

Paper Reference(s)

6663/01

Edexcel GCE

Core Mathematics C1

Advanced Subsidiary

Basic Differentiation

Calculators may NOT be used for these questions.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' might be needed for some questions.

The marks for the parts of questions are shown in round brackets, e.g. (2).

There are 20 questions in this test.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear.

Answers without working may not gain full credit.

1. Given that

$$y = 8x^3 - 4\sqrt{x} + \frac{3x^2 + 2}{x}, \quad x > 0$$

find $\frac{dy}{dx}$.

(Total 6 marks)

2. Given that $y = x^4 + x^{\frac{1}{3}} + 3$, find $\frac{dy}{dx}$.

(Total 3 marks)

3. The curve C has equation

$$y = \frac{(x+3)(x-8)}{x}, \quad x > 0$$

- (a) Find $\frac{dy}{dx}$ in its simplest form.

(4)

- (b) Find an equation of the tangent to C at the point where $x = 2$

(4)

(Total 8 marks)

4. Given that $y = 2x^3 + \frac{3}{x^2}$, $x \neq 0$, find

- (a) $\frac{dy}{dx}$

(3)

- (b) $\int y \, dx$, simplifying each term.

(3)

(Total 6 marks)

5.
$$f(x) = \frac{(3 - 4\sqrt{x})^2}{\sqrt{x}}, \quad x > 0$$

(a) Show that $f(x) = 9x^{-\frac{1}{2}} + Ax^{\frac{1}{2}} + B$, where A and B are constants to be found. (3)

(b) Find $f'(x)$. (3)

(c) Evaluate $f'(9)$. (2)

(Total 8 marks)

6. Given that $\frac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$ can be written in the form $2x^p - x^q$,

(a) write down the value of p and the value of q . (2)

Given that $y = 5x^2 - 3 + \frac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$,

(b) find $\frac{dy}{dx}$, simplifying the coefficient of each term. (4)

(Total 6 marks)

7.
$$f(x) = 3x + x^3, \quad x > 0.$$

(a) Differentiate to find $f'(x)$. (2)

Given that $f'(x) = 15$,

(b) find the value of x . (3)

(Total 5 marks)

8. The curve C has equation $y = kx^3 - x^2 + x - 5$, where k is a constant.

(a) Find $\frac{dy}{dx}$.

(2)

The point A with x -coordinate $-\frac{1}{2}$ lies on C . The tangent to C at A is parallel to the line with equation $2y - 7x + 1 = 0$.

Find

(b) the value of k ,

(4)

(c) the value of the y -coordinate of A .

(2)

(Total 8 marks)

9. (a) Write $\frac{2\sqrt{x} + 3}{x}$ in the form $2x^p + 3x^q$ where p and q are constants.

(2)

Given that $y = 5x - 7 + \frac{2\sqrt{x} + 3}{x}$, $x > 0$,

(b) find $\frac{dy}{dx}$, simplifying the coefficient of each term.

(4)

(Total 6 marks)

10. The curve C has equation

$$y = (x + 3)(x - 1)^2.$$

- (a) Sketch C showing clearly the coordinates of the points where the curve meets the coordinate axes.

(4)

- (b) Show that the equation of C can be written in the form

$$y = x^3 + x^2 - 5x + k,$$

where k is a positive integer, and state the value of k .

(2)

There are two points on C where the gradient of the tangent to C is equal to 3.

- (c) Find the x -coordinates of these two points.

(6)

(Total 12 marks)

11. Given that

$$y = 4x^3 - 1 + 2x^{\frac{1}{2}}, \quad x > 0,$$

find $\frac{dy}{dx}$.

(Total 4 marks)

12. The curve C has equation $y = 4x + 3x^{\frac{3}{2}} - 2x^2$, $x > 0$.

(a) Find an expression for $\frac{dy}{dx}$.

(3)

(b) Show that the point $P(4, 8)$ lies on C .

(1)

(c) Show that an equation of the normal to C at the point P is

$$3y = x + 20.$$

(4)

The normal to C at P cuts the x -axis at the point Q .

(d) Find the length PQ , giving your answer in a simplified surd form.

(3)

(Total 11 marks)

13. Differentiate with respect to x

(a) $x^4 + 6\sqrt{x}$,

(3)

(b) $\frac{(x+4)^2}{x}$

(4)

(Total 7 marks)

14. Given that $y = 2x^2 - \frac{6}{x^3}$, $x \neq 0$,

(a) find $\frac{dy}{dx}$,

(2)

(b) evaluate $\int_1^3 y \, dx$.

(4)

(Total 6 marks)

15. Given that $y = 2x^2 - \frac{6}{x^3}$, $x \neq 0$,

(a) find $\frac{dy}{dx}$,

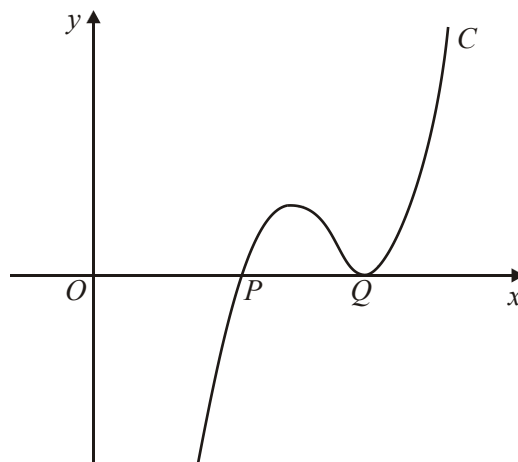
(2)

(b) find $\int y \, dx$.

(3)

(Total 5 marks)

16.



This figure shows part of the curve C with equation $y = f(x)$, where

$$f(x) = x^3 - 13x^2 + 55x - 75.$$

The curve crosses the x -axis at the point P and touches the x -axis at the point Q .

- (a) Show, by using the factor theorem, that $(x - 3)$ is a factor of $f(x)$. (2)
- (b) Factorise $f(x)$ completely. (3)
- (c) Write down the x -coordinate of P and the x -coordinate of Q . (1)
- (d) Find the gradient of the tangent to C at the point P . (3)
- Another point S also lies on C . The tangent to C at S is parallel to the tangent to C at P .
- (e) Find the x -coordinate of S . (4)

(Total 13 marks)

17. (i) Given that $y = 5x^3 + 7x + 3$, find

(a) $\frac{dy}{dx}$,

(3)

(b) $\frac{d^2y}{dx^2}$.

