

# C1 ALGEBRA

## Worksheet A

1 Evaluate

**a**  $\sqrt{49}$       **b**  $\sqrt{121}$       **c**  $\sqrt{\frac{1}{9}}$       **d**  $\sqrt{\frac{4}{25}}$       **e**  $\sqrt{0.01}$       **f**  $\sqrt{0.09}$   
**g**  $\sqrt[3]{8}$       **h**  $\sqrt[3]{1000}$       **i**  $\sqrt[4]{81}$       **j**  $\sqrt[4]{\frac{9}{16}}$       **k**  $\sqrt[3]{0.125}$       **l**  $\sqrt[3]{15\frac{5}{8}}$

2 Simplify

**a**  $\sqrt{7} \times \sqrt{7}$       **b**  $4\sqrt{5} \times \sqrt{5}$       **c**  $(3\sqrt{3})^2$       **d**  $(\sqrt{6})^4$   
**e**  $(\sqrt{2})^5$       **f**  $(2\sqrt{3})^3$       **g**  $\sqrt{2} \times \sqrt{8}$       **h**  $2\sqrt{3} \times \sqrt{27}$   
**i**  $\frac{\sqrt{32}}{\sqrt{2}}$       **j**  $\frac{\sqrt{3}}{\sqrt{12}}$       **k**  $(\sqrt[3]{6})^3$       **l**  $(3\sqrt[3]{2})^3$

3 Express in the form  $k\sqrt{2}$

**a**  $\sqrt{18}$       **b**  $\sqrt{50}$       **c**  $\sqrt{8}$       **d**  $\sqrt{98}$       **e**  $\sqrt{200}$       **f**  $\sqrt{162}$

4 Simplify

**a**  $\sqrt{12}$       **b**  $\sqrt{28}$       **c**  $\sqrt{80}$       **d**  $\sqrt{27}$       **e**  $\sqrt{24}$       **f**  $\sqrt{128}$   
**g**  $\sqrt{45}$       **h**  $\sqrt{40}$       **i**  $\sqrt{75}$       **j**  $\sqrt{112}$       **k**  $\sqrt{99}$       **l**  $\sqrt{147}$   
**m**  $\sqrt{216}$       **n**  $\sqrt{800}$       **o**  $\sqrt{180}$       **p**  $\sqrt{60}$       **q**  $\sqrt{363}$       **r**  $\sqrt{208}$

5 Simplify

**a**  $\sqrt{18} + \sqrt{50}$       **b**  $\sqrt{48} - \sqrt{27}$       **c**  $2\sqrt{8} + \sqrt{72}$   
**d**  $\sqrt{360} - 2\sqrt{40}$       **e**  $2\sqrt{5} - \sqrt{45} + 3\sqrt{20}$       **f**  $\sqrt{24} + \sqrt{150} - 2\sqrt{96}$

6 Express in the form  $a + b\sqrt{3}$

**a**  $\sqrt{3}(2 + \sqrt{3})$       **b**  $4 - \sqrt{3} - 2(1 - \sqrt{3})$       **c**  $(1 + \sqrt{3})(2 + \sqrt{3})$   
**d**  $(4 + \sqrt{3})(1 + 2\sqrt{3})$       **e**  $(3\sqrt{3} - 4)^2$       **f**  $(3\sqrt{3} + 1)(2 - 5\sqrt{3})$

7 Simplify

**a**  $(\sqrt{5} + 1)(2\sqrt{5} + 3)$       **b**  $(1 - \sqrt{2})(4\sqrt{2} - 3)$       **c**  $(2\sqrt{7} + 3)^2$   
**d**  $(3\sqrt{2} - 1)(2\sqrt{2} + 5)$       **e**  $(\sqrt{5} - \sqrt{2})(\sqrt{5} + 2\sqrt{2})$       **f**  $(3 - \sqrt{8})(4 + \sqrt{2})$

8 Express each of the following as simply as possible with a rational denominator.

**a**  $\frac{1}{\sqrt{5}}$       **b**  $\frac{2}{\sqrt{3}}$       **c**  $\frac{1}{\sqrt{8}}$       **d**  $\frac{14}{\sqrt{7}}$       **e**  $\frac{3\sqrt{2}}{\sqrt{3}}$       **f**  $\frac{\sqrt{5}}{\sqrt{15}}$   
**g**  $\frac{1}{3\sqrt{7}}$       **h**  $\frac{12}{\sqrt{72}}$       **i**  $\frac{1}{\sqrt{80}}$       **j**  $\frac{3}{2\sqrt{54}}$       **k**  $\frac{4\sqrt{20}}{3\sqrt{18}}$       **l**  $\frac{3\sqrt{175}}{2\sqrt{27}}$

## C1 ALGEBRA

## Worksheet A continued

9 Simplify

a  $\sqrt{8} + \frac{6}{\sqrt{2}}$

b  $\sqrt{48} - \frac{10}{\sqrt{3}}$

c  $\frac{6-\sqrt{8}}{\sqrt{2}}$

d  $\frac{\sqrt{45}-5}{\sqrt{20}}$

e  $\frac{1}{\sqrt{18}} + \frac{1}{\sqrt{32}}$

f  $\frac{2}{\sqrt{3}} - \frac{\sqrt{6}}{\sqrt{72}}$

10 Solve each equation, giving your answers as simply as possible in terms of surds.

a  $x(x+4) = 4(x+8)$

b  $x - \sqrt{48} = 2\sqrt{3} - 2x$

c  $x\sqrt{18} - 4 = \sqrt{8}$

d  $x\sqrt{5} + 2 = \sqrt{20}(x-1)$

11 a Simplify  $(2 - \sqrt{3})(2 + \sqrt{3})$ .b Express  $\frac{2}{2-\sqrt{3}}$  in the form  $a + b\sqrt{3}$ .

12 Express each of the following as simply as possible with a rational denominator.

a  $\frac{1}{\sqrt{2}+1}$

b  $\frac{4}{\sqrt{3}-1}$

c  $\frac{1}{\sqrt{6}-2}$

d  $\frac{3}{2+\sqrt{3}}$

e  $\frac{1}{2+\sqrt{5}}$

f  $\frac{\sqrt{2}}{\sqrt{2}-1}$

g  $\frac{6}{\sqrt{7}+3}$

h  $\frac{1}{3+2\sqrt{2}}$

i  $\frac{1}{4-2\sqrt{3}}$

j  $\frac{3}{3\sqrt{2}+4}$

k  $\frac{2\sqrt{3}}{7-4\sqrt{3}}$

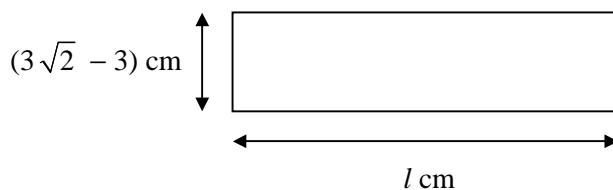
l  $\frac{6}{\sqrt{5}-\sqrt{3}}$

13 Solve the equation

$$3x = \sqrt{5}(x+2),$$

giving your answer in the form  $a + b\sqrt{5}$ , where  $a$  and  $b$  are rational.

14

The diagram shows a rectangle measuring  $(3\sqrt{2} - 3) \text{ cm}$  by  $l \text{ cm}$ .Given that the area of the rectangle is  $6 \text{ cm}^2$ , find the exact value of  $l$  in its simplest form.

15 Express each of the following as simply as possible with a rational denominator.

a  $\frac{\sqrt{2}}{\sqrt{2}+\sqrt{6}}$

b  $\frac{1+\sqrt{3}}{2+\sqrt{3}}$

c  $\frac{1+\sqrt{10}}{\sqrt{10}-3}$

d  $\frac{3-\sqrt{2}}{4+3\sqrt{2}}$

e  $\frac{1-\sqrt{2}}{3-\sqrt{8}}$

f  $\frac{\sqrt{3}-5}{2\sqrt{3}-4}$

g  $\frac{\sqrt{12}+3}{3-\sqrt{3}}$

h  $\frac{3\sqrt{7}-2}{2\sqrt{7}-5}$

**C1 ALGEBRA****Worksheet B****1** Evaluate

**a**  $8^2$       **b**  $6^3$       **c**  $7^0$       **d**  $(-5)^4$       **e**  $(-3)^5$       **f**  $(\frac{1}{2})^4$   
**g**  $(\frac{2}{3})^3$       **h**  $(-\frac{1}{4})^3$       **i**  $(1\frac{1}{3})^2$       **j**  $(1\frac{1}{2})^4$       **k**  $(0.1)^5$       **l**  $(-0.2)^3$

**2** Write in the form  $2^n$ 

**a**  $2^5 \times 2^3$       **b**  $2 \times 2^6$       **c** 1      **d**  $2^6 \div 2^2$       **e**  $2^{15} \div 2^6$       **f**  $(2^7)^2$

