

C1 SEQUENCES AND SERIES

Answers - Worksheet A

- 1 **a** 9, 13, 17, 21, 25 **b** 4, 9, 16, 25, 36 **c** 2, 4, 8, 16, 32 **d** $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$
 e -1, 4, 21, 56, 115 **f** $\frac{2}{3}, \frac{1}{3}, 0, -\frac{1}{3}, -\frac{2}{3}$ **g** $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \frac{9}{10}$ **h** 16, 8, 4, 2, 1
- 2 **a** $u_n = 3n + 1$
 $a = 3, b = 1$ **b** $u_n = 7n - 7$
 $a = 7, b = -7$ **c** $u_n = 18 - 2n$
 $a = -2, b = 18$
 d $u_n = 1.3n - 0.9$ **e** $u_n = 117 - 17n$ **f** $u_n = 8n - 21$
 $a = 1.3, b = -0.9$ $a = -17, b = 117$ $a = 8, b = -21$
- 3 possible answers are
 a $5n - 4$ **b** 3^n **c** $2n^2$
 d $\frac{1}{4} \times 2^n$ **e** $33 - 11n$ **f** $(n - 1)^3$
 g $n^2 + 3$ **h** $\frac{n}{2n+1}$ **i** $2^n - 1$
- 4 **a** $u_3 = c + 3 = 11 \therefore c = 8$
 b $u_6 = 8 + 3^4 = 89$
- 5 **a** $u_4 = 4(8 + k) = 32 + 4k$
 $u_6 = 6(12 + k) = 72 + 6k$
 $\therefore 72 + 6k = 2(32 + 4k) - 2$
 $72 + 6k = 62 + 8k$
 $k = 5$
 b $u_n = n(2n + 5) = 2n^2 + 5n$
 $u_{n-1} = (n - 1)[2(n - 1) + 5] = (n - 1)(2n + 3) = 2n^2 + n - 3$
 $\therefore u_n - u_{n-1} = (2n^2 + 5n) - (2n^2 + n - 3) = 4n + 3$
- 6 **a** $u_1 = k - 3$
 $u_2 = k^2 - 3$
 $\therefore k - 3 + k^2 - 3 = 0$
 $k^2 + k - 6 = 0$
 $(k + 3)(k - 2) = 0$
 $k = -3$ or 2
 b $k = -3 \Rightarrow u_5 = (-3)^5 - 3 = -243 - 3 = -246$
 $k = 2 \Rightarrow u_5 = 2^5 - 3 = 32 - 3 = 29$
- 7 **a** 3, 7, 11, 15 **b** 2, 7, 22, 67
 c -2, 1, 7, 19 **d** 5, 2, 5, 2
 e -1, 14, -46, 194 **f** 10, 3, 2.3, 2.23
 g 6, -1, $1\frac{1}{3}, \frac{5}{9}$ **h** $0, \frac{1}{2}, \frac{2}{5}, \frac{5}{12}$
- 8 possible answers are
 a $u_{n+1} = u_n + 4, u_1 = 5$ **b** $u_{n+1} = 3u_n, u_1 = 1$ **c** $u_{n+1} = u_n - 18, u_1 = 62$
 d $u_{n+1} = \frac{1}{2}u_n, u_1 = 120$ **e** $u_{n+1} = 2u_n + 1, u_1 = 4$ **f** $u_{n+1} = 4u_n - 1, u_1 = 1$

