Question:

Work out the gradients of these lines:

(a) \( y = -2x + 5 \)
(b) \( y = -x + 7 \)
(c) \( y = 4 + 3x \)
(d) \( y = \frac{1}{3}x - 2 \)
(e) \( y = -\frac{2}{3}x \)

(f) \( y = \frac{5}{4}x + \frac{2}{3} \)

(g) \( 2x - 4y + 5 = 0 \)
(h) \( 10x - 5y + 1 = 0 \)
(i) \( -x + 2y - 4 = 0 \)
(j) \( -3x + 6y + 7 = 0 \)
(k) \( 4x + 2y - 9 = 0 \)
(l) \( 9x + 6y + 2 = 0 \)

Solution:

(a) Gradient = -2
(b) Gradient = -1
(c) Gradient = 3
(d) Gradient = \( \frac{1}{3} \)

(e) Gradient = \( -\frac{2}{3} \)

(f) Gradient = \( \frac{5}{4} \)

(g) \( 2x - 4y + 5 = 0 \)
\( 2x + 5 = 4y \)
4y = 2x + 5
y = \frac{1}{4}x + \frac{5}{4}

Gradient = \frac{1}{2}

(h) 10x - 5y + 1 = 0
10x + 1 = 5y
5y = 10x + 1
y = \frac{10}{5}x + \frac{1}{5}

y = 2x + \frac{1}{5}

Gradient = 2

(i) -x + 2y - 4 = 0
2y - 4 = x
2y = x + 4
y = \frac{1}{2}x + 2

Gradient = \frac{1}{2}

(j) -3x + 6y + 7 = 0
6y + 7 = 3x
6y = 3x - 7
y = \frac{3}{6}x - \frac{7}{6}

y = \frac{1}{2}x - \frac{7}{6}

Gradient = \frac{1}{2}

(k) 4x + 2y - 9 = 0
2y - 9 = -4x
2y = -4x + 9
y = -\frac{4}{2}x + \frac{9}{2}

y = -2x + \frac{9}{2}

Gradient = -2

(l) 9x + 6y + 2 = 0
6y + 2 = -9x
6y = -9x - 2
y = -\frac{9}{6}x - \frac{2}{6}

y = -\frac{3}{2}x - \frac{1}{3}

Gradient = -\frac{3}{2}
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Coordinate geometry in the (x, y) plane
Exercise A, Question 2

Question:

These lines intercept the y-axis at (0, c). Work out the value of c in each case.

(a) \( y = -x + 4 \)
(b) \( y = 2x - 5 \)
(c) \( y = \frac{1}{2}x - \frac{2}{3} \)

(d) \( y = -3x \)
(e) \( y = \frac{6}{7}x + \frac{7}{5} \)

(f) \( y = 2 - 7x \)
(g) \( 3x - 4y + 8 = 0 \)
(h) \( 4x - 5y - 10 = 0 \)
(i) \( -2x + y - 9 = 0 \)

(j) \( 7x + 4y + 12 = 0 \)
(k) \( 7x - 2y + 3 = 0 \)
(l) \( -5x + 4y + 2 = 0 \)

Solution:

(a) \( c = 4 \)
(b) \( c = -5 \)
(c) \( c = -\frac{2}{3} \)

(d) \( y = -3x \)
\( y = -3x + 0 \)
\( c = 0 \)

(e) \( c = \frac{7}{5} \)

(f) \( y = 2 - 7x \)
\( y = -7x + 2 \)
\( c = 2 \)

(g) \( 3x - 4y + 8 = 0 \)
\[3x + 8 = 4y\]
\[4y = 3x + 8\]
\[y = \frac{3}{4}x + 2\]
\[c = 2\]

(h) \[4x - 5y - 10 = 0\]
\[4x - 10 = 5y\]
\[5y = 4x - 10\]
\[y = \frac{4}{5}x - \frac{10}{5}\]
\[y = \frac{4}{5}x - 2\]
\[c = -2\]

(i) \[-2x + y - 9 = 0\]
\[y - 9 = 2x\]
\[y = 2x + 9\]
\[c = 9\]

(j) \[7x + 4y + 12 = 0\]
\[4y + 12 = -7x\]
\[4y = -7x - 12\]
\[y = -\frac{7}{4}x - 3\]
\[c = -3\]

(k) \[7x - 2y + 3 = 0\]
\[7x + 3 = 2y\]
\[2y = 7x + 3\]
\[y = \frac{7}{2}x + \frac{3}{2}\]
\[c = \frac{3}{2}\]

(l) \[-5x + 4y + 2 = 0\]
\[4y + 2 = 5x\]
\[4y = 5x - 2\]
\[y = \frac{5}{4}x - \frac{2}{4}\]
\[y = \frac{5}{4}x - \frac{1}{2}\]
\[c = -\frac{1}{2}\]
Question:

Write these lines in the form \( ax + by + c = 0 \).

(a) \( y = 4x + 3 \)

(b) \( y = 3x - 2 \)

(c) \( y = -6x + 7 \)

(d) \( y = \frac{4}{5}x - 6 \)

(e) \( y = \frac{5}{3}x + 2 \)

(f) \( y = \frac{7}{3}x \)

(g) \( y = 2x - \frac{4}{7} \)

(h) \( y = -3x + \frac{2}{9} \)

(i) \( y = -6x - \frac{2}{3} \)

(j) \( y = -\frac{1}{3}x + \frac{1}{2} \)

(k) \( y = \frac{2}{3}x + \frac{5}{6} \)

(l) \( y = \frac{3}{5}x + \frac{1}{2} \)

Solution:

(a) \( y = 4x + 3 \)
\( 0 = 4x + 3 - y \)
\( 4x + 3 - y = 0 \)
\( 4x - y + 3 = 0 \)

(b) \( y = 3x - 2 \)
\( 0 = 3x - 2 - y \)
\( 3x - 2 - y = 0 \)
\( 3x - y - 2 = 0 \)
(c) \( y = -6x + 7 \)
\( 6x + y = 7 \)
\( 6x + y - 7 = 0 \)

(d) \( y = \frac{4}{5}x - 6 \)
Multiply each term by 5:
\( 5y = 4x - 30 \)
\( 0 = 4x - 30 - 5y \)
\( 4x - 30 - 5y = 0 \)
\( 4x - 5y - 30 = 0 \)

(e) \( y = \frac{5}{3}x + 2 \)
Multiply each term by 3:
\( 3y = 5x + 6 \)
\( 0 = 5x + 6 - 3y \)
\( 5x + 6 - 3y = 0 \)
\( 5x - 3y + 6 = 0 \)

(f) \( y = \frac{7}{3}x \)
Multiply each term by 3:
\( 3y = 7x \)
\( 0 = 7x - 3y \)
\( 7x - 3y = 0 \)

(g) \( y = 2x - \frac{4}{7} \)
Multiply each term by 7:
\( 7y = 14x - 4 \)
\( 0 = 14x - 4 - 7y \)
\( 14x - 4 - 7y = 0 \)
\( 14x - 7y - 4 = 0 \)

(h) \( y = -3x + \frac{2}{9} \)
Multiply each term by 9:
\( 9y = -27x + 2 \)
\( 27x + 9y = 2 \)
\( 27x + 9y - 2 = 0 \)

(i) \( y = -6x - \frac{2}{3} \)
Multiply each term by 3:
\( 3y = -18x - 2 \)
\( 18x + 3y = -2 \)
\( 18x + 3y + 2 = 0 \)

(j) \( y = -\frac{1}{3}x + \frac{1}{2} \)
Multiply each term by 6 (6 is divisible by both 3 and 2):
\( 6y = -2x + 3 \)
\( 2x + 6y = 3 \)
\( 2x + 6y - 3 = 0 \)

(k) \( y = \frac{2}{3}x + \frac{5}{6} \)
Multiply each term by 6 (6 is divisible by both 3 and 6):
\( 6y = 4x + 5 \)
\[ 0 = 4x + 5 - 6y \]
\[ 4x + 5 - 6y = 0 \]
\[ 4x - 6y + 5 = 0 \]

(l) \[ y = \frac{3}{5} x + \frac{1}{2} \]

Multiply each term by 10 (10 is divisible by both 5 and 2):
\[ 10y = 6x + 5 \]
\[ 0 = 6x + 5 - 10y \]
\[ 6x + 5 - 10y = 0 \]
\[ 6x - 10y + 5 = 0 \]
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Coordinate geometry in the (x, y) plane
Exercise A, Question 4

Question:
A line is parallel to the line \( y = 5x + 8 \) and its intercept on the \( y \)-axis is \((0, 3)\). Write down the equation of the line.

Solution:
The line is parallel to \( y = 5x + 8 \), so \( m = 5 \).
The line intercepts the \( y \)-axis at \((0, 3)\), so \( c = 3 \).
Using \( y = mx + c \), the equation of the line is \( y = 5x + 3 \).

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A line is parallel to the line $y = -\frac{2}{5}x + 1$ and its intercept on the $y$-axis is $(0, -4)$. Work out the equation of the line. Write your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers.

The line is parallel to $y = -\frac{2}{5}x + 1$, so $m = -\frac{2}{5}$.

The line intercepts the $y$-axis at $(0, -4)$, so $c = -4$.

Using $y = mx + c$, the equation of the line is $y = -\frac{2}{5}x - 4$.

Multiply each term by 5:

$5y = -2x - 20$

$2x + 5y = -20$

$2x + 5y + 20 = 0$
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Coordinate geometry in the (x, y) plane
Exercise A, Question 6

Question:
A line is parallel to the line $3x + 6y + 11 = 0$ and its intercept on the y-axis is $(0, 7)$. Write down the equation of the line.

Solution:

$3x + 6y + 11 = 0$
$6y + 11 = -3x$
$6y = -3x - 11$
$y = -\frac{3}{6}x - \frac{11}{6}$
$y = -\frac{1}{2}x - \frac{11}{6}$

The line is parallel to $y = -\frac{1}{2}x - \frac{11}{6}$, so $m = -\frac{1}{2}$.

The line intercepts the y-axis at $(0, 7)$, so $c = 7$.

Using $y = mx + c$, the equation of the line is $y = -\frac{1}{2}x + 7$

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Question:

A line is parallel to the line $2x - 3y - 1 = 0$ and it passes through the point $(0, 0)$. Write down the equation of the line.

Solution:

$2x - 3y - 1 = 0$
$2x - 1 = 3y$
$3y = 2x - 1$

$y = \frac{2}{3}x - \frac{1}{3}$

The line is parallel to $y = \frac{2}{3}x - \frac{1}{3}$, so $m = \frac{2}{3}$.

The intercept on the $y$-axis is $(0, 0)$, so $c = 0$.

Using $y = mx + c$:

$y = \frac{2}{3}x + 0$

$y = \frac{2}{3}x$
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Coordinate geometry in the (x, y) plane
Exercise A, Question 8

Question:

The line \( y = 6x - 18 \) meets the x-axis at the point \( P \). Work out the coordinates of \( P \).

Solution:

\[ y = 6x - 18 \]

Substitute \( y = 0 \):

\[ 6x - 18 = 0 \]

\[ 6x = 18 \]

\[ x = 3 \]

The line meets the x-axis at \( P (3, 0) \).

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Coordinate geometry in the (x, y) plane
Exercise A, Question 9

Question:
The line $3x + 2y - 5 = 0$ meets the x-axis at the point $R$. Work out the coordinates of $R$.

