CORE 1 Summary Notes

1 Linear Graphs and Equations

$y = mx + c$ (m: gradient, c: y intercept)

$$\text{gradient} = \frac{\text{increase in } y}{\text{increase in } x}$$

Gradient Facts

- Lines that have the same gradient are PARALLEL
- If 2 lines are PERPENDICULAR then $m_1 \times m_2 = -1$ or $m_2 = -\frac{1}{m_1}$

e.g. $2y = 4x - 8$

$y = 2x - 4$ gradient = 2

gradient of perpendicular line = -½

Finding the equation of a straight line

e.g. Find the equation of the line which passes through (2,3) and (4,8)

$$\text{GRADIENT} = \frac{y_1 - y_2}{x_1 - x_2} \qquad \text{GRADIENT} = \frac{3-8}{2-4} = \frac{-5}{-2} = \frac{5}{2}$$

Method 1

$y - y_1 = m(x - x_1)$

Using the point (2,3)

$$y - 3 = \frac{5}{2}(x - 2)$$

$$y = \frac{5}{2}x - 2$$

$$2y = 5x - 4$$

Method 2

$y = mx + c$

Using the point (2,3)

$$3 = \frac{5}{2} \times 2 + c$$

$$c = -2$$

$$y = \frac{5}{2}x - 2$$

$$2y = 5x - 4$$

Finding the Mid-Point

Given the points $(x_1\ y_1)$ and $(x_2\ y_2)$ the midpoint is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Finding the point of Intersection

Treat the equations of the graphs as simultaneous equations and solve

Find the point of intersection of the graphs $y = 2x - 7$ and $5x + 3y = 45$

Substituting $y = 2x - 7$ gives

$$
\begin{aligned}
5x + 3(2x - 7) &= 45 \\
5x + 6x - 21 &= 45 \\
11x &= 66 \\
x &= 6 \qquad y = 2 \times 6 - 7 \\
& \qquad\quad\; y = 5
\end{aligned}
$$

Point of intersection = (6, 5)

2 Surds

- A root such as $\sqrt{5}$ that cannot be written exactly as a fraction is IRRATIONAL
- An expression that involves irrational roots is in SURD FORM *e.g.* $3\sqrt{5}$
- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

e.g $\sqrt{75} - \sqrt{12}$

$$
\begin{aligned}
&= \sqrt{5 \times 5 \times 3} - \sqrt{2 \times 2 \times 3} \\
&= 5\sqrt{3} - 2\sqrt{3} \\
&= 3\sqrt{3}
\end{aligned}
$$

- RATIONALISING THE DENOMINATOR

  $3 + \sqrt{2}$ and $3 \sqrt{2}$ is called a pair of CONJUGATES

  The product of any pair of conjugates is always a rational number

  e.g. $(3 + \sqrt{2})(3 \sqrt{2}) = 9 \;\; 3\sqrt{2} + 3\sqrt{2} 2$

  $= 7$

Rationalise the denominator of $\dfrac{2}{1 - \sqrt{5}}$

$$\frac{2}{1 - \sqrt{5}} \times \frac{1 + \sqrt{5}}{1 + \sqrt{5}} = \frac{2 + 2\sqrt{5}}{1 - 5}$$

$$= \frac{2 + 2\sqrt{5}}{-4}$$

$$= \frac{-1 - \sqrt{5}}{2}$$

3. Quadratic Graphs and Equations

<u>Solution of quadratic equations</u>

- Factorisation

$$
\begin{aligned}
&x^2 - 3x - 4 = 0 \\
&(x + 1)(x - 4) = 0 \\
&x = -1 \text{ or } x = 4
\end{aligned}
$$

- Completing the square

$$x^2 - 4x - 3 = 0$$
$$(x-2)^2 - (2)^2 - 3 = 0$$
$$(x-2)^2 - 7 = 0$$
$$(x-2)^2 = 7$$
$$x - 2 = \pm\sqrt{7}$$
$$x = 2 + \sqrt{7} \text{ or } x = 2 - \sqrt{7}$$

- Using the formula to solve $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

E.g Solve $x^2 - 4x - 3 = 0$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1 \times (-3)}}{2 \times 1}$$
$$= \frac{4 \pm \sqrt{28}}{2}$$
$$= 2 \pm \sqrt{7}$$

- The graph of $y = ax^2 + bx + c$ crosses the y axis at $y = c$
  It crosses or touches the x-axis if the equation has real solutions