(1)

(ii) Find $\int \left(1 + 3\sqrt{x} - \frac{1}{x^2} \right) dx$.

(4)

(Total 8 marks)

18. The curve C with equation $y = f(x)$ is such that

$$\frac{dy}{dx} = 3\sqrt{x} + \frac{12}{\sqrt{x}}, \quad x > 0.$$

(a) Show that, when $x = 8$, the exact value of $\frac{dy}{dx}$ is $9\sqrt{2}$.

(3)

The curve C passes through the point $(4, 30)$.

(b) Using integration, find $f(x)$.

(6)

(Total 9 marks)

19. $y = 7 + 10x^{\frac{3}{2}}$.

(a) Find $\frac{dy}{dx}$.

(2)

(b) Find $\int y \, dx$.

(3)

(Total 5 marks)

20. For the curve C with equation $y = f(x)$,

$$\frac{dy}{dx} = x^3 + 2x - 7.$$

(a) Find $\frac{d^2y}{dx^2}$.

(2)

(b) Show that $\frac{d^2y}{dx^2} \geq 2$ for all values of x .

(1)

Given that the point $P(2, 4)$ lies on C ,

(c) find y in terms of x ,

(5)

(d) find an equation for the normal to C at P in the form $ax + by + c = 0$, where a , b and c are integers.

(5)

(Total 13 marks)

1. $\frac{3x^2 + 2}{x} = 3x + 2x^{-1}$

M1 A1

$(y' =) 24x^2, -2x^{-\frac{1}{2}}, +3 - 2x^{-2}$

M1 A1 A1 A1

$\left[24x^2 - 2x^{-\frac{1}{2}} + 3 - 2x^{-2} \right]$

Note

1st M1 for attempting to divide (one term correct)

1st A1 for both terms correct on the same line, accept $3x^1$ for $3x$ or $\frac{2}{x}$ for $2x^{-1}$

These first two marks may be implied by a correct differentiation at the end.

2nd M1 for an attempt to differentiate $x^n \rightarrow x^{n-1}$ for at least one term of their expression

“Differentiating” $\frac{3x^2 + 2}{x}$ and getting $\frac{6x}{1}$ is M0

2nd A1 for $24x^2$ only

3rd A1 for $-2x^{-\frac{1}{2}}$ allow $\frac{-2}{\sqrt{x}}$. Must be simplified to this, not e.g. $\frac{-4}{2}x^{-\frac{1}{2}}$

4th A1 for $3 - 2x^{-2}$ allow $\frac{-2}{x^2}$. Both terms needed. Condone $3 + (-2)x^{-2}$

If “+c” is included then they lose this final mark

They do not need one line with all terms correct for full marks. Award marks when first seen in this question and apply ISW.

Condone a mixed line of some differentiation and some division

e.g. $24x^2 - 4x^{\frac{1}{2}} + 3x + 2x^{-1}$ can score 1st M1A1 and 2nd M1A1

Quotient / Product Rule

$$\frac{x(6x) - (3x^2 + 2) \times 1}{x^2} \text{ or } 6x(x^{-1}) + (3x^2 + 2)(-x^{-2})$$

$$\frac{3x^2 - 2}{x^2} \text{ or } 3 - \frac{2}{x^2} \text{ (o.e.)}$$

1st M1 for an attempt: $\frac{P-Q}{x^2}$ or $R + (-S)$ with one of P, Q or R, S correct.

1st A1 for a correct expression

4th A1 same rules as above

[6]

2. $x^4 \rightarrow kx^3$ or $x^{1/3} \rightarrow kx^{-2/3}$ or $3 \rightarrow 0$ (k a non-zero constant) M1

$\left(\frac{dy}{dx} = \right) 4x^3$, with ‘3’ differentiated to zero (or ‘vanishing’) A1

$\left(\frac{dy}{dx} = \right)$ + $\frac{1}{3}x^{-2/3}$ or equivalent, e.g. $\frac{1}{3\sqrt[3]{x^2}}$ or $\frac{1}{3(\sqrt[3]{x})^2}$ A1

Note

1st A1 requires $4x^3$, and 3 differentiated to zero.

Having ‘+C’ loses the 1st A mark.

Terms not added, but otherwise correct, e.g. $4x^3, \frac{1}{3}x^{-2/3}$ loses

the 2nd A mark.

[3]

3. (a) $y = \frac{x^2 - 5x - 24}{x} = x - 5 - 24x^{-1}$

(or equiv., e.g. $x + 3 - 8 - \frac{24}{x}$)

M1 A1

$\frac{dy}{dx} = 1 + 24x^{-2}$ or $\frac{dy}{dx} = 1 + \frac{24}{x^2}$

M1 A1 4

Note

1st M: Mult. out to get $x^2 + bx + c$, $b \neq 0$, $c \neq 0$ and dividing by x (not x^2).

Obtaining one correct term, e.g. $x \dots$ is sufficient evidence of a division attempt.

2nd M: Dependent on the 1st M:

Evidence of $x^n \rightarrow kx^{n-1}$ for one x term (i.e. not just the constant term) is sufficient). Note that mark is not given if, for example, the numerator and denominator are differentiated separately.

A mistake in the ‘middle term’, e.g. $x + 5 - 24x^{-1}$, does not invalidate the

2nd A mark, so M1 A0 M1 A1 is possible.

(b) $x = 2: y = -15$ Allow if seen in part (a). B1

$\left(\frac{dy}{dx}\right) 1 + \frac{24}{4} = 7$ Follow-through from

candidate’s non-constant $\frac{dy}{dx}$. B1ft

This must be simplified to a “single value”.

$y + 15 = 7(x - 2)$ (or equiv., e.g. $y = 7x - 29$)

Allow $\frac{y + 15}{x - 2} = 7$ M1 A1 4

Note

B1ft: For evaluation, using $x = 2$, of their $\frac{dy}{dx}$, even if unlabelled or called y .

