Hidden Quadratic Equations (Ch 4)

1. Let $y = x^2$.

$x^4 - 3x^2 - 4 = 0$

$y^2 - 3y - 4 = 0$

$(y-4)(y+1) = 0$

$y = 4$ or $y = -1$

$x^2 = 4$ ~~$x^2 = -1$~~ ← (no real solutions)

$x = 2$ or $x = -2$

2. $t = x^{1/2}$.

$2x + 3 = 7x^{1/2}$

$2t^2 + 3 = 7t$

$2t^2 - 7t + 3 = 0$

$(2t-1)(t-3) = 0$

$2t = 1$ $\quad t = 3$

$t = 1/2$ $\quad x^{1/2} = 3$

$x^{1/2} = 1/2$ $\quad x = 9$

$x = 1/4$

3. (i) $t^{1/4} = y$.

$t^{1/4} + 2t^{-1/4} = 3$

$y + \frac{2}{y} = 3$

$y^2 + 2 = 3y$

$y^2 - 3y + 2 = 0$

(ii)

$(y-2)(y-1) = 0$

$y = 2$ or $y = 1$

$t^{1/4} = 2$ $\quad t^{1/4} = 1$

$t = 16$ $\quad t = 1$

4. $x^4 - 4x^2 + 1 = 0$. Let $y = x^2$.

$y^2 - 4y + 1 = 0$

$y = \frac{4 \pm \sqrt{16 - 4}}{2}$

$= \frac{4 \pm \sqrt{12}}{2}$

$= 2 \pm \sqrt{3}$

$\therefore x^2 = 2 \pm \sqrt{3}$

$x = \pm\sqrt{2+\sqrt{3}}$ or $\pm\sqrt{2-\sqrt{3}}$

$x = 1.93, -1.93, 0.518, -0.518$

5. $x\sqrt{8} - 11 = \frac{3x}{\sqrt{2}}$

$x\sqrt{16} - 11\sqrt{2} = 3x$

$4x - 11\sqrt{2} = 3x$

$x - 11\sqrt{2} = 0$

$x = 11\sqrt{2}$

6. (i) $\sqrt{x} = y$.

$\frac{\sqrt{x} + 10}{\sqrt{x}} = 7$

$\frac{y + 10}{y} = 7$

$y^2 + 10 = 7y$

$y^2 - 7y + 10 = 0$

(ii) $(y-5)(y-2) = 0$

$y = 5$ or $y = 2$

$\sqrt{x} = 5$ $\quad$ $\sqrt{x} = 2$

$x = 25$ $\quad$ $x = 4$

7. (i) $t = x^{1/3}$

$x^{2/3} + (125x)^{1/3} = 14$

$x^{2/3} + 5x^{1/3} - 14 = 0$

$t^2 + 5t - 14 = 0$

(ii) $(t+7)(t-2) = 0$

$t = -7$ or $t = 2$

$x^{1/3} = -7$ $\quad$ $x^{1/3} = 2$

$x = (-7)^3$ $\quad$ $x = 2^3$

$= -343$ $\quad$ $= 8$