  The DISCRIMINANT of $ax^2 + bx + c = 0$ is the expression $b^2 - 4ac$
  If $b^2 - 4ac > 0$ there are 2 real distinct roots
  If $b^2 - 4ac = 0$ there is one repeated root
  If $b^2 - 4ac < 0$ there are no real roots

Graphs of Quadratic Functions

- The graph of any quadratic expression in x is called a PARABOLA

- The graph of $y - q = k(x - p)^2$ is a TRANSLATION of the graph $y = kx^2$

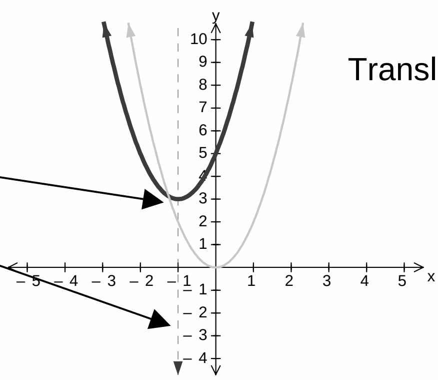
  In VECTOR notation this translation can be described as $\begin{bmatrix} p \\ q \end{bmatrix}$

  The equation can also be written as $y = k(x - p)^2 + q$
  The VERTEX of the graph is (p,q)
  The LINE OF SYMMETRY is x = p

$2x^2 + 4x + 5 = 2(x+1)^2 + 3$

Vertex (-1,3)

Line of symmetry x = -1



Translation of $y=2x^2$ $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$

4 Simultaneous Equations

- Simultaneous equations can be solved by substitution to eliminate one of the variables

Solve the simultanoeus equations $y = 2x - 7$ and $x^2 + xy + 2 = 0$

$y = 7 + 2x$

so $x^2 + x(7 + 2x) + 2 = 0$

$3x^2 + 7x + 2 = 0$

$(3x + 1)(x + 2) = 0$

$x = -\frac{1}{3}$ $y = 6\frac{1}{3}$ or $x = -2$ $y = 3$

- A pair of simultaneous equations can be represented as graphs and the solutions interpreted as points of intersection.
If they lead to a quadratic equation then the DISCRIMINANT tells you the geometrical relationship between the graphs of the functions
$b^2 - 4ac < 0$ no points of intersection
$b^2 - 4ac = 0$ 1 point of intersection
$b^2 - 4ac > 0$ 2 points of intersection

5 Inequalities

Linear Inequality

- Can be solved like a linear equation except
Multiplying or dividing by a negative value reverses the direction of the inequality sign

e.g Solve $-3x + 10 \leq 4$

$-3x + 10 \leq 4$

$-3x \leq -6$

$x \geq 2$

Quadratic Inequality

- Can be solved by either a graphical or algebraic approach.

e.g. solve the inequality $x^2 + 4x - 5 < 0$

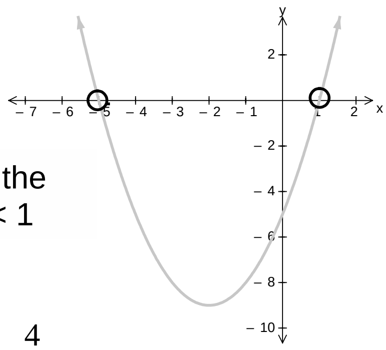
Algebraic $x^2 + 4x - 5 < 0$ factorising gives $(x + 5)(x - 1) < 0$

Using a sign diagram

x + 5	- - - 0 + + + + + + +
x – 1	- - - - - - - 0 + + +
(x + 5)(x – 1)	+ + + 0 - - - 0 + + +

The product is negative for $-5 < x < 1$

Graphical



The curve lies below the x–axis for $-5 < x < 1$

6 Polynomials

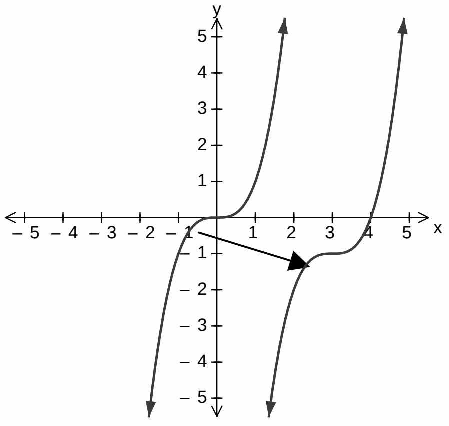
Translation of graphs

To find the equation of a curve after a translation of $\begin{bmatrix} p \\ q \end{bmatrix}$ replace x with (x-p) and replace y with (y - p)

e.g The graph of $y = x^3$ is translated by $\begin{bmatrix} 3 \\ -1 \end{bmatrix}$

