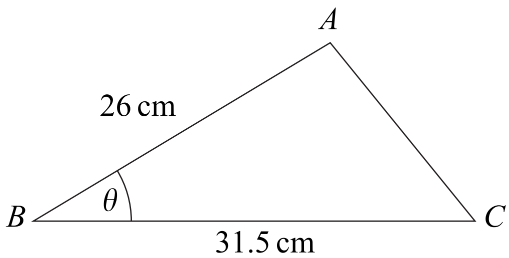
2 The triangle ABC, shown in the diagram, is such that $AB = 26\,\text{cm}$ and $BC = 31.5\,\text{cm}$.



The acute angle ABC is θ, where $\sin\theta = \frac{5}{13}$.

(a) Calculate the area of triangle ABC. *(2 marks)*

(b) Find the exact value of $\cos\theta$. *(1 mark)*

(c) Calculate the length of AC. *(3 marks)*

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3 (a) Expand $\left(x^{\frac{3}{2}} - 1\right)^2$. *(2 marks)*

(b) Hence find $\int \left(x^{\frac{3}{2}} - 1\right)^2 \mathrm{d}x$. *(3 marks)*

(c) Hence find the value of $\int_1^4 \left(x^{\frac{3}{2}} - 1\right)^2 \mathrm{d}x$. *(2 marks)*

4 The nth term of a geometric series is u_n, where $u_n = 48\left(\frac{1}{4}\right)^n$.

(a) Find the value of u_1 and the value of u_2. *(2 marks)*

(b) Find the value of the common ratio of the series. *(1 mark)*

(c) Find the sum to infinity of the series. *(2 marks)*

(d) Find the value of $\sum_{n=4}^{\infty} u_n$. *(3 marks)*

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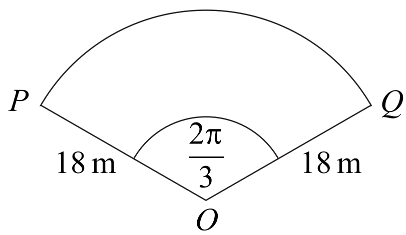
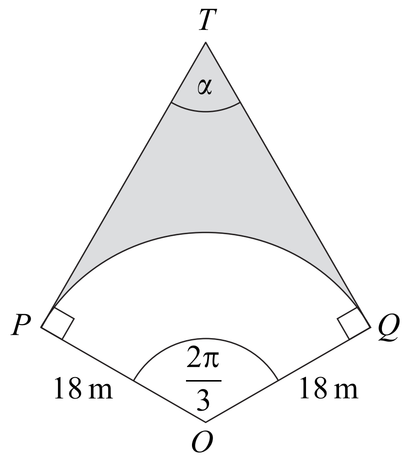
5 The diagram shows a sector OPQ of a circle with centre O.



The radius of the circle is 18 m and the angle POQ is $\frac{2\pi}{3}$ radians.

(a) Find the length of the arc PQ, giving your answer as a multiple of π. *(2 marks)*

(b) The tangents to the circle at the points P and Q meet at the point T, and the angles TPO and TQO are both right angles, as shown in the diagram below.



(i) Angle $PTQ = \alpha$ radians. Find α in terms of π. *(1 mark)*

(ii) Find the area of the shaded region bounded by the arc PQ and the tangents TP and TQ, giving your answer to three significant figures. *(6 marks)*

Turn over ▶

6 At the point (x, y), where $x > 0$, the gradient of a curve is given by

$$\frac{dy}{dx} = 3x^2 - \frac{4}{x^2} - 11$$

The point $P(2, 1)$ lies on the curve.

(a) (i) Verify that $\frac{dy}{dx} = 0$ when $x = 2$. *(1 mark)*

(ii) Find the value of $\frac{d^2y}{dx^2}$ when $x = 2$. *(4 marks)*

(iii) Hence state whether P is a maximum point or a minimum point, giving a reason for your answer. *(1 mark)*

(b) Find the equation of the curve. *(4 marks)*

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7 It is given that $(\tan\theta + 1)(\sin^2\theta - 3\cos^2\theta) = 0$.

(a) Find the possible values of $\tan\theta$. *(4 marks)*

(b) Hence solve the equation $(\tan\theta + 1)(\sin^2\theta - 3\cos^2\theta) = 0$, giving all solutions for θ, in degrees, in the interval $0° \leqslant \theta \leqslant 180°$. *(3 marks)*

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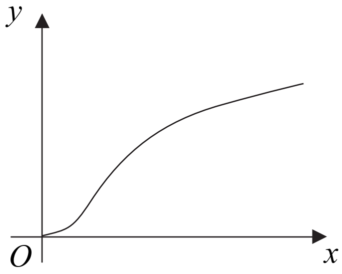
8 (a) Sketch the curve with equation $y = 7^x$, indicating the coordinates of any point where the curve intersects the coordinate axes. *(2 marks)*

(b) The curve C_1 has equation $y = 7^x$.
The curve C_2 has equation $y = 7^{2x} - 12$.

(i) By forming and solving a quadratic equation, prove that the curves C_1 and C_2 intersect at exactly one point. State the y-coordinate of this point. *(4 marks)*

(ii) Use logarithms to find the x-coordinate of the point of intersection of C_1 and C_2, giving your answer to three significant figures. *(2 marks)*

9 The diagram shows part of a curve whose equation is $y = \log_{10}(x^2 + 1)$.



(a) Use the trapezium rule with five ordinates (four strips) to find an approximate value for

$$\int_0^1 \log_{10}(x^2 + 1)\,dx$$

giving your answer to three significant figures. *(4 marks)*

(b) The graph of $y = 2\log_{10} x$ can be transformed into the graph of $y = 1 + 2\log_{10} x$ by means of a translation. Write down the vector of the translation. *(1 mark)*

(c) (i) Show that $\log_{10}(10x^2) = 1 + 2\log_{10} x$. *(2 marks)*

(ii) Show that the graph of $y = 2\log_{10} x$ can also be transformed into the graph of $y = 1 + 2\log_{10} x$ by means of a **stretch**, and describe the stretch. *(4 marks)*

(iii) The curve with equation $y = 1 + 2\log_{10} x$ intersects the curve $y = \log_{10}(x^2 + 1)$ at the point P. Given that the x-coordinate of P is positive, find the gradient of the line OP, where O is the origin. Give your answer in the form $\log_{10}\left(\frac{a}{b}\right)$, where a and b are integers. *(4 marks)*