Solution:

\[3x + 2y - 5 = 0\]

Substitute $y = 0$:

\[3x + 2(0) - 5 = 0\]

\[3x - 5 = 0\]

\[3x = 5\]

\[x = \frac{5}{3}\]

The line meets the x-axis at $R \left( \frac{5}{3}, 0 \right)$.
The line $5x - 4y + 20 = 0$ meets the $y$-axis at the point $A$ and the $x$-axis at the point $B$. Work out the coordinates of the points $A$ and $B$.

**Solution:**

$5x - 4y + 20 = 0$

Substitute $x = 0$:

$5(0) - 4y + 20 = 0$

$-4y + 20 = 0$

$20 = 4y$

$y = 5$

The line meets the $y$-axis at $A(0, 5)$.

Substitute $y = 0$:

$5x - 4(0) + 20 = 0$

$5x + 20 = 0$

$5x = -20$

$x = -4$

The line meets the $x$-axis at $B(-4, 0)$. 
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Coordinate geometry in the (x, y) plane
Exercise B, Question 1

Question:

Work out the gradient of the line joining these pairs of points:

(a) (4, 2), (6, 3)
(b) (−1, 3), (5, 4)
(c) (−4, 5), (1, 2)
(d) (2, −3), (6, 5)
(e) (−3, 4), (7, −6)
(f) (−12, 3), (−2, 8)
(g) (−2, −4), (10, 2)

(h) \left( \frac{1}{2}, 2 \right), \left( \frac{3}{4}, 4 \right)

(i) \left( \frac{1}{4}, \frac{1}{2} \right), \left( \frac{1}{2}, \frac{2}{3} \right)

(j) (−2.4, 9.6), (0, 0)

(k) (1.3, −2.2), (8.8, −4.7)

(l) (0, 5a), (10a, 0)

(m) (3b, −2b), (7b, 2b)

(n) (p, p²), (q, q²)

Solution:

(a) \((x_1, y_1) = (4, 2), (x_2, y_2) = (6, 3)\)

\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 2}{6 - 4} = \frac{1}{2}
\]

(b) \((x_1, y_1) = (−1, 3), (x_2, y_2) = (5, 4)\)

\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 3}{5 - (−1)} = \frac{1}{6}
\]

(c) \((x_1, y_1) = (−4, 5), (x_2, y_2) = (1, 2)\)
\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 5}{1 - (-4)} = \frac{-3}{5}
\]

(d) \((x_1, y_1) = (2, -3), (x_2, y_2) = (6, 5)\)

\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-3)}{6 - 2} = \frac{8}{4} = 2
\]

(e) \((x_1, y_1) = (-3, 4), (x_2, y_2) = (7, -6)\)

\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{-6 - 4}{7 - (-3)} = \frac{-10}{10} = -1
\]

(f) \((x_1, y_1) = (-12, 3), (x_2, y_2) = (-2, 8)\)

\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 3}{-2 - (-12)} = \frac{5}{-2 + 12} = \frac{5}{10} = \frac{1}{2}
\]

(g) \((x_1, y_1) = (-2, -4), (x_2, y_2) = (10, 2)\)

\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-4)}{10 - (-2)} = \frac{6}{12} = \frac{1}{2}
\]

(h) \((x_1, y_1) = (\frac{1}{2}, 2), (x_2, y_2) = (\frac{3}{4}, 4)\)

\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 2}{\frac{3}{4} - \frac{1}{2}} = \frac{2}{\frac{1}{4}} = 8
\]

(i) \((x_1, y_1) = (\frac{1}{4}, \frac{1}{2}), (x_2, y_2) = (\frac{1}{2}, \frac{2}{3})\)

\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{\frac{2}{3} - \frac{1}{2}}{\frac{1}{2} - \frac{1}{4}} = \frac{\frac{1}{6}}{\frac{1}{4}} = \frac{2}{3}
\]

(j) \((x_1, y_1) = (-2.4, 9.6), (x_2, y_2) = (0, 0)\)

\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 9.6}{0 - (-2.4)} = \frac{-9.6}{2.4} = -4
\]

(k) \((x_1, y_1) = (1.3, -2.2), (x_2, y_2) = (8.8, -4.7)\)

\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{-4.7 - (-2.2)}{8.8 - 1.3} = \frac{-2.5}{7.5} = -\frac{1}{3}
\]

(l) \((x_1, y_1) = (0, 5a), (x_2, y_2) = (10a, 0)\)

\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 5a}{10a - 0} = \frac{-5a}{10a} = \frac{-5}{10} = -\frac{1}{2}
\]

(m) \((x_1, y_1) = (3b, -2b), (x_2, y_2) = (7b, 2b)\)
\[ \frac{y_2 - y_1}{x_2 - x_1} = \frac{2b - (-2b)}{7b - 3b} = \frac{4b}{4b} = 1 \]

(n) \( (x_1, y_1) = (p, p^2) \), \( (x_2, y_2) = (q, q^2) \)

\[ \frac{y_2 - y_1}{x_2 - x_1} = \frac{q^2 - p^2}{q - p} = \frac{(q - p)(q + p)}{q - p} = q + p \]
Question:

The line joining $(3, -5)$ to $(6, a)$ has gradient 4. Work out the value of $a$.

Solution:

\[
\frac{y_2 - y_1}{x_2 - x_1} = 4
\]

so

\[
\frac{a - (-5)}{6 - 3} = 4
\]

\Rightarrow \quad \frac{a + 5}{3} = 4

\Rightarrow \quad a + 5 = 12

\Rightarrow \quad a = 7

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Coordinate geometry in the (x, y) plane
Exercise B, Question 3

Question:
The line joining \((5, b)\) to \((8, 3)\) has gradient \(-3\). Work out the value of \(b\).

Solution:
\[
\frac{3 - b}{8 - 5} = -3
\]
\[
\frac{3 - b}{3} = -3
\]
\[
3 - b = -9
\]
\[
b = 12
\]
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Coordinate geometry in the (x, y) plane
Exercise B, Question 4

Question:

The line joining \((c, 4)\) to \((7, 6)\) has gradient \(\frac{3}{4}\). Work out the value of \(c\).

Solution:

\[
\frac{6 - 4}{7 - c} = \frac{3}{4}
\]

\[
\frac{2}{7 - c} = \frac{3}{4}
\]

\[
2 = \frac{3}{4} \left(7 - c\right)
\]

\[
8 = 3 \left(7 - c\right)
\]

\[
8 = 21 - 3c
\]

\[
2 = 21 - 3c
\]

\[
c = \frac{-13}{-3} = \frac{13}{3} = 4 \frac{1}{3}
\]

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The line joining \((-1, 2b)\) to \((1, 4)\) has gradient \(-\frac{1}{4}\). Work out the value of \(d\).

Solution:

\[
\frac{4 - 2b}{1 - (-1)} = -\frac{1}{4}
\]

\[
\frac{4 - 2b}{2} = -\frac{1}{4}
\]

\[
2 - b = -\frac{1}{4}
\]

\[
2 - \frac{1}{4} - b = 0
\]

\[
b = 2\frac{1}{4}
\]
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Coordinate geometry in the (x, y) plane
Exercise B, Question 6

Question:

The line joining \((-3, -2)\) to \((2e, 5)\) has gradient 2. Work out the value of \(e\).

Solution:

\[
\frac{y_2 - y_1}{x_2 - x_1} = 2
\]

\[
\frac{5 - (-2)}{2e - (-3)} = 2
\]

\[
\frac{7}{2e + 3} = 2
\]

\[
7 = 2(2e + 3)
\]

\[
7 = 4e + 6
\]

\[
4e = 1
\]

\[
e = \frac{1}{4}
\]
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Coordinate geometry in the (x, y) plane
Exercise B, Question 7

Question:

The line joining $(7, 2)$ to $(f, 3f)$ has gradient 4. Work out the value of $f$.

Solution:

\[
\begin{align*}
\frac{y_2 - y_1}{x_2 - x_1} &= 4 \\
\frac{3f - 2}{f - 7} &= 4 \\
3f - 2 &= 4(f - 7) \\
3f - 2 &= 4f - 28 \\
-2 &= f - 28 \\
26 &= f
\end{align*}
\]

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Question:

The line joining \((3, -4)\) to \((-g, 2g)\) has gradient \(-3\). Work out the value of \(g\).

Solution:

\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{2g - (-4)}{-g - 3} = -3
\]

\[
\frac{2g + 4}{-g - 3} = -3
\]

\[
2g + 4 = -3(-g - 3)
\]

\[
2g + 4 = 3g + 9
\]

\[
g = -5
\]
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Coordinate geometry in the (x, y) plane
Exercise B, Question 9

Question:
Show that the points \( A(2, 3) \), \( B(4, 4) \), \( C(10, 7) \) can be joined by a straight line. (Hint: Find the gradient of the lines joining the points: i \( A \) and \( B \) and ii \( A \) and \( C \).)

Solution:
The gradient of \( AB \) is \( \frac{4 - 3}{4 - 2} = \frac{1}{2} \)
The gradient of \( AC \) is \( \frac{7 - 3}{10 - 2} = \frac{4}{8} = \frac{1}{2} \)
The gradients are equal so the points can be joined by a straight line.

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Coordinate geometry in the (x, y) plane
Exercise B, Question 10

Question:
Show that the points \((-2a, 5a), (0, 4a), (6a, a)\) are collinear (i.e. on the same straight line).

Solution:
The gradient of the line joining \((-2a, 5a)\) and \(0, 4a\) is
\[\frac{4a - 5a}{0 - (-2a)} = \frac{-a}{2a} = \frac{-1}{2}\]
The gradient of the line joining \((-2a, 5a)\) and \(6a, a\) is
\[\frac{a - 5a}{6a - (-2a)} = \frac{-4a}{8a} = \frac{-1}{2}\]
The gradients are equal so the points can be joined by a straight line (i.e. they are collinear).

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Coordinate geometry in the (x, y) plane
Exercise C, Question 1

Question:
Find the equation of the line with gradient $m$ that passes through the point $(x_1, y_1)$ when:

(a) $m = 2$ and $(x_1, y_1) = (2, 5)$

(b) $m = 3$ and $(x_1, y_1) = (-2, 1)$

(c) $m = -1$ and $(x_1, y_1) = (3, -6)$

(d) $m = -4$ and $(x_1, y_1) = (-2, -3)$

(e) $m = \frac{1}{2}$ and $(x_1, y_1) = (-4, 10)$

(f) $m = -\frac{2}{3}$ and $(x_1, y_1) = (-6, -1)$

(g) $m = 2$ and $(x_1, y_1) = (a, 2a)$

(h) $m = -\frac{1}{2}$ and $(x_1, y_1) = (-2b, 3b)$

Solution:

(a) $y - y_1 = m (x - x_1)$
$y - 5 = 2 (x - 2)$
$y = 2x - 4$

(b) $y - y_1 = m (x - x_1)$
$y - 1 = 3 [x - (-2)]$
$y = 3x + 6$

(c) $y - y_1 = m (x - x_1)$
$y + 6 = -1 (x - 3)$
$y = -x - 3$

(d) $y - y_1 = m (x - x_1)$
$y + 3 = -4 [x - (-2)]$
$y + 3 = -4x - 8$

(e) $y - y_1 = m (x - x_1)$
\[ y - 10 = \frac{1}{2} \left[ x - \left( -4 \right) \right] \]

\[ y - 10 = \frac{1}{2} \left( x + 4 \right) \]

\[ y - 10 = \frac{1}{2} x + 2 \]

\[ y = \frac{1}{2} x + 12 \]

\[ \text{(f)} \quad y - y_1 = m \left( x - x_1 \right) \]

\[ y = \left( -1 \right) = - \frac{2}{3} \left[ x - \left( -6 \right) \right] \]

\[ y + 1 = - \frac{2}{3} \left( x + 6 \right) \]

\[ y + 1 = - \frac{2}{3} x - 4 \]

\[ y = - \frac{2}{3} x - 5 \]

\[ \text{(g)} \quad y - y_1 = m \left( x - x_1 \right) \]

\[ y - 2a = 2 \left( x - a \right) \]

\[ y - 2a = 2x - 2a \]

\[ y = 2x \]

\[ \text{(h)} \quad y - y_1 = m \left( x - x_1 \right) \]

\[ y - 3b = - \frac{1}{2} \left[ x - \left( -2b \right) \right] \]

\[ y - 3b = - \frac{1}{2} \left( x + 2b \right) \]

\[ y - 3b = - \frac{1}{2} x - b \]

\[ y = - \frac{1}{2} x - b + 3b \]

\[ y = - \frac{1}{2} x + 2b \]
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Coordinate geometry in the (x, y) plane
Exercise C, Question 2

Question:

The line \( y = 4x - 8 \) meets the x-axis at the point A. Find the equation of the line with gradient 3 that passes through the point A.