The equation for the new graph is

$y=(x - 3)^3 -1$



Polynomial Functions

A polynomial is an expression which can be written in the form $a + bx + cx^2 + dx^3 + ex^4 + fx^5$ (a, b, c..are constants)

- Polynomials can be divided to give a QUOTIENT and REMAINDER

$x^2 -3x + 7$ — Qutoient

$x + 2 \quad x^3 - x^2 + x + 15$

x^3+2x^2

$-3x^2+ x$

$-3x^2- 6x$

$7x + 15$

$7x + 14$

1 — Remainder

- REMAINDER THEOREM
  When P(x) is divided by (x - a) the remainder is P(a)

- FACTOR THEOREM
  If P(a) = 0 then (x – a) is a factor of P(x)

e.g. The polynomial $f(x) = hx^3 -10x^2 + kx + 26$ has a factor of (x - 2)
When the polynomial is divided by (x+1) the remainder is 15.
Find the values of h and k.

Using the factor theorem f(2) = 0

$8h -40 + 2k + 26 = 0$

$8h +2k = 14$

Using the remainder theorem f(-1) = 15

$-h -10 –k + 26 = 14$

$h + k = 2$

Solving simultaneously $k = 2 –h$ $\quad 8h + 2(2 –h) = 14$

$6h + 4 = 14$

Equation of a Circle

- A circle with centre (0,0) and radius r has the equation $x^2+y^2=r^2$
- A circle with centre (a,b) and radius r has the equation $(x - a)^2+(y - b)^2=r^2$

e.g. A circle has equation $x^2+ y^2 + 2x – 6y= 0$

Find the radius of the circle and the coordinates of its centre.

$$x^2 + 2x + y^2 - 6y = 0$$

$$(x + 1)^2 - 1 + (y - 3)^2 - 9 = 0$$

$$(x + 1)^2 + (y - 3)^2 = 10$$

Centre (1, 3)   radius = √10

- A line from the centre of a circle to where a tangent touches the circle is perpendicular to the tangent. A **perpendicular** to a tangent is called a **NORMAL**.

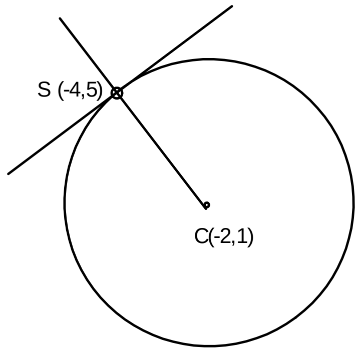
e.g. C(-2,1) is the centre of a circle and S(-4,5) is a point on the circumference. Find the equations of the normal and the tangent to the circle at S.

Gradient of SC is $\dfrac{1 - 5}{-2(-4)} = \dfrac{-4}{2} = -2$

Equation of SC $y = -2x + 7$

Gradient of the tangent $= -\dfrac{1}{-2} = \dfrac{1}{2}$

Equation of $y = \dfrac{1}{2}x + 7$



- Solving simultaneously the equations of a line and a circle results in a quadratic equation.

$b^2- 4ac > 0$ the line intersects the circle
$b^2- 4ac = 0$ the line is a tangent to the circle
$b^2- 4ac < 0$ the line fails to meet the circle

8   Rates of Change

- The gradient of a curve is defined as the gradient of the tangent

Gradient is denoted $\dfrac{dy}{dx}$ if y is given as a function of x

Gradient is denoted by f ’(x) if the function is given as f(x)

- The process of finding $\dfrac{dy}{dx}$ or f’(x) is known as DIFFERENTIATING
- <u>Derivatives</u>

