1 You are given that $f(x) = x^3 + 6x^2 - x - 30$.

(i) Use the factor theorem to find a root of $f(x) = 0$ and hence factorise $f(x)$ completely. **[6]**

(ii) Sketch the graph of $y = f(x)$. **[3]**

(iii) The graph of $y = f(x)$ is translated by $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

Show that the equation of the translated graph may be written as

$$y = x^3 + 3x^2 - 10x - 24.$$ **[3]**

2 You are given that $f(x) = (x + 1)^2(2x - 5)$.

(i) Sketch the graph of $y = f(x)$. **[3]**

(ii) Express $f(x)$ in the form $ax^3 + bx^2 + cx + d$. **[2]**

3 **(i)** You are given that $f(x) = (x + 1)(x - 2)(x - 4)$.

(*A*) Show that $f(x) = x^3 - 5x^2 + 2x + 8$. **[2]**

(*B*) Sketch the graph of $y = f(x)$. **[3]**

(*C*) The graph of $y = f(x)$ is translated by $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$.

State an equation for the resulting graph. You need not simplify your answer.

Find the coordinates of the point at which the resulting graph crosses the y-axis. **[3]**

(ii) Show that 3 is a root of $x^3 - 5x^2 + 2x + 8 = -4$. Hence solve this equation completely, giving the other roots in surd form. **[5]**

4 You are given that $f(x) = 2x^3 + 7x^2 - 7x - 12$.

(i) Verify that $x = -4$ is a root of $f(x) = 0$. [2]

(ii) Hence express $f(x)$ in fully factorised form. [4]

(iii) Sketch the graph of $y = f(x)$. [3]

(iv) Show that $f(x - 4) = 2x^3 - 17x^2 + 33x$. [3]

5 A cubic polynomial is given by $f(x) = 2x^3 - x^2 - 11x - 12$.

(i) Show that $(x - 3)(2x^2 + 5x + 4) = 2x^3 - x^2 - 11x - 12$.

Hence show that $f(x) = 0$ has exactly one real root. [4]

(ii) Show that $x = 2$ is a root of the equation $f(x) = -22$ and find the other roots of this equation. [5]

(iii) Using the results from the previous parts, sketch the graph of $y = f(x)$. [3]