M: For the equation, in any form, of a straight line through

(2, ‘-15’) with candidate’s $\frac{dy}{dx}$ value as gradient.

Alternative is to use (2, ‘-15’) in $y = mx + c$ to find a value for c , in which case $y = 7x + c$ leading to $c = -29$ is sufficient for the A1).

(See general principles for straight line equations at the end of the scheme).

Final A: ‘Unsimplified’ forms are acceptable, but...

$y - (-15) = 7(x - 2)$ is A0 (unresolved ‘minus minus’).

[8]

4. (a) $\frac{dy}{dx} = 6x^2 - 6x^{-3}$ M1 A1 A1 3

Note

M1 for an attempt to differentiate $x^n \rightarrow x^{n-1}$

1st A1 for $6x^2$

2nd A1 for $-6x^{-3}$ or $-\frac{6}{x^3}$ Condone $+ -6x^{-3}$ here. Inclusion of $+c$ scores A0 here.

(b) $\frac{2x^4}{4} + \frac{3x^{-1}}{-1} (+C)$ M1 A1

$\frac{x^4}{2} - 3x^{-1} + C$ A1 3

Note

M1 for some attempt to integrate an x term of the given y . $x^n \rightarrow x^{n+1}$

1st A1 for **both** x terms correct but unsimplified– as printed or better. Ignore $+c$ here

2nd A1 for both x terms correct and simplified and $+c$. Accept $-\frac{3}{x}$ but

NOT $+ -3x^{-1}$

Condone the $+c$ appearing on the first (unsimplified) line but missing on the final (simplified) line

Apply ISW if a correct answer is seen

If part (b) is attempted first and this is clearly labelled then apply the scheme and allow the marks. Otherwise assume the first solution is for part (a).

[6]

5. (a) $(3 - 4\sqrt{x})^2 = 9 - 12\sqrt{x} - 12\sqrt{x} + (-4)^2 x$ M1
 $9x^{\frac{1}{2}} + 16x^{\frac{1}{2}} - 24$ A1, A1 3

Note

M1 for an attempt to expand $(3 - 4\sqrt{x})^2$ with at least 3 terms correct – as printed or better

Or $9 - k\sqrt{x} + 16x$ ($k \neq 0$). See also the MR rule below

1st A1 for their coefficient of $\sqrt{x} = 16$. Condone writing $(\pm) 9x^{(\pm)\frac{1}{2}}$ instead of $9x^{\frac{1}{2}}$

2nd A1 for $B = -24$ or their constant term = -24

(b) $f'(x) = -\frac{9}{2}x^{-\frac{3}{2}} + \frac{16}{2}x^{-\frac{1}{2}}$ M1 A1, A1ft 3

Note

M1 for an attempt to differentiate an x term $x^n \rightarrow x^{n-1}$

1st A1 for $-\frac{9}{2}x^{-\frac{3}{2}}$ and their constant B differentiated to zero. NB

$-\frac{1}{2} \times 9x^{-\frac{3}{2}}$ is A0

2nd A1ft follow through their $Ax^{\frac{1}{2}}$ but can be scored without a value for A , i.e. for $\frac{A}{2}x^{-\frac{1}{2}}$

(c) $f'(9) = -\frac{9}{2} \times \frac{1}{27} + \frac{16}{2} \times \frac{1}{3} = -\frac{1}{6} + \frac{16}{6} = \frac{5}{2}$ M1 A1 2

Note

M1 for some correct substitution of $x = 9$ in their expression for $f'(x)$ including an attempt at $(9)^{\frac{\pm k}{2}}$ (k odd) somewhere that leads to some appropriate multiples of $\frac{1}{3}$ or 3

A1 accept $\frac{15}{6}$ or any exact equivalent of 2.5 e.g. $\frac{45}{18}$, $\frac{135}{54}$ or even $\frac{67.5}{27}$

Misread (MR) Only allow MR of the form $\frac{(3 - k\sqrt{x})^2}{\sqrt{x}}$ N.B. Leads to

answer in (c) of $\frac{k^2 - 1}{6}$ Score as M1A0A0, M1A1A1ft, M1A1ft

[8]

6. (a) $2x^{3/2}$ or $p = \frac{3}{2}$ (Not $2x\sqrt{x}$) B1
 $-x$ or $-x^1$ or $q = 1$ B1 2

Note

1st B1 for $p = 1.5$ or exact equivalent
 2nd B1 for $q = 1$

- (b) $\left(\frac{dy}{dx}\right) 20x^3 + 2 \times \frac{3}{2} x^{1/2} - 1$ M1
 $20x^3 + 3x^{1/2} - 1$ A1A1ftA1ft 4

Note

M1 for an attempt to differentiate $x^n \rightarrow x^{n-1}$ (for any of the 4 terms)

1st A1 for $20x^3$ (the -3 must ‘disappear’)

2nd A1ft for $3x^{1/2}$ or $3\sqrt{x}$. Follow through their p but they must be differentiating $2x^p$, where p is a fraction, and the coefficient must be simplified if necessary.

3rd A1ft for -1 (not the unsimplified $-x^0$), or follow through for correct differentiation of their $-x^q$ (i.e. coefficient of x^q is -1).
 If ft is applied, the coefficient must be simplified if necessary.

‘Simplified’ coefficient means $\frac{a}{b}$ where a and b are integers with no common

factors. Only a single $+$ or $-$ sign is allowed (e.g. $--$ must be replaced by $+$).

If there is a ‘restart’ in part (b) it can be marked independently of part (a), but marks for part (a) cannot be scored for work seen in (b).

Multiplying by \sqrt{x} : (assuming this is a restart)

e.g. $y = 5x^4 \sqrt{x} - 3\sqrt{x} + 2x^2 - x^{3/2}$
 $\left(\frac{dy}{dx}\right) \frac{45}{2}x^{7/2} - \frac{3}{2}x^{-1/2} + 4x - \frac{3}{2}x^{1/2}$ scores M1 A0 A0 (p not a fraction) A1ft.

