GCE 2005

January Series

ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark Scheme

Mathematics

MPC2

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	OE	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

MPC2

Q	Solution	Marks	Total	Comments
1(a)(i)	$y = x + 2x^{-1}$	B1		PI by sight of $-2x^{-2}$
	$\frac{dy}{dx} = 1 - 2x^{-2}$	M1 A1	3	One term correct OE
(ii)	When $x = 2$, $\frac{dy}{dx} = 1 - \frac{2}{4} = \frac{1}{2}$	A1	1	CSO AG (be convinced)
(b)	When $x = 2$, $y = 3$	B1		For $y = 3$
	gradient of normal = − 2	M1		$m \times m' = -1$ used
	Equation normal $y - 3 = -2(x - 2)$	M1		$y - "3" = m(x - 2)$ OE
		A1	4	Award at 1st correct form
	Total		**8**	
2(a)	$32^2 = 24^2 + 24^2 - 2 \times 24 \times 24 \cos\theta$ **or** $\sin\frac{1}{2}\theta = \frac{\frac{1}{2}(32)}{24}$	M1		
	$\cos\theta = \frac{24^2 + 24^2 - 32^2}{2 \times 24 \times 24}$ $\ldots\ldots\ldots = \frac{128}{1152}\{= \frac{1}{9}\}\{= 0.11\}$ **or** $\frac{1}{2}\theta = \sin^{-1}\left(\frac{2}{3}\right)$ (= 0.7297..)	m1		
	$\theta = 1.459\ldots = 1.46$ to 3sf	A1	3	CSO AG (be convinced)
(b)	Arc = $r\theta$	M1		
	= 24 × 1.459… = 35 cm	A1	2	Condone absent cm; 35 to 35.04
(c)(i)	Area of sector = $\frac{1}{2}r^2\theta$	M1		Seen
	= $\frac{1}{2}(24)^2(1.459..)$ = 420.3 = 420 cm^2	A1	2	Condone absent cm^2; 420 to 420.48
(ii)	Area of triangle = $\frac{1}{2}(24)(24)\sin\theta$ [= 286. (….)]	M1		OE
	Shaded area = area of sector – area of triangle	m1		Dep on at least one of the previous two M marks. PI
	$\left[= \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta\right] = 134$ cm^2	A1	3	Condone absent cm^2
	Total		**10**	

MPC2 (cont)

Q	Solution	Marks	Total	Comments
3(a)(i)	$a+19d=181$;	M1		$a+(n-1)d$ used; PI
	$a+4d=46$			
	$\Rightarrow 15d = 181 - 46$	A1		
	$\Rightarrow d = 9$	A1	3	AG (be convinced)
(ii)	$a = 10$	B1	1	
(b)	$S_{20}=\frac{20}{2}\left[2a+(20-1)d\right]$	M1		OE
	$\ldots = 1910$	A1	2	
(c)	$\sum_{n=1}^{50}u_n-\sum_{n=1}^{20}u_n$	M1		OE
	$\ldots = 11525 - \text{“}1910\text{”} = 9615$	A1√	2	ft on 11525 – c's S_{20}
	Total		**8**	
4(a)	$\sqrt{x}=x^{\frac{1}{2}}$	B1	1	Accept $k = 0.5$
(b)	$\sqrt{x}\left(x-1\right)=x^{\frac{1}{2}}x-x^{\frac{1}{2}}=x^{\frac{3}{2}}-x^{\frac{1}{2}}$	M1 A1	2	Accept $p = 1.5$, $q = 0.5$
(c)	$\int\sqrt{x}\left(x-1\right)\mathrm{d}x=\frac{x^{2.5}}{2.5}-\frac{x^{1.5}}{1.5}\ (+c)$	M1 A1√ A1√	3	Increases a power of x by 1 ft non-integer p ft non-integer q
(d)	$\int_1^2 \mathrm{d}x=\left(\frac{2^{2.5}}{2.5}-\frac{2^{1.5}}{1.5}\right)-\left(\frac{1}{2.5}-\frac{1}{1.5}\right)$	M1		Limits; F(2) – F(1)
	$\ldots=\left(\frac{4\sqrt{2}}{2.5}-\frac{2\sqrt{2}}{1.5}\right)-\left(\frac{1}{2.5}-\frac{1}{1.5}\right)$	m1		Fractional power<u>s</u> to surds
	$\left(\frac{24\sqrt{2}}{15}-\frac{20\sqrt{2}}{15}\right)-\left(-\frac{4}{15}\right)$ = pr. ans	A1	3	CSO AG (be convinced)
	Total		**9**	
5(a)	$\log_a x=\log_a 6^3-\log_a 8$	M1		A law of logs used correctly
	$\log_a x=\log_a(6^3\ \div 8)$	M1		A <u>different</u> law of logs used correctly
	$x=6^3\div 8=27$	A1	3	CSO AG (be convinced) **<u>ALT</u>** $\log_a x = 3\log_a 6 - 3\log_a 2$ (M1) $\frac{1}{3}\log_a x=\log_a\frac{6}{2}$ (M1) $x^{\frac{1}{3}}=3 \Rightarrow x = 27$ (A1) CSO
(b)(i)	$\log_4 1=0$	B1		
(ii)	$\log_4 4=1$	B1		SC in (b): For all four answers ¼; 1; ½ ; 2
(iii)	$\log_4 2=0.5$	B1		give 0/4; otherwise mark each
(iv)	$\log_4 8=1.5$	B1	4	independently.
	Total		**7**	