- 9** **a** $-3 = -4a + b$
 $-1 = -3a + b$
subtracting, $2 = a$
 $a = 2, b = 5$
- b** $8 = b$
 $4 = 8a + b$
 $a = -\frac{1}{2}, b = 8$
- c** $4 = \frac{11}{2}a + b$
 $3 = 4a + b$
subtracting, $1 = \frac{3}{2}a$
 $a = \frac{2}{3}, b = \frac{1}{3}$
- 10** **a** $u_2 = 4 + 3k$
 $u_3 = 4(4 + 3k) + 3k = 16 + 15k$
- b** $u_2 = 2k + 5$
 $u_3 = k(2k + 5) + 5 = 2k^2 + 5k + 5$
- c** $u_2 = 4k - k = 3k$
 $u_3 = 4(3k) - k = 11k$
- d** $u_2 = 2 + k$
 $u_3 = 2 - k(2 + k) = 2 - 2k - k^2$
- e** $u_2 = \frac{4}{k}$
 $u_3 = \frac{4}{k} \div k = \frac{4}{k^2}$
- f** $u_2 = \sqrt[3]{61k^3 + 3k^3} = \sqrt[3]{64k^3} = 4k$
 $u_3 = \sqrt[3]{61k^3 + 64k^3} = \sqrt[3]{125k^3} = 5k$
- 11** **a** $u_2 = \frac{1}{2}(k + 6)$
 $u_3 = \frac{1}{2}[k + \frac{3}{2}(k + 6)] = \frac{1}{4}(5k + 18)$
- b** $\frac{1}{4}(5k + 18) = 7$
 $k = 2$
 $u_4 = \frac{1}{2}(2 + 21) = 11\frac{1}{2}$
- 12** **a** $u_4 = 30 - 2 = 28$
 $10 = 3u_2 - 2 \therefore u_2 = 4$
 $4 = 3u_1 - 2 \therefore u_1 = 2$
- b** $u_4 = \frac{15}{4} + 2 = 5\frac{3}{4}$
 $5 = \frac{3}{4}u_2 + 2 \therefore u_2 = 4$
 $4 = \frac{3}{4}u_1 + 2 \therefore u_1 = 2\frac{2}{3}$
- c** $u_4 = 0.2 \times 1.2 = 0.24$
 $-0.2 = 0.2(1 - u_2) \therefore u_2 = 2$
 $2 = 0.2(1 - u_1) \therefore u_1 = -9$
- d** $u_4 = \frac{1}{2}$
 $1 = \frac{1}{2}\sqrt{u_2} \therefore u_2 = 4$
 $4 = \frac{1}{2}\sqrt{u_1} \therefore u_1 = 64$
- 13** **a** $u_5 = 2 + 4c = 30 \therefore c = 7$
b sequence is 2, 9, 16, 23, 30, ...
 $\therefore u_n = 7n - 5$
- 14** **a** $u_2 = 3(-4 - k) = -12 - 3k$
 $u_3 = 3[(-12 - 3k) - k] = -36 - 12k$
- b** $-36 - 12k = 7(-12 - 3k) + 3$
 $9k = -45$
 $k = -5$
- c** $u_3 = -36 + 60 = 24$
 $\therefore u_4 = 3(24 + 5) = 87$
- 15** **a** $t_2 = 1.5k + 2$
 $t_3 = k(1.5k + 2) + 2 = 1.5k^2 + 2k + 2$
- b** $1.5k^2 + 2k + 2 = 12$
 $3k^2 + 4k - 20 = 0$
 $(3k + 10)(k - 2) = 0$
 $k = -3\frac{1}{3}, 2$

