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General Certificate of Education (A-level)
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Mathematics MPC2

(Specification 6360)

Pure Core 2

Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

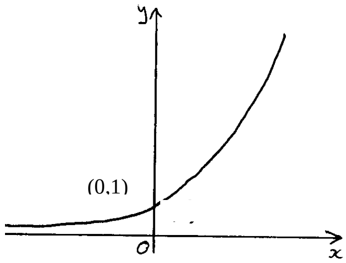
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)	$\frac{10}{\sin\theta}=\frac{9}{\sin 54}$	M1		Sine rule, with $\sin\theta$ being the only unknown
	$\sin\theta=\frac{10\times\sin 54}{9}$ $\left\{=\frac{8.09...}{9}\right\}$ $\left\{=\frac{10}{11.12...}\right\}$	m1		Correct rearrangement to '$\sin\theta=\ldots$' or to '$\theta=\sin^{-1}(\ldots)$'
	$\sin\theta=0.898(9...)$, $\theta=64.01(48\ldots)$ $\theta=64°$ {to nearest degree}	A1	3	AG m1 must have been awarded and must see at least 3sf value either for $\sin\theta$ so that $0.898\le\sin\theta\le 0.8993$ or for θ so that $64.0\le\theta\le 64.1$ as well as seeing 'θ(OE)= 64'
(b)	Angle $C = 180-(54+\theta) = 62$ {to 2sf}	B1		$C = 62$. AWRT 62. PI if '$C = 180-(54+\theta)$' and accurate later work.
	Area = $\frac{1}{2}\times 10\times 9\sin 62$	M1		OE Ft c's value for C ($C\neq 54$, $C\neq\theta$)
	= 39.73.. = 40{cm^2 to nearest sq cm}	A1	3	If not 40 condone a value 39.7 to 39.8 inclusive.
	Total		**6**	

Q	Solution	Marks	Total	Comments
2(a)	{Area of sector =} $\frac{1}{2}r^2\theta = \frac{1}{2}\times 6^2\times 0.5$	M1		$\frac{1}{2}r^2\theta$ seen within (a) or used for the area
	= 9 (cm^2)	A1	2	Condone missing/incorrect units
(b)(i)	{Arc =} $r\theta = 6\times 0.5$	M1		$r\theta$ seen within (b) or used for the arc length
	= 3 (cm)	A1	2	Condone missing/incorrect units
(ii)	Perimeter of sector = 6+6+arc length = 15 (cm) (= 5×3)	M1		PI by value of 12+c's (b)(i) answer
	Perimeter (of sector) = 5×(length of) arc	A1	2	Completion, including concluding statement
	Total		**6**	

