

Paper Reference(s)

**6663/01**

**Edexcel GCE**

**Core Mathematics C1**

**Advanced Subsidiary**

**Integration: Areas**

**Calculators may NOT be used for these questions.**

**Information for Candidates**

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A booklet 'Mathematical Formulae and Statistical Tables' might be needed for some questions.

The marks for the parts of questions are shown in round brackets, e.g. (2).

There are 6 questions in this test.

**Advice to Candidates**

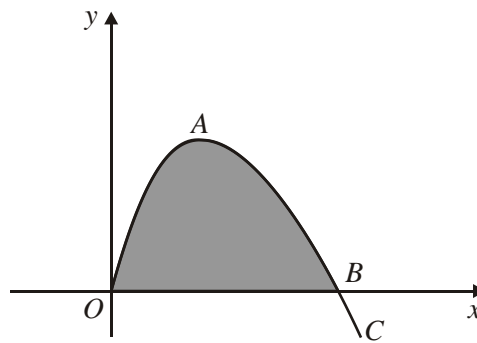
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You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear.

Answers without working may not gain full credit.

1.



The figure above shows part of the curve  $C$  with equation

$$y = 3x^{\frac{1}{2}} - x^{\frac{3}{2}}, \quad x \geq 0.$$

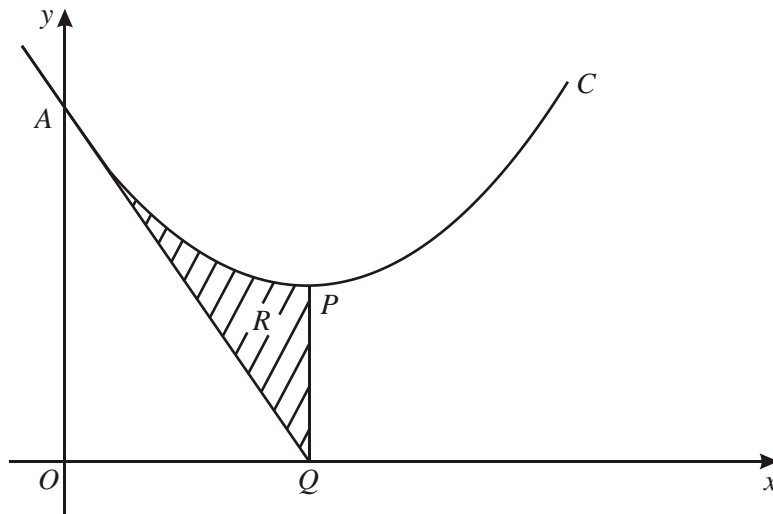
The point  $A$  on  $C$  is a stationary point and  $C$  cuts the  $x$ -axis at the point  $B$ .

- (a) Show that the  $x$ -coordinate of  $B$  is 3. (1)
- (b) Find the coordinates of  $A$ . (5)
- (c) Find the exact area of the finite region enclosed by  $C$  and the  $x$ -axis, shown shaded in the figure above. (5)

**(Total 11 marks)**

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2.



The diagram above shows part of the curve  $C$  with equation  $y = x^2 - 6x + 18$ . The curve meets the  $y$ -axis at the point  $A$  and has a minimum at the point  $P$ .

(a) Express  $x^2 - 6x + 18$  in the form  $(x - a)^2 + b$ , where  $a$  and  $b$  are integers. (3)

(b) Find the coordinates of  $P$ . (2)

(c) Find an equation of the tangent to  $C$  at  $A$ . (4)

The tangent to  $C$  at  $A$  meets the  $x$ -axis at the point  $Q$ .

