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General Certificate of Education (A-level)
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Mathematics **MPC2**

(Specification 6360)

Pure Core 2

Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

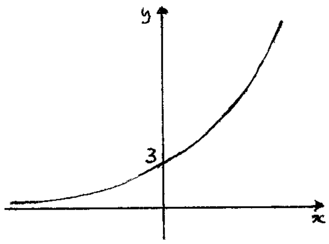
Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)	Arc = $r\theta$ (= 1.25r)	M1		Within (a), $r\theta$ or 15 used for the arc length PI
	P = $r + r + r\theta$ =39	m1		Use of $r + r + r\theta$ for the perimeter. m0 if no indication that '15' comes from $r\theta$.
	3.25r = 39 $r = \dfrac{39}{3.25}$ = 12	A1	3	CSO AG
(b)	{Area of sector =} $\dfrac{1}{2}r^2\theta$	M1		Within (b), $\dfrac{1}{2}r^2\theta$ stated or used for the sector area.
	$= \dfrac{1}{2}\times 12^2 \times 1.25$ = 90 (cm^2)	A1	2	NMS: 90 scores 2 marks
	Total		**5**	
2(a)	$h = 1$ $f(x) = \dfrac{1}{x^2+1}$	B1		PI
	$I \approx \dfrac{h}{2}\{f(1)+f(5)+2[f(2)+f(3)+f(4)]\}$	M1		$\dfrac{h}{2}\{f(1)+f(5)+2[f(2)+f(3)+f(4)]\}$ OE summing of areas of the four 'trapezia'…
	$\dfrac{h}{2}$ with $\{\ldots\} = \dfrac{1}{2}+\dfrac{1}{26}+2\left(\dfrac{1}{5}+\dfrac{1}{10}+\dfrac{1}{17}\right)$ = 0.5 + 0.03(84...)+2[0.2+0.1+0.05(88...)] = 0.538(46…)+2[0.358(82...)]=1.256(108…)	A1		OE Accept 2dp (rounded or truncated) for non-terminating decs. equiv.
	(I ≈) 0.628054… = $\dfrac{694}{1105}$ = 0.628 (to 3sf)	A1	4	CAO Must be 0.628 **SC** for those who use 5 strips, max possible is B0M1A1A0
(b)(i)	$\int \left(x^{-\frac{3}{2}} + 6x^{\frac{1}{2}}\right)dx = \dfrac{x^{-\frac{1}{2}}}{-1/2} + \dfrac{6x^{\frac{3}{2}}}{3/2}$ (+c)	M1 A1		One term correct (even unsimplified) Both terms correct (even unsimplified)
	$= -2x^{-0.5} + 4x^{1.5}$ (+c)	A1	3	Must be simplified.
(ii)	$\int_1^4 \left(x^{-\frac{3}{2}} + 6x^{\frac{1}{2}}\right)dx$ $= [-2(4^{-0.5})+4(4^{1.5})] - [-2(1^{-0.5})+4(1^{1.5})]$	M1		Attempt to calculate F(4)−F(1) where F(x) follows integration and is not just the integrand
	= (−1+32) − (−2+4) = 29	A1	2	Since 'Hence' NMS scores 0/2
	Total		**9**	

Q	Solution	Marks	Total	Comments
3(a)	$\frac{1}{2}\times 5\times 6\sin C = 12.5$	M1		(Area=) $\frac{1}{2}\times 5\times 6\sin C$
	$\sin C = 0.833(3..)$	A1		AWRT 0.83 or 5/6 OE PI by e.g. seeing 56 or better
	(C is obtuse) $C = 123.6°$	A1	3	AWRT 123.6
(b)	$\{AB^2 =\}5^2 + 6^2 - 2\times 5\times 6\cos C$	M1		RHS of cosine rule used
	$= 61 - 60\times(-0.553\ldots) = 94.1(66\ldots)$	m1		Correct ft evaluation, to at least 2 sf, of AB^2 or AB using c's value of C.
	(AB =) 9.7 (cm to 2sf)	A1	3	If not 9.7 accept AWRT 9.70 or AWRT 9.71
	Total		**6**	
4	$\log_a N - \log_a x = \frac{3}{2}$			
	$\log_a \frac{N}{x} = \frac{3}{2}$	M1		A log law used correctly. PI by next line.
	$\frac{N}{x} = a^{\frac{3}{2}}$	m1		Logarithm(s) eliminated correctly
	$x = a^{-\frac{3}{2}}N$	A1	3	ACF of RHS
	Total		**3**	