Q	Solution	Marks	Total	Comments
3(a)	$(2+x^2)^3$ $= [(2)^3] + 3(2)^2(x^2) + 3(2)(x^2)^2 \; [+(x^2)^3]$	M1		For either (1),3,3,(1) OE unsimplified or $\binom{3}{1}2^2x^2 + \binom{3}{2}2(x^2)^2$ OE. PI
	$p = 3(2)^2 = 12$	A1		AG Be convinced. Condone left as $12x^2$
	$q = 6$	B1	3	Accept left as $6x^4$
(b)(i)	$\int \frac{(2+x^2)^3}{x^4}\,dx = \int x^{-4}(8+12x^2+qx^4+x^6)\,dx$ or $\int \left(\frac{8}{x^4}+\frac{12}{x^2}+q+x^2\right)dx$	M1		Uses (a) and either an indication that $\frac{1}{x^n} = x^{-n}$ in a product PI or cancelling to get at least 3 correct ft terms
	$\int (8x^{-4}+12x^{-2}+q+x^2)\,dx$	A1F		Ft on c's non-zero q. PI by next line in solution
	$= \frac{8x^{-3}}{-3} + \frac{12x^{-1}}{-1} + qx + \frac{x^3}{3} \; \{+c\}$	M1		Correct integration of either $8x^{-4}$ or $12x^{-2}$; accept unsimplified
		A1		Correct integration of both $8x^{-4}$ and $12x^{-2}$; accept unsimplified coefficients
	$= \ldots\ldots\ldots\ldots + 6x + \frac{x^3}{3} + c$ $(= -\frac{8}{3}x^{-3} - 12x^{-1} + 6x + \frac{x^3}{3} + c)$	B1F	5	For $\text{"6"}x + \frac{x^3}{3} + c$ simplified. The only ft is "6" replaced by c's value for q where q is a non-zero integer.
(b)(ii)	$\int_1^2 \frac{(2+x^2)^3}{x^4}\,dx = \{-\frac{8}{3}(2)^{-3} - 12(2^{-1}) + 6(2) + \frac{2^3}{3}\} -$ $\{-\frac{8}{3}(1)^{-3} - 12(1)^{-1} + 6(1) + \frac{1^3}{3}\}$	M1		Dealing correctly with limits; F(2)−F(1) (must have attempted integration to get F ie c's F is not just the integrand)
	$= \left(-\frac{1}{3} - 6 + 12 + \frac{8}{3}\right) - \left(-\frac{8}{3} - 12 + 6 + \frac{1}{3}\right)$ $= 16\frac{2}{3}$	A1	2	OE **exact** answer eg 50/3. NMS scores 0
	Total		**10**	

Q	Solution	Marks	Total	Comments
4(a)		B1 B1	 2	Any graph only crossing the y–axis at (0, 1) stated /indicated … (accept 1 on y-axis as equivalent) and not drawn **below** x-axis Correct shaped graph, must clearly go below the intersection pt and an indication of correct behaviour of curve for large positive and large negative values of x. Ignore any scaling on axes.
(b)	Translation; $\begin{bmatrix} 0 \\ -5 \end{bmatrix}$	B1 B1	 2	Accept 'transl…' as equivalent [T or Tr is NOT sufficient] If vector not given, accept **full** equivalent to vector in words provided linked to 'transl../ move/shift' (B0B0 if >1 transformation)
(c)(i)	$4^x = (2^2)^x = 2^{2x} = (2^x)^2 = Y^2$ $2^{x+2} = 2^x \times 2^2 = 4Y$ $4^x - 2^{x+2} - 5 = 0 \Rightarrow Y^2 - 4Y - 5 = 0$	M1 A1	 2	Justifying either $4^x = Y^2$ or $2^{x+2} = 4Y$ with no errors seen AG Be convinced; must have justified both of the above.
(ii)	$(Y - 5)(Y + 1) = 0$ (Since) $2^x>0$ (for all real x,) $2^x=5$ so only one (real) solution $\log 2^x = \log 5 \Rightarrow x \log 2 = \log 5$ $x = 2.3219… = 2.322$ (to 3dp)	M1 E1 M1 A1	 4	Correct factorising or use of quadratic formula or completing sq. PI by both solns 5& −1 seen Rejection of 2^x (condone Y) negative, with justification, (condone "2^x not negative") followed by statement Eqn of form $p^x = q \Rightarrow x \log p = \log q$ provided $p > 0$ & $q > 0$ OE eg $x = \log_2 5$ Condone > 3dp but must see explicit use of logs and must only be the one solution.
	Total		**10**	

