Completing the Square [Ch. 4]

1 Express $2x^2 + 12x + 13$ in the form $a(x+b)^2 + c$. [4]

2 (i) Express $x^2 - 8x + 24$ in the form $(x-a)^2 + b$. [3]

(ii) Hence write down the coordinates of the vertex of the graph of $y = x^2 - 8x + 24$. [2]

3 Express $3x^2 + 12x + 7$ in the form $3(x+a)^2 + b$. [4]

4 (i) Find the constants a, b and c such that, for all values of x,

$$4x^2 + 40x + 97 = a(x+b)^2 + c.$$ [4]

(ii) Hence write down the equation of the line of symmetry of the curve $y = 4x^2 + 40x + 97$. [1]

5 (i) Express $x^2 + 8x + 18$ in the form $(x+a)^2 + b$. [2]

(ii) Sketch the graph of $y = x^2 + 8x + 18$, stating the coordinates of its vertex. [3]

6 (i) Find the constants a and b such that, for all values of x,

$$x^2 + 6x + 20 = (x+a)^2 + b.$$ [3]

(ii) Hence state the least value of $x^2 + 6x + 20$, and state also the value of x for which this least value occurs. [2]

(iii) Write down the greatest value of $\dfrac{1}{x^2 + 6x + 20}$. [1]

7 (i) Express $2x^2 + 4x - 1$ in the form

$$a[(x+p)^2 + q],$$

stating the values of the constants a, p and q. [4]

(ii) Sketch the graph of $y = 2x^2 + 4x - 1$, stating the coordinates of the vertex. [4]

(iii) The graph of $y = 2x^2 + 4x - 1$ is obtained from the graph of $y = x^2$ by a sequence of transformations. Describe such a sequence, specifying each transformation fully, and stating the order in which they are applied. [4]

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8 (i) Express $3x^2 + 4x + 1$ in the form $a(x+b)^2 + c$. [4]

(ii) Hence or otherwise find the coordinates of the vertex of the graph of $y = 3x^2 + 4x + 1$. [2]

9 (i) Express $4x^2 - 16x + 8$ in the form $a(x+b)^2 + c$. [4]

(ii) Hence find the coordinates of the vertex of the graph of $y = 4x^2 - 16x + 8$. [2]

(iii) Sketch the graph of $y = 4x^2 - 16x + 8$, giving the x-coordinates of the points where the graph meets the x-axis. [3]

10 (a) (i) Express the quadratic polynomial $x^2 - (2\sqrt{2})x + 4$ in the form $(x+a)^2 + b$, stating the exact values of the constants a and b. [3]

(ii) Hence write down the equation of the line of symmetry of the curve $y = x^2 - (2\sqrt{2})x + 4$. [1]

(b) The quadratic equation

$$x^2 + (k+1)x + 16 = 0$$

has two distinct real roots. Find the set of possible values of the constant k. [4]