Q	Solution	Marks	Total	Comments
5(a)	$\frac{8}{x^2}=8x^{-2}$	B1		PI by its derivative as $16x^{-3}$ or $-16x^{-3}$
	$\frac{dy}{dx}=2+16x^{-3}$	M1		Differentiating either $6+2x$ correctly or differentiating $-8/x^2$ correctly.
		A1	3	$2+16x^{-3}$ OE
(b)	At $P(2, 8)$, $\frac{dy}{dx}=2+16\times 2^{-3}$ (= 4)	M1		Attempt to find $\frac{dy}{dx}$ when $x=2$
	Gradient of normal at P = $-\frac{1}{4}$	m1		$m \times m' = -1$ used
	Eqn. of normal at P: $y-8=-\frac{1}{4}(x-2) \Rightarrow x+4y=34$	A1	3	CSO AG
(c)(i)	At St. Pt $\frac{dy}{dx}=0$, $2+16x^{-3}=0$	M1		Equating c's $\frac{dy}{dx}$ to 0. Accept '$\frac{dy}{dx}$ =0 so $x=-2$' stated with no errors seen
	$(16x^{-3}=-2)$ $x=-2$	A1		$x=-2$
	When $x=-2$, $y=6-4-2=0$; $M(-2,0)$ lies on x-axis	A1	3	Need statement and correct coords.
(c)(ii)	Tangent at M has equation $y=0$	B1	1	$y=0$ OE
(d)	Intersects normal at P when $x+0=34$	M1		PI Solving c's eqn. of tangent with ans (b) as far as correctly eliminating one variable.
	T (34, 0)	A1	2	Accept $x=34$, $y=0$
	Total		**12**	

	Solution	Marks	Total	Comments
6(a)(i)	$r = \dfrac{294}{420} = 0.7$	B1	1	AG. Accept any valid justification to the given answer
(ii)	$\{S_\infty =\}\ \dfrac{a}{1-r} = \dfrac{420}{1-0.7}$	M1		$\dfrac{a}{1-r}$ <u>used</u>
	$\{S_\infty =\}\ 1400$	A1	2	1400 NMS mark as 2/2 or 0/2
(iii)	nth term = $600\times(0.7)^n$	B2	2	If not B2 award B1 for $420 \times (0.7)^{n-1}$ OE
(b)(i)	$\{u_n =\} 248 - 8n$	B1	1	Accept ACF
(ii)	$u_k = 0 \Rightarrow 8k = 248$	M1		$248-8k=0$ OE e.g. $240+(k-1)(-8)=0$ ft if no recovery, on c's (b)(i) answer
	$k = 31$	A1		
	$\sum_{n=1}^{k} u_n = 240+232+\ldots+0 = \dfrac{k}{2}[240+0]$	M1		For $\dfrac{k}{2}[240+0]$ or for $\dfrac{k}{2}[\text{c's } u_1 + 0]$ OE e.g. $\dfrac{k}{2}[2\times \text{c's } u_1 + (k-1)(-8)]$
	$\sum_{n=1}^{k} u_n \quad (= 15.5\times 240) = 3720$	A1	4	3720
	Total		**10**	

Q	Solution	Marks	Total	Comments
7(a)	Stretch(**I**) in y-direction(**II**) scale factor 3(**III**)	M1		OE Need (**I**) **and** either (**II**) or (**III**)
		A1	2	All correct. Need (**I**) and (**II**) and (**III**) [>1 transformation scores 0/2]
(b)		B1		Shape with indication of correct asymptotic behaviour in 2nd quadrant below pt of intersection with y-axis
		B1	2	Only intersection is with y-axis, and only intercept is 3 stated/indicated
(c)	$3\times 4^{x} = 4^{-x}$	M1		OE eqn. in x
	$\log 3 + \log 4^{x} = \log 4^{-x}$	m1		Log Law 1 (or Law 2 applied to $\frac{4^{x}}{4^{-x}} = 3$ or $\frac{1}{3}$ OE) used correctly or correct rearrangement to $4^{2x}=1/3$ OE simplified e.g.$16^{x}=3^{-1}$ or $4^{x}=(1/\sqrt{3})$
	$\log 3 + x\log 4 = -x\log 4$	m1		Log Law 3 applied correctly twice (dependent on both M1 & m1) or a correct method using logs to solve an eqn. of form $a^{kx}=b$, $b>0$ (including case $k=1$) (dependent on M1 and valid method to a^{kx})
	$x = \frac{-\log 3}{2\log 4}$ $\left(= \frac{-\log 3}{\log 16}\right)$	A1		Correct expression for x or for $-x$ e.g. $x = \frac{1}{2}\log_4\left(\frac{1}{3}\right)$ PI by correct 3sf value or better
	$x = -0.396(2406\ldots)$ $= -0.396$ (to 3sf)	A1	5	If logs not used explicitly then max of M1m1m0.
	Total		**9**	