**3** Simplify

**a**  $2p^2 \times 4p^5$       **b**  $x^2 \times x^3 \times x^5$       **c**  $12n^7 \div 2n^2$       **d**  $(y^3)^4$   
**e**  $(2b)^3 \div 4b^2$       **f**  $p^3q \times pq^2$       **g**  $x^4y^3 \div xy^2$       **h**  $2r^2s \times 3s^2$   
**i**  $6x^5y^8 \div 3x^2y$       **j**  $6a^4b^5 \times \frac{2}{3}ab^3$       **k**  $(5rs^2)^3 \div (10rs)^2$       **l**  $3p^4q^3 \div \frac{1}{5}pq^2$

**4** Evaluate

**a**  $3^{-2}$       **b**  $(\frac{2}{5})^0$       **c**  $(-2)^{-6}$       **d**  $(\frac{1}{6})^{-2}$       **e**  $(1\frac{1}{2})^{-3}$       **f**  $9^{\frac{1}{2}}$   
**g**  $16^{\frac{1}{4}}$       **h**  $(-27)^{\frac{1}{3}}$       **i**  $(\frac{1}{49})^{\frac{1}{2}}$       **j**  $125^{\frac{1}{3}}$       **k**  $(\frac{4}{9})^{\frac{1}{2}}$       **l**  $36^{-\frac{1}{2}}$   
**m**  $81^{-\frac{1}{4}}$       **n**  $(-64)^{-\frac{1}{3}}$       **o**  $(\frac{1}{32})^{-\frac{1}{5}}$       **p**  $(-\frac{8}{125})^{\frac{1}{3}}$       **q**  $(2\frac{1}{4})^{\frac{1}{2}}$       **r**  $(3\frac{3}{8})^{-\frac{1}{3}}$

**5** Evaluate

**a**  $4^{\frac{3}{2}}$       **b**  $27^{\frac{2}{3}}$       **c**  $16^{\frac{3}{4}}$       **d**  $(-125)^{\frac{2}{3}}$       **e**  $9^{\frac{5}{2}}$       **f**  $8^{-\frac{2}{3}}$   
**g**  $36^{-\frac{3}{2}}$       **h**  $(\frac{1}{8})^{\frac{4}{3}}$       **i**  $(\frac{4}{9})^{\frac{3}{2}}$       **j**  $(\frac{1}{216})^{-\frac{2}{3}}$       **k**  $(\frac{9}{16})^{-\frac{3}{2}}$       **l**  $(-\frac{27}{64})^{\frac{4}{3}}$   
**m**  $(0.04)^{\frac{1}{2}}$       **n**  $(2.25)^{-\frac{3}{2}}$       **o**  $(0.064)^{\frac{2}{3}}$       **p**  $(1\frac{9}{16})^{-\frac{3}{2}}$       **q**  $(5\frac{1}{16})^{\frac{3}{4}}$       **r**  $(2\frac{10}{27})^{-\frac{4}{3}}$

**6** Work out

**a**  $4^{\frac{1}{2}} \times 27^{\frac{1}{3}}$       **b**  $16^{\frac{1}{4}} + 25^{\frac{1}{2}}$       **c**  $8^{-\frac{1}{3}} \div 36^{\frac{1}{2}}$       **d**  $(-64)^{\frac{1}{3}} \times 9^{\frac{3}{2}}$   
**e**  $(\frac{1}{3})^{-2} - (-8)^{\frac{1}{3}}$       **f**  $(\frac{1}{25})^{\frac{1}{2}} \times (\frac{1}{4})^{-2}$       **g**  $81^{\frac{3}{4}} - (\frac{1}{49})^{-\frac{1}{2}}$       **h**  $(\frac{1}{27})^{-\frac{1}{3}} \times (\frac{4}{9})^{-\frac{3}{2}}$   
**i**  $(\frac{1}{9})^{-\frac{1}{2}} \times (-32)^{\frac{3}{5}}$       **j**  $(121)^{0.5} + (32)^{0.2}$       **k**  $(100)^{0.5} \div (0.25)^{1.5}$       **l**  $(16)^{-0.25} \times (243)^{0.4}$

**7** Simplify

**a**  $x^8 \times x^{-6}$       **b**  $y^{-2} \times y^{-4}$       **c**  $6p^3 \div 2p^7$       **d**  $(2x^{-4})^3$   
**e**  $y^3 \times y^{-\frac{1}{2}}$       **f**  $2b^{\frac{2}{3}} \times 4b^{\frac{1}{4}}$       **g**  $x^{\frac{3}{5}} \div x^{\frac{1}{3}}$       **h**  $a^{\frac{1}{2}} \div a^{\frac{4}{3}}$   
**i**  $p^{\frac{1}{4}} \div p^{-\frac{1}{5}}$       **j**  $(3x^{\frac{2}{5}})^2$       **k**  $y \times y^{\frac{5}{6}} \times y^{-\frac{3}{2}}$       **l**  $4t^{\frac{3}{2}} \div 12t^{\frac{1}{2}}$   
**m**  $\frac{b^2 \times b^{\frac{1}{4}}}{b^{\frac{1}{2}}}$       **n**  $\frac{y^{\frac{1}{2}} \times y^{\frac{1}{3}}}{y}$       **o**  $\frac{4x^{\frac{2}{3}} \times 3x^{-\frac{1}{6}}}{6x^{\frac{3}{4}}}$       **p**  $\frac{2a \times a^{\frac{3}{4}}}{8a^{-\frac{1}{2}}}$