Solution:

\[ y = 4x - 8 \]
Substitute \( y = 0 \):
\[ 4x - 8 = 0 \]
\[ 4x = 8 \]
\[ x = 2 \]
So A has coordinates \((2, 0)\).

\[ y - y_1 = m(x - x_1) \]
\[ y - 0 = 3(x - 2) \]
\[ y = 3x - 6 \]
The equation of the line is \( y = 3x - 6 \).

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Question:

The line $y = -2x + 8$ meets the y-axis at the point $B$. Find the equation of the line with gradient 2 that passes through the point $B$.

Solution:

\[
y = -2x + 8
\]

Substitute $x = 0$:

\[
y = -2(0) + 8 \\
y = 8
\]

So $B$ has coordinates $(0, 8)$.

\[
y - y_1 = m (x - x_1) \\
y - 8 = 2(x - 0) \\
y - 8 = 2x \\
y = 2x + 8
\]

The equation of the line is $y = 2x + 8$. 
Question:

The line \( y = \frac{1}{2}x + 6 \) meets the x-axis at the point C. Find the equation of the line with gradient \( \frac{2}{3} \) that passes through the point C. Write your answer in the form \( ax + by + c = 0 \), where \( a, b \) and \( c \) are integers.

Solution:

\[ y = \frac{1}{2}x + 6 \]

Substitute \( y = 0 \):

\[ \frac{1}{2}x + 6 = 0 \]

\[ \frac{1}{2}x = -6 \]

\[ x = -12 \]

So C has coordinates \((-12, 0)\).

\[ y - y_1 = m(x - x_1) \]

\[ y - 0 = \frac{2}{3} \left[ x - \left( \begin{array}{c} -12 \\ \end{array} \right) \right] \]

\[ y = \frac{2}{3} \left( x + 12 \right) \]

\[ y = \frac{2}{3}x + 8 \]

Multiply each term by 3:

\[ 3y = 2x + 24 \]

\[ 0 = 2x + 24 - 3y \]

\[ 2x - 3y + 24 = 0 \]

The equation of the line is \( 2x - 3y + 24 = 0 \).
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Coordinate geometry in the (x, y) plane
Exercise C, Question 5

Question:

The line \( y = \frac{1}{4}x + 2 \) meets the y-axis at the point \( B \). The point \( C \) has coordinates \((-5, 3)\). Find the gradient of the line joining the points \( B \) and \( C \).

Solution:

\[ y = \frac{1}{4}x + 2 \]

Substitute \( x = 0 \):

\[ y = \frac{1}{4} \left( 0 \right) + 2 \]

\[ y = 2 \]

So \( B \) has coordinates \((0, 2)\).

\[ \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 2}{-5 - 0} = \frac{1}{-5} = -\frac{1}{5} \]

The gradient of the line joining \( B \) and \( C \) is \(-\frac{1}{5}\).

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Coordinate geometry in the (x, y) plane
Exercise C, Question 6

Question:

The lines \( y = x \) and \( y = 2x - 5 \) intersect at the point \( A \). Find the equation of the line with gradient \( \frac{2}{5} \) that passes through the point \( A \). (Hint: Solve \( y = x \) and \( y = 2x - 5 \) simultaneously.)

Solution:

Substitute \( y = x \):

\[
x = 2x - 5
0 = x - 5
x = 5
\]

\( y = x \)

Substitute \( x = 5 \):

\( y = 5 \)

The coordinates of \( A \) are \( (5, 5) \).

\[
y - y_1 = m (x - x_1)
\]

\[
y - 5 = \frac{2}{5} (x - 5)
\]

\[
y - 5 = \frac{2}{5}x - 2
\]

\[
y = \frac{2}{5}x + 3
\]

The equation of the line is \( y = \frac{2}{5}x + 3 \).
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Coordinate geometry in the (x, y) plane
Exercise C, Question 7

Question:

The lines $y = 4x - 10$ and $y = x - 1$ intersect at the point $T$. Find the equation of the line with gradient $-rac{2}{3}$ that passes through the point $T$. Write your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers.

Solution:

Substitute $y = x - 1$:

$x - 1 = 4x - 10$
$-1 = 3x - 10$
$9 = 3x$
$x = 3$

$y = x - 1$
Substitute $x = 3$:

$y = 3 - 1 = 2$

The coordinates of $T$ are $(3, 2)$.

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{-2}{3}(x - 3)$

$y - 2 = \frac{-2}{3}x + 2$

$\frac{2}{3}x + y = 2$

$\frac{2}{3}x + y - 4 = 0$

$2x + 3y - 12 = 0$

The equation of the line is $2x + 3y - 12 = 0$. 

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The line $p$ has gradient $\frac{2}{3}$ and passes through the point $(6, -12)$. The line $q$ has gradient $-1$ and passes through the point $(5, 5)$. The line $p$ meets the $y$-axis at $A$ and the line $q$ meets the $x$-axis at $B$. Work out the gradient of the line joining the points $A$ and $B$.

Solution:

The equation of $p$ is

$$y - (-12) = \frac{2}{3}(x - 6)$$

$$y + 12 = \frac{2}{3}x - 4$$

$$y = \frac{2}{3}x - 16$$

The equation of $q$ is

$$y - 5 = -1(x - 5)$$

$$y - 5 = -x + 5$$

$$y = -x + 10$$

For the coordinates of $A$ substitute $x = 0$ into

$$y = \frac{2}{3}x - 16$$

$$y = \frac{2}{3}(0) - 16$$

$$y = -16$$

Coordinates are $A(0, -16)$

For the coordinates of $B$ substitute $y = 0$ into

$$y = -x + 10$$

$$0 = -x + 10$$

$$x = 10$$

Coordinates are $B(10, 0)$

Gradient of $AB$ is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-16 - 0}{10 - 10} = \frac{-16}{-10} = \frac{8}{5}$$

The gradient of the line joining $A$ and $B$ is $\frac{8}{5}$. 

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Solution:

\( y = -2x + 6 \)

Substitute \( y = 0 \):

\[ 0 = -2x + 6 \]

\[ 2x = 6 \]

\[ x = 3 \]

\( P \) has coordinates \((3, 0)\).

\[ y = \frac{3}{2}x - 4 \]

Substitute \( x = 0 \):

\[ y = \frac{3}{2} \left( 0 \right) - 4 \]

\[ y = -4 \]

\( Q \) has coordinates \((0, -4)\).

Gradient of \( PQ \) is

\[ \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - (-4)}{3 - 0} = \frac{4}{3} \]

Equation of \( PQ \) is

\[ y - y_1 = m(x - x_1) \]

Substitute \((3, 0)\):

\[ y - 0 = \frac{4}{3} \left( x - 3 \right) \]

\[ y = \frac{4}{3}x - 4 \]

The equation of the line through \( P \) and \( Q \) is \( y = \frac{4}{3}x - 4 \).
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Coordinate geometry in the (x, y) plane
Exercise C, Question 10

Question:

The line \( y = 3x - 5 \) meets the x-axis at the point \( M \). The line \( y = -\frac{2}{3}x + \frac{2}{3} \) meets the y-axis at the point \( N \). Find the equation of the line joining the points \( M \) and \( N \). Write your answer in the form \( ax + by + c = 0 \), where \( a \), \( b \) and \( c \) are integers.

Solution:

\( y = 3x - 5 \)
Substitute \( y = 0 \):
\[
3x - 5 = 0 \\
3x = 5 \\
x = \frac{5}{3}
\]

\( M \) has coordinates \( \left( \frac{5}{3}, 0 \right) \).

\( y = -\frac{2}{3}x + \frac{2}{3} \)
Substitute \( x = 0 \):
\[
y = -\frac{2}{3} \left( 0 \right) + \frac{2}{3} = \frac{2}{3}
\]

\( N \) has coordinates \( \left( 0, \frac{2}{3} \right) \).

Gradient of \( MN \) is
\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - \frac{2}{3}}{\frac{5}{3} - 0} = -\frac{2}{3} = -\frac{2}{5}
\]

Equation of \( MN \) is
\[ y - y_1 = m \left( x - x_1 \right) \]
Substitute \( \left( \frac{5}{3}, 0 \right) \):
\[
y - 0 = -\frac{2}{5} \left( x - \frac{5}{3} \right)
\]
\[
y = -\frac{2}{5}x + \frac{2}{3}
\]

Multiply each term by 15:
\[
15y = -6x + 10 \\
6x + 15y = 10 \\
6x + 15y - 10 = 0
\]

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Coordinate geometry in the (x, y) plane
Exercise D, Question 1

Question:

Find the equation of the line that passes through these pairs of points:

(a) (2, 4) and (3, 8)
(b) (0, 2) and (3, 5)
(c) (−2, 0) and (2, 8)
(d) (5, −3) and (7, 5)
(e) (3, −1) and (7, 3)
(f) (−4, −1) and (6, 4)
(g) (−1, −5) and (−3, 3)
(h) (−4, −1) and (−3, −9)

(i) \(\left(\frac{1}{3}, \frac{2}{5}\right)\) and \(\left(\frac{2}{3}, \frac{4}{5}\right)\)

(j) \(\left(-\frac{3}{4}, \frac{1}{7}\right)\) and \(\left(\frac{1}{4}, \frac{3}{7}\right)\)

Solution:

(a) \((x_1, y_1) = (2, 4), \ (x_2, y_2) = (3, 8)\)

\[
\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}
\]

\[
\frac{y - 4}{8 - 4} = \frac{x - 2}{3 - 2}
\]

\[
\frac{y - 4}{4} = x - 2
\]

Multiply each side by 4:

\[
4 \times \frac{y - 4}{4} = 4 \left(\frac{x - 2}{1}\right)
\]

\[
y - 4 = 4 (x - 2)
\]

\[
y = 4x - 4
\]

(b) \((x_1, y_1) = (0, 2), \ (x_2, y_2) = (3, 5)\)
\[
\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}
\]

\[
\frac{y - 2}{5 - 2} = \frac{x - 0}{3 - 0}
\]

\[
\frac{y - 2}{3} = \frac{x}{3}
\]