Q	Solution	Marks	Total	Comments
8(a)	$\left(1+\frac{4}{x}\right)^2 = 1+\frac{8}{x}+\frac{16}{x^2}$ (or $1+8x^{-1}+16x^{-2}$)	B1	1	Unsimplified equivalent answers, e.g. $1+\frac{4}{x}+\frac{4}{x}+\left(\frac{4}{x}\right)^2$ etc. must be correctly simplified in part (c) to one of the two forms in 'solution' to retrospectively score the B1 here
(b)	$\left(1+\frac{x}{4}\right)^8 = \{1+\}\binom{8}{1}\left(\frac{x}{4}\right)+\binom{8}{2}\left(\frac{x}{4}\right)^2+\binom{8}{3}\left(\frac{x}{4}\right)^3+\ldots$	M1		Any valid method. PI by a correct value for either *a* or *b* or *c*
	$=\{1+\}2x+\frac{7}{4}x^2+\frac{7}{8}x^3+\ldots$	A1A1A1		A1 for each of *a, b, c*
	$\{a=2,\ b=1.75 \text{ OE},\ c=0.875 \text{ OE}\}$		4	**SC** $a=8,\ b=28,\ c=56$ or $a=32,\ b=448,\ c=3584$ either explicitly or within expn (M1A0)
(c)	$(1+\frac{8}{x}+\frac{16}{x^2})\left(1+2x+\frac{7}{4}x^2+\frac{7}{8}x^3\right)$	M1		Product of c's two expansions either stated explicitly or used
	x terms from expansion of $\left(1+\frac{4}{x}\right)^2\left(1+\frac{x}{4}\right)^8$ are ax and '8'bx and '16'cx	m1		Any **two** of the three, **ft** from products of non-zero terms using c's two expansions. May just use the coefficients.
	ax + '8'bx + '16'cx	A1F		Ft on c's non-zero **values** for *a, b* **and** *c* and also ft on c's non-zero coeffs. of $1/x$ and $1/x^2$ in part (a). Accept x's missing i.e. sum of coeffs. PI by the correct final answer.
	Coefficient of x is $2+14+14=30$	A1	4	OE Condone answer left as $30x$. Ignore terms in other powers of x in the expansion.
	Total		**9**	

Q	Solution	Marks	Total	Comments
9(a)	$x + 30^{\circ} = 79^{\circ}$, $x + 30^{\circ} = 180^{\circ} + 79^{\circ}$			
	$x = 49^{\circ}$	B1		49 as the only solution in the interval $0^\circ \le x < 90^\circ$
	$x = 229^{\circ}$	B1	2	AWRT 229. Not given if any other soln. in the interval $90^\circ \le x \le 360^\circ$. Ignore anything outside $0^{\circ} \le x \le 360^{\circ}$
(b)	Translation;	B1		Accept 'translat…' as equivalent. [T or Tr is NOT sufficient]
	$\begin{bmatrix} -30^\circ \\ 0 \end{bmatrix}$	B1	2	OE Accept **full** equivalent to vector in words provided linked to 'translation/ move/shift' and **correct** direction. (0/2 if >1 transformation).
(c)(i)	$5+\sin^2\theta = (5+3\cos\theta)\cos\theta$ $\Rightarrow 5+\sin^2\theta = 5\cos\theta + 3\cos^2\theta$	B1		Correct RHS.
	$5+1-\cos^2\theta = 5\cos\theta + 3\cos^2\theta$	M1		$\sin^2\theta = 1-\cos^2\theta$ used to get a quadratic in cos .
	$6 = 5\cos\theta + 4\cos^2\theta$ or $4\cos^2\theta + 5\cos\theta - 6\ (=0)$	A1		ACF with like terms collected.
	$\Rightarrow$ $(4\cos\theta - 3)(\cos\theta + 2)$ $(=0)$	m1		**Correct** quadratic and $(4c\pm3)(c\pm2)$ or by formula OE PI by 'correct' 2 values for $\cos\theta$.
	Since $\cos\theta \ne -2$, $\cos\theta = \dfrac{3}{4}$	A1	5	CSO AG. Must show that the 'soln' $\cos\theta = -2$ has been considered and rejected
(ii)	$5+\sin^2 2x = (5+3\cos 2x)\cos 2x$ $\Rightarrow \cos 2x = \dfrac{3}{4}$	M1		Using (c)(i) to reach $\cos 2x = ¾$ **or** finding at least 3 solutions of $\cos\theta = 3/4$ **and** dividing them by 2.
	$2x = 0.722(7…)$, $2\pi - 0.722(7…)$, $2\pi + 0.722(7…)$, $4\pi - 0.722(7...)$,	m1		Valid method to find all four 'positions' of solutions.
	$x = 0.361$, 2.78, 3.50, 5.92	A1	3	CAO Must be these four 3sf values but ignore any values outside the interval $0 < x < 2\pi$.
	Total		**12**	
	TOTAL		**75**	