## C1 ALGEBRA

## Worksheet B continued

8 Solve each equation.

a  $x^{\frac{1}{2}} = 6$

b  $x^{\frac{1}{3}} = 5$

c  $x^{-\frac{1}{2}} = 2$

d  $x^{-\frac{1}{4}} = \frac{1}{3}$

e  $x^{\frac{3}{2}} = 8$

f  $x^{\frac{2}{3}} = 16$

g  $x^{\frac{4}{3}} = 81$

h  $x^{-\frac{3}{2}} = 27$

9 Express in the form  $x^k$

a  $\sqrt{x}$

b  $\frac{1}{\sqrt[3]{x}}$

c  $x^2 \times \sqrt{x}$

d  $\frac{\sqrt[4]{x}}{x}$

e  $\sqrt{x^3}$

f  $\sqrt{x} \times \sqrt[3]{x}$

g  $(\sqrt{x})^5$

h  $\sqrt[3]{x^2} \times (\sqrt{x})^3$

10 Express each of the following in the form  $ax^b$ , where  $a$  and  $b$  are rational constants.

a  $\frac{4}{\sqrt{x}}$

b  $\frac{1}{2x}$

c  $\frac{3}{4x^3}$

d  $\frac{1}{(3x)^2}$

e  $\frac{2}{5\sqrt[3]{x}}$

f  $\frac{1}{\sqrt{9x^3}}$

11 Express in the form  $2^k$

a  $8^2$

b  $(\frac{1}{4})^{-2}$

c  $(\frac{1}{2})^{\frac{1}{3}}$

d  $16^{-\frac{1}{6}}$

e  $8^{\frac{2}{5}}$

f  $(\frac{1}{32})^{-3}$

12 Express each of the following in the form  $3^y$ , where  $y$  is a function of  $x$ .

a  $9^x$

b  $81^{x+1}$

c  $27^{\frac{x}{4}}$

d  $(\frac{1}{3})^x$

e  $9^{2x-1}$

f  $(\frac{1}{27})^{x+2}$

13 Given that  $y = 2^x$ , express each of the following in terms of  $y$ .

a  $2^{x+1}$

b  $2^{x-2}$

c  $2^{2x}$

d  $8^x$

e  $2^{4x+3}$

f  $(\frac{1}{2})^{x-3}$

14 Find the value of  $x$  such that

a  $2^x = 64$

b  $5^{x-1} = 125$

c  $3^{x+4} - 27 = 0$

d  $8^x - 2 = 0$

e  $3^{2x-1} = 9$

f  $16 - 4^{3x-2} = 0$

g  $9^{x-2} = 27$

h  $8^{2x+1} = 16$

i  $49^{x+1} = \sqrt{7}$

j  $3^{3x-2} = \sqrt[3]{9}$

k  $(\frac{1}{6})^{x+3} = 36$

l  $(\frac{1}{2})^{3x-1} = 8$

15 Solve each equation.

a  $2^{x+3} = 4^x$

b  $5^{3x} = 25^{x+1}$

c  $9^{2x} = 3^{x-3}$

d  $16^x = 4^{1-x}$

e  $4^{x+2} = 8^x$

f  $27^{2x} = 9^{3-x}$

g  $6^{3x-1} = 36^{x+2}$

h  $8^x = 16^{2x-1}$

i  $125^x = 5^{x-3}$

j  $(\frac{1}{3})^x = 3^{x-4}$

k  $(\frac{1}{2})^{1-x} = (\frac{1}{8})^{2x}$

l  $(\frac{1}{4})^{x+1} = 8^x$

16 Expand and simplify

a  $x(x^2 - x^{-1})$

b  $2x^3(x^{-1} + 3)$

c  $x^{-1}(3x - x^3)$

d  $4x^{-2}(3x^5 + 2x^3)$

e  $\frac{1}{2}x^2(6x + 4x^{-1})$

f  $3x^{\frac{1}{2}}(x^{-\frac{1}{2}} - x^{\frac{3}{2}})$

g  $x^{-\frac{3}{2}}(5x^2 + x^{\frac{7}{2}})$

h  $x^{\frac{1}{3}}(3x^{\frac{5}{3}} - x^{-\frac{4}{3}})$

i  $(x^2 + 1)(x^4 - 3)$

j  $(2x^5 + x)(x^4 + 3)$

k  $(x^2 - 2x^{-1})(x - x^{-2})$

l  $(x^2 - x^{\frac{3}{2}})(x - x^{\frac{1}{2}})$

17 Simplify

a  $\frac{x^3 + 2x}{x}$

b  $\frac{4t^5 - 6t^3}{2t^2}$

c  $\frac{x^{\frac{3}{2}} - 3x}{x^{\frac{1}{2}}}$

d  $\frac{y^2(y^3 - 6)}{3y}$

e  $\frac{p + p^{\frac{3}{2}}}{p^{\frac{3}{4}}}$

f  $\frac{8w - 2w^{\frac{1}{2}}}{4w^{-\frac{1}{2}}}$

g  $\frac{x+1}{x^{\frac{1}{2}} + x^{-\frac{1}{2}}}$

h  $\frac{2t^3 - 4t}{t^{\frac{3}{2}} - 2t^{-\frac{1}{2}}}$

**C1 ALGEBRA****Worksheet C**

1 Express each of the following in the form  $a\sqrt{2} + b\sqrt{3}$ , where  $a$  and  $b$  are integers.

a  $\sqrt{27} + 2\sqrt{50}$

b  $\sqrt{6}(\sqrt{3} - \sqrt{8})$

2 Given that  $x > 0$ , find in the form  $k\sqrt{3}$  the value of  $x$  such that

$$x(x - 2) = 2(6 - x).$$

3 Solve the equation

$$25^x = 5^{4x+1}.$$

4 a Express  $\sqrt[3]{24}$  in the form  $k\sqrt[3]{3}$ .

b Find the integer  $n$  such that

$$\sqrt[3]{24} + \sqrt[3]{81} = \sqrt[3]{n}.$$

5 Show that

$$\frac{10\sqrt{3}}{\sqrt{15}} + \frac{4}{\sqrt{5}-\sqrt{7}}$$

can be written in the form  $k\sqrt{7}$ , where  $k$  is an integer to be found.

6 Showing your method clearly,

a express  $\sqrt{37.5}$  in the form  $a\sqrt{6}$ ,

b express  $\sqrt{9\frac{3}{5}} - \sqrt{6\frac{2}{3}}$  in the form  $b\sqrt{15}$ .

7 Given that  $x = 2^{t-1}$  and  $y = 2^{3t}$ ,

a find expressions in terms of  $t$  for

i  $xy$                       ii  $2y^2$

b Hence, or otherwise, find the value of  $t$  for which

$$2y^2 - xy = 0.$$

8 Solve the equation

$$\sqrt{2}(3x - 1) = 2(2x + 3),$$

giving your answer in the form  $a + b\sqrt{2}$ , where  $a$  and  $b$  are integers.

9 Given that  $6^{y+1} = 36^{x-2}$ ,

a express  $y$  in the form  $ax + b$ ,

b find the value of  $4^{x - \frac{1}{2}y}$ .

10 Express each of the following in the form  $a + b\sqrt{2}$ , where  $a$  and  $b$  are integers.

a  $(3 - \sqrt{2})(1 + \sqrt{2})$

b  $\frac{\sqrt{2}}{\sqrt{2}-1}$

## C1 ALGEBRA

## Worksheet C continued

- 11 Solve the equation

$$16^{x+1} = 8^{2x+1}.$$

- 12 Given that

$$(a - 2\sqrt{3})^2 = b - 20\sqrt{3},$$

find the values of the integers  $a$  and  $b$ .

- 13 a Find the value of
- $t$
- such that

$$\left(\frac{1}{4}\right)^{t-3} = 8.$$

- b Solve the equation

$$\left(\frac{1}{3}\right)^y = 27^{y+1}.$$

- 14 Express each of the following in the form
- $a + b\sqrt{5}$
- , where
- $a$
- and
- $b$
- are integers.

a  $\sqrt{20}(\sqrt{5} - 3)$

b  $(1 - \sqrt{5})(3 + 2\sqrt{5})$

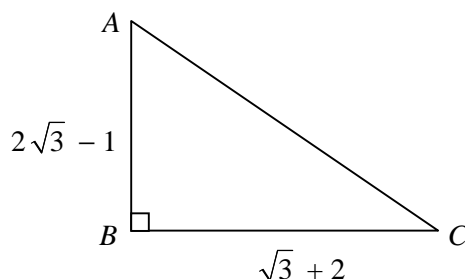
c  $\frac{1+\sqrt{5}}{\sqrt{5}-2}$

- 15 Given that
- $a^{\frac{1}{3}} = b^{\frac{3}{4}}$
- , and that
- $a > 0$
- and
- $b > 0$
- ,

- a find an expression for
- $a^{\frac{1}{2}}$
- in terms of
- $b$
- ,

- b find an expression for
- $b^{\frac{1}{2}}$
- in terms of
- $a$
- .

- 16

In triangle  $ABC$ ,  $AB = 2\sqrt{3} - 1$ ,  $BC = \sqrt{3} + 2$  and  $\angle ABC = 90^\circ$ .

- a Find the exact area of triangle
- $ABC$
- in its simplest form.

- b Show that
- $AC = 2\sqrt{5}$
- .

- c Show that
- $\tan(\angle ACB) = 5\sqrt{3} - 8$
- .

- 17 a Given that
- $y = 2^x$
- , express each of the following in terms of
- $y$
- .

i  $2^{x+2}$

ii  $4^x$

- b Hence, or otherwise, find the value of
- $x$
- for which

$$4^x - 2^{x+2} = 0.$$

- 18 Given that the point with coordinates
- $(1 + \sqrt{3}, 5\sqrt{3})$
- lies on the curve with the equation

$$y = 2x^2 + px + q,$$

find the values of the rational constants  $p$  and  $q$ .

**C1 ALGEBRA****Worksheet D**

**1** Giving your answers in descending powers of  $x$ , simplify

- |  |   |
|--|---|
| <b>a</b> $(x^2 + 3x + 2) + (2x^2 + 5x + 1)$                | <b>b</b> $(x^3 + 4x^2 + x - 6) + (x^2 - 3x + 7)$            |
| <b>c</b> $(4 - x + 2x^3) + (3 - x + 6x^2 - 5x^3)$          | <b>d</b> $(x^5 + 8x^3 - 5x^2 - 9) + (-x^4 - 4x + 1)$        |
| <b>e</b> $(3x^3 - 7x^2 + 2) - (x^3 + 2x^2 + x - 6)$        | <b>f</b> $(x^5 + 3x^4 - x^2 - 3) - (x^4 + 2x^3 - 3x + 2)$   |
| <b>g</b> $(2x^7 - 9x^5 + x^3 + x) - (3x^6 - 4x^3 + x + 5)$ | <b>h</b> $2(x^4 + 4x^2 - 3) + (x^4 + 3x^3 - 8)$             |
| <b>i</b> $3(7 + 4x - x^2 - 2x^3) + 5(-2 - 3x + x^3)$       | <b>j</b> $6(x^3 + 5x^2 - 2) - 3(2x^3 - x^2 - x)$            |
| <b>k</b> $8(x^4 + 2x^2 - 4x - 1) - 2(5 - 3x + x^3)$        | <b>l</b> $7(x^6 + 3x^3 + x^2 - 4) - 4(2x^6 + x^5 - 3x - 7)$ |

**2** Simplify

- a**  $(3y^2 + 2y + 1) + (y^3 - 4y^2 + 7y) + (2y^3 - y^2 - 8y + 5)$   
**b**  $3(t^4 - t^3 + 4t) + (6 - t - 3t^3) + 2(t^4 - 2t^2 + 4)$   
**c**  $(x^3 - 6x^2 + 8) + (5x^2 - x + 1) - (2x^3 + 3x^2 + x - 7)$   
**d**  $2(3 + m + 7m^2 - 3m^5) + 6(1 - m^2 + 2m^4) - 5(m^5 + 3m^3 - m^2 + 2)$   
**e**  $\frac{1}{3}(1 - 2u + \frac{3}{5}u^2 + 3u^4) - \frac{1}{2}(2 - u + \frac{2}{3}u^2 - \frac{1}{2}u^3)$