Multiply each side by 3:

\[
3 \times \frac{y - 2}{3} = 3 \times \frac{x}{3}
\]

\[
y - 2 = x
\]

\[
y = x + 2
\]

(c) \((x_1, y_1) = (-2, 0), \ (x_2, y_2) = (2, 8)\)

\[
\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}
\]

\[
\frac{y - 0}{8 - 0} = \frac{x - (-2)}{2 - (-2)}
\]

\[
\frac{y}{8} = \frac{x + 2}{4}
\]

Multiply each side by 8:

\[
8 \times \frac{y}{8} = 8 \times \frac{x + 2}{4}
\]

\[
y = 2(x + 2)
\]

\[
y = 2x + 4
\]

(d) \((x_1, y_1) = (5, -3), \ (x_2, y_2) = (7, 5)\)

\[
\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}
\]

\[
\frac{y - (-3)}{5 - (-3)} = \frac{x - 5}{7 - 5}
\]

\[
\frac{y + 3}{8} = \frac{x - 5}{2}
\]

Multiply each side by 8:

\[
8 \times \frac{y + 3}{8} = 8 \times \frac{x - 5}{2}
\]

\[
y + 3 = 4(x - 5)
\]

\[
y + 3 = 4x - 20
\]

\[
y = 4x - 23
\]

(e) \((x_1, y_1) = (3, -1), \ (x_2, y_2) = (7, 3)\)

\[
\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}
\]

\[
\frac{y - (-1)}{3 - (-1)} = \frac{x - 3}{7 - 3}
\]

\[
\frac{y + 1}{4} = \frac{x - 3}{4}
\]

Multiply each side by 4:

\[
y + 1 = x - 3
\]

\[
y = x - 4
\]

(f) \((x_1, y_1) = (-4, -1), \ (x_2, y_2) = (6, 4)\)

\[
\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}
\]
Multiply each side by 10:
\[ 2(y + 1) = x + 4 \]
\[ 2y + 2 = x + 4 \]
\[ 2y = x + 2 \]
Divide each term by 2:
\[ y = \frac{1}{2}x + 1 \]

(g) \((x_1, y_1) = (-1, -5), (x_2, y_2) = (-3, 3)\)
\[ \frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1} \]
\[ \frac{y - (-5)}{3 - (-5)} = \frac{x - (-1)}{-3 - (-1)} \]
\[ \frac{y + 5}{8} = \frac{x + 1}{-2} \]

Multiply each side by 8:
\[ y + 5 = -4(x + 1) \ (\text{Note: } \frac{8}{-2} = -4) \]
\[ y + 5 = -4x - 4 \]
\[ y = -4x - 9 \]

(h) \((x_1, y_1) = (-4, -1), (x_2, y_2) = (-3, -9)\)
\[ \frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1} \]
\[ \frac{y - (-1)}{-9 - (-1)} = \frac{x - (-4)}{-3 - (-4)} \]
\[ \frac{y + 1}{-8} = \frac{x + 4}{1} \]

Multiply each side by -8:
\[ y + 1 = -8(x + 4) \]
\[ y + 1 = -8x - 32 \]
\[ y = -8x - 33 \]

(i) \((x_1, y_1) = \left(\frac{1}{3}, \frac{2}{5}\right), (x_2, y_2) = \left(\frac{2}{3}, \frac{4}{5}\right)\)
\[ \frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1} \]
\[ \frac{y - \frac{2}{5}}{\frac{4}{5} - \frac{2}{5}} = \frac{x - \frac{1}{3}}{\frac{2}{3} - \frac{1}{3}} \]
\[ \frac{y - \frac{2}{5}}{\frac{2}{5}} = \frac{x - \frac{1}{3}}{\frac{1}{3}} \]
\[ \frac{2}{5} = \frac{1}{3} \]
\[
\frac{5}{2} \left( y - \frac{2}{5} \right) = 3 \left( x - \frac{1}{3} \right) \quad \text{(Note:} \quad \frac{1}{2} = \frac{5}{2} \text{and} \quad \frac{1}{3} = 3) \\
\]
\[
\frac{5}{2} y - 1 = 3x - 1 \\
\frac{5}{2} y = 3x \\
5y = 6x \\
y = \frac{6}{5}x
\]
\[
(j) \quad \begin{pmatrix} x_1, y_1 \end{pmatrix} = \begin{pmatrix} \frac{-3}{4}, \frac{1}{7} \end{pmatrix}, \quad \begin{pmatrix} x_2, y_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{4}, \frac{3}{7} \end{pmatrix}
\]
\[
\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}
\]
\[
y - \frac{1}{7} = x - \left( -\frac{3}{4} \right)
\]
\[
\frac{\frac{3}{7} - \frac{1}{7}}{\frac{3}{4} - \left( -\frac{3}{4} \right)} = \frac{\frac{2}{7} = \frac{x + \frac{3}{4}}{1}}
\]
Multiply each side by \(\frac{2}{7}\):
\[
y - \frac{1}{7} = \frac{2}{7} \left( x + \frac{3}{4} \right)
\]
\[
y - \frac{1}{7} = \frac{2}{7}x + \frac{3}{14}
\]
\[
y = \frac{2}{7}x + \frac{3}{14} + \frac{1}{7}
\]
\[
y = \frac{2}{7}x + \frac{5}{14}
\]

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Coordinate geometry in the (x, y) plane
Exercise D, Question 2

Question:
The line that passes through the points \((2, -5)\) and \((-7, 4)\) meets the x-axis at the point \(P\). Work out the coordinates of the point \(P\).

Solution:

\[
\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}
\]

\[
\frac{y - (-5)}{4 - (-5)} = \frac{x - 2}{-7 - 2}
\]

\[
\frac{y + 5}{9} = \frac{x - 2}{-9}
\]

Multiply each side by 9:

\[
y + 5 = -1(x - 2) \quad \text{(Note:} \quad \frac{9}{-9} = -1\text{)}
\]

\[
y + 5 = -x + 2
\]

\[
y = -x - 3
\]

Substitute \(y = 0\):

\[
0 = -x - 3
\]

\[
x = -3
\]

So the line meets the x-axis at \(P( -3, 0)\).
Question:

The line that passes through the points \((-3, -5)\) and \((4, 9)\) meets the y-axis at the point \(G\). Work out the coordinates of the point \(G\).

Solution:

\[
\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}
\]

\[
\frac{y - (-5)}{9 - (-5)} = \frac{x - (-3)}{4 - (-3)}
\]

\[
\frac{y + 5}{14} = \frac{x + 3}{7}
\]

Multiply each side by 14:

\[
y + 5 = 2 (x + 3)
\]

\[
y + 5 = 2x + 6
\]

\[
y = 2x + 1
\]

Substitute \(x = 0\):

\[
y = 2 (0) + 1 = 1
\]

The coordinates of \(G\) are \((0, 1)\).
The line that passes through the points \( (3, 2 \frac{1}{2}) \) and \( (-1 \frac{1}{2}, 4) \) meets the y-axis at the point \( J \). Work out the coordinates of the point \( J \).

\[
\frac{y - 2 \frac{1}{2}}{-1 \frac{1}{2} - 3} = \frac{x - 3}{-4 \frac{1}{2}}
\]

Multiply top and bottom of each fraction by 2:

\[
\frac{2y - 5}{3} = \frac{2x - 6}{-9}
\]

Multiply each side by 9:

\[
3 (2y - 5) = -1 (2x - 6) \quad \text{(Note:} \quad \frac{9}{-9} = -1)\]

\[
6y - 15 = -2x + 6
\]

\[
6y = -2x + 21
\]

\[
y = -\frac{2}{6}x + \frac{21}{6}
\]

\[
y = -\frac{1}{3}x + \frac{7}{2}
\]

Substitute \( x = 0 \):

\[
y = -\frac{1}{3}(0) + \frac{7}{2} = \frac{7}{2}
\]

The coordinates of \( J \) are \( \left( 0, \frac{7}{2} \right) \) or \( \left( 0, 3 \frac{1}{2} \right) \).
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Coordinate geometry in the (x, y) plane
Exercise D, Question 5

Question:
The line \( y = 2x - 10 \) meets the \( x \)-axis at the point \( A \). The line \( y = -2x + 4 \) meets the \( y \)-axis at the point \( B \). Find the equation of the line joining the points \( A \) and \( B \). (Hint: First work out the coordinates of the points \( A \) and \( B \).)

Solution:

\( y = 2x - 10 \)

Substitute \( y = 0 \):
\[ 2x - 10 = 0 \]
\[ 2x = 10 \]
\[ x = 5 \]
The coordinates of \( A \) are \((5, 0)\).

\( y = -2x + 4 \)

Substitute \( x = 0 \):
\[ y = -2(0) + 4 = 4 \]
The coordinates of \( B \) are \((0, 4)\).

Equation of \( AB \):
\[
\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}
\]
\[ y - 0 = \frac{x - 5}{0 - 5} \]
\[ y = \frac{x - 5}{-5} \]

Multiply each side by 4:
\[ y = 4 \left( \frac{x - 5}{-5} \right) = -\frac{4}{5} \left( x - 5 \right) = -\frac{4}{5}x + 4 \]

The equation of the line is \( y = -\frac{4}{5}x + 4 \).

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Question:

The line \( y = 4x + 5 \) meets the y-axis at the point C. The line \( y = -3x - 15 \) meets the x-axis at the point D. Find the equation of the line joining the points C and D. Write your answer in the form \( ax + by + c = 0 \), where \( a, b \) and \( c \) are integers.

Solution:

\( y = 4x + 5 \)

Substitute \( x = 0 \):

\[ y = 4(0) + 5 = 5 \]

The coordinates of \( C \) are \((0, 5)\).

\( y = -3x - 15 \)

Substitute \( y = 0 \):

\[ 0 = -3x - 15 \]

\[ 3x = -15 \]

\[ x = -5 \]

The coordinates of \( D \) are \((-5, 0)\).

Equation of \( CD \):

\[ \frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1} \]

\[ \frac{y - 5}{0 - 5} = \frac{x - 0}{-5 - 0} \]

\[ \frac{y - 5}{-5} = \frac{x}{-5} \]

Multiply each side by \(-5\):

\[ y - 5 = x \]

\[ -5 = x - y \]

\[ 0 = x - y + 5 \]

The equation of the line is \( x - y + 5 = 0 \).
Question:
The lines \( y = x - 5 \) and \( y = 3x - 13 \) intersect at the point \( S \). The point \( T \) has coordinates \((-4, 2)\). Find the equation of the line that passes through the points \( S \) and \( T \).

Solution:

\[
\begin{align*}
y &= 3x - 13 \\
y &= x - 5 \\
\text{So } 3x - 13 &= x - 5 \\
\Rightarrow \quad 3x &= x + 8 \\
\Rightarrow \quad 2x &= 8 \\
\Rightarrow \quad x &= 4 \\
\text{when } x = 4, \quad y &= 4 - 5 = -1 \\
\text{The coordinates of } S \text{ are } (4, -1). \\
\end{align*}
\]

Equation of \( ST \):

\[
\begin{align*}
\frac{y - y_1}{y_2 - y_1} &= \frac{x - x_1}{x_2 - x_1} \\
\frac{2 - (-1)}{3 - 4} &= \frac{x - 4}{-8} \\
y + 1 &= \frac{x - 4}{-8} \\
\end{align*}
\]

Multiply each side by 3:

\[
\begin{align*}
y + 1 &= 3 \times \left( \frac{x - 4}{-8} \right) \\
y + 1 &= \frac{3}{-8} \times (x - 4) \\
y + 1 &= \frac{-3}{8} (x - 4) \\
y + 1 &= \frac{-3}{8} x + \frac{3}{2} \\
y &= \frac{-3}{8} x + \frac{1}{2} \\
\end{align*}
\]
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Coordinate geometry in the $(x, y)$ plane
Exercise D, Question 8

Question:

The lines $y = -2x + 1$ and $y = x + 7$ intersect at the point $L$. The point $M$ has coordinates $(-3, 1)$. Find the equation of the line that passes through the points $L$ and $M$.

