
CORE 1 MODULE REVISION SHEET

The C1 exam is 1 hour 30 minutes long and is in two sections.

Section A (36 marks) 8 – 10 short questions worth no more than 5 marks each.

Section B (36 marks) 3 questions worth 12 marks each.

You are **not** allowed **any** calculator.

Before you go into the exam make sure you are fully aware of the contents of the formula booklet you receive. Also be sure not to panic; it is not uncommon to get stuck on a question (I've been there!). Just continue with what you can do and return at the end to the question(s) you have found hard. If you have time check all your work, especially the first question you attempted... always an area prone to error.

J M S

1. Basic Algebra

- Changing the subject of an equation. For example in $y = \sqrt{x+6}$, y is the subject of the equation. To change the subject to x we merely need to re-arrange to $x = y^2 - 6$. A harder example is make to make g the subject of $T = 2\pi\sqrt{\frac{l}{g}}$.

$$2\pi\sqrt{\frac{l}{g}} = T \quad \Rightarrow \quad \frac{l}{g} = \left(\frac{T}{2\pi}\right)^2 \quad \Rightarrow \quad g = l \left(\frac{2\pi}{T}\right)^2 = \frac{4\pi^2 l}{T^2}.$$

- Factorising quadratics is a *must!* When the x^2 coefficient (the number in front of the x^2) is one this is easy. Just spot two numbers which multiply to the constant and add to the x coefficient. For example with $x^2 + 8x + 15$ we need to find two numbers which multiply to 15 and sum to 8; clearly 3 and 5. So we split up the $8x$ into $5x + 3x$.

$$\begin{aligned}x^2 + 8x + 15 &= x^2 + 5x + 3x + 15 \\ &= x(x + 5) + 3(x + 5) \quad \dagger \\ &= (x + 3)(x + 5).\end{aligned}$$

Notice that the brackets in the \dagger line are the same. If this is not the case then you have made a mistake. In the simple case when the x^2 coefficient is one, you should be able to do this without any working, because the 3 and 5 appear unaltered in the factorisation.

- If the x^2 coefficient is not one then more work is required. You need to multiply the x^2 coefficient by the constant term and then find 2 numbers which multiply to this and sum to the x coefficient. For example with $6x^2 + x - 12$ we calculate $6 \times -12 = -72$ so the two numbers are clearly 9 and -8 . So

$$\begin{aligned}6x^2 + x - 12 &= 6x^2 + 9x - 8x - 12 &= 6x^2 - 8x + 9x - 12 \\ &= 3x(2x + 3) - 4(2x + 3) &= 2x(3x - 4) + 3(3x - 4) \\ &= (3x - 4)(2x + 3) &= (2x + 3)(3x - 4).\end{aligned}$$

Notice that it does not matter which way round we write the $9x$ and $-8x$.

- For quadratics that cannot be factorised we need to use the formula. For $ax^2 + bx + c = 0$ the solution is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \quad 1$$

$b^2 - 4ac$ is the *discriminant*. If it is positive then there are two *distinct* roots. If it is zero then there exists only one root and it is *repeated*. If it is negative then there are no roots.²

- Completing the square. All about halving the x coefficient into the bracket and then correcting the constant term. For example $x^2 - 6x + 10 = (x - 3)^2 - 9 + 10 = (x - 3)^2 + 1$. If the x^2 coefficient isn't one then need to factorise it out. For example

$$\begin{aligned} -2x^2 + 4x - 8 &= -2[x^2 - 2x + 4] \\ &= -2[(x - 1)^2 - 1 + 4] \\ &= -2[(x - 1)^2 + 3] \\ &= -2(x - 1)^2 - 6. \end{aligned}$$

From this we can find the maximum or minimum of the quadratic. For $y = -2(x - 1)^2 - 6$ it is when $x = 1$ (to make the bracket 0) and therefore $y = -6$. In this case $(1, -6)$ is a maximum due to negative x^2 coefficient.

- Simultaneous equations. Use either substitution or elimination. For example $\begin{matrix} 2x+3y=7; \\ 3x+y=7 \end{matrix}$;
 1. *Elimination*; $\begin{matrix} 2x+3y=7 \\ 3x+y=7 \end{matrix} \Rightarrow \begin{matrix} 2x+3y=7 \\ 9x+3y=21 \end{matrix}$. Subtracting to get $7x = 14 \Rightarrow x = 2$. Then $6 + y = 7 \Rightarrow y = 1$.
 2. *Substitution*; $y = 7 - 3x$. Put this into first equation to get $2x + 3(7 - 3x) = 7 \Rightarrow -7x = -14 \Rightarrow x = 2 \Rightarrow y = 7 - 6 = 1$.
- You must also know how to handle algebraic fractions and how to write two algebraic fractional expressions as one fraction. The general rules are

$$\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd} \quad \text{and} \quad \frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}.$$