**3** Giving your answers in ascending powers of  $x$ , simplify

- |  |  |
|--|--|
| <b>a</b> $x(2 - 3x + x^2) + 4(1 + 2x^2 - x^3)$       | <b>b</b> $x(x^4 + 7x^2 - 5x + 9) - 2(x^4 - 4x^3 - 3)$        |
| <b>c</b> $2x(-5 + 4x - x^3) + 7(2 - 3x^2 + x^4)$     | <b>d</b> $x^2(8 + 2x + x^2) - 3(5 + 4x^2 + x^3)$             |
| <b>e</b> $3x^2(x + 3) - x(x^3 + 4x^2) + 5(x^3 - 2x)$ | <b>f</b> $x^2(6 - x + 5x^2) + 7x(2 - x^3) + 4(1 - 3x - x^2)$ |

**4** Show that

- a**  $(3x + 1)(x^2 - 2x + 4) \equiv 3x^3 - 5x^2 + 10x + 4$   
**b**  $(1 + 2x - x^2)(1 - 2x + x^2) \equiv 1 - 4x^2 + 4x^3 - x^4$   
**c**  $(3 - x)^3 \equiv 27 - 27x + 9x^2 - x^3$

**5** Giving your answers in descending powers of  $x$ , expand and simplify

- |   |   |
|---|---|
| <b>a</b> $(x + 1)(x^2 + 5x - 6)$            | <b>b</b> $(2x - 5)(x^2 - 3x + 7)$               |
| <b>c</b> $(4 - 7x)(2 + 5x - x^2)$           | <b>d</b> $(3x - 2)^3$                           |
| <b>e</b> $(x^2 + 3)(2x^2 - x + 9)$          | <b>f</b> $(4x - 1)(x^4 - 3x^2 + 5x + 2)$        |
| <b>g</b> $(x^2 + 2x + 5)(x^2 + 3x + 1)$     | <b>h</b> $(x^2 + x - 3)(2x^2 - x + 4)$          |
| <b>i</b> $(3x^2 - 5x + 2)(2x^2 - 4x - 8)$   | <b>j</b> $(x^2 + 2x - 6)^2$                     |
| <b>k</b> $(x^3 + 4x^2 + 1)(2x^4 + x^2 + 3)$ | <b>l</b> $(6 - 2x + x^3)(3 + x^2 - x^3 + 2x^4)$ |

**6** Simplify

- a**  $(p^2 - 1)(p + 4)(2p + 3)$   
**b**  $(t + 2)(t^2 + 3t + 5) + (t + 4)(t^2 + t + 7)$   
**c**  $2(x^2 - 3)(x^2 + x - 4) + (3x - 1)(4x^3 + 2x^2 - x + 6)$   
**d**  $(u^3 - 4u^2 - 3)(u + 2) - (2u^3 + u - 1)(u^2 + 5u - 3)$

