Completing the Square (ch 4)

1. $2x^2+12x+13 = 2(x^2+6x)+13$
$= 2[(x+3)^2-9]+13$
$= 2(x+3)^2-5$

2. (i) $x^2-8x+24 = (x-4)^2+8$
(ii) vertex: $(4, 8)$

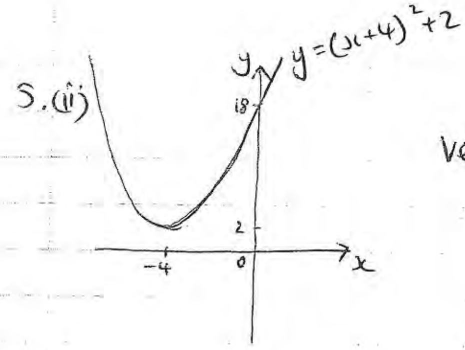
3. $3x^2+12x+7 = 3(x+2)^2-5$

4. (i) $4x^2+40x+97 = 4(x+5)^2-3$ $\quad a=4,\ b=5,\ c=-3$

(ii) Line of symmetry is $x=-5$

5. (i) $x^2+8x+18 = (x+4)^2+2$
(ii) next page.

5. (ii)



vertex has co-ordinates $(-4, 2)$.

6. (i) $x^2+6x+20 = (x+3)^2+11$

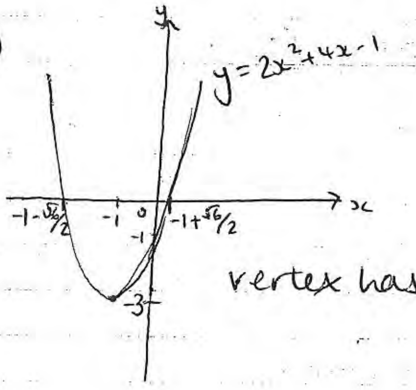
(ii) Least value is 11, which occurs when $x=-3$.

(iii) $\dfrac{1}{x^2+6x+20} = \dfrac{1}{(x+3)^2+11} < \dfrac{1}{11}$.

7. (i) $2x^2+4x-1 = 2\left(x^2+2x-\tfrac{1}{2}\right)$

$= 2\left[(x+1)^2-\tfrac{3}{2}\right]$ $\quad a=2,\ p=1,\ q=-\tfrac{3}{2}$.

(ii)



crosses x-axis when $(x+1)^2-\tfrac{3}{2}=0$

$x+1 = \pm\sqrt{\tfrac{3}{2}}$

$x=-1\pm\frac{\sqrt{6}}{2}$

vertex has co-ordinates $(-1,-3)$

(iii) $y=x^2 \longrightarrow y=2\left[(x+1)^2-\tfrac{3}{2}\right]$

Transformations are: translation $\begin{pmatrix}-1\\-\tfrac{3}{2}\end{pmatrix}$ followed by a stretch SF 2 in the y-direction.

8. (i) $3x^2+4x+1 = 3\left[x^2+\tfrac{4}{3}x\right]+1$

$= 3\left[\left(x+\tfrac{2}{3}\right)^2-\tfrac{4}{9}\right]+1$

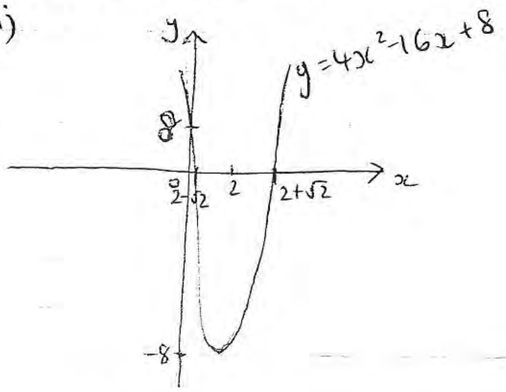
$= 3\left(x+\tfrac{2}{3}\right)^2-\tfrac{1}{3}$

(ii) vertex is $\left(-\tfrac{2}{3}, -\tfrac{1}{3}\right)$

9. (i) $4x^2-16x+8 = 4(x-2)^2-8$

(ii) vertex is $(2,-8)$

(iii)



graph crosses x-axis when $4(x-2)^2-8=0$

$4(x-2)^2=8$

$x-2=\pm\sqrt{2}$

$x=2\pm\sqrt{2}$

10. (a) (i) $x^2 - (2\sqrt{2})x + 4 = (x - \sqrt{2})^2 + 2$ $\quad a = -\sqrt{2},\ b = 2$

(ii) Line of symmetry is $x = \sqrt{2}$

(b) $x^2 + (k+1)x + 16 = 0$

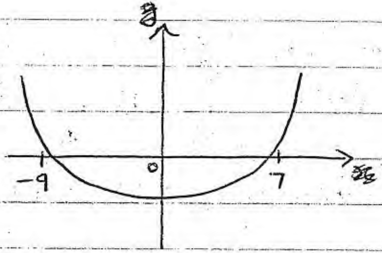
Two real roots $\Rightarrow b^2 - 4ac > 0$

$(k+1)^2 - 4 \times 16 > 0$

$k^2 + 2k + 1 - 64 > 0$

$k^2 + 2k - 63 > 0$

$(k+9)(k-7) > 0$



$k < -9$ or $k > 7$