C1 SEQUENCES AND SERIES

Answers - Worksheet B

- 1** **a** $d = 6$
 $u_{40} = 4 + (39 \times 6) = 238$ **b** $d = -3$
 $u_{40} = 30 + (39 \times -3) = -87$ **c** $d = 2.3$
 $u_{40} = 8.9 + (39 \times 2.3) = 98.6$
- 2** **a** $a = 7, d = 2$
 $u_n = 7 + 2(n - 1) = 5 + 2n$ **b** $a = \frac{1}{6}, d = \frac{4}{3}$
 $u_n = \frac{1}{6} + \frac{4}{3}(n - 1) = -\frac{7}{6} + \frac{4}{3}n$ **c** $a = 17, d = -8$
 $u_n = 17 - 8(n - 1) = 25 - 8n$
- 3** **a** $a = 8, d = 4, n = 30$
 $S_{30} = \frac{30}{2} [16 + (29 \times 4)]$
 $= 1980$ **b** $a = 60, d = -7, n = 30$
 $S_{30} = \frac{30}{2} [120 + (29 \times -7)]$
 $= -1245$ **c** $a = 7\frac{1}{4}, d = 1\frac{1}{2}, n = 30$
 $S_{30} = \frac{30}{2} [14\frac{1}{2} + (29 \times 1\frac{1}{2})]$
 $= 870$
- 4** **a** $S_{20} = \frac{20}{2} (60 + 136)$
 $= 1960$ **b** $S_{32} = \frac{32}{2} (100 + 84.5)$
 $= 2952$ **c** $S_{17} = \frac{17}{2} [28 + (-20)]$
 $= 68$
- 5** **a** $S_{48} = \frac{48}{2} [4 + (47 \times 9)]$
 $= 10\,248$ **b** $S_{36} = \frac{36}{2} [200 + (35 \times -5)]$
 $= 450$ **c** $S_{55} = \frac{55}{2} [38 + (54 \times 13)]$
 $= 20\,350$
- 6** **a** $8 + 3(n - 1) = 65$
 $n = 20$
 $S_{20} = \frac{20}{2} (8 + 65)$
 $= 730$ **b** $3.4 + 1.2(n - 1) = 23.8$
 $n = 18$
 $S_{18} = \frac{18}{2} (3.4 + 23.8)$
 $= 244.8$ **c** $22 - 8(n - 1) = -226$
 $n = 32$
 $S_{32} = \frac{32}{2} [22 + (-226)]$
 $= -3264$
- 7** **a** $a = 21$
 $21 + 2d = 27$
 $\therefore d = 3$
b $S_{40} = \frac{40}{2} [42 + (39 \times 3)] = 3180$
- 8** $n = 1$, first term $= 7 + 16 = 23$
 $d = 7$
 $S_{35} = \frac{35}{2} [46 + (34 \times 7)] = 4970$
- 9** **a** $a + d = 13$
 $a + 4d = 46$
b subtracting, $3d = 33$
 $d = 11$
sub. $a = 2$
c $u_{40} = 2 + (39 \times 11) = 431$
- 10** **a** $a + 2d = 72$
 $a + 7d = 37$
subtracting, $5d = -35$
 $d = -7$
sub. $a = 86$
b $S_{25} = \frac{25}{2} [172 + (24 \times -7)] = 50$
- 11** **a** $a + 4d = 23$ (1)
 $\frac{10}{2} (2a + 9d) = 240 \Rightarrow 2a + 9d = 48$
 $2 \times (1) \Rightarrow 2a + 8d = 46$
subtracting, $d = 2$
sub. $a = 15$
b $S_{60} = \frac{60}{2} [30 + (59 \times 2)] = 4440$
- 12** **a** $S_n = 1 + 2 + 3 + \dots + (n - 1) + n$
write in reverse
 $S_n = n + (n - 1) + \dots + 3 + 2 + 1$
adding, $2S_n = n \times (n + 1)$
 $S_n = \frac{1}{2} n(n + 1)$
b $= S_{100} - S_{29}$
 $= \frac{1}{2} \times 100 \times 101 - \frac{1}{2} \times 29 \times 30$
 $= 5050 - 435 = 4615$