**C1 ALGEBRA****Worksheet E****1** Factorise

**a**  $x^2 + 4x + 3$

**b**  $x^2 + 7x + 10$

**c**  $y^2 - 3y + 2$

**d**  $x^2 - 6x + 9$

**e**  $y^2 - y - 2$

**f**  $a^2 + 2a - 8$

**g**  $x^2 - 1$

**h**  $p^2 + 9p + 14$

**i**  $x^2 - 2x - 15$

**j**  $16 - 10m + m^2$

**k**  $t^2 + 3t - 18$

**l**  $y^2 - 13y + 40$

**m**  $r^2 - 16$

**n**  $y^2 - 2y - 63$

**o**  $121 + 22a + a^2$

**p**  $x^2 + 6x - 72$

**q**  $26 - 15x + x^2$

**r**  $s^2 + 23s + 120$

**s**  $p^2 + 14p - 51$

**t**  $m^2 - m - 90$

**2** Factorise

**a**  $2x^2 + 3x + 1$

**b**  $2 + 7p + 3p^2$

**c**  $2y^2 - 5y + 3$

**d**  $2 - m - m^2$

**e**  $3r^2 - 2r - 1$

**f**  $5 - 19y - 4y^2$

**g**  $4 - 13a + 3a^2$

**h**  $5x^2 - 8x - 4$

**i**  $4x^2 + 8x + 3$

**j**  $9s^2 - 6s + 1$

**k**  $4m^2 - 25$

**l**  $2 - y - 6y^2$

**m**  $4u^2 + 17u + 4$

**n**  $6p^2 + 5p - 4$

**o**  $8x^2 + 19x + 6$

**p**  $12r^2 + 8r - 15$

**3** Using factorisation, solve each equation.

**a**  $x^2 - 4x + 3 = 0$

**b**  $x^2 + 6x + 8 = 0$

**c**  $x^2 + 4x - 5 = 0$

**d**  $x^2 - 7x = 8$

**e**  $x^2 - 25 = 0$

**f**  $x(x - 1) = 42$

**g**  $x^2 = 3x$

**h**  $27 + 12x + x^2 = 0$

**i**  $60 - 4x - x^2 = 0$

**j**  $5x + 14 = x^2$

**k**  $2x^2 - 3x + 1 = 0$

**l**  $x(x - 1) = 6(x - 2)$

**m**  $3x^2 + 11x = 4$

**n**  $x(2x - 3) = 5$

**o**  $6 + 23x - 4x^2 = 0$

**p**  $6x^2 + 10 = 19x$

**q**  $4x^2 + 4x + 1 = 0$

**r**  $3(x^2 + 4) = 13x$

**s**  $(2x + 5)^2 = 5 - x$

**t**  $3x(2x - 7) = 2(7x + 3)$

**4** Factorise fully

**a**  $2y^2 - 10y + 12$

**b**  $x^3 + x^2 - 2x$

**c**  $p^3 - 4p$

**d**  $3m^3 + 21m^2 + 18m$

**e**  $a^4 + 4a^2 + 3$

**f**  $t^4 + 3t^2 - 10$

**g**  $12 + 20x - 8x^2$

**h**  $6r^2 - 9r - 42$

**i**  $6x^3 - 26x^2 + 8x$

**j**  $y^4 + 3y^3 - 18y^2$

**k**  $m^4 - 1$

**l**  $p^5 - 4p^3 + 4p$

**5** Sketch each curve showing the coordinates of any points of intersection with the coordinate axes.

**a**  $y = x^2 - 3x + 2$

**b**  $y = x^2 + 5x + 6$

**c**  $y = x^2 - 9$

**d**  $y = x^2 - 2x$

**e**  $y = x^2 - 10x + 25$

**f**  $y = 2x^2 - 14x + 20$

**g**  $y = -x^2 + 5x - 4$

**h**  $y = 2 + x - x^2$

**i**  $y = 2x^2 - 3x + 1$

**j**  $y = 2x^2 + 13x + 6$

**k**  $y = 3 - 8x + 4x^2$

**l**  $y = 2 + 7x - 4x^2$

**m**  $y = 5x^2 - 17x + 6$

**n**  $y = -6x^2 + 7x - 2$

**o**  $y = 6x^2 + x - 5$

**6** Solve each of the following equations.

**a**  $x - 5 + \frac{4}{x} = 0$

**b**  $x - \frac{10}{x} = 3$

**c**  $2x^3 - x^2 - 3x = 0$

**d**  $x^2(10 - x^2) = 9$

**e**  $\frac{5}{x^2} + \frac{4}{x} - 1 = 0$

**f**  $\frac{x-6}{x-4} = x$

**g**  $x + 5 = \frac{3}{x+3}$

**h**  $x^2 - \frac{4}{x^2} = 3$

**i**  $4x^4 + 7x^2 = 2$

**j**  $\frac{2x}{3-x} = \frac{1}{x+2}$

**k**  $\frac{2x+1}{x+3} = \frac{2}{x}$

**l**  $\frac{7}{x+2} - 3x = 2$



# C1 ALGEBRA

## Worksheet F

- 1** Express in the form  $(x + a)^2 + b$
- |                            |                                  |                                   |   |
|----------------------------|----------------------------------|-----------------------------------|---|
| <b>a</b> $x^2 + 2x + 4$    | <b>b</b> $x^2 - 2x + 4$          | <b>c</b> $x^2 - 4x + 1$           | <b>d</b> $x^2 + 6x$                         |
| <b>e</b> $x^2 + 4x + 8$    | <b>f</b> $x^2 - 8x - 5$          | <b>g</b> $x^2 + 12x + 30$         | <b>h</b> $x^2 - 10x + 25$                   |
| <b>i</b> $x^2 + 6x - 9$    | <b>j</b> $18 - 4x + x^2$         | <b>k</b> $x^2 + 3x + 3$           | <b>l</b> $x^2 + x - 1$                      |
| <b>m</b> $x^2 - 18x + 100$ | <b>n</b> $x^2 - x - \frac{1}{2}$ | <b>o</b> $20 + 9x + x^2$          | <b>p</b> $x^2 - 7x - 2$                     |
| <b>q</b> $5 - 3x + x^2$    | <b>r</b> $x^2 - 11x + 37$        | <b>s</b> $x^2 + \frac{2}{3}x + 1$ | <b>t</b> $x^2 - \frac{1}{2}x - \frac{1}{4}$ |
- 2** Express in the form  $a(x + b)^2 + c$
- |                            |                           |                            |  |
|----------------------------|---------------------------|----------------------------|--|
| <b>a</b> $2x^2 + 4x + 3$   | <b>b</b> $2x^2 - 8x - 7$  | <b>c</b> $3 - 6x + 3x^2$   | <b>d</b> $4x^2 + 24x + 11$                             |
| <b>e</b> $-x^2 - 2x - 5$   | <b>f</b> $1 + 10x - x^2$  | <b>g</b> $2x^2 + 2x - 1$   | <b>h</b> $3x^2 - 9x + 5$                               |
| <b>i</b> $3x^2 - 24x + 48$ | <b>j</b> $3x^2 - 15x$     | <b>k</b> $70 + 40x + 5x^2$ | <b>l</b> $2x^2 + 5x + 2$                               |
| <b>m</b> $4x^2 + 6x - 7$   | <b>n</b> $-2x^2 + 4x - 1$ | <b>o</b> $4 - 2x - 3x^2$   | <b>p</b> $\frac{1}{3}x^2 + \frac{1}{2}x - \frac{1}{4}$ |
- 3** Solve each equation by completing the square, giving your answers as simply as possible in terms of surds where appropriate.
- |                             |                               |                             |                              |
|-----------------------------|-------------------------------|-----------------------------|------------------------------|
| <b>a</b> $y^2 - 4y + 2 = 0$ | <b>b</b> $p^2 + 2p - 2 = 0$   | <b>c</b> $x^2 - 6x + 4 = 0$ | <b>d</b> $7 + 10r + r^2 = 0$ |
| <b>e</b> $x^2 - 2x = 11$    | <b>f</b> $a^2 - 12a - 18 = 0$ | <b>g</b> $m^2 - 3m + 1 = 0$ | <b>h</b> $9 - 7t + t^2 = 0$  |
| <b>i</b> $u^2 + 7u = 44$    | <b>j</b> $2y^2 - 4y + 1 = 0$  | <b>k</b> $3p^2 + 18p = -23$ | <b>l</b> $2x^2 + 12x = 9$    |
| <b>m</b> $-m^2 + m + 1 = 0$ | <b>n</b> $4x^2 + 49 = 28x$    | <b>o</b> $1 - t - 3t^2 = 0$ | <b>p</b> $2a^2 - 7a + 4 = 0$ |
- 4** By completing the square, find the maximum or minimum value of  $y$  and the value of  $x$  for which this occurs. State whether your value of  $y$  is a maximum or a minimum in each case.
- |                               |                               |                               |
|-------------------------------|-------------------------------|-------------------------------|
| <b>a</b> $y = x^2 - 2x + 7$   | <b>b</b> $y = x^2 + 2x - 3$   | <b>c</b> $y = 1 - 6x + x^2$   |
| <b>d</b> $y = x^2 + 10x + 35$ | <b>e</b> $y = -x^2 + 4x + 4$  | <b>f</b> $y = x^2 + 3x - 2$   |
| <b>g</b> $y = 2x^2 + 8x + 5$  | <b>h</b> $y = -3x^2 + 6x$     | <b>i</b> $y = 7 - 5x - x^2$   |
| <b>j</b> $y = 4x^2 - 12x + 9$ | <b>k</b> $y = 4x^2 + 20x - 8$ | <b>l</b> $y = 17 - 2x - 2x^2$ |
- 5** Sketch each curve showing the exact coordinates of its turning point and the point where it crosses the  $y$ -axis.
- |                              |                                |                                |
|------------------------------|--------------------------------|--------------------------------|
| <b>a</b> $y = x^2 - 4x + 3$  | <b>b</b> $y = x^2 + 2x - 24$   | <b>c</b> $y = x^2 - 2x + 5$    |
| <b>d</b> $y = 30 + 8x + x^2$ | <b>e</b> $y = x^2 + 2x + 1$    | <b>f</b> $y = 8 + 2x - x^2$    |
| <b>g</b> $y = -x^2 + 8x - 7$ | <b>h</b> $y = -x^2 - 4x - 7$   | <b>i</b> $y = x^2 - 5x + 4$    |
| <b>j</b> $y = x^2 + 3x + 3$  | <b>k</b> $y = 3 + 8x + 4x^2$   | <b>l</b> $y = -2x^2 + 8x - 15$ |
| <b>m</b> $y = 1 - x - 2x^2$  | <b>n</b> $y = 25 - 20x + 4x^2$ | <b>o</b> $y = 3x^2 - 4x + 2$   |
- 6** **a** Express  $x^2 - 4\sqrt{2}x + 5$  in the form  $a(x + b)^2 + c$ .  
**b** Write down an equation of the line of symmetry of the curve  $y = x^2 + 4\sqrt{2}x + 5$ .
- 7**  $f(x) \equiv x^2 + 2kx - 3$ .  
 By completing the square, find the roots of the equation  $f(x) = 0$  in terms of the constant  $k$ .

# C1 ALGEBRA

## Worksheet G

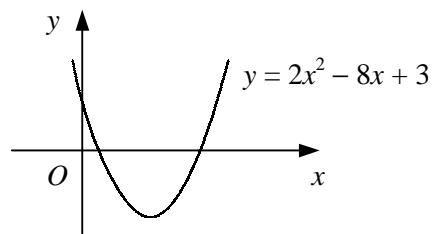
- 1 By completing the square, show that the roots of the equation  $ax^2 + bx + c = 0$  are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

- 2 Use the quadratic formula to solve each equation, giving your answers as simply as possible in terms of surds where appropriate.

<b>a</b> $x^2 + 4x + 1 = 0$	<b>b</b> $4 + 8t - t^2 = 0$	<b>c</b> $y^2 - 20y + 91 = 0$	<b>d</b> $r^2 + 2r - 7 = 0$
<b>e</b> $6 + 18a + a^2 = 0$	<b>f</b> $m(m - 5) = 5$	<b>g</b> $x^2 + 11x + 27 = 0$	<b>h</b> $2u^2 + 6u + 3 = 0$
<b>i</b> $5 - y - y^2 = 0$	<b>j</b> $2x^2 - 3x = 2$	<b>k</b> $3p^2 + 7p + 1 = 0$	<b>l</b> $t^2 - 14t = 14$
<b>m</b> $0.1r^2 + 1.4r = 0.9$	<b>n</b> $6u^2 + 4u = 1$	<b>o</b> $\frac{1}{2}y^2 - 3y = \frac{2}{3}$	<b>p</b> $4x(x - 3) = 11 - 4x$

- 3



The diagram shows the curve with equation  $y = 2x^2 - 8x + 3$ .

Find and simplify the exact coordinates of the points where the curve crosses the  $x$ -axis.

- 4 State the condition for which the roots of the equation  $ax^2 + bx + c = 0$  are

**a** real and distinct                      **b** real and equal                      **c** not real

- 5 Sketch the curve  $y = ax^2 + bx + c$  and the  $x$ -axis in the cases where

<b>a</b> $a > 0$ and $b^2 - 4ac > 0$	<b>b</b> $a < 0$ and $b^2 - 4ac < 0$
<b>c</b> $a > 0$ and $b^2 - 4ac = 0$	<b>d</b> $a < 0$ and $b^2 - 4ac > 0$

- 6 By evaluating the discriminant, determine whether the roots of each equation are real and distinct, real and equal or not real.

<b>a</b> $x^2 + 2x - 7 = 0$	<b>b</b> $x^2 + x + 3 = 0$	<b>c</b> $x^2 - 4x + 5 = 0$	<b>d</b> $x^2 - 6x + 3 = 0$
<b>e</b> $x^2 + 14x + 49 = 0$	<b>f</b> $x^2 - 9x + 17 = 0$	<b>g</b> $x^2 + 3x = 11$	<b>h</b> $2 + 3x + 2x^2 = 0$
<b>i</b> $5x^2 + 8x + 3 = 0$	<b>j</b> $3x^2 - 7x + 5 = 0$	<b>k</b> $9x^2 - 12x + 4 = 0$	<b>l</b> $13x^2 + 19x + 7 = 0$
<b>m</b> $4 - 11x + 8x^2 = 0$	<b>n</b> $x^2 + \frac{2}{3}x = \frac{1}{4}$	<b>o</b> $x^2 - \frac{3}{4}x + \frac{1}{8} = 0$	<b>p</b> $\frac{2}{5}x^2 + \frac{3}{5}x + \frac{1}{3} = 0$

- 7 Find the value of the constant  $p$  such that the equation  $x^2 + x + p = 0$  has equal roots.

- 8 Given that  $q \neq 0$ , find the value of the constant  $q$  such that the equation  $x^2 + 2qx - q = 0$  has a repeated root.

- 9 Given that the  $x$ -axis is a tangent to the curve with the equation

$$y = x^2 + rx - 2x + 4,$$

find the two possible values of the constant  $r$ .

# C1 ALGEBRA

## Worksheet H

- 1 a Factorise fully the expression

$$20x - 2x^2 - 6x^3.$$

- b Hence, find all solutions to the equation

$$20x - 2x^2 - 6x^3 = 0.$$

- 2 A is the point  $(-2, 1)$  and B is the point  $(6, k)$ .

