Co-ordinate Geometry ch 1.

1. (i) $4y - 3x + 2 = 0$

$4y = 3x - 2$

$y = \frac{3}{4}x - \frac{1}{2}$ gradient $= \frac{3}{4}$

(ii) $m_1 = \frac{3}{4}$, $m_2 = -\frac{4}{3}$

$y - y_1 = m(x - x_1)$

$y - 0 = -\frac{4}{3}(x - 0)$

$y = -\frac{4}{3}x$

2. $A(-2,3)$ $B(2,5)$ $C(4,1)$

(i) AB: $m_1 = \frac{5-3}{2--2} = \frac{2}{4} = \frac{1}{2}$

BC: $m_2 = \frac{1-5}{4-2} = \frac{-4}{2} = -2$

CA: $m_3 = \frac{1-3}{4--2} = \frac{-2}{6} = \frac{-1}{3}$

(ii) $m_1 \times m_2 = \frac{1}{2} \times -2 = -1$ ∴ AB and BC are perpendicular and the △ is right-angled.

3. midpoint $= \left(\frac{2-3}{2}, \frac{4+1}{2}\right) = \left(-\frac{1}{2}, \frac{5}{2}\right)$

4. $P\left(\frac{1+3}{2}, \frac{7+1}{2}\right) = (2,4)$

$5y + x = 7$

$5y = -x + 7$

$y = -\frac{1}{5}x + \frac{7}{5}$ ∴ $m_1 = -\frac{1}{5}$, $m_2 = 5$

$y - y_1 = m(x - x_1)$

$y - 4 = 5(x - 2)$

$\underline{y = 5x - 6}$

5. (i) $2x+3y=7$

$3y=-2x+7$

$y=-\frac{2}{3}x+\frac{7}{3}$ $\qquad M_1=-\frac{2}{3}$.

$y-y_1=m(x-x_1)$

$y-1=-\frac{2}{3}(x-5)$

$3y-3=-2x+10$

$\underline{2x+3y-13=0}$

(ii) $\left(\frac{a+7}{2}, \frac{-3+b}{2}\right)=(5,1)$

$a+7=10 \qquad -3+b=2$

$a=3 \qquad b=5$

6. $A(1,2) \quad B(3,4) \quad C(9,-2)$

(i) gradient of $AB = m_1 = \frac{4-2}{3-1} = \frac{2}{2} = 1$

(ii) gradient of $BC = m_2 = \frac{-2-4}{9-3} = \frac{-6}{6} = -1$

$m_1 m_2 = 1 \times -1 = -1$ $\qquad \therefore$ AB and BC are perpendicular.

(iii) $y-4=-(x-3)$

$y=-x+7$

(iv) $AB=\sqrt{(3-1)^2+(4-2)^2}=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$ $\qquad \therefore p=2$

(v) $BC=\sqrt{(-2-4)^2+(9-3)^2}=\sqrt{36+36}=\sqrt{72}=6\sqrt{2}$

Area of $ABC = \frac{1}{2}AB \times BC = \frac{2\sqrt{2}\times 6\sqrt{2}}{2} = 12$

7. (i) $2x+y=5$

$y=-2x+5$

$y-5=-2(x-2)$

$\underline{y=-2x+9}$

(ii)(a) $m=\frac{1}{2}$

$y-k=\frac{1}{2}(x-1)$

$y=\frac{1}{2}x-\frac{1}{2}+k$

(b) $0=-\frac{1}{2}+k$

$\therefore k=\frac{1}{2}$ when l passes through the origin.

8. (i) $A(2,3)$ $l_1: x+4y=31$

$y=-\frac{1}{4}x+\frac{31}{4}$

$m_1=-\frac{1}{4}$ $m_2=4$

$l_2: y-3=4(x-2)$

$y=4x-5$

(ii) $-\frac{1}{4}x+\frac{31}{4}=4x-5$

$\frac{51}{4}=\frac{17}{4}x$

$x=3, y=7$ $\therefore$ l_1 and l_2 intersect at $(3,7)$

(iii) M is the midpoint of AC.