Solution:

\[
y = x + 7 \\
y = -2x + 1
\]

So $x + 7 = -2x + 1$

$\Rightarrow 3x + 7 = 1$

$\Rightarrow 3x = -6$

$\Rightarrow x = -2$

when $x = -2$, $y = (-2) + 7 = 5$

The coordinates of $L$ are $(-2, 5)$.

Equation of $LM$:

\[
\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}
\]

\[
\frac{y - 5}{1 - 5} = \frac{x - (-2)}{-3 - (-2)}
\]

\[
y - 5\frac{1}{4} = \frac{x + 2}{-1}
\]

Multiply each side by $-4$:

\[
y - 5 = 4(x + 2) \quad (\text{Note: } \frac{-4}{-1} = 4)
\]

\[
y - 5 = 4x + 8
\]

\[
y = 4x + 13
\]

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Coordinate geometry in the (x, y) plane
Exercise D, Question 9

Question:
The vertices of the triangle \(ABC\) have coordinates \(A(3, 5)\), \(B(-2, 0)\) and \(C(4, -1)\). Find the equations of the sides of the triangle.

Solution:

(1) Equation of \(AB\):
\[ (x_1, y_1) = (3, 5), \ (x_2, y_2) = (-2, 0) \]
\[ \frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1} \]
\[ \frac{y-5}{0-5} = \frac{x-3}{-2-3} \]
\[ y-5 = \frac{x-3}{-5} \]
Multiply each side by \(-5\):
\[ y = x - 3 \]
\[ y = x + 2 \]

(2) Equation of \(AC\):
\[ (x_1, y_1) = (3, 5), \ (x_2, y_2) = (4, -1) \]
\[ \frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1} \]
\[ \frac{y-5}{-1-5} = \frac{x-3}{4-3} \]
\[ y-5 = \frac{x-3}{1} \]
Multiply each side by \(-6\):
\[ y - 5 = -6(x - 3) \]
\[ y = -6x + 18 \]
\[ y = -6x + 23 \]

(3) Equation of \(BC\):
\[ (x_1, y_1) = (-2, 0), \ (x_2, y_2) = (4, -1) \]
\[ \frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1} \]
\[ \frac{y-0}{-1-0} = \frac{x-(-2)}{4-(-2)} \]
\[ y-0 = \frac{x+2}{6} \]
Multiply each side by \(-1\):
\[ y = -1 \cdot \frac{x+2}{6} \]
\[ y = -\frac{1}{6}(x+2) \]
\[ y = -\frac{1}{6}x - \frac{1}{3} \]
Question:

The line $V$ passes through the points $(-5, 3)$ and $(7, -3)$ and the line $W$ passes through the points $(2, -4)$ and $(4, 2)$. The lines $V$ and $W$ intersect at the point $A$. Work out the coordinates of the point $A$.

Solution:

(1) The equation of $V$:

For line $V$, using the points $(-5, 3)$ and $(7, -3)$,

\[
\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}
\]

\[
\frac{y - 3}{-3 - 3} = \frac{x - (-5)}{7 - (-5)}
\]

Multiply each side by $-6$:

\[
y - 3 = -\frac{1}{2} (x + 5)
\]

(Note: $\frac{-6}{12} = -\frac{1}{2}$)

\[
y = -\frac{1}{2}x - \frac{5}{2}
\]

(2) The equation of $W$:

For line $W$, using the points $(2, -4)$ and $(4, 2)$,

\[
\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}
\]

\[
\frac{y - (-4)}{2 - (-4)} = \frac{x - 2}{4 - 2}
\]

Multiply each side by 6:

\[
y + 4 = 3(x - 2)\quad (Note: \frac{6}{2} = 3)
\]

\[
y + 4 = 3x - 6
\]

\[
y = 3x - 10
\]

Solving simultaneously:

\[
y = -\frac{1}{2}x + \frac{1}{2}
\]

\[
y = 3x - 10
\]

So $3x - 10 = -\frac{1}{2}x + \frac{1}{2}$

\[
\Rightarrow \quad \frac{7}{2}x - 10 = \frac{1}{2}
\]

\[
\Rightarrow \quad \frac{7}{2}x = \frac{21}{2}
\]
\[ 7x = 21 \]
\[ \Rightarrow x = 3 \]

When \( x = 3 \), \( y = 3(3) - 10 = 9 - 10 = -1 \)

The lines intersect at \( A(3, -1) \).
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Coordinate geometry in the (x, y) plane
Exercise E, Question 1

Question:
Work out if these pairs of lines are parallel, perpendicular or neither:

(a) \( y = 4x + 2 \)
   \( y = -\frac{1}{4}x - 7 \)

(b) \( y = \frac{2}{3}x - 1 \)
   \( y = \frac{2}{3}x - 11 \)

(c) \( y = \frac{1}{5}x + 9 \)
   \( y = 5x + 9 \)

(d) \( y = -3x + 2 \)
   \( y = \frac{1}{3}x - 7 \)

(e) \( y = \frac{3}{5}x + 4 \)
   \( y = -\frac{5}{3}x - 1 \)

(f) \( y = \frac{5}{7}x \)
   \( y = \frac{5}{7}x - 3 \)

(g) \( y = 5x - 3 \)
   \( 5x - y + 4 = 0 \)

(h) \( 5x - y - 1 = 0 \)
   \( y = -\frac{1}{5}x \)

(i) \( y = -\frac{3}{2}x + 8 \)
   \( 2x - 3y - 9 = 0 \)

(j) \( 4x - 5y + 1 = 0 \)
   \( 8x - 10y - 2 = 0 \)

(k) \( 3x + 2y - 12 = 0 \)
   \( 2x + 3y - 6 = 0 \)

(l) \( 5x - y + 2 = 0 \)
2x + 10y − 4 = 0

Solution:

(a) The gradients of the lines are 4 and \( \frac{1}{4} \).
\[ 4 \times -\frac{1}{4} = -1 \]
The lines are \textbf{perpendicular}.

(b) The gradients of the lines are \( \frac{2}{3} \) and \( \frac{2}{3} \), i.e. they have the same gradient.
The lines are \textbf{parallel}.

(c) The gradients of the lines are \( \frac{1}{7} \) and 5.
\[ \frac{1}{7} \times 5 = 1 \]
The lines are \textbf{neither} perpendicular nor parallel.

(d) The gradients of the lines are \(-3\) and \(\frac{1}{3}\).
\[ -3 \times \frac{1}{3} = -1 \]
The lines are \textbf{perpendicular}.

(e) The gradients of the lines are \( \frac{3}{5} \) and \( \frac{5}{3} \).
\[ \frac{3}{5} \times -\frac{5}{3} = -1 \]
The lines are \textbf{perpendicular}.

(f) The gradients of the lines are \( \frac{5}{7} \) and \( \frac{5}{7} \), i.e. they have the same gradient.
The lines are \textbf{parallel}.

(g) The gradient of \( y = 5x - 3 \) is 5.
\[ 5x - y + 4 = 0 \]
\[ 5x + 4 = y \]
\[ y = 5x + 4 \]
The gradient of \( 5x - y + 4 = 0 \) is 5.
The lines have the same gradient.
The lines are \textbf{parallel}.

(h) \( 5x - y - 1 = 0 \)
\[ 5x - 1 = y \]
\[ y = 5x - 1 \]
The gradient of \( 5x - y - 1 = 0 \) is 5.
The gradient of \( y = -\frac{1}{5}x \) is \( -\frac{1}{5} \).
The product of the gradients is \( 5 \times -\frac{1}{5} = -1 \)
So the lines are \textbf{perpendicular}.

(i) The gradient of \( y = -\frac{3}{2}x + 8 \) is \( -\frac{3}{2} \).
\[ 2x - 3y - 9 = 0 \]
\[2x - 9 = 3y\]
\[3y = 2x - 9\]
\[y = \frac{2}{3}x - 3\]

The gradient of \(2x - 3y - 9 = 0\) is \(\frac{2}{3}\).

The product of the gradients is \(\frac{2}{3} \times -\frac{2}{3} = -1\)

So the lines are perpendicular.

(j) \(4x - 5y + 1 = 0\)
\[4x + 1 = 5y\]
\[5y = 4x + 1\]
\[y = \frac{4}{5}x + \frac{1}{5}\]

The gradient of \(4x - 5y + 1 = 0\) is \(\frac{4}{5}\).

\[8x - 10y - 2 = 0\]
\[8x - 2 = 10y\]
\[10y = 8x - 2\]
\[y = \frac{8}{10}x - \frac{2}{10}\]
\[y = \frac{4}{5}x - \frac{1}{5}\]

The gradient of \(8x - 10y - 2 = 0\) is \(\frac{4}{5}\).

The lines have the same gradient, they are parallel.

(k) \(3x + 2y - 12 = 0\)
\[3x + 2y = 12\]
\[2y = -3x + 12\]
\[y = -\frac{3}{2}x + 6\]

The gradient of \(3x + 2y - 12 = 0\) is \(-\frac{3}{2}\).

\[2x + 3y - 6 = 0\]
\[2x + 3y = 6\]
\[3y = -2x + 6\]
\[y = -\frac{2}{3}x + 2\]

The gradient of \(2x + 3y - 6 = 0\) is \(-\frac{2}{3}\).

The product of the gradient is
\[-\frac{3}{2} \times -\frac{2}{3} = 1\]

So the lines are neither parallel nor perpendicular.

(l) \(5x - y + 2 = 0\)
\[5x + 2 = y\]
\[y = 5x + 2\]

The gradient of \(5x - y + 2 = 0\) is 5.
\[2x + 10y - 4 = 0\]
\[2x + 10y = 4\]
\[10y = -2x + 4\]
\[y = -\frac{2}{10}x + \frac{4}{10}\]
\[ y = - \frac{1}{5}x + \frac{2}{5} \]

The gradient of \(2x + 10y - 4 = 0\) is \(-\frac{1}{5}\).

The product of the gradients is
\[ 5 \times -\frac{1}{5} = -1 \]

So the lines are **perpendicular**.
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Coordinate geometry in the (x, y) plane
Exercise E, Question 2

Question:

Find an equation of the line that passes through the point (6, -2) and is perpendicular to the line y = 3x + 5.

Solution:

The gradient of y = 3x + 5 is 3.
The gradient of a line perpendicular to y = 3x + 5 is $-\frac{1}{3}$.

\[
y - y_1 = m(x - x_1)
\]

\[
y - (-2) = -\frac{1}{3}(x - 6)
\]

\[
y + 2 = -\frac{1}{3}x + 2
\]

\[
y = -\frac{1}{3}x
\]

The equation of the line is $y = -\frac{1}{3}x$.