Extra term included: This invalidates the final mark.

e.g. $y = 5x^4 - 3 + 2x^2 - x^{3/2} - x^{1/2}$
 $\left(\frac{dy}{dx}\right) 20x^3 + 4x - \frac{3}{2}x^{1/2} - \frac{1}{2}x^{-1/2}$ scores M1 A1 A0 (p not a

fraction) A0.

Numerator and denominator differentiated separately:

For this, neither of the last two (ft) marks should be awarded.

Quotient/product rule:

Last two terms must be correct to score the last 2 marks. (If the M mark has not already been earned, it can be given for the quotient/product rule attempt.)

[6]

7. (a) $[f'(x) =] 3 + 3x^2$ M1A1 2

M1 for attempting to differentiate $x^n \rightarrow x^{n-1}$.

Just one term will do.

A poor integration attempt that gives $3x^2 + \dots$ (or similar) scores M0A0

A1 for a fully correct expression. Must be 3 not $3x^0$.
If there is a + c they score A0.

(b) $3 + 3x^2 = 15$ and start to try and simplify M1
 $x^2 = k \rightarrow x = \sqrt{k}$ (ignore \pm) M1
 $x = 2$ (ignore $\times = -2$) A1 3

1st M1 for forming a correct equation and trying to rearrange their $f'(x) = 15$ e.g. collect terms.

e.g. $3x^2 = 15 - 3$ or $1 + x^2 = 5$ or even $3 + 3x^2 \rightarrow 3x^2 = \frac{15}{3}$

or $3x^{-1} + 3x^2 = 15 \rightarrow 6x = 15$

(i.e algebra can be awful as long as they try to collect terms in their $f'(x) = 15$ equation)

2nd M1 this is dependent upon their $f'(x)$ being of the form

$a + bx^2$ and attempting to solve $a + bx^2 = 15$

For correct processing leading to $x = \dots$

Can condone arithmetic slips but processes should be correct so

e.g. $3 + 3x^2 = 15 \rightarrow 3x^2 = \frac{15}{3} \rightarrow x = \frac{\sqrt{15}}{3}$ scores M1M0A0

$3 + 3x^2 = 15 \rightarrow 3x^2 = 12 \rightarrow x^2 = 9 \rightarrow x = 3$ scores M1M0A0

$3 + 3x^2 = 15 \rightarrow 3x^2 = 12 \rightarrow 3x = \sqrt{12} \rightarrow x = \frac{\sqrt{12}}{3}$ scores M1M0A0

[5]

8. (a) $\left[\frac{dy}{dx} = \right] 3kx^2 - 2x + 1$ M1A1 2

M1 for attempting to differentiate $x^n \rightarrow x^{n-1}$ (or -5 going to 0 will do)

A1 all correct. A “+ c” scores A0

(b) Gradient of line is $\frac{7}{2}$ B1

When $x = -\frac{1}{2}$: $3k \times \left(\frac{1}{4}\right) - 2 \times \left(-\frac{1}{2}\right) + 1, = \frac{7}{2}$ M1, M1

$\frac{3k}{4} = \frac{3}{2} \Rightarrow k = 2$ A1 4

B1 for $m = \frac{7}{2}$. Rearranging the line into $y = \frac{7}{2}x + c$ does not score this mark until you are sure they are using $\frac{7}{2}$ as the gradient of the line or state $m = \frac{7}{2}$

1st M1 for substituting $x = -\frac{1}{2}$ into their $\frac{dy}{dx}$, some correct substitution seen

2nd M1 for forming a suitable equation in k and attempting to solve leading to $k = \dots$

Equation must use their $\frac{dy}{dx}$ and their gradient of line.

Assuming the gradient is 0 or 7 scores

M0 unless they have clearly stated that this is the gradient of the line.

A1 for $k = 2$

(c) $x = -\frac{1}{2} \Rightarrow y = k \times \left(-\frac{1}{8}\right) - \left(\frac{1}{4}\right) - \frac{1}{2} - 5, = -6$ M1, A1 2

M1 for attempting to substitute their k (however it was found or can still be a letter) and

$x = \frac{1}{2}$ into y (some correct substitution)

A1 for -6

[8]

9. (a) $\left(2x^{\frac{1}{2}} + 3x^{-1}\right)$ $p = -\frac{1}{2}, q = -1$ B1, B1 2

(b) $\left(y = 5x - 7 + 2x^{\frac{1}{2}} + 3x^{-1}\right)$
 $\left(\frac{dy}{dx} =\right) 5 \text{ (or } 5x^0)$ (5x – 7 correctly differentiated) B1

Attempt to differentiate either $2x^p$ with a fractional p , giving kx^{p-1} ($k \neq 0$), M1 (the fraction p could be in decimal form)

or $3x^q$ with a negative q , giving kx^{q-1} ($k \neq 0$). A1ft, A1ft 4

$\left(-\frac{1}{2} \times 2x^{\frac{3}{2}} - 1 \times 3x^{-2} =\right) -x^{\frac{3}{2}}, -3x^{-2}$

N.B. It is possible to ‘start again’ in (b), so the p and q may be different from those seen in (a), but note that the M mark is for the attempt to differentiate $2x^p$ or $3x^q$.

However, marks for part (a) cannot be earned in part (b).

1st A1ft:ft their $2x^p$, but p must be a fraction and coefficient must be simplified (the fraction p could be in decimal form).

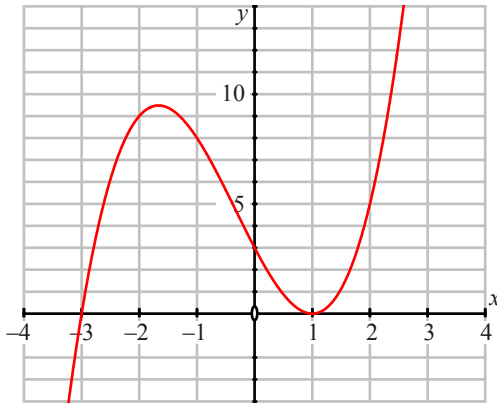
2nd A1ft: ft their $3x^q$, but q must be negative and coefficient must be simplified.


‘Simplified’ coefficient means $\frac{a}{b}$ where a and b are integers with no common factors. Only a single + or – sign is allowed (e.g. – – must be replaced by +).

Having +C loses the B mark.