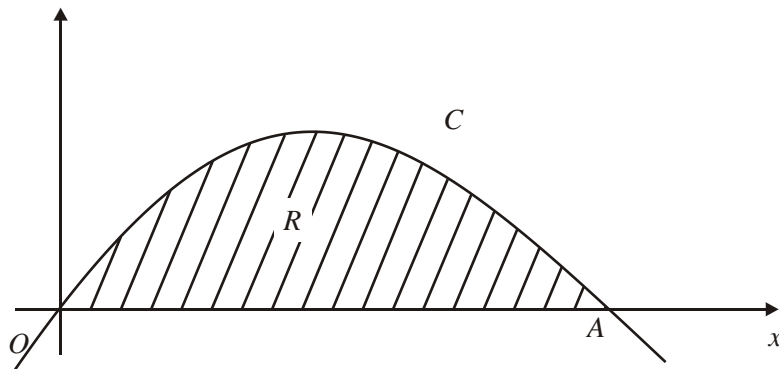
(d) Verify that  $PQ$  is parallel to the  $y$ -axis. (1)

The shaded region  $R$  in the diagram is enclosed by  $C$ , the tangent at  $A$  and the line  $PQ$ .

(e) Use calculus to find the area of  $R$ . (5)

**(Total 15 marks)**

3.



The curve  $C$ , with equation  $y = x(4 - x)$ , intersects the  $x$ -axis at the origin  $O$  and at the point  $A$ , as shown in the diagram above. At the point  $P$  on  $C$  the gradient of the tangent is  $-2$ .

(a) Find the coordinates of  $P$ .

(4)

The region  $R$ , enclosed between  $C$  and  $OA$ , is shown shaded.

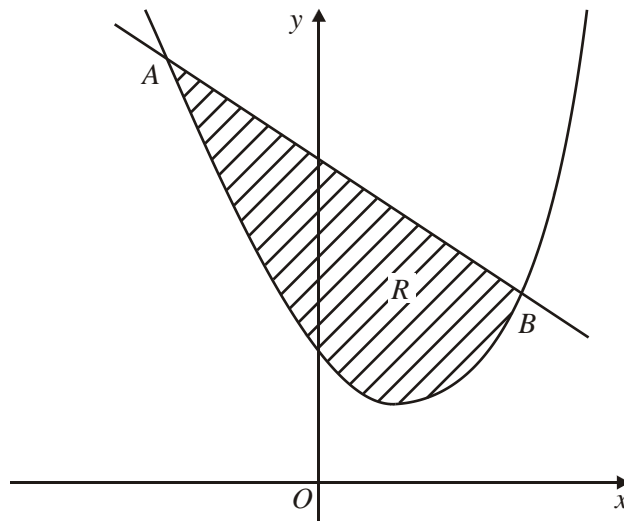
(b) Find the exact area of  $R$ .

(5)

(Total 9 marks)

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4.



The diagram above shows the line with equation  $y = 9 - x$  and the curve with equation  $y = x^2 - 2x + 3$ . The line and the curve intersect at the points  $A$  and  $B$ , and  $O$  is the origin.

- (a) Calculate the coordinates of  $A$  and the coordinates of  $B$ .

(5)

The shaded region  $R$  is bounded by the line and the curve.

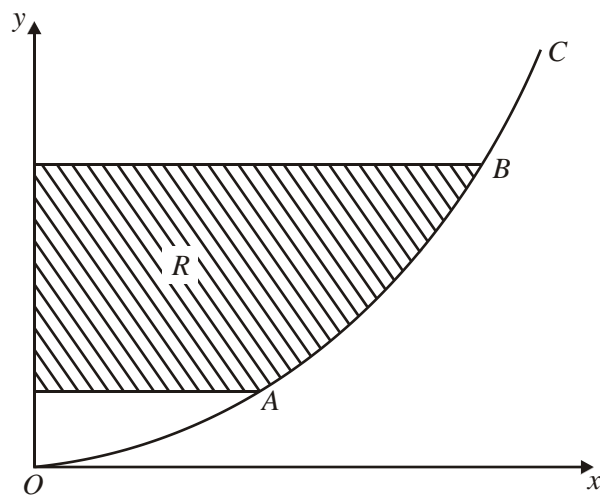
- (b) Calculate the area of  $R$ .