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Coordinate geometry in the (x, y) plane
Exercise E, Question 3

Question:
Find an equation of the line that passes through the point (−2, 7) and is parallel to the line \( y = 4x + 1 \). Write your answer in the form \( ax + by + c = 0 \).

Solution:
The gradient of a line parallel to \( y = 4x + 1 \) is 4.

\[
\begin{align*}
y - 7 &= 4(x - (−2)) \\
y - 7 &= 4(x + 2) \\
y - 7 &= 4x + 8 \\
y &= 4x + 15 \\
o &= 4x + 15 - y \\
4x - y + 15 &= 0
\end{align*}
\]
The equation of the line is \( 4x - y + 15 = 0 \).
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Coordinate geometry in the \((x, y)\) plane
Exercise E, Question 4

Question:

Find an equation of the line:

(a) parallel to the line \(y = -2x - 5\), passing through \((-\frac{1}{2}, \frac{3}{2})\).

(b) parallel to the line \(x - 2y - 1 = 0\), passing through \((0, 0)\).

(c) perpendicular to the line \(y = x - 4\), passing through \((-1, -2)\).

(d) perpendicular to the line \(2x + y - 9 = 0\), passing through \((4, -6)\).

Solution:

(a) The gradient of a line parallel to \(y = -2x - 5\) is \(-2\).

\[
y - y_1 = m (x - x_1) \\
y - \frac{3}{2} = -2 \left[ x - \left( -\frac{1}{2} \right) \right] \\
y - \frac{3}{2} = -2 \left( x + \frac{1}{2} \right) \\
y - \frac{3}{2} = -2x + 1 \\
y = -2x + \frac{3}{2}
\]

(b) \(x - 2y - 1 = 0\)

\[
x - 1 = 2y \\
2y = x - 1 \\
y = \frac{1}{2}x - \frac{1}{2}
\]

The gradient of \(x - 2y - 1 = 0\) is \(\frac{1}{2}\).

\[
y - y_1 = m (x - x_1) \\
y - 0 = \frac{1}{2} \left( x - 0 \right) \\
y = \frac{1}{2}x
\]

(c) The gradient of \(y = x - 4\) is \(1\).

The gradient of a line perpendicular to \(y = x - 4\) is \(-\frac{1}{1} = -1\).

\[
y - y_1 = m (x - x_1) \\
y - (-2) = -1 \left[ x - (-1) \right] \\
y + 2 = -1 (x + 1) \\
y + 2 = -x - 1
\]


\[ y = -x - 3 \]

(d) \[ 2x + y - 9 = 0 \]
\[ 2x + y = 9 \]
\[ y = -2x + 9 \]

The gradient of \[ 2x + y - 9 = 0 \] is \(-2\).

The gradient of a line perpendicular to \[ 2x + y - 9 = 0 \] is \(-\frac{1}{-2} = \frac{1}{2}\).

\[
\begin{align*}
y - y_1 &= m(x - x_1) \\
y - \left(-\frac{6}{2}\right) &= \frac{1}{2}\left(x - 4\right) \\
y + 6 &= \frac{1}{2}\left(x - 4\right) \\
y + 6 &= \frac{1}{2}x - 2 \\
y &= \frac{1}{2}x - 8
\end{align*}
\]
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Coordinate geometry in the (x, y) plane
Exercise E, Question 5

Question:

Find an equation of the line:

(a) parallel to the line \( y = 3x + 6 \), passing through \((-2, 5)\).

(b) perpendicular to the line \( y = 3x + 6 \), passing through \((-2, 5)\).

(c) parallel to the line \( 4x - 6y + 7 = 0 \), passing through \((3, 4)\).

(d) perpendicular to the line \( 4x - 6y + 7 = 0 \), passing through \((3, 4)\).

Solution:

(a) The gradient of a line parallel to \( y = 3x + 6 \) is 3.
\[
y - y_1 = m(x - x_1)
\]
\[
y - 5 = 3[x - (-2)]
\]
\[
y - 5 = 3(x + 2)
\]
\[
y - 5 = 3x + 6
\]
\[
y = 3x + 11
\]

(b) The gradient of a line perpendicular to \( y = 3x + 6 \) is \(-\frac{1}{3}\).
\[
y - y_1 = m(x - x_1)
\]
\[
y - 5 = -\frac{1}{3}[x - (-2)]
\]
\[
y - 5 = -\frac{1}{3}(x + 2)
\]
\[
y - 5 = -\frac{1}{3}x - \frac{2}{3}
\]
\[
y = -\frac{1}{3}x + \frac{13}{3}
\]

(c) \( 4x - 6y + 7 = 0 \)
\( 4x + 7 = 6y \)
\( 6y = 4x + 7 \)
\( y = \frac{4}{6}x + \frac{7}{6} \)
\( y = \frac{2}{3}x + \frac{7}{6} \)

The gradient of a line parallel to \( 4x - 6y + 7 = 0 \) is \(\frac{2}{5}\).
\[
y - y_1 = m(x - x_1)
\]
\[
y - 4 = \frac{2}{5}[x - 3]
\]
\[
y - 4 = \frac{2}{5}x - 2
\]
\[ y = \frac{2}{3}x + 2 \]

(d) The gradient of the line \(4x - 6y + 7 = 0\) is \(\frac{2}{3}\) [see part (c)].

The gradient of a line perpendicular to \(4x - 6y + 7 = 0\) is \(-\frac{1}{\frac{2}{3}} = -\frac{3}{2}\).

\[
y - y_1 = m(x - x_1) \\
y - 4 = -\frac{3}{2}\left(x - 3\right) \\
y - 4 = -\frac{3}{2}x + \frac{9}{2} \\
y = -\frac{3}{2}x + \frac{17}{2}
\]
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Coordinate geometry in the \((x, y)\) plane
Exercise E, Question 6

Question:

Find an equation of the line that passes through the point \((5, -5)\) and is perpendicular to the line \(y = \frac{2}{3}x + 5\). Write your answer in the form \(ax + by + c = 0\), where \(a\), \(b\) and \(c\) are integers.

Solution:

The gradient of a line perpendicular to \(y = \frac{2}{3}x + 5\) is \(-\frac{3}{2}\).

\[
y - y_1 = m(x - x_1)
\]

\[
y - (-5) = -\frac{3}{2}(x - 5)
\]

\[
y + 5 = -\frac{3}{2}(x - 5)
\]

Multiply each term by 2:

\[
2y + 10 = -3(x - 5)
\]

\[
2y + 10 = -3x + 15
\]

\[
3x + 2y + 10 = 15
\]

\[
3x + 2y - 5 = 0
\]

The equation of the line is \(3x + 2y - 5 = 0\).

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Coordinate geometry in the (x, y) plane
Exercise E, Question 7

Question:

Find an equation of the line that passes through the point \((-2, -3)\) and is perpendicular to the line \(y = -\frac{4}{7}x + 5\).
Write your answer in the form \(ax + by + c = 0\), where \(a\), \(b\) and \(c\) are integers.

Solution:

The gradient of a line perpendicular to \(y = -\frac{4}{7}x + 5\) is \(-\frac{7}{4}\).

\[
y - y_1 = m(x - x_1) \\
y - (-3) = \frac{7}{4} \left[ x - (-2) \right] \\
y + 3 = \frac{7}{4} \left( x + 2 \right)
\]

Multiply each term by 4:

\[
4y + 12 = 7(x + 2) \\
4y + 12 = 7x + 14 \\
v = 7x + 2 \\
0 = 7x + 2 - 4y \\
7x - 4y + 2 = 0
\]

The equation of the line is \(7x - 4y + 2 = 0\).
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Coordinate geometry in the (x, y) plane
Exercise E, Question 8

Question:

The line \( r \) passes through the points \( (1, 4) \) and \( (6, 8) \) and the line \( s \) passes through the points \( (5, -3) \) and \( (20, 9) \). Show that the lines \( r \) and \( s \) are parallel.

Solution:

The gradient of \( r \) is
\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 4}{6 - 1} = \frac{4}{5}
\]

The gradient of \( s \) is
\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - (-3)}{20 - 5} = \frac{12}{15} = \frac{4}{5}
\]

The gradients are equal, so the lines are parallel.
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Coordinate geometry in the (x, y) plane
Exercise E, Question 9

Question:

The line \( l \) passes through the points \((-3, 0)\) and \((3, -2)\) and the line \( n \) passes through the points \((1, 8)\) and \((-1, 2)\). Show that the lines \( l \) and \( n \) are perpendicular.

Solution:

The gradient of \( l \) is

\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 0}{3 - (-3)} = -\frac{2}{6} = -\frac{1}{3}
\]

The gradient of \( n \) is

\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 8}{-1 - 1} = -\frac{6}{-2} = 3
\]

The product of the gradients is

\[-\frac{1}{3} \times 3 = -1\]

So the lines are perpendicular.
Question:
The vertices of a quadrilateral $ABCD$ has coordinates $A(-1, 5)$, $B(7, 1)$, $C(5, -3)$, $D(-3, 1)$. Show that the quadrilateral is a rectangle.

Solution:

(1) The gradient of $AB$ is
\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 1}{-1 - 7} = \frac{4}{-8} = -\frac{1}{2}
\]

(2) The gradient of $DC$ is
\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 1}{5 - (-3)} = \frac{-4}{8} = -\frac{1}{2}
\]
The gradient of $AB$ is the same as the gradient of $DC$, so the lines are parallel.

(3) The gradient of $AD$ is
\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 1}{-1 - (-3)} = \frac{4}{-1 + 3} = \frac{4}{2} = 2
\]

(4) The gradient of $BC$ is
\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 1}{5 - 7} = \frac{-4}{-2} = 2
\]
The gradient of $AD$ is the same as the gradient of $BC$, so the lines are parallel.

The line $AD$ is perpendicular to the line $AB$ as
\[
2 \times -\frac{1}{2} = -1
\]
So $ABCD$ is a rectangle.
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Coordinate geometry in the (x, y) plane  
Exercise F, Question 1

Question:

The points A and B have coordinates (−4, 6) and (2, 8) respectively. A line p is drawn through B perpendicular to AB to meet the y-axis at the point C.

(a) Find an equation of the line p.

(b) Determine the coordinates of C. [4]  

Solution:

(a) The gradient of AB is

\[ \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 6}{2 - (-4)} = \frac{2}{6} = \frac{1}{3} \]

The gradient of a line perpendicular to AB is

\[ -\frac{1}{\frac{1}{3}} = -3 \]

The equation of p is

\[ y - y_1 = m ( x - x_1 ) \]
\[ y - 8 = -3 ( x - 2 ) \]
\[ y - 8 = -3x + 6 \]
\[ y = -3x + 14 \]

(b) Substitute x = 0:

\[ y = -3 ( 0 ) + 14 = 14 \]

The coordinates of C are (0, 14).

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Coordinate geometry in the (x, y) plane
Exercise F, Question 2

Question:

The line \( l \) has equation \( 2x - y - 1 = 0 \).
The line \( m \) passes through the point \( A (0, 4) \) and is perpendicular to the line \( l \).

(a) Find an equation of \( m \) and show that the lines \( l \) and \( m \) intersect at the point \( P (2, 3) \).

The line \( n \) passes through the point \( B (3, 0) \) and is parallel to the line \( m \).

(b) Find an equation of \( n \) and hence find the coordinates of the point \( Q \) where the lines \( l \) and \( n \) intersect.