[6]

10. (a)



Shape  (drawn anywhere)

B1

Minimum at (1, 0) (perhaps labelled 1 on x-axis)

B1

(-3, 0) (or -3 shown on -ve x-axis)

B1

(0, 3) (or 3 shown on +ve y-axis)

B1

4

N.B. The max. can be anywhere.

The individual marks are independent, but the 2nd, 3rd and 4th B's are dependent upon a sketch having been attempted.

B marks for coordinates: Allow (0, 1), etc. (coordinates the wrong way round) if marked in the correct place on the sketch.

(b) $y = (x + 3)(x^2 - 2x + 1)$ (*)

M1

$= x^3 + x^2 - 5x + 3$ (*) ($k = 3$)

A1cso

2

(*)Marks can be awarded if this is seen in part (a)

M: Attempt to multiply out $(x - 1)^2$ and write as a product with $(x + 3)$, or attempt to multiply out $(x + 3)(x - 1)$ and write as a product with $(x - 1)$, or attempt to expand $(x + 3)(x - 1)(x - 1)$ directly (at least 7 terms).

The $(x - 1)^2$ or $(x + 3)(x - 1)$ expansion must have 3 (or 4) terms, so should not, for example, be just $x^2 + 1$.

A: It is not necessary to state explicitly ' $k = 3$ '.

Condone missing brackets if the intention seems clear and a fully correct expansion is seen.

(c) $\frac{dy}{dx} = 3x^2 + 2x - 5$

M1A1

$3x^2 + 2x - 5 = 3$ or $3x^2 + 2x - 8 = 0$

M1

$(3x - 4)(x + 2) = 0$ $x = \dots$

M1

$x = \frac{4}{3}$ (or exact equiv.), $x = -2$

A1, A1

6

1st M: Attempt to differentiate (correct power of x in at least one term).

2nd M: Setting their derivative equal to 3.

3rd M: Attempt to solve a 3-term quadratic based on their derivative.

The equation could come from $\frac{dy}{dx} = 0$.

N.B. After an incorrect k value in (b), full marks are still possible in (c).

[12]

11. $4x^3 \rightarrow kx^2$ or $2x^{\frac{1}{2}} \rightarrow kx^{-\frac{1}{2}}$ (k a non-zero constant) M1
 $12x^2, +x^{-\frac{1}{2}}$..., ($-1 \rightarrow 0$) A1, A1, B1 4

Accept equivalent alternatives to $x^{-\frac{1}{2}}$, e.g. $\frac{1}{x^{\frac{1}{2}}}$, $\frac{1}{\sqrt{x}}$, $x^{-0.5}$.

M1: $4x^3$ ‘differentiated’ to give kx^2 , or...
 $2x^{\frac{1}{2}}$ ‘differentiated’ to give $kx^{-\frac{1}{2}}$ (but not for just $-1 \rightarrow 0$).

1st A1: $12x^2$ (Do not allow just $3 \times 4x^2$)

2nd A1: $x^{-\frac{1}{2}}$ or equivalent. (Do not allow just $\frac{1}{2} \times 2x^{-\frac{1}{2}}$, but allow $1x^{-\frac{1}{2}}$ or $\frac{2}{2}x^{-\frac{1}{2}}$).

B1: -1 differentiated to give zero (or ‘disappearing’).
 Can be given provided that at least one of the other terms has been changed.
 Adding an extra term, e.g. $+ C$, is B0.

[4]

12. (a) $4x \rightarrow k$ or $3x^{\frac{3}{2}} \rightarrow k^{\frac{1}{2}}$ or $-2x^2 \rightarrow kx$ M1
 $\frac{dy}{dx} = 4 + \frac{9}{2}x^{\frac{1}{2}} - 4x$ A1A1 3

For the 2 A marks coefficients need not be simplified, but powers must be simplified. For example, $\frac{3}{2} \times 3x^{\frac{1}{2}}$ is acceptable.

All 3 terms correct: A1 A1
 Two terms correct: A1 A0
 Only one term correct: A0 A0

- (b) For $x = 4, y = (4 \times 4) + (3 \times 4\sqrt{4}) - (2 \times 16) = 16 + 24 - 32 = 8$ (*) B1 1
 There must be some evidence of the “24” value.

- (c) $\frac{dy}{dx} = 4 + 9 - 16 = -3$ M: Evaluate their $\frac{dy}{dx}$ at $x = 4$ M1
 Gradient of normal = $\frac{1}{3}$ A1ft

Equation of normal: $y - 8 = \frac{1}{3}(x - 4), \quad 3y = x + 20$ (*) M1, A1 4

In this part, beware ‘working backwards’ from the given answer.

A1ft: Follow through is just from the candidate’s value of $\frac{dy}{dx}$.

2nd M: Is not given if an m value appears “from nowhere”.

2nd M: Must be an attempt at a normal equation, not a tangent.

2nd M: Alternative is to use $(4, 8)$ in $y = mx + c$ to find a value for c .

- (d) $y = 0: x = \dots (-20)$ and use $(x_2 - x_1)^2 + (y_2 - y_1)^2$ M1
 $PQ = \sqrt{24^2 + 8^2}$ or $PQ^2 = 24^2 + 8^2$ Follow through from $(k, 0)$ A1ft
 May also be scored with $(-24)^2$ and $(-8)^2$.
 $= 8\sqrt{10}$ A1 3

M: Using the normal equation to attempt coordinates of Q ,
 (even if using $x = 0$ instead of $y = 0$), and using Pythagoras
 to attempt PQ or PQ^2 .
 Follow through from $(k, 0)$, but not from $(0, k)$...

A common wrong answer is to use $x = 0$ to give $\frac{20}{3}$.

This scores M1 A0 A0.

For final answer, accept other simplifications of $\sqrt{640}$,
 e.g. $2\sqrt{160}$ or $4\sqrt{40}$.

[11]

13. (a) $(y = x^4 + 6x^{\frac{1}{2}} \Rightarrow y' =)$ or $4x^3 + \frac{3}{\sqrt{x}}$ M1A1A1 3

M1 For some attempt to differentiate $x^n \rightarrow x^{n-1}$

1st A1 For one correct term as printed.