- a Show that  $AB^2 = k^2 - 2k + 65$ .

Given also that  $AB = 10$ ,

- b find the possible values of  $k$ .

- 3 Solve the equations

a  $x - \frac{5}{x} = 4$

b  $\frac{9}{5-x} - 1 = 2x$

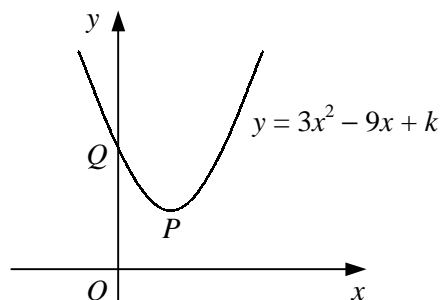
- 4 a Find the coordinates of the turning point of the curve with equation  $y = 3 - 5x - 2x^2$ .

- b Sketch the curve  $y = 3 - 5x - 2x^2$ , showing the coordinates of any points of intersection with the coordinate axes.

- 5 Find in the form  $k\sqrt{2}$  the solutions of the equation

$$2x^2 + 5\sqrt{2}x - 6 = 0.$$

- 6



The diagram shows the curve with equation  $y = 3x^2 - 9x + k$  where  $k$  is a constant.

- a Find the  $x$ -coordinate of the turning point of the curve,  $P$ .

Given that the  $y$ -coordinate of  $P$  is  $\frac{17}{4}$ ,

- b find the coordinates of the point  $Q$  where the curve crosses the  $y$ -axis.

- 7 By letting  $y = 2^x$ , or otherwise, solve the equation

$$2^{2x} - 10(2^x) + 16 = 0.$$

- 8 Given that the equation

$$kx^2 - 2x + 3 - 2k = 0$$

has equal roots, find the possible values of the constant  $k$ .

- 9  $f(x) \equiv 3 + 4x - x^2$ .
- Express  $f(x)$  in the form  $a(x + b)^2 + c$ .
  - State the coordinates of the turning point of the curve  $y = f(x)$ .
  - Solve the equation  $f(x) = 2$ , giving your answers in the form  $d + e\sqrt{5}$ .
- 10 Giving your answers in terms of surds, solve the equations
- $3x^2 - 5x + 1 = 0$
  - $\frac{x}{x+2} = \frac{3}{x-1}$
- 11 **a** By completing the square, find, in terms of  $k$ , the solutions of the equation
- $$x^2 - 4kx + 6 = 0.$$
- b** Using your answers to part **a**, solve the equation
- $$x^2 - 12x + 6 = 0.$$
- 12 **a** Find in the form  $a + b\sqrt{3}$ , where  $a$  and  $b$  are integers, the values of  $x$  such that
- $$2x^2 - 12x = 6.$$
- b** Solve the equation
- $$2y^3 + y^2 - 15y = 0.$$
- 13 Labelling the coordinates of any points of intersection with the coordinate axes, sketch the curves
- $y = (x + 1)(x - p)$  where  $p > 0$ ,
  - $y = (x + q)^2$  where  $q < 0$ .
- 14  $f(x) \equiv 2x^2 - 6x + 5$ .
- Find the values of  $A$ ,  $B$  and  $C$  such that
- $$f(x) \equiv A(x + B)^2 + C.$$
- b** Hence deduce the minimum value of  $f(x)$ .
- 15 **a** Given that  $t = x^{\frac{1}{3}}$  express  $x^{\frac{2}{3}}$  in terms of  $t$ .
- b** Hence, or otherwise, solve the equation
- $$2x^{\frac{2}{3}} + x^{\frac{1}{3}} - 6 = 0.$$
- 16 **a** Express  $k^2 - 8k + 20$  in the form  $a(k + b)^2 + c$ , where  $a$ ,  $b$  and  $c$  are constants.
- b** Hence prove that the equation
- $$x^2 - kx + 2k = 5$$
- has real and distinct roots for all real values of  $k$ .
- 17 **a** Show that
- $$(x^2 + 2x - 3)(x^2 - 3x - 4) \equiv x^4 - x^3 - 13x^2 + x + 12.$$
- b** Hence solve the equation
- $$x^4 - x^3 - 13x^2 + x + 12 = 0.$$

**C1 ALGEBRA****Worksheet 1**

**1** Solve each pair of simultaneous equations.

**a**  $y = 3x$

$y = 2x + 1$

**b**  $y = x - 6$

$y = \frac{1}{2}x - 4$

**c**  $y = 2x + 6$

$y = 3 - 4x$

**d**  $x + y - 3 = 0$

$x + 2y + 1 = 0$

**e**  $x + 2y + 11 = 0$

$2x - 3y + 1 = 0$

**f**  $3x + 3y + 4 = 0$

$5x - 2y - 5 = 0$

**2** Find the coordinates of the points of intersection of the given straight line and curve in each case.

**a**  $y = x + 2$

$y = x^2 - 4$

**b**  $y = 4x + 11$

$y = x^2 + 3x - 1$

**c**  $y = 2x - 1$

$y = 2x^2 + 3x - 7$

**3** Solve each pair of simultaneous equations.

**a**  $x^2 - y + 3 = 0$

$x - y + 5 = 0$

**b**  $2x^2 - y - 8x = 0$

$x + y + 3 = 0$

**c**  $x^2 + y^2 = 25$

$2x - y = 5$

**d**  $x^2 + 2xy + 15 = 0$

$2x - y + 10 = 0$

**e**  $x^2 - 2xy - y^2 = 7$

$x + y = 1$

**f**  $3x^2 - x - y^2 = 0$

$x + y - 1 = 0$

**g**  $2x^2 + xy + y^2 = 22$

$x + y = 4$

**h**  $x^2 - 4y - y^2 = 0$

$x - 2y = 0$

**i**  $x^2 + xy = 4$

$3x + 2y = 6$

**j**  $2x^2 + y - y^2 = 8$

$2x - y = 3$

**k**  $x^2 - xy + y^2 = 13$

$2x - y = 7$

**l**  $x^2 - 5x + y^2 = 0$

$3x + y = 5$

**m**  $3x^2 - xy + y^2 = 36$

$x - 2y = 10$

**n**  $2x^2 + x - 4y = 6$

$3x - 2y = 4$

**o**  $x^2 + x + 2y^2 - 52 = 0$

$x - 3y + 17 = 0$

**4** Solve each pair of simultaneous equations.

**a**  $x - \frac{1}{y} - 4y = 0$

$x - 6y - 1 = 0$

**b**  $xy = 6$

$x - y = 5$

**c**  $\frac{3}{x} - 2y + 4 = 0$

$4x + y - 7 = 0$

**5** The line  $y = 5 - x$  intersects the curve  $y = x^2 - 3x + 2$  at the points  $P$  and  $Q$ .

Find the length  $PQ$  in the form  $k\sqrt{2}$ .

**6** Solve the simultaneous equations

$$3^{x-1} = 9^{2y}$$

$$8^{x-2} = 4^{1+y}$$

**7** Given that

$$(A + 2\sqrt{3})(B - \sqrt{3}) \equiv 9\sqrt{3} - 1,$$

find the values of the integers  $A$  and  $B$ .

**C1 ALGEBRA****Worksheet J**

1 Find the set of values of  $x$  for which

- a**  $2x + 1 < 7$       **b**  $3x - 1 \geq 20$       **c**  $2x - 5 > 3$       **d**  $6 + 3x \leq 42$   
**e**  $5x + 17 \geq 2$       **f**  $\frac{1}{3}x + 7 < 8$       **g**  $9x - 4 \geq 50$       **h**  $3x + 11 < 7$   
**i**  $18 - x > 4$       **j**  $10 + 4x \leq 0$       **k**  $12 - 3x < 10$       **l**  $9 - \frac{1}{2}x \geq 4$

2 Solve each inequality.

- a**  $2y - 3 > y + 4$       **b**  $5p + 1 \leq p + 3$       **c**  $x - 2 < 3x - 8$   
**d**  $a + 11 \geq 15 - a$       **e**  $17 - 2u < 2 + u$       **f**  $5 - b \geq 14 - 3b$   
**g**  $4x + 23 < x + 5$       **h**  $12 + 3y \geq 2y - 1$       **i**  $16 - 3p \leq 36 + p$   
**j**  $5(r - 2) > 30$       **k**  $3(1 - 2t) \leq t - 4$       **l**  $2(3 + x) \geq 4(6 - x)$   
**m**  $7(y + 3) - 2(3y - 1) < 0$       **n**  $4(5 - 2x) > 3(7 - 2x)$       **o**  $3(4u - 1) - 5(u - 3) < 9$

