

Core 1 Coordinate Geometry Questions

- 2 The point A has coordinates $(1, 1)$ and the point B has coordinates $(5, k)$.

The line AB has equation $3x + 4y = 7$.

- (a) (i) Show that $k = -2$. *(1 mark)*
- (ii) Hence find the coordinates of the mid-point of AB . *(2 marks)*
- (b) Find the gradient of AB . *(2 marks)*
- (c) The line AC is perpendicular to the line AB .
- (i) Find the gradient of AC . *(2 marks)*
- (ii) Hence find an equation of the line AC . *(1 mark)*
- (iii) Given that the point C lies on the x -axis, find its x -coordinate. *(2 marks)*
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- (b) The line L has equation $y + 2x = 12$ and the curve C has equation $y = x^2 - 4x + 9$.

- (i) Show that the x -coordinates of the points of intersection of L and C satisfy the equation

$$x^2 - 2x - 3 = 0 \quad (1 \text{ mark})$$

- (ii) Hence find the coordinates of the points of intersection of L and C . *(4 marks)*
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5 A circle with centre C has equation $x^2 + y^2 - 8x + 6y = 11$.

(a) By completing the square, express this equation in the form

$$(x - a)^2 + (y - b)^2 = r^2 \quad (3 \text{ marks})$$

(b) Write down:

(i) the coordinates of C ; (1 mark)

(ii) the radius of the circle. (1 mark)

(c) The point O has coordinates $(0, 0)$.

(i) Find the length of CO . (2 marks)

(ii) Hence determine whether the point O lies inside or outside the circle, giving a reason for your answer. (2 marks)

1 The point A has coordinates $(1, 7)$ and the point B has coordinates $(5, 1)$.

(a) (i) Find the gradient of the line AB . (2 marks)

(ii) Hence, or otherwise, show that the line AB has equation $3x + 2y = 17$. (2 marks)

(b) The line AB intersects the line with equation $x - 4y = 8$ at the point C . Find the coordinates of C . (3 marks)

(c) Find an equation of the line through A which is perpendicular to AB . (3 marks)

7 A circle has equation $x^2 + y^2 - 4x - 14 = 0$.

(a) Find:

(i) the coordinates of the centre of the circle; (3 marks)

(ii) the radius of the circle in the form $p\sqrt{2}$, where p is an integer. (3 marks)

(b) A chord of the circle has length 8. Find the perpendicular distance from the centre of the circle to this chord. (3 marks)

(c) A line has equation $y = 2k - x$, where k is a constant.

(i) Show that the x -coordinate of any point of intersection of the line and the circle satisfies the equation

$$x^2 - 2(k + 1)x + 2k^2 - 7 = 0 \quad (3 \text{ marks})$$

(ii) Find the values of k for which the equation

$$x^2 - 2(k + 1)x + 2k^2 - 7 = 0$$

has equal roots. (4 marks)

(iii) Describe the geometrical relationship between the line and the circle when k takes either of the values found in part (c)(ii). (1 mark)

2 The line AB has equation $3x + 5y = 8$ and the point A has coordinates $(6, -2)$.

(a) (i) Find the gradient of AB . (2 marks)

(ii) Hence find an equation of the straight line which is perpendicular to AB and which passes through A . (3 marks)

(b) The line AB intersects the line with equation $2x + 3y = 3$ at the point B . Find the coordinates of B . (3 marks)

(c) The point C has coordinates $(2, k)$ and the distance from A to C is 5. Find the **two** possible values of the constant k . (3 marks)

4 A circle with centre C has equation $x^2 + y^2 + 2x - 12y + 12 = 0$.

(a) By completing the square, express this equation in the form

$$(x - a)^2 + (y - b)^2 = r^2 \quad (3 \text{ marks})$$

(b) Write down:

(i) the coordinates of C ; (1 mark)

(ii) the radius of the circle. (1 mark)

(c) Show that the circle does **not** intersect the x -axis. (2 marks)

(d) The line with equation $x + y = 4$ intersects the circle at the points P and Q .