2nd A1 For both terms correct as printed

$4x^3 + 3x^{-\frac{1}{2}} + c$ scores M1 A1A0

(b) $(x + 4)^2 = x^2 + 8x + 16$ M1

$\frac{(x+4)^2}{x} = x + 8 + 16x^{-1}$ (allow 4 + 4 for 8) A1

$(y = \frac{(x+4)^2}{x} \Rightarrow y' =)$ $1 - 16x^{-2}$ o.e. M1A1 4

1st M1 For attempt to expand $(x + 4)^2$, must have x^2 , x , x^0 terms and at least 2 correct

e.g. $x^2 + 8x + 8$ or $x^2 + 2x + 16$

1st A1 Correct expression for $\frac{(x+4)^2}{x}$.

As printed but allow $\frac{16}{x}$ and $8x^0$.

2nd M1 For some correct differentiation, any term. Can follow through their simplification.

N.B. $\frac{x^2 + 8x + 16}{x}$ giving rise to $(2x + 8) / 1$ is

M0A0

ALT Product or Quotient rule (If in doubt send to review)

M2 For correct use of product or quotient rule. Apply usual rules on formulae.

1st A1 For $\frac{2(x+4)}{x}$ or $\frac{2x(x+4)}{x^2}$

2nd A1 for $-\frac{(x+4)}{x^2}$

[7]

14. (a) $\frac{dy}{dx} = 4x + 18x^{-4}$ $x^n \mapsto x^{n-1}$ M1 A1 2
- (b) $\int (2x^2 - 6x^{-3}) dx = \frac{2}{3}x^3 + 3x^{-2}$ $x^n \mapsto x^{n+1}$ M1 A1
- $[\dots]_1^3 = \frac{2}{3} \times 3^3 + \frac{3}{9} - \left(\frac{2}{3} + 3 \right)$ M1
- $= 14\frac{2}{3}$ $\frac{44}{3}, \frac{132}{9}$ or equivalent A1 4

[6]

15. (a) $\frac{dy}{dx} = 4x + 18x^{-4}$ M1: $x^2 \rightarrow x$ or $x^{-3} \rightarrow x^{-4}$ M1 A1 2
- (b) $\frac{2x^3}{3} - \frac{6x^{-2}}{-2} + C$ M1: $x^2 \rightarrow x^3$ or $x^{-3} \rightarrow x^{-2}$ or + CM1 A1 A1 3
- $\left(= \frac{2x^3}{3} + 3x^{-2} + C \right)$ First A1: $\frac{2x^3}{3} + C$
- Second A1: $-\frac{6x^{-2}}{-2}$

In both parts, accept any correct version, simplified or not. Accept $4x^1$ for $4x$.

+ C in part (a) instead of part (b):

Penalise only once, so if otherwise correct scores M1 A0, M1 A1 A1.

[5]

16. (a) $f(3) = 27 - 117 + 165 - 75$ (some working needed) M1
- $= 0$, so $(x - 3)$ is a factor of $f(x)$ A1 2
- M1 is for substituting $x = 3$ in $f(x)$*
- A1 requires $f(3) = 0$ and statement*
- (b) $(x - 3)(x^2 - 10x + 25)$ M1 A1
- $(x - 3)(x - 5)(x - 5)$ A1 3
- M1 for quadratic factor $(x^2 + ax \pm 25)$*
- SC: M1 for one other linear factor found*
- (c) 3 and 5 B1 1

(d) $f(x) = 3x^2 - 26x + 55$ M1 A1
 $f(3) = 27 - 78 + 55 = 4$ A1 3
M1 for differentiation: at least one term has power of x reduced by 1

(e) “ $3x^2 - 26x + 55 = 4$ ” M1
 $3x^2 - 26x + 51 = 0 \Rightarrow (3x - 17)(x - 3) = 0 \quad x = \dots$ depM1 A1ft
 x-coordinate of S is $\frac{17}{3} \left(\frac{34}{6} \text{ or } 5\frac{2}{3} \text{ or } 5.\dot{6} \text{ or } 5.67 \right)$ A1 4

First M1: equating their gradient function to their answer to (d)

Second M1: Solving by factors requires

$(ax^2 + bx + c) = (mx + p)(nx + q)$
 where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

Solving by quadratic formula requires attempt to use correct formula with candidate’s values of a, b and c used.

A1 f.t. only for correct (real) answers to their quadratic

Final A1 for a correct exact form.

[13]

17. (i) (a) $15x^2 + 7$ M1 A1 A1 3
 (i) (b) $30x$ B1ft 1
 (ii) $x + 2x^{\frac{3}{2}} + x^{-1} + C$ A1: $x + C$, A1: $+2x^{\frac{3}{2}}$, A1: $+x^{-1}$ M1 A1 A1 A1 4

[8]

18. (a) $\sqrt{8} = 2\sqrt{2}$ seen or used somewhere (possibly implied). B1
 $\frac{12}{\sqrt{8}} = \frac{12\sqrt{8}}{8}$ or $\frac{12}{2\sqrt{2}} = \frac{12\sqrt{2}}{4}$
 Direct statement, e.g. $\frac{6}{\sqrt{2}} = 3\sqrt{2}$ (no indication of method) is M0. M1
 At $x = 8$, $\frac{dy}{dx} = 3\sqrt{8} + \frac{12}{\sqrt{8}} = 6\sqrt{2} + 3\sqrt{2} = 9\sqrt{2}$ (*) A1 3

(b) Integrating: $\frac{3x^{\frac{3}{2}}}{(\frac{3}{2})} + \frac{12x^{\frac{1}{2}}}{(\frac{1}{2})} + C$ (C not required) M1 A1 A1
 At (4, 30), $\frac{3 \times 4^{\frac{3}{2}}}{(\frac{3}{2})} + \frac{12 \times 4^{\frac{1}{2}}}{(\frac{1}{2})} + C = 30$ (C required) M1
 $(f(x) =) 2x^{\frac{3}{2}} + 24x^{\frac{1}{2}}, -34$ A1, A1 6

[9]

19. (a) $\frac{dy}{dx} = 10 \times \frac{3}{2} x^{\frac{1}{2}} \left(= 15x^{\frac{1}{2}} \right)$ M1 A1