3 Find the set of values of  $x$  for which

- a**  $x^2 - 4x + 3 < 0$       **b**  $x^2 - 4 \leq 0$       **c**  $15 + 8x + x^2 < 0$       **d**  $x^2 + 2x \leq 8$   
**e**  $x^2 - 6x + 5 > 0$       **f**  $x^2 + 4x > 12$       **g**  $x^2 + 10x + 21 \geq 0$       **h**  $22 + 9x - x^2 > 0$   
**i**  $63 - 2x - x^2 \leq 0$       **j**  $x^2 + 11x + 30 > 0$       **k**  $30 + 7x - x^2 > 0$       **l**  $x^2 + 91 \geq 20x$

4 Solve each inequality.

- a**  $2x^2 - 9x + 4 \leq 0$       **b**  $2r^2 - 5r - 3 < 0$       **c**  $2 - p - 3p^2 \geq 0$   
**d**  $2y^2 + 9y - 5 > 0$       **e**  $4m^2 + 13m + 3 < 0$       **f**  $9x - 2x^2 \leq 10$   
**g**  $a^2 + 6 < 8a - 9$       **h**  $x(x + 4) \leq 7 - 2x$       **i**  $y(y + 9) > 2(y - 5)$   
**j**  $x(2x + 1) > x^2 + 6$       **k**  $u(5 - 6u) < 3 - 4u$       **l**  $2t + 3 \geq 3t(t - 2)$   
**m**  $(y - 2)^2 \leq 2y - 1$       **n**  $(p + 2)(p + 3) \geq 20$       **o**  $2(13 + 2x) < (6 + x)(1 - x)$

5 Giving your answers in terms of surds, find the set of values of  $x$  for which

- a**  $x^2 + 2x - 1 < 0$       **b**  $x^2 - 6x + 4 > 0$       **c**  $11 - 6x - x^2 > 0$       **d**  $x^2 + 4x + 1 \geq 0$

6 Find the value or set of values of  $k$  such that

- a** the equation  $x^2 - 6x + k = 0$  has equal roots,  
**b** the equation  $x^2 + 2x + k = 0$  has real and distinct roots,  
**c** the equation  $x^2 - 3x + k = 0$  has no real roots,  
**d** the equation  $x^2 + kx + 4 = 0$  has real roots,  
**e** the equation  $kx^2 + x - 1 = 0$  has equal roots,  
**f** the equation  $x^2 + kx - 3k = 0$  has no real roots,  
**g** the equation  $x^2 + 2x + k - 2 = 0$  has real and distinct roots,  
**h** the equation  $2x^2 - kx + k = 0$  has equal roots,  
**i** the equation  $x^2 + kx + 2k - 3 = 0$  has no real roots,  
**j** the equation  $3x^2 + kx - x + 3 = 0$  has real roots.

# C1 ALGEBRA

## Worksheet K

1 Solve each of the following inequalities.

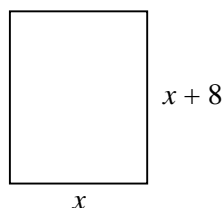
a  $\frac{1}{2}y + 3 > 2y - 1$

b  $x^2 - 8x + 12 \geq 0$

2 Find the set of integers,  $n$ , for which

$$2n^2 - 5n < 12.$$

3



The diagram shows a rectangular birthday card which is  $x$  cm wide and  $(x + 8)$  cm tall.

Given that the height of the card is to be at least 50% more than its width,

a show that  $x \leq 16$ .

Given also that the area of the front of the card is to be at least  $180 \text{ cm}^2$ ,

b find the set of possible values of  $x$ .

4 Find the set of values of  $x$  for which

$$(3x - 1)^2 < 5x - 1.$$

5 Given that  $x - y = 8$ ,

and that  $xy \leq 240$ ,

find the maximum value of  $(x + y)$ .

6 Solve the inequality

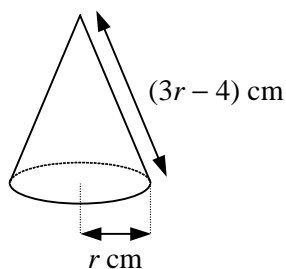
$$(3t + 1)(t - 4) \geq 2t(t - 7).$$

7 Given that the equation  $2x(x + 1) = kx - 8$  has real and distinct roots,

a show that  $k^2 - 4k - 60 > 0$ ,

b find the set of possible values of  $k$ .

8



A party hat is designed in the shape of a right circular cone of base radius  $r$  cm and slant height  $(3r - 4)$  cm.

Given that the height of the cone must not be more than 24 cm, find the maximum value of  $r$ .

# C1 ALGEBRA

## Worksheet L

- 1 a Find the value of  $x$  such that  

$$2^{x-1} = 16. \quad (3)$$
- b Find the value of  $y$  such that  

$$2(3^y - 10) = 34. \quad (2)$$
- 2 a Express  $x^2 - 6x + 11$  in the form  $(x + a)^2 + b$ . (2)
- b Sketch the curve  $y = x^2 - 6x + 11$ , and show the coordinates of the turning point of the curve. (3)
- 3 a Express  $(12\frac{1}{4})^{-\frac{1}{2}}$  as an exact fraction in its simplest form. (2)
- b Solve the equation  

$$3x^{-3} = 7\frac{1}{9}. \quad (3)$$
- 4 Solve the equation  

$$x\sqrt{12} + 9 = x\sqrt{3},$$
giving your answer in the form  $k\sqrt{3}$ , where  $k$  is an integer. (4)
- 5 a Solve the equation  

$$x^2 + 10x + 13 = 0,$$
giving your answers in the form  $a + b\sqrt{3}$ , where  $a$  and  $b$  are integers. (4)
- b Hence find the set of values of  $x$  for which  

$$x^2 + 10x + 13 > 0. \quad (2)$$
- 6 Solve the equations
- a  $7(6x - 7) = 9x^2$  (3)
- b  $\frac{2}{y+1} + 1 = 2y$  (4)
- 7 Solve the simultaneous equations  

$$x - y + 3 = 0$$

$$3x^2 - 2xy + y^2 - 17 = 0 \quad (6)$$
- 8 a Find the value of  $x$  such that  

$$x^{\frac{3}{2}} = 64. \quad (2)$$
- b Given that  

$$\frac{\sqrt{3}+1}{2\sqrt{3}-3} \equiv a + b\sqrt{3},$$
find the values of the rational constants  $a$  and  $b$ . (4)
- 9 The point  $P(2k, k)$  lies within a circle of radius 3, centre  $(2, 4)$ .
- a Show that  $5k^2 - 16k + 11 < 0$ . (4)
- b Hence find the set of possible values of  $k$ . (3)



## C1 ALGEBRA

## Worksheet L continued

10 Solve each of the following inequalities.

a  $4x - 1 \leq 2x + 6$  (2)

b  $x(2x + 1) < 1$  (4)

11  $f(x) = 2x^2 - 8x + 5$ .

a Express  $f(x)$  in the form  $a(x + b)^2 + c$ , where  $a$ ,  $b$  and  $c$  are integers. (3)

b Write down the coordinates of the turning point of the curve  $y = f(x)$ . (1)

c Solve the equation  $f(x) = 0$ , giving your answers in the form  $p + q\sqrt{6}$ , where  $p$  and  $q$  are rational. (3)

12 Simplify

a  $\sqrt{12} - \frac{5}{\sqrt{3}}$  (3)

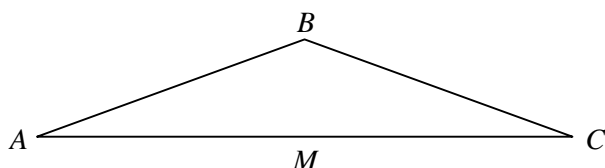
b  $\frac{(4\sqrt{x})^3}{16x}$  (2)

13 Given that the equation

$$x^2 - 2kx + k + 6 = 0$$

has no real roots, find the set of possible values of the constant  $k$ . (6)

14



The diagram shows triangle  $ABC$  in which  $AB = BC = 4 + \sqrt{3}$  and  $AC = 4 + 4\sqrt{3}$ .

Given that  $M$  is the mid-point of  $AC$ ,

a find the exact length  $BM$ , (4)

b show that the area of triangle  $ABC$  is  $6 + 2\sqrt{3}$ . (2)

15 Solve the equation

$$4^{2y+7} = 8^{y+3}. \quad (4)$$

16 Show that

$$(x^2 - x + 3)(2x^2 - 3x - 9) \equiv Ax^4 + Bx^3 + C,$$

where  $A$ ,  $B$  and  $C$  are constants to be found. (4)

17  $f(x) = x^2 + 4x + k$ .

a By completing the square, find in terms of the constant  $k$  the roots of the equation  $f(x) = 0$ . (4)

b State the set of values of  $k$  for which the equation  $f(x) = 0$  has real roots. (1)

c Use your answers to part a to solve the equation

$$x^2 + 4x - 4 = 0,$$

giving your answers in the form  $a + b\sqrt{2}$ , where  $a$  and  $b$  are integers. (2)

# C1 ALGEBRA

## Worksheet M

- 1 Solve the inequality

$$(x + 1)(x + 2) \leq 12. \quad (5)$$

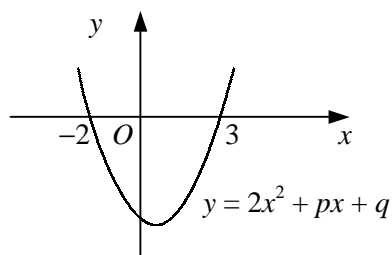
- 2 a Express  $2^{\frac{7}{2}} - 2^{\frac{3}{2}}$  in the form  $k\sqrt{2}$ . (2)

- b Show that

$$(\sqrt{x} + 6)^2 + (2\sqrt{x} - 3)^2$$

can be written in the form  $ax + b$  where  $a$  and  $b$  are integers to be found. (3)

- 3



The diagram shows the curve with equation  $y = 2x^2 + px + q$ , where  $p$  and  $q$  are constants, which crosses the  $x$ -axis at the points with coordinates  $(-2, 0)$  and  $(3, 0)$ .