(i) Show that the x -coordinates of P and Q satisfy the equation

$$x^2 + 3x - 10 = 0 \quad (3 \text{ marks})$$

(ii) Given that P has coordinates $(2, 2)$, find the coordinates of Q . (2 marks)

(iii) Hence find the coordinates of the midpoint of PQ . (2 marks)

1 The points A and B have coordinates $(6, -1)$ and $(2, 5)$ respectively.

(a) (i) Show that the gradient of AB is $-\frac{3}{2}$. (2 marks)

(ii) Hence find an equation of the line AB , giving your answer in the form $ax + by = c$, where a , b and c are integers. (2 marks)

(b) (i) Find an equation of the line which passes through B and which is perpendicular to the line AB . (2 marks)

(ii) The point C has coordinates $(k, 7)$ and angle ABC is a right angle.

Find the value of the constant k . (2 marks)

5 A circle with centre C has equation $(x + 3)^2 + (y - 2)^2 = 25$.

(a) Write down:

(i) the coordinates of C ; *(2 marks)*

(ii) the radius of the circle. *(1 mark)*

(b) (i) Verify that the point $N(0, -2)$ lies on the circle. *(1 mark)*

(ii) Sketch the circle. *(2 marks)*

(iii) Find an equation of the normal to the circle at the point N . *(3 marks)*

(c) The point P has coordinates $(2, 6)$.

(i) Find the distance PC , leaving your answer in surd form. *(2 marks)*

(ii) Find the length of a tangent drawn from P to the circle. *(3 marks)*

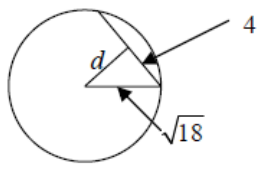
Core 1 Coordinate Geometry Answers

2(a)(i)	$15 + 4k = 7 \Rightarrow 4k = -8 \Rightarrow k = -2$	B1	1	AG (condone verification or $y = -2$)
(ii)	$\frac{1}{2}(x_1 + x_2)$ or $\frac{1}{2}(y_1 + y_2)$	M1		
	Midpoint coordinates $\left(3, -\frac{1}{2}\right)$	A1	2	One coordinate correct implies M1
(b)	Attempt at $\Delta y / \Delta x$ or $y = -\frac{3}{4}x + \frac{7}{4}$	M1		(Not x over y)(may use M instead of A/B)
	Gradient $AB = -\frac{3}{4}$	A1	2	-0.75 etc any correct equivalent
(c)(i)	$m_1 m_2 = -1$ used or stated	1		
	Hence gradient $AC = \frac{4}{3}$	A1✓	2	Follow through their gradient of AB from part (b)
(ii)	$y - 1 = \frac{4}{3}(x - 1)$ or $3y = 4x - 1$ etc	B1✓	1	Follow through their gradient of AC from part (c) (i) must be normal & (1,1) used
(iii)	$y = 0 \Rightarrow x - 1 = -\frac{3}{4}$	M1		Putting $y = 0$ in their AC equation and attempting to find x
	$x = \frac{1}{4}$	A1	2	CSO. C has coordinates $\left(\frac{1}{4}, 0\right)$
Total			10	

(b)(i)	$12 - 2x = x^2 - 4x + 9$ $\Rightarrow x^2 - 2x - 3 = 0$	B1	1	Or $x^2 - 4x + 9 + 2x = 12$ AG (be convinced) (must have = 0)
(ii)	$(x - 3)(x + 1) = 0$	M1		Attempt at factors or quadratic formula or one value spotted
	$x = 3, -1$	A1		Both values correct & simplified
	Substitute one value of x to find y	M1		May substitute into equation for L or C
	Points are (3, 6) and (-1, 14)	A1	4	y -coordinates correct linked to x values

5(a)	$(x-4)^2 + (y+3)^2$ $(11+16+9=36)$ RHS = 6^2	B2 B1	3	B1 for one term correct Condone 36
(b)(i)	Centre $(4, -3)$	B1✓	1	Ft their a and b from part (a)
(ii)	Radius = 6	B1✓	1	Ft their r from part (a)
(c)(i)	$CO^2 = (-4)^2 + 3^2$ $CO = 5$	M1 A1✓	2	Accept + or - with numbers but must add Full marks for answer only
(ii)	Considering CO and radius $CO < r \Rightarrow O$ is inside the circle	M1 A1✓	2	Ft outside circle when 'their CO ' > r or on the circle when 'their CO ' = r SC B1✓ if no explanation given
Total			9	