(b) $7x + 4x^{\frac{5}{2}} + C$ M1 A2 (1, 0)

[5]

20. (a) $\frac{d^2y}{dx^2} = 3x^2 + 2$ M1 A1 2

(b) Since x^2 is always positive, $\frac{d^2y}{dx^2} \geq 2$ for all x . B1 1

(c) $y = \frac{x^4}{4} + x^2 - 7x + (k)$ [k not required here] M1 A2 (1, 0)

$4 = \frac{2^4}{4} + 2^2 - 14 + k \quad k = 10 \quad y = \frac{x^4}{4} + x^2 - 7x + 10$ M1 A1 5

(d) $x = 2: \frac{dy}{dx} = 8 + 4 - 7 = 5$ M1 A1

Gradient of normal = $-\frac{1}{5}$ M1

$y - 4 = -\frac{1}{5}(x - 2) \quad x + 5y - 22 = 0$ M1 A1 5

[13]

1. There were many perfect answers here with candidates securing their marks in 3 or 4 clear lines of working. The division caused problems for some but even these candidates could differentiate $8x^3$ correctly and sometimes $-4\sqrt{x}$ as well.

There were two common approaches to the division with splitting and dividing usually proving more successful than multiplying by x^{-1} as this often resulted in a $3x^{-2}$.

A minority of candidates had problems in writing \sqrt{x} as $x^{\frac{1}{2}}$ choosing x^{-1} instead and it was encouraging to see very few cases of integration or correct differentiation with a $+c$ this year.
2. Many candidates differentiated correctly, scoring full marks. The most common mistake was to give $\frac{1}{3}x^{\frac{1}{3}}$ as the derivative of $x^{\frac{1}{3}}$. Just a few candidates integrated or included $+C$ in their answer.
3. This was a successful question for many candidates, although for some the required division by x in part (a) proved too difficult. Sometimes the numerator was multiplied by x , or x^{-1} was added to the numerator. Occasionally the numerator and denominator were differentiated separately.

In part (b), most candidates substituted $x = 2$ into their $\frac{dy}{dx}$, but in finding the equation of the tangent numerical mistakes were common and there was sometimes confusion between the value of $\frac{dy}{dx}$ and the value of y .
4. This question was answered very well with most candidates knowing and applying the rules for differentiation and integration correctly although the use of the notation for this topic, especially the integration sign, is still poor.

Most differentiated $2x^3$ correctly in part (a) and although many wrote $\frac{3}{x^2}$ as $3x^{-2}$, some still thought the derivative was $-6x^{-1}$ or $+6x^{-3}$. Similar problems arose with the integration in part (b) and some lost marks through failing to simplify their expressions and of course others forgot the $+c$.

5. Most candidates made a good attempt at expanding the brackets but some struggled with $-4\sqrt{x} \times -4\sqrt{x}$ with answers such as $-4\sqrt{x}$ or $\pm 16\sqrt{x}$ or $\pm 16x^{\frac{1}{4}}$ being quite common. The next challenge was the division by \sqrt{x} and some thought that $\frac{x}{\sqrt{x}} = 1$. Many, but not all, who had difficulties in establishing the first part made use of the given expression and there were plenty of good attempts at differentiating. Inevitably some did not interpret $f'(x)$ correctly and a few attempted to integrate but with a follow through mark here many scored all 3 marks. In part (c) the candidates were expected to evaluate $9^{\frac{3}{2}}$ or $9^{\frac{1}{2}}$ correctly and then combine the fractions - two significant challenges but many completed both tasks very efficiently.
6. Good candidates generally had no difficulty with the division in part (a) of this question, but others were often unable to cope with the required algebra and produced some very confused solutions. A common mistake was to “multiply instead of divide”, giving $2x^2 \div \sqrt{x} = 2x^{\frac{5}{2}}$, and sometimes \sqrt{x} was interpreted as x^{-1} . Examiners saw a wide variety of wrong answers for p and q . Most candidates were able to pick up at least two marks in part (b), where follow-through credit was available in many cases. While the vast majority used the answers from part (a), a few differentiated the numerator and denominator of the fraction term separately, then divided.
7. Most candidates differentiated in part (a) and usually scored both marks. Sometimes the coefficient of the second term was incorrect (values of 2, $\frac{1}{2}$ or $\frac{1}{3}$ were seen). In part (b) most candidates were able to form a suitable equation and start to collect terms but a few simply evaluated $f'(15)$. There were a number of instances of poor algebraic processing from a correct equation with steps such as $3x^2 = 12 \Rightarrow 3x = \sqrt{12}$ or $3x^2 = 12 \Rightarrow x^2 = 9 \Rightarrow x = 3$ appearing far too often.
8. Most candidates could start this question and there were many fully correct solutions to part (a) although some weaker candidates were confused by the k and answers such as $2kx^2$ or $3k^2$ were seen. Part (b) though required some careful thought and proved quite discriminating. Many candidates identified the gradient of the line as 3.5 and sometimes they equated this to their answer to part (a). Those who realised that they needed to use $x = -0.5$ in the resulting equation often went on to find k correctly but there were many who failed to give a convincing argument that $k = 2$. Some found $f(0.5)$ but they set this equal to 7, -7 or 0 and a few, who confused tangents and normals, used $-\frac{2}{7}$. In part (c) there were attempts to substitute $x = -0.5$ into the equation of the line or the differential and those who did substitute into the equation of the curve along with their value of k (even when this was correct) often floundered with the resulting arithmetic and so completely correct solutions to this question were rare.

9. Most candidates were successful in finding at least one correct term from their division in part (a), but mistakes here included dividing only one term in the numerator by x and multiplying by x instead of dividing. Just a few seemed unaware that \sqrt{x} was equivalent to $x^{\frac{1}{2}}$. The vast majority who scored both marks (a) went on to score full marks in part (b). Much of the differentiation seen in part (b) was correct, but occasionally terms were not simplified. Some candidates failed to cope with the differentiation of the ‘easier’ part of the expression, $5x - 7$.

