General Certificate of Education
Advanced Subsidiary Examination
January 2012

Mathematics MPC2

Unit Pure Core 2

Friday 13 January 2012 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

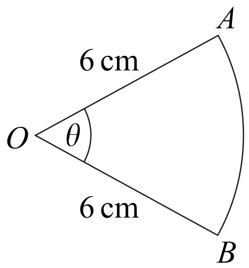
Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 The diagram shows a sector OAB of a circle with centre O and radius 6 cm.



The angle between the radii OA and OB is θ radians.

The area of the sector OAB is $21.6\,\text{cm}^2$.

(a) Find the value of θ. *(2 marks)*

(b) Find the length of the arc AB. *(2 marks)*

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2 (a) Use the trapezium rule with five ordinates (four strips) to find an approximate value for

$$\int_0^4 \frac{2^x}{x+1}\,\mathrm{d}x$$

giving your answer to three significant figures. *(4 marks)*

(b) State how you could obtain a better approximation to the value of the integral using the trapezium rule. *(1 mark)*

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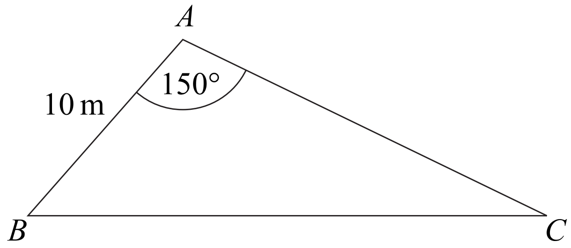
3 (a) Write $\sqrt[4]{x^3}$ in the form x^k. *(1 mark)*

(b) Write $\dfrac{1-x^2}{\sqrt[4]{x^3}}$ in the form $x^p - x^q$. *(2 marks)*

4 The triangle ABC, shown in the diagram, is such that AB is 10 metres and angle BAC is 150°.



The area of triangle ABC is $40\,\text{m}^2$.

(a) Show that the length of AC is 16 metres. *(2 marks)*

(b) Calculate the length of BC, giving your answer, in metres, to two decimal places. *(3 marks)*

(c) Calculate the smallest angle of triangle ABC, giving your answer to the nearest 0.1°. *(3 marks)*

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5 (a) (i) Describe the geometrical transformation that maps the graph of $y = \left(1 + \frac{x}{3}\right)^6$ onto the graph of $y = (1 + 2x)^6$. *(2 marks)*

(ii) The curve $y = \left(1 + \frac{x}{3}\right)^6$ is translated by the vector $\begin{bmatrix} 3 \\ 0 \end{bmatrix}$ to give the curve $y = \mathrm{g}(x)$. Find an expression for $\mathrm{g}(x)$, simplifying your answer. *(2 marks)*

(b) The first four terms in the binomial expansion of $\left(1 + \frac{x}{3}\right)^6$ are $1 + ax + bx^2 + cx^3$.

Find the values of the constants a, b and c, giving your answers in their simplest form. *(4 marks)*

6 An arithmetic series has first term a and common difference d.

The sum of the first 25 terms of the series is 3500.

(a) Show that $a + 12d = 140$. *(3 marks)*

(b) The fifth term of this series is 100.

Find the value of d and the value of a. *(4 marks)*

(c) The nth term of this series is u_n. Given that

$$33\left(\sum_{n=1}^{25} u_n - \sum_{n=1}^{k} u_n\right) = 67\sum_{n=1}^{k} u_n$$

find the value of $\sum_{n=1}^{k} u_n$. *(3 marks)*

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7 (a) Sketch the graph of $y = \dfrac{1}{2^x}$, indicating the value of the intercept on the y-axis. *(2 marks)*

(b) Use logarithms to solve the equation $\dfrac{1}{2^x} = \dfrac{5}{4}$, giving your answer to three significant figures. *(3 marks)*

(c) Given that

$$\log_a(b^2) + 3\log_a y = 3 + 2\log_a\left(\frac{y}{a}\right)$$

express y in terms of a and b.

Give your answer in a form not involving logarithms. *(5 marks)*

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8 (a) Given that $2\sin\theta = 7\cos\theta$, find the value of $\tan\theta$. *(2 marks)*

(b) (i) Use an appropriate identity to show that the equation

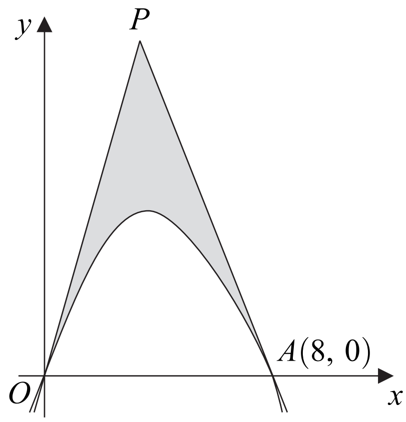
$$6\sin^2 x = 4 + \cos x$$

can be written as

$$6\cos^2 x + \cos x - 2 = 0$$ *(2 marks)*

(ii) Hence solve the equation $6\sin^2 x = 4 + \cos x$ in the interval $0° < x < 360°$, giving your answers to the nearest degree. *(6 marks)*

9 The diagram shows part of a curve crossing the x-axis at the origin O and at the point $A(8, 0)$. Tangents to the curve at O and A meet at the point P, as shown in the diagram.



The curve has equation

$$y = 12x - 3x^{\frac{5}{3}}$$

(a) Find $\dfrac{dy}{dx}$. *(2 marks)*

(b) (i) Find the value of $\dfrac{dy}{dx}$ at the point O and hence write down an equation of the tangent at O. *(2 marks)*

(ii) Show that the equation of the tangent at $A(8, 0)$ is $y + 8x = 64$. *(3 marks)*

(c) Find $\displaystyle\int \left(12x - 3x^{\frac{5}{3}}\right) dx$. *(3 marks)*

(d) Calculate the area of the shaded region bounded by the curve from O to A and the tangents OP and AP. *(7 marks)*