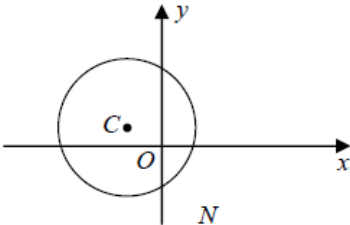
1(a)(i)	Gradient $AB = \frac{1-7}{5-1}$ $= -\frac{6}{4} = -\frac{3}{2} = -1.5$	M1 A1	2	Must be y on top and subtr'n of cords Any correct equivalent
(ii)	$y-7 = m(x-1)$ or $y-1 = m(x-5)$ leading to $3x+2y = 17$	M1 A1	2	Verifying 2 points or $y = -\frac{3}{2}x + c$ AG (or grad & 1 point verified)
(b)	Attempt to eliminate x or y : $7x = 42$ etc $x = 6$ $y = -\frac{1}{2}$	M1 A1 A1	3	Solving $x-4y = 8$; $3x+2y = 17$ C is point $(6, -\frac{1}{2})$
(c)	Grad of perp = $-1 / \text{their gradient } AB$ $= \frac{2}{3}$ $y-7 = \frac{2}{3}(x-1)$ or $3y - 2x = 19$	M1 A1✓ A1	3	Or $m_1m_2 = -1$ used or stated ft their gradient AB CSO Any correct form of equation
Total			10	

7(a)(i)	$(x-2)^2$ centre has x -coordinate = 2 and y -coordinate = 0	M1 A1 B1	3	Attempt to complete square for x M1 implied if value correct or -2 Centre (2,0)
(ii)	RHS = 18 Radius = $\sqrt{18}$ Radius = $3\sqrt{2}$	B1 M1 A1	3	Withhold if circle equation RHS incorrect Square root of RHS of equation (if > 0)
(b)	Perpendicular bisects chord so need to use Length of 4 $d^2 = (\text{radius})^2 - 4^2$ $d^2 = 18 - 16$ so perpendicular distance = $\sqrt{2}$	B1 M1 A1	3	
(c)(i)	$x^2 + (2k-x)^2 - 4x - 14 = 0$ $(2k-x)^2 = 4k^2 - 4kx + x^2$ $\Rightarrow 2x^2 + 4k^2 - 4kx - 4x - 14 = 0$ $(\Rightarrow x^2 + 2k^2 - 2kx - 2x - 7 = 0)$ $\Rightarrow x^2 - 2(k+1)x + 2k^2 - 7 = 0$	M1 B1 A1	3	AG (be convinced about algebra and = 0)
(ii)	$4(k+1)^2 - 4(2k^2 - 7)$ $4k^2 - 8k - 32 = 0$ or $k^2 - 2k - 8 = 0$	M1 A1		" $b^2 - 4ac$ " in terms of k (either term correct) $b^2 - 4ac = 0$ correct quadratic equation in k
	$(k-4)(k+2) = 0$ $k = -2, k = 4$	m1 A1	4	Attempt to factorise, solve equation SC B1, B1 for -2, 4 (if M0 scored)
(iii)	Line is a tangent to the circle	E1	1	Line touches circle at one point etc
Total			17	

2(a)(i)	$y = -\frac{3}{5}x + \dots$; Gradient $AB = -\frac{3}{5}$	M1 A1	2	Attempt to find $y =$ or $\Delta y / \Delta x$ or $\frac{3}{5}$ or $3x/5$ Gradient correct – condone slip in $y = \dots$
(ii)	$m_1 m_2 = -1$ Gradient of perpendicular = $\frac{5}{3}$ $\Rightarrow y + 2 = \frac{5}{3}(x - 6)$	M1 A1 ✓	3	Stated or used correctly ft gradient of AB CSO Any correct form eg $y = \frac{5}{3}x - 12$, $5x - 3y = 36$ etc Must use $3x + 5y = 8$; $2x + 3y = 3$
(b)	Eliminating x or y (unsimplified) $x = -9$ $y = 7$	M1 A1 A1	3	$B(-9, 7)$
(c)	$4^2 + (k+2)^2 (= 25)$ or $16 + d^2 = 25$ $k = 1$ or $k = -5$	M1 A1 A1	3	Diagram with 3,4, 5 triangle Condone slip in one term (or $k + 2 = 3$) SC1 with no working for spotting one correct value of k . Full marks if both values spotted with no contradictory work
Total			11	