MPC2 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$(2+x)^3 =$			Any valid method; must contain all
	$(2^3)+3(2^2)(x)+3(2)(x^2)+(x^3)$	M1		components
	… = 8 + 12x + 6x2 + x3 (*)	A1		Accept a = 12
		A1	3	Accept b = 6
(ii)	$(2-x)^3 = 8-12x+6x^2-x^3$ (**)	M1 A1√	2	Clear $x \rightarrow -x$ in (i) OE ft numerical a and b
(b)	$(2+x)^3-(2-x)^3 = (*)-(**)$ …… = $24x + 2x^3$.	M1 A1	2	Subtracts the 2 expressions in (a) CSO AG (be convinced)
(c)	$\frac{dy}{dx} = 24+6x^2$	M1		A power of x decreased by 1
	For st. pt. $24+6x^2=0$	A1		
	Not possible since $24+6x^2>0$	E1	3	Any valid explanation
	Total		**10**	
7(a)	$A(0^\circ, 1)$	B1		Condone radians
	$B(45^\circ, 0)$	B1		Condone (0.785,0) or better.
	$C(270^\circ, -1)$	B1, B1	4	B1 for 270; B1 for –1
(b)	Stretch **(I)** in x-direction **(II)** with a scale factor $\frac{1}{2}$ **(III)**	M1A1	2	More than one transformation is M0 M1 for (I) **and** either (II) or (III)
(c)	$\cos^{-1}0.37$ = "68.284..." $(=\alpha)$	M1		$\text{Cos}^{-1}0.37$ (PI eg by 68.3 or 1.19)
	$x=\frac{\alpha}{2} = 34.1(42.)^\circ$	A1		Condone 34.2°, 34° or 0.596 rads
	$x = 180-\frac{\alpha}{2}$	m1		OE eg $2x = 360-\alpha$
	$x = 180+\frac{\alpha}{2}$ **and** $x = 180+180-\frac{\alpha}{2}$	m1		OE Need both (OE for $2x =$) with no extras (quadrants) within the given interval
	$2x = 68.284\ldots; 291.715\ldots; 428.284\ldots; 651.715\ldots$			
	$x = (34.1^\circ;)$ $145.9^\circ;\ 214.1^\circ;\ 325.9^\circ$	A1	5	Dep. on **all** three method marks. Must be in degrees
	Total		**11**	

MPC2 (cont)

Q	Solution	Marks	Total	Comments
8(a)	{y-coordinate of A is} 2	B1	1	
(b)(i)	$h = 0.25$	B1		
	Integral $= \frac{h}{2}$ {…}			
	{…} = $\mathrm{f}(0) + 2[\mathrm{f}\left(\frac{1}{4}\right) + \mathrm{f}\left(\frac{1}{2}\right) + \mathrm{f}\left(\frac{3}{4}\right)] + \mathrm{f}(1)$	M1		
	{…}= 2 + 4 + 2[(2.316..+ 2.732..+ 3.279(5.)] {= 6 + 2 × 8.3276..} {= 22.65(5…)}	A1√		Condone one numerical slip Accept values to 3sf (rnd or trunc) ft answer from (a) if not "2"
	Integral = 0.125 × 22.655.. = 2.8319.. Integral = 2.83 to 3 sf	A1	4	CAO Must be 2.83 (NMS scores 0/4)
(ii)	Relevant trapezia drawn on a copy of given graph	M1		Accept relevant single trapezium with its sloping side above the curve
	{Approximation is an}overestimate	A1	2	
(c)	$5 = 3^x + 1 \Rightarrow 3^x = 4$	B1		
	$\log_{10} 3^x = \log_{10} 4$	M1		Takes ln or $\log_{10}$ on both sides of $3^x = k$, where $k > 0$
	$x\log_{10} 3 = \log_{10} 4$	m1		Use of $\log 3^x = x\log 3$
	$x = \frac{\lg 4}{\lg 3} = 1.26185... = 1.2619$ to 4dp	A1	4	Accept 4dp or better [If using T&I a full justification is required; else M0m0A0]
(d)	$\mathrm{f}(x) = 3^{-x} + 1$	B1	1	
	Total		**12**	
	TOTAL		**75**	