Solution:

(a) \( 2x - y - 1 = 0 \)
\( 2x = y + 1 \)
The gradient of \( 2x - y - 1 = 0 \) is 2.
The gradient of a line perpendicular to \( 2x - y - 1 = 0 \) is \( -\frac{1}{2} \).
The equation of the line \( m \) is
\[ y - y_1 = m (x - x_1) \]
\[ y - 4 = -\frac{1}{2} (x - 0) \]
\[ y - 4 = -\frac{1}{2} x \]
\[ y = -\frac{1}{2} x + 4 \]

To find \( P \) solve \( y = -\frac{1}{2} x + 4 \) and \( 2x - y - 1 = 0 \) simultaneously.

Substitute:
\[ 2x = \left( -\frac{1}{2} x + 4 \right) - 1 = 0 \]
\[ 2x + \frac{1}{2} x - 4 - 1 = 0 \]
\[ \frac{5}{2} x - 5 = 0 \]
\[ \frac{5}{2} x = 5 \]
\[ 5x = 10 \]
\[ x = 2 \]

Substitute \( x = 2 \) into \( y = -\frac{1}{2} x + 4 \):
\[ y = -\frac{1}{2} \left( 2 \right) + 4 = -1 + 4 = 3 \]
The lines intersect at \( P (2, 3) \), as required.

(b) A line parallel to the line \( m \) has gradient \( -\frac{1}{2} \).
The equation of the line \( n \) is

\[
y - y_1 = m \left( x - x_1 \right)
\]

\[
y - 0 = - \frac{1}{2} \left( x - 3 \right)
\]

\[
y = - \frac{1}{2}x + \frac{3}{2}
\]

To find \( Q \) solve \( 2x - y - 1 = 0 \) and \( y = - \frac{1}{2}x + \frac{3}{2} \) simultaneously.

Substitute:

\[
2x - \left( - \frac{1}{2}x + \frac{3}{2} \right) - 1 = 0
\]

\[
2x + \frac{1}{2}x - \frac{3}{2} - 1 = 0
\]

\[
\frac{5}{2}x - \frac{5}{2} = 0
\]

\[
\frac{5}{2}x = \frac{5}{2}
\]

\[
x = 1
\]

Substitute \( x = 1 \) into \( y = - \frac{1}{2}x + \frac{3}{2} \):

\[
y = - \frac{1}{2} \left( 1 \right) + \frac{3}{2} = - \frac{1}{2} + \frac{3}{2} = 1
\]

The lines intersect at \( Q \left( 1, 1 \right) \).
Question:

The line $L_1$ has gradient $\frac{1}{7}$ and passes through the point $A(2, 2)$. The line $L_2$ has gradient $-1$ and passes through the point $B(4, 8)$. The lines $L_1$ and $L_2$ intersect at the point $C$.

(a) Find an equation for $L_1$ and an equation for $L_2$.

(b) Determine the coordinates of $C$. [E]

Solution:

(a) The equation of $L_1$ is

$$y - y_1 = m (x - x_1)$$

$$y - 2 = \frac{1}{7} \left( x - 2 \right)$$

$$y - 2 = \frac{1}{7}x - \frac{2}{7}$$

$$y = \frac{1}{7}x + \frac{12}{7}$$

The equation of $L_2$ is

$$y - y_1 = m (x - x_1)$$

$$y - 8 = -1 (x - 4)$$

$$y - 8 = -x + 4$$

$$y = -x + 12$$

(b) Solve $y = \frac{1}{7}x + \frac{12}{7}$ and $y = -x + 12$ simultaneously.

Substitute:

$$-x + 12 = \frac{1}{7}x + \frac{12}{7}$$

$$12 = \frac{8}{7}x + \frac{12}{7}$$

$$10 \frac{2}{7} = \frac{8}{7}x$$

$$x = \frac{10 \frac{2}{7}}{\frac{8}{7}} = 9$$

Substitute $x = 9$ into $y = -x + 12$:

$$y = -9 + 12 = 3$$

The lines intersect at $C(9, 3)$. 

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Question:

The straight line passing through the point \( P(2, 1) \) and the point \( Q(k, 11) \) has gradient \( -\frac{5}{12} \).

(a) Find the equation of the line in terms of \( x \) and \( y \) only.

(b) Determine the value of \( k \). [E]

Solution:

(a) \( m = -\frac{5}{12}, \quad (x_1, y_1) = (2, 1) \)

The equation of the line is

\[ y - y_1 = m(x - x_1) \]

\[ y - 1 = -\frac{5}{12}(x - 2) \]

\[ y = -\frac{5}{12}x + \frac{5}{6} \]

(b) Substitute \( (k, 11) \) into \( y = -\frac{5}{12}x + \frac{11}{6} \):

\[ 11 = -\frac{5}{12}k + \frac{11}{6} \]

\[ 11 - \frac{11}{6} = -\frac{5}{12}k \]

\[ \frac{55}{6} = -\frac{5}{12}k \]

Multiply each side by 12:

\[ 110 = -5k \]

\[ k = -22 \]

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Question:

(a) Find an equation of the line \( l \) which passes through the points \( A(1, 0) \) and \( B(5, 6) \).

The line \( m \) with equation \( 2x + 3y = 15 \) meets \( l \) at the point \( C \).

(b) Determine the coordinates of the point \( C \). [E]

Solution:

(a) The equation of \( l \) is

\[
\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}
\]

\[
\frac{y - 0}{6 - 0} = \frac{x - 1}{5 - 1}
\]

\[
\frac{y}{6} = \frac{x - 1}{4}
\]

Multiply each side by 6:

\[
y = 6 \left( \frac{x - 1}{4} \right)
\]

\[
y = \frac{3}{2} \left( x - 1 \right)
\]

\[
y = \frac{3}{2}x - \frac{3}{2}
\]

(b) Solve \( 2x + 3y = 15 \) and \( y = \frac{3}{2}x - \frac{3}{2} \) simultaneously.

Substitute:

\[
2x + 3 \left( \frac{3}{2}x - \frac{3}{2} \right) = 15
\]

\[
2x + \frac{9}{2}x - \frac{9}{2} = 15
\]

\[
\frac{13}{2}x - \frac{9}{2} = 15
\]

\[
\frac{13}{2}x = \frac{39}{2}
\]

\[
x = 3
\]

Substitute \( x = 3 \) into \( y = \frac{3}{2}x - \frac{3}{2} \):

\[
y = \frac{3}{2} \left( 3 \right) - \frac{3}{2} = \frac{9}{2} - \frac{3}{2} = \frac{6}{2} = 3
\]

The coordinates of \( C \) are \( (3, 3) \).
Question:

The line \( L \) passes through the points \( A(1, 3) \) and \( B(-19, -19) \).

Find an equation of \( L \) in the form \( ax + by + c = 0 \), where \( a, b \) and \( c \) are integers. [E]

Solution:

\[
\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}
\]

\[
\frac{y - 3}{-19 - 3} = \frac{x - 1}{-19 - 1}
\]

\[
\frac{y - 3}{-22} = \frac{x - 1}{-20}
\]

Multiply each side by \(-22\):

\[
y - 3 = \frac{-22}{-20} \begin{pmatrix} x - 1 \end{pmatrix}
\]

\[
y - 3 = \frac{11}{10} \begin{pmatrix} x - 1 \end{pmatrix}
\]

Multiply each term by 10:

\[
10y - 30 = 11(x - 1)
\]

\[
10y - 30 = 11x - 11
\]

\[
10y = 11x + 19
\]

\[
0 = 11x - 10y + 19
\]

The equation of \( L \) is \( 11x - 10y + 19 = 0 \).
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Coordinate geometry in the (x, y) plane
Exercise F, Question 7

Question:

The straight line $l_1$ passes through the points $A$ and $B$ with coordinates $(2, 2)$ and $(6, 0)$ respectively.

(a) Find an equation of $l_1$.

The straight line $l_2$ passes through the point $C$ with coordinates $(-9, 0)$ and has gradient $\frac{1}{4}$.

(b) Find an equation of $l_2$. [E]

Solution:

(a) The equation of $l_1$ is

$$
\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}
$$

$$
\frac{y - 2}{0 - 2} = \frac{x - 2}{6 - 2}
$$

Multiply each side by $-2$:

$$
y - 2 = -\frac{1}{2} \left( x - 2 \right) \quad \text{(Note: } -\frac{2}{4} = -\frac{1}{2})
$$

$$
y - 2 = -\frac{1}{2} x + 1
$$

$$
y = -\frac{1}{2} x + 3
$$

(b) The equation of $l_2$ is

$$
y - y_1 = m (x - x_1)
$$

$$
y - 0 = \frac{1}{4} \left[ x - \left( -9 \right) \right]
$$

$$
y = \frac{1}{4} \left( x + 9 \right)
$$

$$
y = \frac{1}{4} x + \frac{9}{4}
$$

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The straight line $l_1$ passes through the points $A$ and $B$ with coordinates $(0, -2)$ and $(6, 7)$ respectively.

(a) Find the equation of $l_1$ in the form $y = mx + c$.

The straight line $l_2$ with equation $x + y = 8$ cuts the $y$-axis at the point $C$. The lines $l_1$ and $l_2$ intersect at the point $D$.

(b) Calculate the coordinates of the point $D$.

(c) Calculate the area of $\triangle ACD$. [E]

Solution:

(a) The equation of $l_1$ is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - (-2)}{7 - (-2)} = \frac{x - 0}{6 - 0}$$

$$\frac{y + 2}{9} = \frac{x}{6}$$

Multiply each term by 9:

$$y + 2 = \frac{3}{2} x$$

$$y = \frac{3}{2} x - 2$$

(b) Solve $x + y = 8$ and $y = \frac{3}{2} x - 2$ simultaneously.

Substitute:

$$x + \left( \frac{3}{2} x - 2 \right) = 8$$

$$x + \frac{3}{2} x - 2 = 8$$

$$\frac{5}{2} x - 2 = 8$$

$$\frac{5}{2} x = 10$$

$$5x = 20$$

$$x = 4$$

Substitute $x = 4$ into $x + y = 8$:

$$y = 4$$

The coordinates of $D$ are $(4, 4)$.

(c) $x + y = 8$ cuts the $y$-axis when $x = 0$.

Substitute $x = 0$: 
\[ 0 + y = 8 \]
\[ y = 8 \]

The coordinates of \( C \) are \((0, 8)\)

\[ AC = 10 \]
\[ h = 4 \]

\[ \text{Area} = \frac{1}{2} \times 10 \times 4 = 20 \]
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Edexcel Modular Mathematics for AS and A-Level
Coordinate geometry in the (x, y) plane
Exercise F, Question 9

Question:

The points A and B have coordinates (2, 16) and (12, -4) respectively. A straight line \(l_1\) passes through A and B.

(a) Find an equation for \(l_1\) in the form \(ax + by = c\).

The line \(l_2\) passes through the point C with coordinates (-1, 1) and has gradient \(\frac{1}{3}\).