$$f(x) = x^n \quad f'(x) = nx^{n-1}$$

$$f(x) = a \quad f'(x) = 0$$

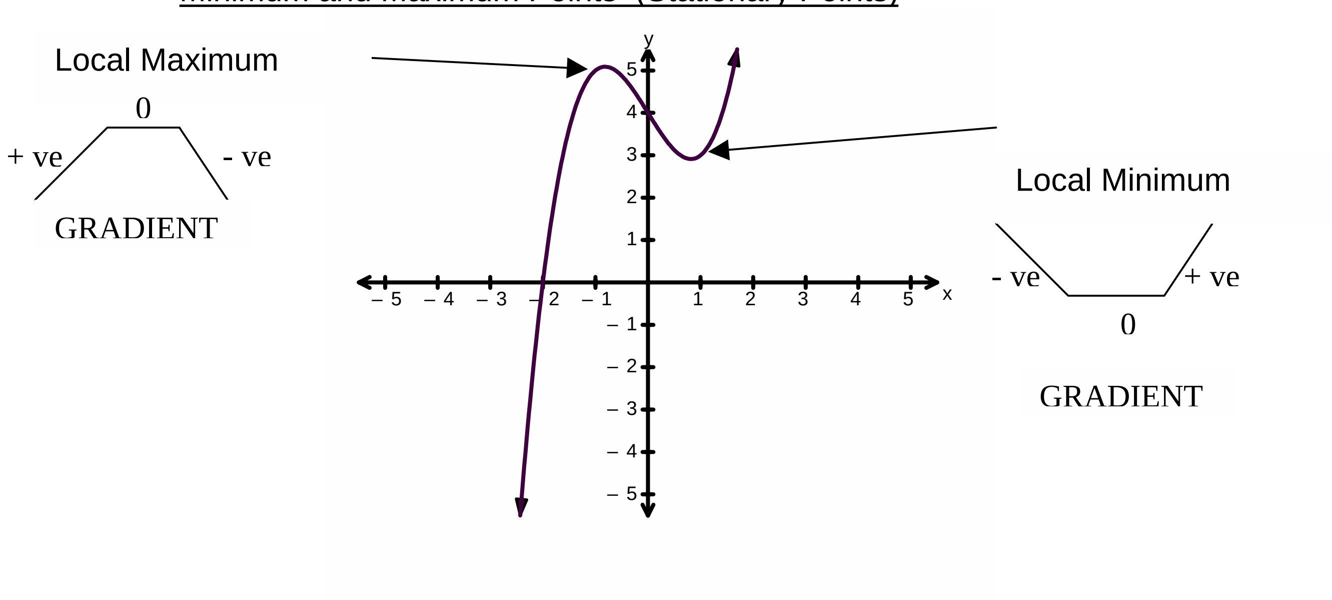
$$y = x^3 + 4x^2 - 3x + 6$$

$$\frac{dy}{dx} = 3x^2 + 8x - 3$$

9 Using Differentiation

- If the value of $\frac{dy}{dx}$ is positive at x = a, then y is increasing at x = a
- If the value of $\frac{dy}{dx}$ is negative at x = a, then y is decreasing at x = a
- Points where $\frac{dy}{dx} = 0$ are called stationary points

<u>Minimum and Maximum Points (Stationary Points)</u>



Stationary points can be investigated

- by calculating the gradients close to the point (see above)
- by differentiating again to find $\frac{d^2y}{dx^2}$ or f "(x)
  - o $\frac{d^2y}{dx^2} > 0$ then the point is a local minimum
  - o $\frac{d^2y}{dx^2} < 0$ then the point is a local maximum

<u>Optimisation Problems</u>

Optimisation means getting the best result. It might mean maximising (e.g. profit) or minimising (e.g. costs)

10 Integration

- Integration is the reverse of differentiation

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$$ ← Constant of integration

e.g. *Given that*

$f'(x) = 8x^3 - 6x$ and that $f(2) = 9$ find $f(x)$

$$f(x) = \int 8x^3 - 6x \, dx$$

$$= \frac{8x^4}{4} - \frac{6x^2}{2} + c$$

$$= 2x^4 - 3x^2 + c$$

To find c use f(2) = 9

$32 - 12 + c = 9$

$c = -11$

So $f(x) = 2x^4 - 3x^2 - 11$

11 Area Under a Graph

- The are under the graph of $y = f(x)$ between $x = a$ and $x = b$ is found by evaluating the definite integral

$$\int_a^b f(x)\, dx$$

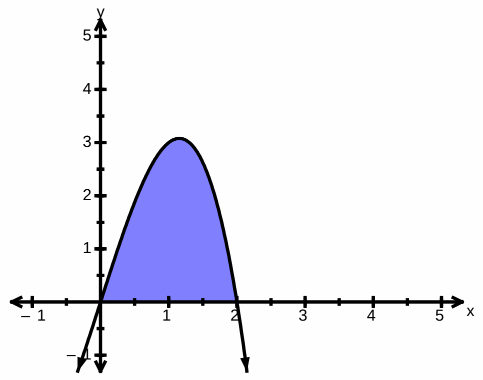
e.g. Calculate the area under the graph of $y = 4x - x^3$ between the lines $x = 0$ and $x = 2$

$$\int_0^2 4x - x^3 \, dx =$$

$$= 2x^2 - \frac{x^4}{4}$$

$$= (8 - 4) - (0 - 0)$$

$$= 4$$



- An area BELOW the x–axis has a NEGATIVE VALUE