- 13** a $5 + 7 + 9 + 11 + 13$
 b $15 + 12 + 9 + 6 + 3 + 0 - 3 - 6 - 9$
 c $15 + 19 + 23 + 27 + 31 + 35 + 39$
 d $4\frac{1}{2} + 4 + 3\frac{1}{2} + 3 + 2\frac{1}{2} + 2 + 1\frac{1}{2} + 1$
- 14** a AP: $a = 4$,
 $l = 61, n = 20$
 $S_{20} = \frac{20}{2}(4 + 61)$
 $= 650$
 b AP: $a = 88$,
 $l = 0, n = 45$
 $S_{45} = \frac{45}{2}(88 + 0)$
 $= 1980$
 c AP: $a = 19$,
 $l = 127, n = 28$
 $S_{28} = \frac{28}{2}(19 + 127)$
 $= 2044$
 d AP: $a = 3$,
 $l = 13, n = 41$
 $S_{41} = \frac{41}{2}(3 + 13)$
 $= 328$
- 15** AP: $a = -2, l = 4n - 6$
 $S_n = \frac{n}{2}[-2 + (4n - 6)] = 720$
 $\therefore n(4n - 8) = 1440$
 $n^2 - 2n - 360 = 0$
 $(n + 18)(n - 20) = 0$
 $n > 0 \therefore n = 20$
- 16** a AP: $a = 2, l = 160, n = 80$
 $S_{80} = \frac{80}{2}(2 + 160) = 6480$
 b AP: $a = 3, l = 198, n = 66$
 $S_{66} = \frac{66}{2}(3 + 198) = 6633$
 c AP: $a = 30, l = 300, d = 6$
 $30 + 6(n - 1) = 300 \therefore n = 46$
 $S_{46} = \frac{46}{2}(30 + 300) = 7590$
- 17** a $a + (9 \times -11) = 101$
 $a = 200$
 b $S_{30} = \frac{30}{2}[400 + (29 \times -11)] = 1215$
- 18** a $a = 17, 17 + 4d = 27 \therefore d = 2.5$
 b $17 + 2.5(r - 1) = 132$
 $r = 47$
 c $S_{47} = \frac{47}{2}(17 + 132) = 3501.5$
- 19** a $\frac{6}{2}(2a + 5d) = 213 \Rightarrow 2a + 5d = 71$
 $\frac{10}{2}(2a + 9d) = 295 \Rightarrow 2a + 9d = 59$
 subtracting, $4d = -12$
 $d = -3$
 sub. $a = 43$
 b $43 - 3(n - 1) > 0$
 $n < \frac{46}{3} \therefore 15$ positive terms
 c max S_n when $n = 15$
 $S_{15} = \frac{15}{2}[86 + (14 \times -3)] = 330$
- 20** a $S_8 = (2 \times 8^2) + (5 \times 8) = 168$
 b $S_7 = (2 \times 7^2) + (5 \times 7) = 133$
 $u_8 = S_8 - S_7 = 35$
 c $S_{n-1} = 2(n-1)^2 + 5(n-1)$
 $= 2n^2 + n - 3$
 $u_n = S_n - S_{n-1}$
 $= (2n^2 + 5n) - (2n^2 + n - 3)$
 $= 4n + 3$
- 21** a $(2k + 3) - (k + 2) = (4k - 2) - (2k + 3)$
 $k + 1 = 2k - 5$
 $k = 6$
 b $a = 8, a + d = 15 \therefore d = 7$
 $S_{25} = \frac{25}{2}[16 + (24 \times 7)] = 2300$
- 22** a $2t - (5 - t) = (6t - 3) - 2t$
 $3t - 5 = 4t - 3$
 $t = -2$
 b $u_5 = 7, u_6 = -4 \therefore d = -11$
 $a + (4 \times -11) = 7 \therefore a = 51$
 $S_{18} = \frac{18}{2}[102 + (17 \times -11)] = -765$