Therefore to write $x - \frac{x}{x+1}$ as a single fraction we do the following

$$x - \frac{x}{x+1} = \frac{x}{1} - \frac{x}{x+1} = \frac{x(x+1) - x}{x+1} = \frac{x^2}{x+1}.$$

2. Co-ordinate Geometry

Lines

- Mid point of $(x_1, y_1), (x_2, y_2)$ is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.
- Length from (x_1, y_1) to (x_2, y_2) is (by Pythagoras) $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Be careful about negatives! Remember $(2 - (-3))^2 = (2 + 3)^2$.
- Gradient is defined to be $\frac{\text{difference in } y}{\text{difference in } x} = \frac{y_2 - y_1}{x_2 - x_1}$.
- Two lines with gradients m_1 and m_2 are at right angles (perpendicular) if $m_1 \times m_2 = -1$. So if a line has gradient -3 then the line perpendicular to it has gradient $\frac{1}{3}$.

¹This is *not* (as one student thinks) $x = -b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$.

²Actually there are no *real* roots but there are two *complex/imaginary* roots (Further Mathematicians only!).

- Lines can be written in many forms, the most common being $y = mx + c$ and $ax + by = c$. Any form can be converted to any other. For example write $3x - 2y = 4$ in the form $y = mx + c$.

$$\begin{aligned} 3x - 2y &= 4 \\ 2y &= 3x - 4 \\ y &= \frac{3}{2}x - 2. \end{aligned}$$

- Given two points (x_1, y_1) and (x_2, y_2) the line connecting them is given by $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$.
- Given one point (x_1, y_1) and a gradient m the line is given by $y - y_1 = m(x - x_1)$.

Circles

- Circles with centre $(0, 0)$ and radius r are expressed by $x^2 + y^2 = r^2$.
- Circles with centre (a, b) and radius r are expressed by $(x - a)^2 + (y - b)^2 = r^2$.
- By ‘completing the square’ you can convert circles of the form $x^2 + y^2 + \alpha x + \beta y + \gamma = 0$ into the form $(x - a)^2 + (y - b)^2 = r^2$. For example

$$\begin{aligned} x^2 + y^2 + 6x - 4y + 9 &= 0 \\ x^2 + 6x + y^2 - 4y + 9 &= 0 \\ (x + 3)^2 - 9 + (y - 2)^2 - 4 + 9 &= 0 \\ (x + 3)^2 + (y - 2)^2 &= 4. \end{aligned}$$

- When finding the intersection of a line and a circle it is easiest to substitute in the value of y from the line into the circle and solve the resulting quadratic. For example; find where the line $y = 2x - 1$ intersects to circle $(x - 3)^2 + (y - 2)^2 = 25$.

$$\begin{aligned} (x - 3)^2 + (y - 2)^2 &= 25 \\ (x - 3)^2 + (2x - 3)^2 &= 25 \\ x^2 - 6x + 9 + 4x^2 - 12x + 9 - 25 &= 0 \\ 5x^2 - 18x - 7 &= 0. \end{aligned}$$

Solve the quadratic (in this case by the formula) and then find the y values by substituting both x values into $y = 2x - 1$ (the original line). There will usually be 2 points of intersection except if the line doesn’t intersect the circle at all *or*...

- If a line touches the circle only once then it is a tangent to the circle at that point. The gradient of this tangent is perpendicular to the radius of the circle at that point.

3. Polynomials

- Must be able to visualise a polynomial curve quickly. This is determined by two things (summed up at top of page 80);
 1. Whether the largest power of x is odd or even. x^{1000} has very different shape from x^{1001} .

3. Calculate binomial coefficients either on calculator or by drawing a mini Pascal's Triangle to give $(a + b)^5 = 1a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + 1b^5$.
 4. Next notice that in our case $a = 2x$ and $b = -y$ and substitute in to get $(2x - y)^5 = 1(2x)^5 + 5(2x)^4(-y) + 10(2x)^3(-y)^2 + 10(2x)^2(-y)^3 + 5(2x)(-y)^4 + 1(-y)^5$.
 5. Tidying³ up we get $(2x - y)^5 = 32x^5 - 80x^4y + 80x^3y^2 - 40x^2y^3 + 10xy^4 - y^5$.
- Also of note is the way each *individual* component is constructed. For example; find the x^5 coefficient in the expansion of $(2 - 3x)^7$. The component with x^5 is given by $\binom{7}{5}(2)^2(-3x)^5 = -20412x^5$, so the coefficient is -20412 .
 - $\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$. For example $\binom{5}{2} = \frac{5!}{2!3!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{(2 \times 1) \times (3 \times 2 \times 1)} = 10$.
 - Know what $(1 + x)^n$ expands to.