(7)

(Total 12 marks)

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5.



The curve  $C$ , shown in the diagram above, represents the graph of

$$y = \frac{x^2}{25}, \quad x \geq 0.$$

The points  $A$  and  $B$  on the curve  $C$  have  $x$ -coordinates 5 and 10 respectively.

(a) Write down the  $y$ -coordinates of  $A$  and  $B$ . (1)

(b) Find an equation of the tangent to  $C$  at  $A$ . (4)

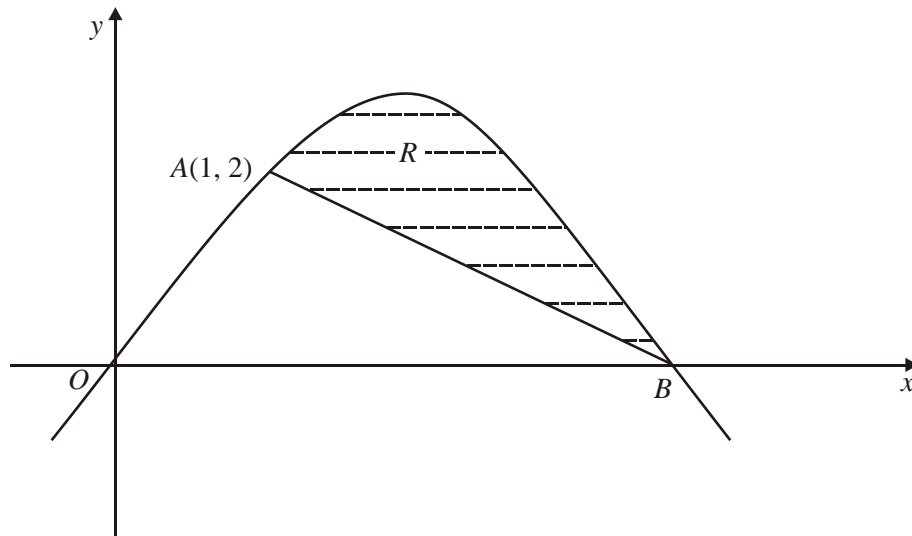
The finite region  $R$  is enclosed by  $C$ , the  $y$ -axis and the lines through  $A$  and  $B$  parallel to the  $x$ -axis.

(c) For points  $(x, y)$  on  $C$ , express  $x$  in terms of  $y$ . (2)

(d) Use integration to find the area of  $R$ . (5)

**(Total 12 marks)**

6.



The curve  $C$ , shown above, has equation  $y = 3x - x^2$ . It passes through the origin  $O$  and the point  $B$  on the  $x$ -axis.

- (a) Find, in the form  $ax + by + c = 0$ , an equation of the normal to  $C$  at the point  $A(1, 2)$ .

(5)

- (b) Show that this normal passes through the point  $B$ .

(2)

The shaded region  $R$  is bounded by the curve and the line  $AB$ .

- (c) Find, by integration, the area of  $R$ .

(7)

**(Total 14 marks)**

1. (a)  $y = 0 \Rightarrow x^{\frac{1}{2}}(3-x) = 0 \Rightarrow x = 3$  B1 1  
 or  $3\sqrt{3} - 3^{\frac{3}{2}} = 3\sqrt{3} - 3\sqrt{3} = 0$

(b)  $\frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}}$   $x^n \mapsto x^{n-1}$  M1 A1  
 $\frac{dy}{dx} = 0 \Rightarrow x^{\frac{1}{2}} = x^{-\frac{1}{2}}$  Use of  $\frac{dy}{dx} = 0$  M1  
 $\Rightarrow x = 1$  A1  
 A: (1, 2) A1 5