C1 SEQUENCES AND SERIES

Answers - Worksheet C

- 1 a** $a + 2d = -10$ (1)
 $\frac{8}{2}(2a + 7d) = 16 \Rightarrow 2a + 7d = 4$
 $2 \times (1) \Rightarrow 2a + 4d = -20$
 subtracting, $3d = 24$
 $d = 8$
 sub. $a = -26$
- b** $-26 + 8(n - 1) > 300$
 $n > 41\frac{3}{4} \therefore$ smallest $n = 42$
- 3 a** $\frac{9}{2}(2a + 8d) = 126$
 $9(a + 4d) = 126$
 $a + 4d = 14$
- b** $\frac{15}{2}(2a + 14d) = 277.5$
 $a + 7d = 18.5$
 subtracting, $3d = 4.5$
 $d = 1.5$
 sub. $a = 8$
- c** $S_{32} = \frac{32}{2} [16 + (31 \times 1.5)] = 1000$
- 5 a** AP: $a = 4, l = 120, n = 30$
 $S_{30} = \frac{30}{2} (4 + 120) = 1860$
- b i** $= \sum_{r=1}^{30} 4r + 30 = 1890$
- ii** $= 2 \times \sum_{r=1}^{30} 4r - (30 \times 5)$
 $= (2 \times 1860) - 150 = 3570$
- 7 a** $S_n = 2 + 4 + 6 + \dots + (2n - 2) + 2n$
 write in reverse
 $S_n = 2n + (2n - 2) + \dots + 6 + 4 + 2$
 adding, $2S_n = n \times (2n + 2)$
 $S_n = n(n + 1)$
- b** integers 200 to 800, AP: $n = 601$
 $S_{601} = \frac{601}{2} (200 + 800) = 300\,500$
 integers 200 to 800 divisible by 4
 AP: $a = 200, l = 800$
 $200 + 4(n - 1) = 800 \Rightarrow n = 151$
 $S_{151} = \frac{151}{2} (200 + 800) = 75\,500$
 required sum = $300\,500 - 75\,500$
 $= 225\,000$
- 2 a** $a + 2d = \frac{5}{6}$
 $a + 6d = 2\frac{1}{3}$
 subtracting, $4d = 1\frac{1}{2}$
 $d = \frac{3}{8}$
 sub. $a = \frac{1}{12}$
- b** $S_n = \frac{n}{2} [\frac{1}{6} + \frac{3}{8}(n - 1)]$
 $= \frac{1}{48} n[4 + 9(n - 1)]$
 $= \frac{1}{48} n(9n - 5) \quad [k = \frac{1}{48}]$
- 4 a** $(5k + 3) - (7k - 1) = (4k + 1) - (5k + 3)$
 $-2k + 4 = -k - 2$
 $k = 6$
- b** given terms = 41, 33, 25
 $d = -8$
 smallest +ve term = $25 + (3 \times -8) = 1$
- c** consider series of +ve terms in reverse
 $a = 1, d = 8$
 $S_r = \frac{r}{2} [2 + 8(r - 1)] = r(4r - 3)$
- 6 a** $500 + (7 \times 40) = \text{£}780$
- b** AP: $a = 500, d = 40$
 $S_n = \frac{n}{2} [1000 + 40(n - 1)] = 20n(n + 24)$
- c** AP: $a = 400, d = 60$
 $S_n = \frac{n}{2} [800 + 60(n - 1)] = 10n(3n + 37)$
 $\therefore 20n(n + 24) = 10n(3n + 37)$
 $n \neq 0 \therefore 2(n + 24) = (3n + 37)$
 $n = 11 \therefore 11$ years
- 8 a** $S_n = \frac{1}{2} n[2a + (n - 1)d]$
- b** $S_2 = \frac{2}{2} (2a + d) = 2a + d$
 $S_6 = \frac{6}{2} (2a + 5d) = 6a + 15d$
 $S_8 = \frac{8}{2} (2a + 7d) = 8a + 28d$
 $2(S_6 - S_2) = 2[(6a + 15d) - (2a + d)]$
 $= 2(4a + 14d)$
 $= 8a + 28d = S_8$
- c** for +ve terms $40 - 3(n - 1) > 0$
 $n < \frac{43}{3} \therefore 14$ terms
 $\therefore S_{14} = \frac{14}{2} [80 + (13 \times -3)] = 287$