Q	Solution	Marks	Total	Comments
5(a)		B1		For either 6 or $6x^0$
	$\frac{dy}{dx} = 6 - 3x^{\frac{1}{2}}$	M1		$Ax^{\frac{3}{2}-1}$, $A \neq 0$ OE
		A1	3	$6 - 3x^{\frac{1}{2}}$ or $6 - 3\sqrt{x}$ with no '+c' [If unsimplified here, A1 can be awarded retrospectively if correct simplified expression is seen explicitly in (b)(i).]
(b)(i)	$6 - 3x^{\frac{1}{2}} = 0$	M1		Equating c's $\frac{dy}{dx}$ to 0 PI by correct ft rearrangement of c's $dy/dx=0$
	$x^{\frac{1}{2}} = 2 \Rightarrow x = 2^2$	m1		$x^{\frac{1}{2}} = k$ (k>0), to $x = k^2$. PI by correct value of x if no error seen
	$M(4, 8)$	A1	3	SC If M0 award B1 for (4, 8)
(ii)	Eqn of normal at M is $x = 4$	B1F	1	Ft on x = c's x_M
(c)(i)	When $x = \frac{9}{4}$, $\frac{dy}{dx} = 6 - 3 \times \frac{3}{2} = \frac{3}{2}$	M1		Attempt to find $\frac{dy}{dx}$ when $x = \frac{9}{4}$
	Gradient of normal at $P = -\frac{2}{3}$	m1		$m \times m' = -1$ used
	Eqn of normal: $y - \frac{27}{4} = -\frac{2}{3}\left(x - \frac{9}{4}\right)$	A1		ACF eg $y = -\frac{2}{3}x + \frac{33}{4}$
	$12y - 81 = -8x + 18 \Rightarrow 8x + 12y = 99$	A1	4	Coeffs and constant must now be positive integers, but accept different order eg $12y + 8x = 99$
(ii)	$8(4) + 12y = 99$	M1		Solving c's answer (b)(ii), (must be in form x = positive const), with c's answer (c)(i). PI by correct earlier work and <u>correct</u> coordinates for R.
	$R\left(4, \frac{67}{12}\right)$	A1	2	Accept 5.58 or better as equivalent to $\frac{67}{12}$
	Total		**13**	

Q	Solution	Marks	Total	Comments
6(a)	$h = 0.5$	B1		$h = 0.5$ stated or used. (PI by x-values 0, 0.5, 1, 1.5, 2 provided no contradiction)
	$f(x) = \sin x$ $I \approx h/2\{\ldots\}$ $\{.\}=f(0)+f(2)+2[f(0.5)+f(1)+f(1.5)]$	M1		OE summing of areas of 'trapezia'..
	$\{.\}=$ $0+0.90929..+2[0.4794..+0.84147..+0.99749..]$ $\{=0.90929..+2[2.318..]\}=\{0.90929..+4.636..\}$	A1		Min. of 2dp values rounded or truncated. Can be implied by later correct work provided >1 term or a single term which rounds to 1.39
	$(I \approx)$ 0.25[5.546..] = 1.3865.. = 1.39 (to 3sf)	A1	4	CAO Must be 1.39
(b)	Stretch(**I**) in y-direction(**II**) scale factor 2(**III**)	M1 A1	 2	Need (**I**) **and** either (**II**) or (**III**) All correct. Need (**I**) and (**II**) and (**III**) [>1 transformation scores 0/2]
(c)	$\frac{\sin x}{\cos x} = \frac{1}{2}$; $\tan x = 0.5$	M1		$\frac{\sin x}{\cos x} = \tan x$ used to get $\tan x = k$ or identity $\cos^2 x + \sin^2 x = 1$ used to get either $\sin^2 x = p$ or to get $\cos^2 x = q$, (p and q must be between 0 and 1)
	$\tan x = 0.5$	A1		Either $\tan x = \frac{1}{2}$ or $\cos x = \pm\sqrt{\frac{4}{5}}$ (=±0.894..) or $\sin x = \pm\sqrt{\frac{1}{5}}$ (=±0.447..)
	$x = \alpha$ or $\pi + \alpha$ where $\alpha = \tan^{-1}(k)$	m1		Correct method to find 2nd angle. Any in wrong ft quadrants then m0. In case of squaring method candidates must also have rejected the extra 'quadrants' for the m1. Condone degrees or mixture
	$x = 0.464, 3.61$	A1	4	Both. Condone>3sf [0.463(6..), 3.60(5..or 6.)] Accept **pair** of truncated values [0.463, 3.60] Ignore any answers outside interval 0 to 6.28
	Total		**10**	