(c)  $\int \left( 3x^{\frac{1}{2}} - x^{\frac{3}{2}} \right) dx = 2x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}$  M1  $x^n \mapsto x^{n+1}$  M1 A1+A1  
 Accept unsimplified expressions for As  
 Area =  $[...]_0^3 = 2 \times 3\sqrt{3} - \frac{2}{5} \times 9\sqrt{3}$  Use of correct limits M1  
 Area is  $\frac{12}{5}\sqrt{3}$  (units<sup>2</sup>) A1 5

For final A1, terms must be collected together but accept exact equivalents, e.g.  $\frac{4}{5}\sqrt{27}$

[11]

2. (a)  $(x-3)^2, +9$  isw .  $a = 3$  and  $b = 9$  may just be written down with no method shown. B1, M1 A1 3

(b) P is (3, 9) B1

(c) A = (0, 18) B1

$\frac{dy}{dx} = 2x - 6$ , at A  $m = -6$  M1 A1  
 Equation of tangent is  $y - 18 = -6x$  (in any form) A1ft 4

(d) Showing that line meets x axis directly below P, i.e. at  $x = 3$ . A1cso 1

(e)  $A = \int x^2 - 6x + 18x = \left[ \frac{1}{3}x^3 - 3x^2 + 18x \right]$  M1 A1

Substituting  $x = 3$  to find area A under curve A [=36] M1

Area of R = A - area of triangle =  $A - \frac{1}{2} \times 18 \times 3 = 9$  M1 A1 5

Alternative:  $\int x^2 - 6x + 18 - (18 - 6x) dx$  M1  
 $= \frac{1}{3}x^3$  M1 A1 ft  
 Use  $x = 3$  to give answer 9 M1 A1

[13]



**C1 Integration - Areas**

3. (a)  $y = 4x - x^2 \quad \frac{dy}{dx} = 4 - 2x$  M1 A1  
 $"4 - 2x" = -2, \quad x = \dots$  M1  
 $x = 3, y = 3$  A1 4

(b)  $x$ -coordinate of  $A$  is 4 B1  
 $\int (4x - x^2) dx = \left[ \frac{4x^2}{2} - \frac{x^3}{3} \right]$  M1 A1ft  
 $\left[ \frac{4x^2}{2} - \frac{x^3}{3} \right]_0^4 = 32 - \frac{64}{3} = \frac{32}{3} \quad \left( = 10\frac{2}{3} \right)$  (or exact equivalent) M1 A1 5

[9]

4. (a)  $x^2 - 2x + 3 = 9 - x$  M1  
 $x^2 - x - 6 = 0 \quad (x + 2)(x - 3) = 0 \quad x = -2, 3$  M1 A1  
 $y = 11, 6$  M1 A1 ft 5

(b)  $\int (x^2 - 2x + 3) dx = \frac{x^3}{3} - x^2 + 3x$  M1 A1  
 $\left[ \frac{x^3}{3} - x^2 + 3x \right]_{-2}^3 = (9 - 9 + 9) - \left( \frac{-8}{3} - 4 - 6 \right) \quad \left( = 21\frac{2}{3} \right)$  M1 A1  
 Trapezium:  $\frac{1}{2}(11 + 6) \times 5 \quad \left( = 42\frac{1}{2} \right)$  B1 ft  
 Area =  $42\frac{1}{2} - 21\frac{2}{3} = 20\frac{5}{6}$  M1 A1 7

Alternative:  $(9 - x) - (x^2 - 2x + 3) = 6 + x - x^2$  M1 A1  
 $\int (6 + x - x^2) dx = 6x + \frac{x^2}{2} - \frac{x^3}{3}$  M1 A1 ft  
 $\left[ 6x + \frac{x^2}{2} - \frac{x^3}{3} \right]_{-2}^3 = \left( 18 + \frac{9}{2} - 9 \right) - \left( -12 + 2 + \frac{8}{3} \right) = 20\frac{5}{6}$  M1 A1, A1

[12]