(b) Find an equation for \(l_2\). [E]

Solution:

(a) The equation of \(l_1\) is

\[
\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}
\]

\[
\frac{y - 16}{-4 - 16} = \frac{x - 2}{12 - 2}
\]

\[
y - 16 = \frac{x - 2}{10}
\]

Multiply each side by -20:

\[
y - 16 = -2 (x - 2) \quad \text{(Note: } - \frac{20}{10} = -2\text{)}
\]

\[
y = -2x + 4
\]

\[
y = -2x + 20
\]

\[
2x + y = 20
\]

(b) The equation of \(l_2\) is

\[
y - y_1 = m(x - x_1)
\]

\[
y - 1 = \frac{1}{3} \left[ x - \left( -1 \right) \right]
\]

\[
y - 1 = \frac{1}{3} \left[ x + 1 \right]
\]

\[
y - 1 = \frac{1}{3}x + \frac{1}{3}
\]

\[
y = \frac{1}{3}x + \frac{4}{3}
\]

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Coordinate geometry in the \((x, y)\) plane
Exercise F, Question 10

Question:

The points \(A(-1, -2)\), \(B(7, 2)\) and \(C(k, 4)\), where \(k\) is a constant, are the vertices of \(\triangle ABC\). Angle \(ABC\) is a right angle.

(a) Find the gradient of \(AB\).

(b) Calculate the value of \(k\).

(c) Find an equation of the straight line passing through \(B\) and \(C\). Give your answer in the form \(ax + by + c = 0\), where \(a, b\) and \(c\) are integers. [IF]

Solution:

(a) The gradient of \(AB\) is

\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-2)}{7 - (-1)} = \frac{4}{8} = \frac{1}{2}
\]

(b) The gradient of \(BC\) is

\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 2}{k - 7} = -2
\]

So

\[
\frac{2}{k - 7} = -2
\]

\[
\Rightarrow 2 = -2(k - 7)
\]

\[
\Rightarrow 2 = -2k + 14
\]

\[
\Rightarrow 12 = -2k
\]

\[
k = 6
\]

(c) The equation of the line passing through \(B\) and \(C\) is

\[
\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}
\]

\[
\frac{y - 2}{x - 7} = \frac{4 - 2}{6 - 7}
\]

\[
\frac{y - 2}{x - 2} = \frac{2}{-1}
\]

Multiply each side by 2:

\[
y - 2 = -2(x - 7) \quad (\text{Note: } \frac{2}{-1} = -2)
\]

\[
y - 2 = -2x + 14
\]

\[
y = -2x + 16
\]

\[
2x + y = 16
\]

\[
2x + y - 16 = 0
\]

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Coordinate geometry in the (x, y) plane
Exercise F, Question 11

Question:

The straight line $l$ passes through $A (1, 3\sqrt{3})$ and $B (2 + \sqrt{3}, 3 + 4\sqrt{3})$.

(a) Calculate the gradient of $l$ giving your answer as a surd in its simplest form.

(b) Give the equation of $l$ in the form $y = mx + c$, where constants $m$ and $c$ are surds given in their simplest form.

(c) Show that $l$ meets the x-axis at the point $C (−2, 0)$.

Solution:

(a) The gradient of $l$ is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{(3 + 4\sqrt{3}) - 3\sqrt{3}}{(2 + \sqrt{3}) - 1} = \frac{3 + \sqrt{3}}{1 + \sqrt{3}}$$

Rationalise the denominator:

$$\frac{3 + \sqrt{3}}{1 + \sqrt{3}} \times \frac{1 - \sqrt{3}}{1 - \sqrt{3}} = \frac{3 - 3\sqrt{3} + \sqrt{3} - 3}{1 - 3} = \frac{-2\sqrt{3}}{-2} = \sqrt{3}$$

(b) The equation of $l$ is

$$y - y_1 = m (x - x_1)$$

$$y - 3\sqrt{3} = \sqrt{3} (x - 1)$$

$$y = \sqrt{3}x - \sqrt{3}$$

$$y = \sqrt{3}x + 2\sqrt{3}$$

(c) Substitute $y = 0$:

$$0 = \sqrt{3}x + 2\sqrt{3}$$

$$\sqrt{3}x = -2\sqrt{3}$$

$$x = \frac{-2\sqrt{3}}{\sqrt{3}} = -2$$

The coordinates of $C$ are $(−2, 0)$.

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Coordinate geometry in the (x, y) plane
Exercise F, Question 12

Question:

(a) Find an equation of the straight line passing through the points with coordinates (−1, 5) and (4, −2), giving your answer in the form ax + by + c = 0, where a, b and c are integers.
The line crosses the x-axis at the point A and the y-axis at the point B, and O is the origin.

(b) Find the area of △OAB. [E]

Solution:

(a) The equation of the line is

\[ \frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1} \]

\[ \frac{y-5}{-2-5} = \frac{x-(-1)}{4-(-1)} \]

\[ \frac{y-5}{-7} = \frac{x+1}{5} \]

Multiply each side by −35:

\[ 5(y-5) = -7(x+1) \]  \( \text{Note:} \ \frac{-35}{-7} = 5 \text{ and } \frac{-35}{5} = -7 \)

5y − 25 = −7x − 7
7x + 5y − 25 = −7
7x + 5y − 18 = 0

(b) For the coordinates of A substitute y = 0:
7x + 5(0) − 18 = 0
7x − 18 = 0
7x = 18
x = \frac{18}{7}

The coordinates of A are \( \left( \frac{18}{7}, 0 \right) \).

For the coordinates of B substitute x = 0:
7(0) + 5y − 18 = 0
5y − 18 = 0
5y = 18
y = \frac{18}{5}

The coordinates of B are \( \left( 0, \frac{18}{5} \right) \).

The area of △OAB is

\[ \frac{1}{2} \times \frac{18}{7} \times \frac{18}{5} = \frac{162}{35} \]
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Coordinate geometry in the (x, y) plane
Exercise F, Question 13

Question:

The points $A$ and $B$ have coordinates $(k, 1)$ and $(8, 2k - 1)$ respectively, where $k$ is a constant. Given that the gradient of $AB$ is $\frac{1}{3}$.

(a) Show that $k = 2$.

(b) Find an equation for the line through $A$ and $B$. [E]

Solution:

(a) The gradient of $AB$ is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1}{3}$$

$$\frac{(2k - 1) - 1}{8 - k} = \frac{1}{3}$$

$$\frac{2k - 2}{8 - k} = \frac{1}{3}$$

Multiply each side by $(8 - k)$:

$$2k - 2 = \frac{1}{3} \left( 8 - k \right)$$

Multiply each term by 3:

$6k - 6 = 8 - k$

$7k = 14$

$k = 2$

(b) $k = 2$

So $A$ and $B$ have coordinates $(2, 1)$ and $(8, 3)$.

The equation of the line is

$$\frac{y - y_1}{x - x_1} = \frac{x_2 - x_1}{y_2 - y_1}$$

$$\frac{y - 1}{x - 2} = \frac{8 - 2}{6}$$

$$\frac{y - 1}{2} = \frac{x - 2}{6}$$

Multiply each side by 2:

$$y - 1 = \frac{1}{3} \left( x - 2 \right)$$

$$y - 1 = \frac{1}{3}x - \frac{2}{3}$$

$$y = \frac{1}{3}x + \frac{1}{3}$$

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Question:

The straight line \( l_1 \) has equation \( 4y + x = 0 \).
The straight line \( l_2 \) has equation \( y = 2x - 3 \).

(a) On the same axes, sketch the graphs of \( l_1 \) and \( l_2 \). Show clearly the coordinates of all points at which the graphs meet the coordinate axes.
The lines \( l_1 \) and \( l_2 \) intersect at the point \( A \).

(b) Calculate, as exact fractions, the coordinates of \( A \).

(c) Find an equation of the line through \( A \) which is perpendicular to \( l_1 \). Give your answer in the form \( ax + by + c = 0 \), where \( a \), \( b \) and \( c \) are integers. [E]

Solution:

(a) (1) Rearrange \( 4y + x = 0 \) into the form \( y = mx + c \):
\[
4y = -x \\
y = -\frac{1}{4}x
\]
\( l_1 \) has gradient \( -\frac{1}{4} \) and it meets the coordinate axes at \( (0, 0) \).

(2) \( l_2 \) has gradient 2 and it meets the \( y \)-axis at \( (0, -3) \).
\( l_2 \) meets the \( x \)-axis when \( y = 0 \).
Substitute \( y = 0 \):
\[
0 = 2x - 3 \\
2x = 3 \\
x = \frac{3}{2}
\]
\( l_2 \) meets the \( x \)-axis at \( \left( \frac{3}{2}, 0 \right) \).

(b) Solve \( 4y + x = 0 \) and \( y = 2x - 3 \) simultaneously.
Substitute:
\[
4 \left( 2x - 3 \right) + x = 0 \\
8x - 12 + x = 0 \\
9x = 12 \\
x = \frac{12}{9} = \frac{4}{3}
\]
\[ x = \frac{12}{9} \]
\[ x = \frac{4}{3} \]

Substitute \( x = \frac{4}{3} \) into \( y = 2x - 3 \):
\[ y = 2 \left( \frac{4}{3} \right) - 3 = \frac{8}{3} - 3 = -\frac{1}{3} \]

The coordinates of \( A \) are \( \left( \frac{4}{3}, -\frac{1}{3} \right) \).

(c) The gradient of \( l_1 \) is \( -\frac{1}{4} \).

The gradient of a line perpendicular to \( l_1 \) is \( \frac{1}{-\frac{1}{4}} = 4 \).

The equation of the line is
\[ y - y_1 = m \left( x - x_1 \right) \]
\[ y - \left( -\frac{1}{3} \right) = 4 \left( x - \frac{4}{3} \right) \]
\[ y + \frac{1}{3} = 4x - \frac{16}{3} \]
\[ y = 4x - \frac{17}{3} \]

Multiply each term by 3:
\[ 3y = 12x - 17 \]
\[ 0 = 12x - 3y - 17 \]
The equation of the line is \( 12x - 3y - 17 = 0 \).

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Coordinate geometry in the \((x, y)\) plane
Exercise F, Question 15

Question:

The points \(A\) and \(B\) have coordinates \((4, 6)\) and \((12, 2)\) respectively.

The straight line \(l_1\) passes through \(A\) and \(B\).

(a) Find an equation for \(l_1\) in the form \(ax + by + c = 0\), where \(a, b\) and \(c\) are integers.

The straight line \(l_2\) passes through the origin and has gradient \(-4\).

(b) Write down an equation for \(l_2\).

The lines \(l_1\) and \(l_2\) intersect at the point \(C\).

(c) Find the coordinates of \(C\).  \([E]\)

Solution:

(a) The equation of \(l_1\) is

\[
\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}
\]

\[
\frac{y - 6}{2 - 6} = \frac{x - 4}{12 - 4}
\]

\[
\frac{y - 6}{-4} = \frac{x - 4}{8}
\]

Multiply each side by 8:

\[-2(y - 6) = x - 4 \quad \text{(Note:} \quad \frac{8}{-4} = -2)\]

\[-2y + 12 = x - 4\]

\[-2y + 16 = x\]

\[16 = x + 2y\]

\[0 = x + 2y - 16\]

The equation of the line is \(x + 2y - 16 = 0\)

(b) The equation of \(l_2\) is

\[y - y_1 = m(\ x - x_1\ )\]

\[y - 0 = -4(\ x - 0\ )\]

\[y = -4x\]

(c) Solve \(y = -4x\) and \(x + 2y = 16\) simultaneously.

Substitute:

\[x + 2(-4x) = 16\]

\[x - 8x = 16\]

\[-7x = 16\]

\[x = \frac{16}{-7}\]

\[x = -\frac{16}{7}\]

Substitute \(x = -\frac{16}{7}\) in \(y = -4x\):
The coordinates of \( C \) are \( \left( -\frac{16}{7}, \frac{64}{7} \right) \).