$\therefore \left(\frac{2+x}{2}, \frac{3+y}{2}\right)=(3,7)$

$x=4, y=11$ $\therefore C=(4,11)$.

9. (i) $4x-3y+5=0$ (l_1)

$3y=4x+5$

$y=\frac{4}{3}x+\frac{5}{3}$ $m_1=\frac{4}{3}$

(ii) $m_2=-\frac{3}{4}$

$y-2=-\frac{3}{4}(x-1)$

$4y-8=-3x+3$

$3x+4y-11=0$ (l_2)

(iii) $l_1: 4x-3y+5=0$

At x-axis: $4x+5=0$

$x=-\frac{5}{4}$ $\therefore P=\left(-\frac{5}{4}, 0\right)$

$l_2: 3x+4y-11=0$

At y-axis: $4y-11=0$

$y=\frac{11}{4}$ $\therefore Q=\left(0, \frac{11}{4}\right)$

midpoint of PQ $=\left(\frac{-5/4}{2}, \frac{11/4}{2}\right)=\left(\frac{-5}{8}, \frac{11}{8}\right)$

10. (i) $r = \frac{1}{2}\sqrt{(10-2)^2+(3-9)^2} = \frac{1}{2}\sqrt{64+36} = \frac{1}{2}\sqrt{100} = 5$

(ii) $m_1 = \frac{3-9}{10-2} = \frac{-6}{8} = \frac{-3}{4}$

$y - 9 = -\frac{3}{4}(x-2)$

$4y - 36 = -3x + 6$

$3x + 4y - 42 = 0$

(iii) $m_2 = \frac{4}{3}$

$y - 3 = \frac{4}{3}(x-10)$

$3y - 9 = 4x - 40$

$4x - 3y - 31 = 0$

11. $A(3,4)$ $B(1,k)$ $C(4,-3)$

(i) midpoint of $AC = \left(\frac{3+4}{2}, \frac{4-3}{2}\right) = \left(\frac{7}{2}, \frac{1}{2}\right)$

(ii) $m_1 = \frac{k-4}{1-3} = \frac{k-4}{-2} = \frac{4-k}{2}$

(iii) AB is perpendicular to BC, so BC has gradient $\frac{2}{k-4}$.

gradient of $BC = \frac{k--3}{1-4} = \frac{k+3}{-3}$

$\therefore \frac{k+3}{-3} = \frac{2}{k-4}$

$(k+3)(k-4) = -6$

$k^2 - k - 12 = -6$

$k^2 - k - 6 = 0$

$(k+2)(k-3) = 0$

$k = -2$ or $k = 3$

(iv) $k = 3$, Area of $ABCD = AB \times BC$

$= \sqrt{(3-1)^2+(4-3)^2} \times \sqrt{(1-4)^2+(3-3)^2}$

$= \sqrt{5} \times \sqrt{45}$

$= \sqrt{5} \times 3\sqrt{5}$

$= 15$

12. $A(-3,5)$ $B(9,-3)$ $C(7,7)$

(i) $m_1 = \frac{-3-5}{9--3} = \frac{-8}{12} = \frac{-2}{3}$

$y--3 = \frac{-2}{3}(x-9)$

$3y+9 = -2x+18$

$2x+3y-9=0$

(ii) $D = \left(\frac{-3+9}{2}, \frac{5-3}{2}\right) = (3,1)$

$m_2 = \frac{7-1}{7-3} = \frac{6}{4} = \frac{3}{2}$

$m_1 m_2 = \frac{-2}{3} \times \frac{3}{2} = -1$ $\quad\therefore$ AB and CD are perpendicular.

(iii) CD is the radius of the circle.

$r = CD = \sqrt{(7-3)^2+(7-1)^2} = \sqrt{16+36} = \sqrt{52} = 2\sqrt{13}$

area of circle $= \pi r^2 = \pi \times (2\sqrt{13})^2 = 52\pi$