**C1 Integration - Areas**

5. (a) A:  $y = 1$     B:  $y = 4$  B1

(b)  $\frac{dy}{dx} = \frac{2x}{25} = \frac{2}{5}$  where  $x = 5$  M1 A1

Tangent:  $y - 1 = \frac{2}{5}(x - 5)$     ( $5y = 2x - 5$ ) M1 A1

(c)  $x = 5y^{\frac{1}{2}}$  B1 B1

(d) Integrate:  $\frac{5y^{\frac{3}{2}}}{\frac{3}{2}} \left( = \frac{10y^{\frac{3}{2}}}{3} \right)$  M1 A1 ft

$[ ]^4 - [ ]_1 = \left( \frac{10 \times 4^{\frac{3}{2}}}{3} \right) - \left( \frac{10 \times 1^{\frac{3}{2}}}{3} \right), = \frac{70}{3} \left( 23\frac{1}{3}, 23.3 \right)$  M1 A1, A1

Alternative for (d): Integrate:  $\frac{x^3}{75}$  M1 A1

Area =  $(10 \times 4) - (5 \times 1) - \left( \frac{1000}{75} - \frac{125}{75} \right), = \frac{70}{3} \left( 23\frac{1}{3}, 23.3 \right)$  M1 A1, A1

In both (d) schemes, final M is scored using candidate's "4" and "1".

**[12]**

6. (a)  $y = 3x - x^2$
- $$\frac{dy}{dx} = 3 - 2x \quad \text{M1}$$
- At (1, 2)  $\frac{dy}{dx} = 1$  so gradient of normal = -1 A1 A1 ft
- Equation of normal  $y - 2 = -1(x - 1)$  M1
- i.e.  $x + y - 3 = 0$  A1 ft 5
- (b) From curve equation,  $B$  is (3, 0) B1
- and normal  $x + y - 3 = 0$  is satisfied by (3, 0) and  $\therefore$  passes through  $B$  B1 2
- (c) Area of  $R$  = Area under curve – Area of triangle M1
- Area of triangle =  $\frac{1}{2} \times 2 \times 2 = 2 \text{ units}^2$  B1
- Area under curve =  $\int (3x - x^2) dx = \frac{3x^2}{2} - \frac{x^3}{3}$  M1 A1
- $$\left[ \frac{3x^2}{2} - \frac{x^3}{3} \right]_1^3 = 13\frac{1}{2} - 9 - \left( \frac{3}{2} - \frac{1}{3} \right) = \frac{10}{3}$$
- Area of  $R = \left( \frac{10}{3} - 2 \right) \text{ units}^2 = \frac{4}{3} \text{ units}^2$  A1 7