4. Uncertainty

- If a number is stated to be $x = 24.7$ to 1 decimal place then $24.65 \leq x < 24.75$.
- Absolute error is the difference (by convention positive) between the value obtained and the true value.
- Relative error = $\frac{\text{absolute error}}{\text{true value}}$.
- Percentage error = relative error $\times 100$.
- Treat linear inequalities like equations except when multiplying or dividing by a negative number when you reverse the sign. For example

$$\begin{aligned} 2x + 4 &< 3x + 2 \\ -x &< -2 \\ x &> 2. \end{aligned}$$

- To solve quadratic inequalities:
 1. Get all terms over one side so that quadratic > 0 or quadratic < 0 .
 2. Is the quadratic \cup or \cap shaped?
 3. Solve the quadratic $= 0$ to find where it crosses x axis.
 4. Sketch the graph and read off solution. If it is quadratic > 0 then it is the region(s) above the x axis, and if quadratic < 0 then it is region(s) below the x axis.
 5. If one region then express as one triple inequality (e.g. $-2 < x < 5$) and if two regions then two *separate* inequalities $x > 5$ or $x < -2$.

5. Indices

- $(ab)^m = a^m \times b^m$. For example $6^5 = 2^5 \times 3^5$.
- When multiplying a number raised to different powers the powers *add*. Therefore $a^m \times a^n = a^{m+n}$. You can think of this as follows $2^2 \times 2^4 = (2 \times 2) \times (2 \times 2 \times 2 \times 2) = 2^6$.

³When the expansion is of the form (something – another thing)ⁿ, then the signs will alternate.

- Know that $a^{-m} = \frac{1}{a^m}$. Remember this by the standard result that $2^{-1} = \frac{1}{2}$. “When moving something from the bottom line of a fraction to the top (or vice versa), the sign changes.”
- From the above two results we can obtain the result $\frac{a^m}{a^n} = a^{m-n}$. This is derived thus; $\frac{a^m}{a^n} = a^m \times a^{-n} = a^{m-n}$ as required.
- We can also derive the important result $a^0 = 1$ for any $a \neq 0$. Derived by considering something like this; $a^0 = a^{1-1} = \frac{a^1}{a^1} = \frac{a}{a} = 1$.
- Know that $(a^m)^n = a^{mn}$. Think about it like this; $(a^3)^4 = a^3 \times a^3 \times a^3 \times a^3 = a^{12}$.
- The n^{th} root of a number can be expressed as a power thus; $\sqrt[n]{a} = a^{\frac{1}{n}}$.
- A few examples:
 1. Write 8 as a power of 4; well $8 = 2^3 = \left(4^{\frac{1}{2}}\right)^3 = 4^{\frac{3}{2}}$.
 2. Simplify $\sqrt[4]{16^3} = \left((2^4)^3\right)^{\frac{1}{4}} = (2^{12})^{\frac{1}{4}} = 2^3 = 8$.
 3. Simplify $\frac{12x^8y^{\frac{3}{2}}}{6x^6y^{\frac{5}{2}}} = 2x^{8-6}y^{\frac{3}{2}-\frac{5}{2}} = 2x^2y^{-1}$.
 4. Simplify $\sqrt{x^6y^4} \times \sqrt[3]{x^3y^{-6}} = (x^6y^4)^{\frac{1}{2}} \times (x^3y^{-6})^{\frac{1}{3}} = x^3y^2x^1y^{-2} = x^4$.
- You also need to be able to rationalise the denominator of certain types of surd expressions. For example to rationalise $\frac{9}{\sqrt{3}}$ is easy; just multiply by $\frac{\sqrt{3}}{\sqrt{3}}$ to obtain $\frac{9\sqrt{3}}{3} = 3\sqrt{3}$. In harder examples you must multiply the top and bottom of the fraction by the denominator with the sign ‘flipped’. For example

$$\frac{2 + 2\sqrt{3}}{5 - 2\sqrt{3}} = \frac{2 + 2\sqrt{3}}{5 - 2\sqrt{3}} \times \frac{5 + 2\sqrt{3}}{5 + 2\sqrt{3}} = \frac{10 + 4\sqrt{3} + 10\sqrt{3} + 12}{25 + 10\sqrt{3} - 10\sqrt{3} - 12} = \frac{22 + 14\sqrt{3}}{13}$$