4(a)	$(x+1)^2 + (y-6)^2$ $(1+36-12=25) \quad \text{RHS} = 5^2$	B2 B1	3	B1 for one term correct or missing + sign Condone 25
(b)(i)	Centre $(-1, 6)$	B1✓	1	FT their a and b from part (a) or correct
(ii)	Radius = 5	B1✓	1	FT their r from part (a) RHS must be > 0
(c)	Attempt to solve "their" $x^2 + 2x + 12 = 0$ (all working correct) so no real roots or statement that does not intersect	M1 A1	2	Or comparing "their" $y_c = 6$ and their $r = 5$ may use a diagram with values shown $\left\{ \begin{array}{l} r < y_c \text{ so does not intersect} \\ \text{condone } \pm 1 \text{ or } \pm 6 \text{ in centre for A1} \end{array} \right.$
(d)(i)	$(4-x)^2 = 16 - 8x + x^2$ $x^2 + (4-x)^2 + 2x - 12(4-x) + 12 = 0$ or $(x+1)^2 + (-2-x)^2 = 25$ $\Rightarrow 2x^2 + 6x - 20 = 0 \Rightarrow x^2 + 3x - 10 = 0$	B1 M1 A1	3	Or $(-2-x)^2 = 4 + 4x + x^2$ Sub $y = 4-x$ in circle eqn (condone slip) or "their" circle equation AG CSO (must have = 0)
(ii)	$(x+5)(x-2) = 0 \Rightarrow x = -5, x = 2$ Q has coordinates $(-5, 9)$	M1 A1	2	Correct factors or unsimplified solution to quadratic (give credit if factorised in part (i)) SC2 if Q correct. Allow $x = -5 \quad y = 9$
(iii)	Mid point of 'their' $(-5, 9)$ and $(2, 2)$ $\left(-1\frac{1}{2}, 5\frac{1}{2}\right)$	M1 A1	2	Arithmetic mean of either x or y coords Must follow from correct value in (ii)
Total			14	

1(a)(i)	Gradient $AB = \frac{-1-5}{6-2}$ or $\frac{5-1}{2-6}$ $= \frac{-6}{4} = -\frac{3}{2}$	M1 A1	2	$\pm \frac{6}{4}$ implies M1 AG
(ii)	$\left. \begin{array}{l} y-5 \\ y+1 \end{array} \right\} = -\frac{3}{2} \left\{ \begin{array}{l} (x-2) \\ (x-6) \end{array} \right.$ $\Rightarrow 3x + 2y = 16$	M1 A1	2	or $y = -\frac{3}{2}x + c$ and attempt to find c OE: must have integer coefficients
(b)(i)	Gradient of perpendicular = $\frac{2}{3}$ $\Rightarrow y - 5 = \frac{2}{3}(x - 2)$	M1 A1	2	or use of $m_1 m_2 = -1$ $3y - 2x = 11$ (no misreads permitted)
(ii)	Substitute $x = k, y = 7$ into their (b)(i) $\Rightarrow 2 = \frac{2}{3}(k - 2) \Rightarrow k = 5$	M1 A1	2	or grads $\frac{7-5}{k-2} \times \frac{-3}{2} = -1$ or Pythagoras $(k-2)^2 = (k-6)^2 + 8$
Total			8	

5(a)(i)	Centre $(-3, 2)$	M1		± 3 or ± 2
		A1	2	correct
(ii)	Radius = 5	B1	1	accept $\sqrt{25}$ but not $\pm\sqrt{25}$
(b)(i)	$3^2 + (-4)^2 = 9 + 16 = 25$ $\Rightarrow N$ lies on circle	B1	1	must have $9 + 16 = 25$ or a statement
(ii)		M1		must draw axes; fit their centre in correct quadrant
		A1	2	correct (reasonable freehand circle enclosing origin)
(iii)	Attempt at gradient of CN $\text{grad } CN = -\frac{4}{3}$ $y = -\frac{4}{3}x - 2$ (or equivalent)	M1		withhold if subsequently finds tangent
		A1		CSO
		A1✓	3	fit their grad CN
(c)(i)	$P(2, 6)$ Hence $PC^2 = 5^2 + 4^2$	M1		“their” PC^2
	$\Rightarrow PC = \sqrt{41}$	A1	2	
(ii)	Use of Pythagoras correctly $PT^2 = PC^2 - r^2 = 41 - 25,$ where T is a point of contact of tangent $\Rightarrow PT = 4$	M1		
		A1✓		fit their PC^2 and r^2
		A1	3	Alternative sketch with vertical tangent M1 showing that tangent touches circle at point $(2, 2)$ A1 hence $PT = 4$ A1
Total			14	