**[14]**

1. The first part of this question gave difficulty to many. There are a number of possible approaches but many just wrote down  $3x^{\frac{1}{2}} - x^{\frac{3}{2}} = 3\sqrt{3} - 3^{\frac{3}{2}} = 0$  and this was thought inadequate unless they could show that  $3^{\frac{3}{2}}$  or  $\sqrt{27}$  was  $3\sqrt{3}$ .  $3^{\frac{3}{2}} = 3 \times 3^{\frac{1}{2}}$  would have been sufficient demonstration of this. In part (b), not all could solve  $\frac{3}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} = 0$  and a few found the second derivative and equated that to zero. In part (c), most could gain the first four of the five marks available but cleaning up the final answer to the single surd,  $\frac{12}{5}\sqrt{3}$  or its equivalent, proved difficult and many had recourse to their calculators, which did not fulfil the condition of the question that an exact answer is to be given.
2. (a) This was generally answered very well. Many candidates scored full marks, with others gaining B1, M1, A0, for answers such as  $(x - 3)^2 - 9$ , or  $-27$ , or  $+27$
- (b) The majority of candidates did not use their (a) to get the answer in (b) but used differentiation. This meant that most candidates had this part correct, even if they had (a) incorrect or didn't attempt it.
- (c) Again this part was well answered, especially by those who had done the differentiation in (b) as they went on to get the gradient of  $-6$ , and used  $(0, 18)$  to get the equation of the line. Unfortunately, some assumed the coordinates of  $Q$  at  $(3, 0)$ , and found the gradient using the points  $A$  and  $Q$ , so didn't gain many marks at all in this part.
- (d) Generally if candidates had answered part (c) correctly, they were able to do part (d) as well, although quite a few lost credibility because they stated that the gradient was 0. Many compared the  $x$  coordinates and deduced the line was parallel to the  $y$  axis and gained the credit.
- (e) This was very well answered. Many candidates had full marks in this part even if they hadn't scored full marks earlier. A few candidates made the mistake of using the  $y$  value of 9 instead of the  $x$  value of 3 in the integral. Other errors included using a trapezium instead of a triangle, and some candidates made small slips such as the 18 being copied down as an 8 or integrating the  $6x$  to get  $6x/2$ , or  $6x^2$ . It is possible that these candidates were short of time.
3. In part (a) of this question, some candidates failed to appreciate the need to differentiate and therefore made little progress. Such candidates often tried, in various ways, to use an equation of a straight line with gradient  $-2$ , perhaps inappropriately passing through the point  $A$ . Those who did differentiate were often successful, although some ignored the  $-2$  and equated their derivative to zero.

There were many completely correct solutions to part (b). Integration techniques were usually sound, but a few candidates had difficulty in finding the correct limits, especially where they looked for a link between parts (a) and (b).

4. The vast majority of candidates coped well with part (a), solving the equations simultaneously to find the correct coordinates of  $A$  and  $B$ . A few made it more difficult for themselves initially by finding a quadratic in  $y$  rather than  $x$ . Others simply used a table of values of  $x$  and  $y$  to find the intersection points.

The most popular method in part (b) was to find the area between the curve and the  $x$ -axis  $\left(21\frac{2}{3}\right)$ , and then to subtract this from the area of the trapezium, and many

candidates completed this accurately to score full marks. Quite often the area of the trapezium was not considered at all, the answer being left as  $21\frac{2}{3}$ . Those who used integration to find the area under the straight line were more likely to make numerical mistakes. The alternative approach of subtracting first  $(9 - x) - (x^2 - 2x + 3)$  and then integrating  $(6 + x - x^2)$  was sometimes used successfully, but was less than fully convincing when the initial subtraction was performed “the wrong way round”. Occasionally  $y$  limits were wrongly used for the integration instead of  $x$  limits.

5. Although many candidates found this question difficult, those who were competent in algebra and calculus often produced excellent, concise solutions.

Part (a) caused very little difficulty, almost everyone scoring the one available mark. In part (b), however, attempts at finding the equation of the tangent were disappointing. Some candidates made no attempt to differentiate, while others differentiated correctly to get  $\frac{2x}{25}$ , but failed to find the gradient at  $x = 5$ , using instead  $\frac{2}{25}$ , or even  $\frac{2x}{25}$ , as the gradient  $m$  in the equation of the tangent.

Although some had no idea what to do in part (c), the majority were able to express  $x$  as  $\sqrt{25y}$  or equivalent. Here, it was not always clear whether the square root extended to the  $y$  as well as the 25, but a correct expression in part (d) gained the marks retrospectively.

The expression of  $x$  in terms of  $y$  was, of course, intended as a hint for part (d), but many candidates happily ignored this and integrated  $y$  with respect to  $x$  rather than  $x$  with respect to  $y$ . Using this approach, just a few were able to find the area of the required region by a process of subtracting the appropriate areas from a rectangle of area 40, but it was very rare to see a complete method here. Some candidates wrongly used 5 and 10 as  $y$  limits (or 1 and 4 as  $x$  limits). Those integrating  $5\sqrt{y}$  with respect to  $y$  were very often successful in reaching the correct answer.

6. No Report available for this question.