10. For the sketch in part (a), most candidates produced a cubic graph but many failed to appreciate that the minimum was at $(1, 0)$. Often three different intersections with the x -axis were seen. More often than not the intersections with the x -axis were labelled but the intersection at $(0, 3)$ was frequently omitted. A sizeable minority of candidates drew a parabola. Many unnecessarily expanded the brackets for the function at this stage (perhaps gaining credit for the work required in part (b)).

The majority of candidates scored at least one mark in part (b), where the required form of the expansion was given. The best approach was to evaluate the product of two of the linear brackets and then to multiply the resulting quadratic with the third linear factor. Some tried, often unsuccessfully, to multiply out all three linear brackets at the same time. Again, as in Q7, the omission of brackets was common.

Although weaker candidates sometimes failed to produce any differentiation in part (c), others usually did well. Occasional mistakes included not equating the gradient to 3 and slips in the solution of the quadratic equation. Some candidates wasted time in unnecessarily evaluating the y coordinates of the required points.

11. For most candidates, this was a straightforward question and full marks were very common. The fractional power caused some problems with $x^{\frac{3}{2}}$ occasionally appearing instead of $x^{-\frac{1}{2}}$. Apart from this the most common mistake was to differentiate -1 incorrectly to give $-x$. A few candidates integrated throughout, or added a constant to their differentiated expression.

12. Apart from those who made arithmetic slips, many candidates did well on the first three parts of this question. Some, however, managed the differentiation in part (a) but were unable to make further progress. Mistakes in the differentiation inevitably led to problems in part (c), since it was not then legitimately possible to obtain the given normal equation.

Part (b) was accessible to most candidates, although some did attempt to substitute $x = 4$ into the derivative from part (a) rather than into the equation of the curve. The inability to calculate the value of $4^{3/2}$ prevented some candidates from being able to show the given result convincingly.

Candidates often found part (c) difficult. Where the need to evaluate the derivative was not realised, many resorted to working backwards from the given equation and were unlikely to score any marks.

In part (d), a few candidates made no progress at all, not appearing to understand the demands of the question or not knowing how to find the coordinates of Q . In many cases the intersection with the y -axis instead of the x -axis was found. Those that used a sketch were generally more successful. There was sometimes careless arithmetic in the use of Pythagoras' theorem, and then those candidates who did correctly reach $24^2 + 8^2$ sometimes found the subsequent calculation difficult. Of those who reached $\sqrt{640}$, most made a good attempt to simplify the surd.

13. Many students answered this question very well but there were the usual crop of errors as well as some unusual misinterpretations of the notation.

In part (a) the first term was nearly always differentiated correctly but some interpreted $6\sqrt{x}$ as $x^{1/6}$ and some integrated one or both terms. In part (b) most were able to multiply out the numerator correctly, although a few still gave it as $x^2 + 16$, but then problems arose. A number simply differentiated numerator and denominator but many did attempt some sort of division. A common mistake though was to multiply by x instead of divide whilst others simply added or subtracted x^{-1} to their expression. Those who did complete the division correctly often went on to complete the problem but some forgot to differentiate the x term and the 1 was missing from their answer. Some, presumably A2 candidates, used the product or quotient rule to answer part (b) with varied degrees of success.

14. The general standard of calculus displayed throughout the paper was excellent and full marks were common on this question. A few candidates took the negative index in the wrong direction, differentiating x^{-3} to obtain $-2x^{-2}$ and integrating x^{-3} to obtain

$$-\frac{x^{-4}}{4}.$$

15. This was a standard test of candidates' ability to differentiate and integrate, and many completely correct solutions were seen. Answers to part (a) were usually correct, although

$18x^{-2}$ appeared occasionally as the derivative of $-6x^{-3}$. Mistakes in the integration in part (b) were more common, particularly with the negative power, and inevitably the integration constant was frequently omitted. Sometimes the answer to part (a) was integrated, rather than the original function.

16. This was another good source of marks for the majority of candidates, although part (e) was a discriminator at the top end, with only the better candidates able to correctly interpret the request.

Part (a) did require the use of the factor theorem, which was not understood by many candidates, who just showed the factorisation of $f(x)$. Such candidates, however, had a head start in part (b), which was generally well answered. Some candidates made heavy weather of gaining the one mark in part (c), thinking that they needed to find the stationary values to identify Q , and it was quite surprising to see the number of candidates who did not gain the single mark in this part for some reason. The majority of candidates knew what was required in part (d) and any errors tended to be slips in the main, although some candidates were confused between tangent and normal and others felt they had to solve $\frac{dy}{dx} = 0$. Many candidates went on to give the equation of the tangent at P unnecessarily.

Part (e) proved too taxing for the majority of candidates, who were not able to make a start, and although there were many good solutions these were clearly from the more able candidates.

17. This was a standard test of candidates' ability to differentiate and integrate. Answers to part (i)(a) were almost always correct, but in (i)(b) a few candidates seemed unfamiliar with the idea of a second derivative. Before integrating in part (ii), it was necessary to consider \sqrt{x} as $x^{\frac{1}{2}}$ and $\frac{1}{x^2}$ as x^{-2} , and this step defeated some candidates. Apart from this, other common mistakes were to integrate x^{-2} to give $\frac{x^{-3}}{-3}$, to interpret $3\sqrt{x}$ as $x^{\frac{1}{3}}$ and to omit the constant of integration.

- 18.** The manipulation of surds in part (a) was often disappointing in this question. While most candidates appreciated the significance of the “exact value” demand and were not tempted to use decimals from their calculators, the inability to rationalise the denominator was a common problem. Various alternative methods were seen, but for all of these, since the answer was given, it was necessary to show the relevant steps in the working to obtain full marks.

Integration techniques in part (b) were usually correct, despite some problems with fractional indices, but the lack of an integration constant limited candidates to 3 marks out of 6. Sometimes the (4, 30) coordinates were used as limits for an attempted “definite integration”.

- 19.** Most candidates were proficient in the techniques of differentiation and integration, gaining good marks in this question. Answers to part (a) were nearly always correct, but in part (b) it was very common for the constant of integration to be omitted. Also in (b), many candidates were unwilling or unable to simplify $\frac{10x^{\frac{5}{2}}}{\frac{5}{2}}$. Just a few candidates integrated their answer to part (a) rather than the given function.

- 20.** No Report available for this question.