- a Show that  $p = -2$  and find the value of  $q$ . (4)

- b Find the coordinates of the turning point of the curve. (3)

- 4 Solve the equation

$$2(x - \sqrt{32}) = \sqrt{98} - x,$$

giving your answer in the form  $k\sqrt{2}$ . (4)

- 5 Given that the equation

$$kx^2 - 4kx + 3 = 0,$$

where  $k$  is a constant, has real and distinct roots,

- a show that  $k(4k - 3) > 0$ , (3)

- b find the set of possible values of  $k$ . (2)

- 6 Solve the simultaneous equations

$$4^{2x} = 2^{y-1}$$

$$9^{4x} = 3^{y+1} \quad (7)$$

- 7 a Find the values of the constants  $a$  and  $b$  such that

$$x^2 - 7x + 9 \equiv (x + a)^2 + b. \quad (3)$$

- b Hence, write down an equation of the line of symmetry of the curve  $y = x^2 - 7x + 9$ . (1)

- 8 a Solve the inequality

$$y^2 - 2y - 15 < 0. \quad (3)$$

- b Find the exact values of  $x$  for which

$$\frac{x}{x-3} = \frac{4}{2-x}. \quad (5)$$

## C1 ALGEBRA

## Worksheet M continued

- 9 Solve the equation

$$2^{x^2+2} = 8^x. \quad (5)$$

- 10 Giving your answers in terms of surds, solve the equations

a  $t(1 - 2t) = 3(t - 5)$  (4)

b  $x^4 - x^2 - 6 = 0$  (4)

- 11 Find the set of values of  $x$  for which

$$21 - 4x - x^2 \leq 0. \quad (4)$$

- 12 a Given that  $y = 3^x$  express  $3^{2x+2}$  in terms of  $y$ . (2)

- b Hence, or otherwise, solve the equation

$$3^{2x+2} - 10(3^x) + 1 = 0. \quad (4)$$

- 13 a Express  $5\sqrt{3}$  in the form  $\sqrt{k}$ . (2)

- b Hence find the integer  $n$  such that

$$n < 5\sqrt{3} < n + 1. \quad (3)$$

- 14 Solve the simultaneous equations

$$\begin{aligned} 2x^2 - y^2 - 7 &= 0 \\ 2x - 3y + 7 &= 0 \end{aligned} \quad (8)$$

- 15 Express each of the following in the form  $a + b\sqrt{2}$ , where  $a$  and  $b$  are integers.

a  $\frac{\sqrt{48} - \sqrt{600}}{\sqrt{12}}$  (3)

b  $\frac{\sqrt{2}}{4 + 3\sqrt{2}}$  (4)

- 16 Given that  $5^{x+1} = 25^{y-3}$ ,

- a find an expression for  $y$  in terms of  $x$ . (4)

Given also that  $16^{x-1} = 4^z$ ,

- b find an expression for  $z$  in terms of  $y$ . (4)

- 17 a By completing the square, find in terms of the constant  $k$  the roots of the equation

$$x^2 - 2kx - k = 0. \quad (4)$$

- b Hence, find the set of values of  $k$  for which the equation has real roots. (3)

- 18 a Given that  $y = x^{\frac{1}{5}}$ , show that the equation

$$x^{-\frac{1}{5}} - x^{\frac{1}{5}} = \frac{3}{2}$$

can be written as

$$2y^2 + 3y - 2 = 0. \quad (3)$$

- b Hence find the values of  $x$  for which

$$x^{-\frac{1}{5}} - x^{\frac{1}{5}} = \frac{3}{2}. \quad (4)$$

**C1 ALGEBRA****Worksheet N**

1 a Express  $(\frac{2}{3})^{-2}$  as an exact fraction in its simplest form. (2)

b Solve the equation

$$x^{\frac{3}{2}} - 27 = 0. \quad (3)$$

2 Solve the simultaneous equations

$$x + 3y = 16$$

$$x^2 - xy + 2y^2 = 46 \quad (7)$$

3 Simplify

a  $\sqrt{192} - 2\sqrt{12} + \sqrt{75}$  (4)

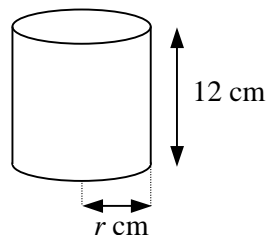
b  $(2 + \sqrt{3})(5 - 2\sqrt{3})$  (3)

4  $f(x) \equiv x^2 - 4\sqrt{2}x + 11.$

a Express  $f(x)$  in the form  $a(x + b)^2 + c$  stating the exact values of the constants  $a$ ,  $b$  and  $c$ . (4)

b Sketch the curve  $y = f(x)$ , showing the coordinates of the turning point of the curve and of any points of intersection of the curve with the coordinate axes. (3)

5



A sealed metal container for food is a cylinder of height 12 cm and base radius  $r$  cm.

Given that the surface area of the container must be at most  $128\pi \text{ cm}^2$ ,

a show that  $r^2 + 12r - 64 \leq 0$ . (3)

b Hence find the maximum value of  $r$ . (4)

6 Find the non-zero value of  $x$  for which

$$(2\sqrt{x})^3 = 4x. \quad (4)$$

7 a Write down the value of  $x$  such that  $2^x = 32$ . (1)

b Solve the equation

$$32^{y+1} = 4^y. \quad (3)$$

8 a Given that  $t = \sqrt{x}$ , express  $x - 5\sqrt{x}$  in terms of  $t$ . (1)

b Hence, or otherwise, solve the equation

$$x - 5\sqrt{x} + 6 = 0. \quad (4)$$

9 Prove, by completing the square, that there is no real value of the constant  $k$  for which the equation  $x^2 + kx + 3 + k^2 = 0$  has real roots. (6)

## C1 ALGEBRA

## Worksheet N continued

- 10 a Find the value of  $x$  such that  

$$8^{2x-1} = 32. \quad (3)$$
- b Find the value of  $y$  such that  

$$\left(\frac{1}{3}\right)^{y-2} = 81. \quad (3)$$
- 11 Solve the inequality  

$$x(2x - 7) < (x - 2)^2. \quad (5)$$
- 12 Express  

$$\frac{2}{3\sqrt{2}-4} - \frac{3-\sqrt{2}}{\sqrt{2}+1}$$
in the form  $a + b\sqrt{2}$ , where  $a$  and  $b$  are integers. (6)
- 13 a Solve the equation  

$$6y^2 + 25y - 9 = 0. \quad (3)$$
- b Find the values of the constant  $k$  such that the equation  

$$x^2 + kx + 16 = 0$$
has equal roots. (3)
- 14 a Given that  $y = 2^x$ ,  
i show that  $4^x = y^2$ ,  
ii express  $2^{x-1}$  in terms of  $y$ . (4)
- b By using your answers to part a, or otherwise, find the values of  $x$  for which  

$$4^x - 9(2^{x-1}) + 2 = 0. \quad (4)$$
- 15 Find the pairs of values  $(x, y)$  which satisfy the simultaneous equations  

$$x^2 + 2xy + y^2 = 9$$

$$x - 3y = 1 \quad (7)$$
- 16 a Prove, by completing the square, that the roots of the equation  $x^2 + ax + b = 0$  are given by  

$$x = \frac{-a \pm \sqrt{a^2 - 4b}}{2}. \quad (6)$$
- b Hence, find an expression for  $b$  in terms of  $a$  such that the equation  $x^2 + ax + b = 0$  has a repeated root. (2)
- 17 
$$f(x) \equiv 2x^2 - 12x + 19.$$
- a Prove that  $f(x) \geq 1$  for all real values of  $x$ . (5)
- b Find the set of values of  $x$  for which  $f(x) < 9$ . (4)
- 18 a Express  $(1 - \sqrt{5})^2$  in the form  $a + b\sqrt{5}$ . (2)
- b Hence, or otherwise, solve the equation  

$$y^2 = 3 - \sqrt{5},$$
giving your answers in the form  $c\sqrt{2} + d\sqrt{10}$ , where  $c$  and  $d$  are exact fractions. (6)