Hidden Quadratic Equations [Ch. 4]

1 By substituting $y = x^2$, or otherwise, find the real roots of the equation $x^4 - 3x^2 - 4 = 0$. [4]

2 By substituting $t = x^{\frac{1}{2}}$, find the values of x for which $2x + 3 = 7x^{\frac{1}{2}}$. [5]

3 (i) Given that $t^{\frac{1}{4}} = y$, show that the equation $t^{\frac{1}{4}} + 2t^{-\frac{1}{4}} = 3$ may be written as $y^2 - 3y + 2 = 0$. [3]

(ii) Hence solve the equation $t^{\frac{1}{4}} + 2t^{-\frac{1}{4}} = 3$. [3]

4 Find, correct to 3 significant figures, all the roots of the equation

$$x^4 - 4x^2 + 1 = 0.$$ [5]

5 Solve the equation

$$x\sqrt{8} - 11 = \frac{3x}{\sqrt{2}},$$

giving your answer in the form $k\sqrt{2}$, where k is an integer. [3]

6 (i) Given that $\sqrt{x} = y$, show that the equation

$$\sqrt{x} + \frac{10}{\sqrt{x}} = 7$$

may be written as

$$y^2 - 7y + 10 = 0.$$ [3]

(ii) Hence solve the equation

$$\sqrt{x} + \frac{10}{\sqrt{x}} = 7.$$ [3]

7 (i) By letting $t = x^{\frac{1}{3}}$, show that the equation

$$x^{\frac{2}{3}} + (125x)^{\frac{1}{3}} = 14$$

may be written as

$$t^2 + 5t - 14 = 0.$$ [1]

(ii) Hence solve the equation

$$x^{\frac{2}{3}} + (125x)^{\frac{1}{3}} = 14.$$ [4]