9 a i $u_4 - u_1 = x + 3$

$$u_7 = u_4 + (x + 3) = 3x + 6$$

ii $3d = x + 3$

$$d = \frac{1}{3}x + 1$$

iii $S_{10} = \frac{10}{2} [2x + 9(\frac{1}{3}x + 1)]$

$$= 5[2x + 3x + 9] = 25x + 45$$

b $x + 19(\frac{1}{3}x + 1) = 52$

$$3x + 19x + 57 = 156$$

$$x = \frac{99}{22} = \frac{9}{2} \text{ or } 4\frac{1}{2}$$

10 $S_{20} = \frac{20}{2} (2a + 19d) = 20a + 190d$

$$S_{30} = \frac{30}{2} (2a + 29d) = 30a + 435d$$

$$S_{30} - S_{20} = 10a + 245d$$

$$\therefore 20a + 190d = 10a + 245d$$

$$10a = 55d$$

$$2a = 11d$$

$$\therefore a : d = 11 : 2$$

11 a $S_6 = 12(16 - 6) = 120$

$$S_5 = 10(16 - 5) = 110$$

$$u_6 = S_6 - S_5 = 10$$

b $S_n = 2n(16 - n) = 32n - 2n^2$

$$S_{n-1} = 2(n-1)[16 - (n-1)]$$

$$= 2(n-1)(17 - n)$$

$$= -2n^2 + 36n - 34$$

$$u_n = S_n - S_{n-1}$$

$$= (32n - 2n^2) - (-2n^2 + 36n - 34)$$

$$= 34 - 4n$$

c $u_{n-1} = 34 - 4(n-1) = 38 - 4n$

$$u_n - u_{n-1} = (34 - 4n) - (38 - 4n) = -4$$

$$u_n - u_{n-1} \text{ constant } \therefore \text{ arithmetic series}$$

12 a i $2400 + (5 \times 250) = 3650$

ii AP: $a = 2400, d = 250$

$$S_{10} = \frac{10}{2} [4800 + (9 \times 250)]$$

$$= 35\,250$$

b AP: $a = 2400, d = C$

$$\frac{10}{2} [4800 + (9 \times C)] = 40\,000$$

$$C = \frac{3200}{9} = 356 \text{ (nearest unit)}$$

13 a let common difference be d

$$S_r = a + (a + d) + (a + 2d) + \dots + (l - 2d) + (l - d) + l$$

write in reverse

$$S_r = l + (l - d) + (l - 2d) + \dots + (a + 2d) + (a + d) + a$$

adding, $2S_r = r \times (a + l)$

$$S_r = \frac{1}{2} r(a + l)$$

b $n = 18, l = 68, S_{18} = 153$

$$\therefore 153 = \frac{18}{2} (a + 68)$$

$$a = 17 - 68 = -51$$

C1 SEQUENCES AND SERIES

Answers - Worksheet D

- 1** **a** $a + d = 40$, $a + 4d = 121$
 subtracting, $3d = 81$
 $d = 27$
 sub. $a = 13$
b $S_{25} = \frac{25}{2} [26 + (24 \times 27)] = 8425$
- 2** **a** 3, 7, 11, 15, 19
b AP: $a = 3$, $d = 4$, $n = 20$
 $S_{20} = \frac{20}{2} [6 + (19 \times 4)]$
 $= 820$
- 3** **a** $(2t - 5) - t = 8.6 - (2t - 5)$
 $t = 6.2$
b $u_1 = 6.2$, $u_2 = 12.4 - 5 = 7.4$
 $a = 6.2$, $d = 7.4 - 6.2 = 1.2$
 $u_{16} = 6.2 + (15 \times 1.2) = 24.2$
c $S_{20} = \frac{20}{2} [12.4 + (19 \times 1.2)] = 352$
- 4** **a** $S_n = \frac{1}{2}n(n + 1)$
b $= S_{400} - S_{199}$
 $= \frac{1}{2} \times 400 \times 401 - \frac{1}{2} \times 199 \times 200$
 $= 80\,200 - 19\,900 = 60\,300$
c $\frac{1}{2}N(N + 1) = 4950$
 $N^2 + N - 9900 = 0$
 $(N + 100)(N - 99) = 0$
 $N > 0 \therefore N = 99$
- 5** **a** $u_2 = k + 1$
 $u_3 = k + (k + 1)^2 = k^2 + 3k + 1$
b $k^2 + 3k + 1 = 1$
 $k(k + 3) = 0$
 $k \neq 0 \therefore k = -3$
c $u_{25} = 1$
 $u_1 = 1 \Rightarrow u_3 = 1$
 $\therefore u_n = 1$ for all odd values of n
- 6** **a** AP: $a = 3$, $d = 3$
 $500 \div 3 = 166\frac{2}{3} \therefore n = 166$
 $S_{166} = \frac{166}{2} [6 + (165 \times 3)]$
 $= 41\,583$
b AP: $a = 14$, $l = 99$, $n = 18$
 $S_{18} = \frac{18}{2} (14 + 99)$
 $= 1017$
- 7** **a** $S_n = a + (a + d) + (a + 2d) + \dots$
 $+ [a + (n - 2)d] + [a + (n - 1)d]$
 write in reverse
 $S_n = [a + (n - 1)d] + [a + (n - 2)d] + \dots$
 $+ (a + 2d) + (a + d) + a$
 adding $2S_n = n \times \{a + [a + (n - 1)d]\}$
 $S_n = \frac{1}{2}n[2a + (n - 1)d]$
b $S_{26} = \frac{26}{2} [-2 + (25 \times 6)] = 1924$
 $S_{27} = \frac{27}{2} [-2 + (26 \times 6)] = 2079$
 \therefore largest $n = 26$
- 8** $t_2 = 4 + 2k$
 $t_3 = 4 - k(4 + 2k)$
 $\therefore 4 - 4k - 2k^2 = 3$
 $2k^2 + 4k - 1 = 0$
 $k = \frac{-4 \pm \sqrt{16 + 8}}{4} = \frac{-4 \pm 2\sqrt{6}}{4}$
 $k > 0 \therefore k = -1 + \frac{1}{2}\sqrt{6}$
- 9** **a** $= 6 + (19 \times 3) = 63$
b $S_n = \frac{n}{2} [12 + 3(n - 1)] = 270$
 $\therefore n(3n + 9) = 540$
 $n^2 + 3n - 180 = 0$
 $(n + 15)(n - 12) = 0$
 $n > 0 \therefore n = 12$
- 10** **a** $= 3 \times 570 = 1710$
b $= 570 + (2 \times 30) = 630$
c $= 570 + (\frac{1}{2} \times 30 \times 31) = 1035$

