General Certificate of Education
January 2008
Advanced Subsidiary Examination

MATHEMATICS **MPC2**
Unit Pure Core 2

ASSESSMENT and QUALIFICATIONS ALLIANCE

Wednesday 9 January 2008 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The diagrams show a rectangle of length 6 cm and width 3 cm, and a sector of a circle of radius 6 cm and angle θ radians.

The area of the rectangle is twice the area of the sector.

(a) Show that $\theta = 0.5$. *(3 marks)*

(b) Find the perimeter of the sector. *(3 marks)*

2 The arithmetic series

$$51 + 58 + 65 + 72 + \ldots + 1444$$

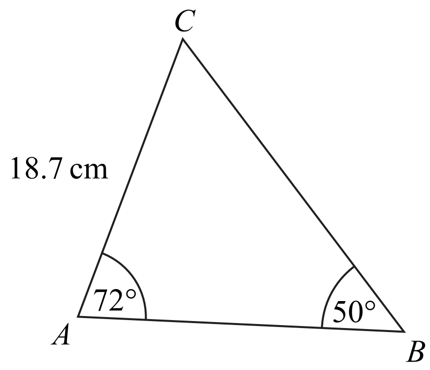
has 200 terms.

(a) Write down the common difference of the series. *(1 mark)*

(b) Find the 101st term of the series. *(2 marks)*

(c) Find the sum of **the last** 100 terms of the series. *(2 marks)*

3 The diagram shows a triangle *ABC*. The length of *AC* is 18.7 cm, and the sizes of angles *BAC* and *ABC* are 72° and 50° respectively.



(a) Show that the length of $BC = 23.2$ cm, correct to the nearest 0.1 cm. *(3 marks)*

(b) Calculate the area of triangle *ABC*, giving your answer to the nearest cm^2. *(3 marks)*

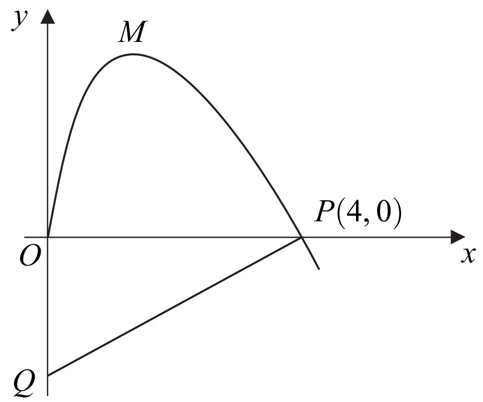
4 Use the trapezium rule with four ordinates (three strips) to find an approximate value for

$$\int_0^3 \sqrt{x^2+3}\, \mathrm{d}x$$

giving your answer to three decimal places. *(4 marks)*

5 A curve, drawn from the origin O, crosses the x-axis at the point $P(4, 0)$.

The normal to the curve at P meets the y-axis at the point Q, as shown in the diagram.



The curve, defined for $x \geqslant 0$, has equation

$$y = 4x^{\frac{1}{2}} - x^{\frac{3}{2}}$$

(a) (i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. *(3 marks)*

(ii) Show that the gradient of the curve at $P(4, 0)$ is -2. *(2 marks)*

(iii) Find an equation of the normal to the curve at $P(4, 0)$. *(3 marks)*

(iv) Find the y-coordinate of Q and hence find the area of triangle OPQ. *(3 marks)*

(v) The curve has a maximum point M. Find the x-coordinate of M. *(3 marks)*

(b) (i) Find $\displaystyle\int \left(4x^{\frac{1}{2}} - x^{\frac{3}{2}}\right) \mathrm{d}x$. *(3 marks)*

(ii) Find the total area of the region bounded by the curve and the lines PQ and QO. *(3 marks)*

6 (a) Using the binomial expansion, or otherwise:

(i) express $(1+x)^3$ in ascending powers of x; *(2 marks)*

(ii) express $(1+x)^4$ in ascending powers of x. *(2 marks)*

(b) Hence, or otherwise:

(i) express $(1+4x)^3$ in ascending powers of x; *(2 marks)*

(ii) express $(1+3x)^4$ in ascending powers of x. *(2 marks)*

(c) Show that the expansion of

$$(1+3x)^4 - (1+4x)^3$$

can be written in the form

$$px^2 + qx^3 + rx^4$$

where p, q and r are integers. *(2 marks)*

7 (a) Given that

$$\log_a x = \log_a 16 - \log_a 2$$

write down the value of x. *(1 mark)*

(b) Given that

$$\log_a y = 2\log_a 3 + \log_a 4 + 1$$

express y in terms of a, giving your answer in a form **not** involving logarithms. *(3 marks)*

8 (a) Sketch the graph of $y = 3^x$, stating the coordinates of the point where the graph crosses the y-axis. *(2 marks)*

(b) Describe a single geometrical transformation that maps the graph of $y = 3^x$:

(i) onto the graph of $y = 3^{2x}$; *(2 marks)*

(ii) onto the graph of $y = 3^{x+1}$. *(2 marks)*

(c) (i) Using the substitution $Y = 3^x$, show that the equation

$$9^x - 3^{x+1} + 2 = 0$$

can be written as

$$(Y - 1)(Y - 2) = 0$$ *(2 marks)*

(ii) Hence show that the equation $9^x - 3^{x+1} + 2 = 0$ has a solution $x = 0$ and, by using logarithms, find the other solution, giving your answer to four decimal places. *(4 marks)*

9 (a) Given that

$$\frac{3 + \sin^2 \theta}{\cos \theta - 2} = 3 \cos \theta$$

show that

$$\cos \theta = -\frac{1}{2}$$ *(4 marks)*

(b) Hence solve the equation

$$\frac{3 + \sin^2 3x}{\cos 3x - 2} = 3 \cos 3x$$

giving all solutions in degrees in the interval $0° < x < 180°$. *(4 marks)*

END OF QUESTIONS

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