Q	Solution	Marks	Total	Comments
7(a)	$48 = 60p + q$ $12 = 12p + q$	M1 M1		M1 for each equation in ACF (Condone embedded values for the M1M1)
	$36 = 48p$ or $p = \frac{36}{48}$	m1		Valid method to solve the correct two simultaneous eqns in *p* and *q* to at least the stage $ap = b$ or $cq = d$
	$p = \frac{3}{4}$	A1		AG (condone if left as equiv. decimal)
	$q = 3$	B1	5	Can award if seen **explicitly** in (b) and no contradiction [ie not attempted in (a)]
(b)	$u_3 = 36 + q = 39$	B1F	1	If not 39, ft on (36 + c's *q*)
	Total		**6**	

Q	Solution	Marks	Total	Comments
8	$\ldots = 9\sin^2 x + 6\sin x\cos x + \cos^2 x +$ $\sin^2 x - 6\sin x\cos x + 9\cos^2 x$	M1		Attempt at expanding both sets of brackets. Minimum requirement either one of the two expansions correct or 4 of these 6 terms seen. Expanding and simplifying the given expression in one step to get the correct two terms scores this M1 and next A1
	$\ldots = 10\cos^2 x + 10\sin^2 x$	A1		Either correct pair of expansions and simplification to remove sin*x*cos*x* terms or full collecting of like terms within the original correct expansion
	$= 10(1 - \sin^2 x) + 10\sin^2 x$	M1		$\cos^2 x + \sin^2 x = 1$ clearly used. If identity is applied correctly, but not directly, it must be stated at the relevant point in the proof.
	= 10 (which is an integer)	A1	4	CSO [all previous 3 marks must have been scored] Condone absence of statement after 10 obtained correctly
	Total		**4**	

Q	Solution	Marks	Total	Comments
9(a)	$\{S_\infty =\}\ \dfrac{a}{1-r}=\dfrac{12}{1-\frac{3}{8}}$	M1		$\dfrac{a}{1-r}$ used
	$\{S_\infty =\}\ 19.2$	A1	2	19.2 OE NMS mark as 2/2 or 0/2
(b)	{6th term = } ar^{6-1}	M1		Stated or used
	$= 12\times\left(\dfrac{3}{8}\right)^5 = 2\times2\times3\times\left(\dfrac{3}{2\times2\times2}\right)^5$	m1		Changing 8 and 12 in correct expression to correct products/powers of 2 and 3
	$=\dfrac{2\times2\times3\times3^5}{(2^3)^5} = \dfrac{2^2\times3^6}{2^{15}}=\dfrac{3^6}{2^{13}}$	A1	3	AG Be convinced
(c)(i)	$\{u_n =\}\ 12\times\left(\dfrac{3}{8}\right)^{n-1}$	B1	1	OE. eg $32(3/8)^n$
(ii)	$\log u_n = \log 12+\log\left(\dfrac{3}{8}\right)^{n-1}$ $\log u_n = \log 12+(n-1)\log\left(\dfrac{3}{8}\right)$			Log laws log(PQ)=logP + logQ ; $\log\left(\dfrac{P}{Q}\right)$ = log P − log Q $\log(P)^k = k$ logP
	$\log u_n = \log 12+(n-1)[\log 3-\log 8]$	M1		Using (c)(i) and taking logs: one log law used correctly, on a correct expression for u_n.
		M1		a second different log law used correctly, indep of prev M error and ft on cand's (c)(i) provided cand's u_n expression has a power involving n.
	$\log u_n = \log 3+2\log 2+(n-1)[\log 3-3\log 2]$	m1		A third different log law used correctly (or equivalent valid step) to reach a correct RHS whose terms are all multiples of log2 and log3. Dep on both prev two Ms
	$\log u_n = n\log 3-3n\log 2+5\log 2$ $\log_a u_n = n\log_a 3-(3n-5)\log_a 2$	A1	4	CSO AG Be convinced, no slips although we will condone the absence of the bases a even in the final line.
	Total		**10**	