- 11 a** 2 years = 8×3 months
total = $3 \times S_8$ [AP: $a = 40, d = 2$]
 $= 3 \times \frac{8}{2} [80 + (7 \times 2)]$
 $= 3 \times 376 = \text{£}1128$
- b** n years = $4n \times 3$ months
total = $3 \times S_{4n}$
 $= 3 \times \frac{4n}{2} \{80 + [(4n - 1) \times 2]\}$
 $= 6n(80 + 8n - 2)$
 $= 12n(4n + 39)$
- 12** AP: $a = 80, d = -3, n = 45$
 $S_{45} = \frac{45}{2} [160 + (44 \times -3)] = 630$
- 13 a** $a + 2d = 298, a + 7d = 263$
subtracting, $5d = -35$
 $d = -7$
- b** sub. $a = 312$
 $312 - 7(n - 1) > 0$
 $n < \frac{319}{7} \therefore 45$ positive terms
- c** max S_n when $n = 45$
 $S_{45} = \frac{45}{2} [624 + (44 \times -7)] = 7110$
- 14 a** AP: $a = 10, d = 6$
 $S_n = \frac{n}{2} [20 + 6(n - 1)]$
 $= n(3n + 7)$
- b** $S_{2n} = 2n[(3 \times 2n) + 7]$
 $= 12n^2 + 14n$
required sum = $S_{2n} - S_n$
 $= (12n^2 + 14n) - (3n^2 + 7n)$
 $= 9n^2 + 7n = n(9n + 7)$
- 15 a** $u_2 = k^2 - 2, u_4 = k^4 - 4$
 $\therefore k^2 - 2 + k^4 - 4 = 6$
 $k^4 + k^2 - 12 = 0$
 $(k^2 + 4)(k^2 - 3) = 0$
 $k^2 = -4$ [no solutions] or 3
 $k > 0 \therefore k = \sqrt{3}$
- b** $u_1 = \sqrt{3} - 1$
 $u_3 = (\sqrt{3})^3 - 3 = 3(\sqrt{3} - 1) = 3u_1$
- 16 a** $(4k - 2) - (k + 4) = (k^2 - 2) - (4k - 2)$
 $3k - 6 = k^2 - 4k$
 $k^2 - 7k + 6 = 0$
- b** $(k - 1)(k - 6) = 0$
 $k = 1$ or 6
 $d = 3k - 6$
 $d > 0 \therefore k = 6$
 $a = 10, d = 12$
 $u_{15} = 10 + (14 \times 12) = 178$