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Algebra and functions
Exercise A, Question 1

Question:

Simplify this expression:

$4x - 5y + 3x + 6y$

Solution:

$4x - 5y + 3x + 6y$
$= 4x + 3x - 5y + 6y$
$= 7x + y$

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Algebra and functions
Exercise A, Question 2

Question:

Simplify this expression:

$3r + 7t - 5r + 3t$

Solution:

$3r + 7t - 5r + 3t$
$= 3r - 5r + 7t + 3t$
$= -2r + 10t$

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Algebra and functions
Exercise A, Question 3

Question:

Simplify this expression:

$3m - 2n - p + 5m + 3n - 6p$

Solution:

$3m - 2n - p + 5m + 3n - 6p$
$= 3m + 5m - 2n + 3n - p - 6p$
$= 8m + n - 7p$

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Algebra and functions
Exercise A, Question 4

Question:

Simplify this expression:

$3ab - 3ac + 3a - 7ab + 5ac$

Solution:

$3ab - 3ac + 3a - 7ab + 5ac$
$= 3ab - 7ab - 3ac + 5ac + 3a$
$= 3a - 4ab + 2ac$

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Algebra and functions
Exercise A, Question 5

Question:

Simplify this expression:

$7x^2 - 2x^2 + 5x^2 - 4x^2$

Solution:

$7x^2 - 2x^2 + 5x^2 - 4x^2$
$= 6x^2$

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Algebra and functions
Exercise A, Question 6

Question:

Simplify this expression:

$4m^2n + 5mn^2 - 2m^2n + mn^2 - 3mn^2$

Solution:

$4m^2n + 5mn^2 - 2m^2n + mn^2 - 3mn^2$
$= 4m^2n - 2m^2n + 5mn^2 + mn^2 - 3mn^2$
$= 2m^2n + 3mn^2$

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Algebra and functions
Exercise A, Question 7

Question:

Simplify this expression:

$5x^2 + 4x + 1 - 3x^2 + 2x + 7$

Solution:

$5x^2 + 4x + 1 - 3x^2 + 2x + 7$
$= 5x^2 - 3x^2 + 4x + 2x + 1 + 7$
$= 2x^2 + 6x + 8$

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Algebra and functions
Exercise A, Question 8

Question:

Simplify this expression:

$6x^2 + 5x - 12 + 3x^2 - 7x + 11$

Solution:

$6x^2 + 5x - 12 + 3x^2 - 7x + 11$
$= 6x^2 + 3x^2 + 5x - 7x - 12 + 11$
$= 9x^2 - 2x - 1$

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Algebra and functions
Exercise A, Question 9

Question:

Simplify this expression:

$3x^2 - 5x + 2 + 3x^2 - 7x - 12$

Solution:

$3x^2 - 5x + 2 + 3x^2 - 7x - 12$
$= 3x^2 + 3x^2 - 5x - 7x + 2 - 12$
$= 6x^2 - 12x - 10$

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Algebra and functions
Exercise A, Question 10

Question:

Simplify this expression:

$4c^2d + 5cd^2 - c^2d + 3cd^2 + 7c^2d$

Solution:

$4c^2d + 5cd^2 - c^2d + 3cd^2 + 7c^2d$
$= 4c^2d - c^2d + 7c^2d + 5cd^2 + 3cd^2$
$= 10c^2d + 8cd^2$

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Algebra and functions
Exercise A, Question 11

Question:

Simplify this expression:

$2x^2 + 3x + 1 + 2(3x^2 + 6)$

Solution:

$2x^2 + 3x + 1 + 2(3x^2 + 6)$
$= 2x^2 + 3x + 1 + 6x^2 + 12$
$= 8x^2 + 3x + 13$

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Algebra and functions
Exercise A, Question 12

Question:

Simplify this expression:

$4(a + a^2b) - 3(2a + a^2b)$

Solution:

$4(a + a^2b) - 3(2a + a^2b)$
$= 4a + 4a^2b - 6a - 3a^2b$
$= a^2b - 2a$

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Algebra and functions
Exercise A, Question 13

Question:

Simplify this expression:

$2(3x^2 + 4x + 5) - 3(x^2 - 2x - 3)$

Solution:

$2(3x^2 + 4x + 5) - 3(x^2 - 2x - 3)$
$= 6x^2 + 8x + 10 - 3x^2 + 6x + 9$
$= 3x^2 + 14x + 19$

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Algebra and functions
Exercise A, Question 14

Question:

Simplify this expression:

$7(1 - x^2) + 3(2 - 3x + 5x^2)$

Solution:

$7(1 - x^2) + 3(2 - 3x + 5x^2)$
$= 7 - 7x^2 + 6 - 9x + 15x^2$
$= 8x^2 - 9x + 13$

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Algebra and functions
Exercise A, Question 15

Question:

Simplify this expression:

$4 (a + b + 3c) - 3a + 2c$

Solution:

$4 (a + b + 3c) - 3a + 2c$
$= 4a + 4b + 12c - 3a + 2c$
$= a + 4b + 14c$

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Algebra and functions
Exercise A, Question 16

Question:

Simplify this expression:

$4(c + 3d^2) - 3(2c + d^2)$

Solution:

$4(c + 3d^2) - 3(2c + d^2)$
$= 4c + 12d^2 - 6c - 3d^2$
$= -2c + 9d^2$

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Algebra and functions
Exercise A, Question 17

Question:

Simplify this expression:

$5 - 3(x^2 + 2x - 5) + 3x^2$

Solution:

$5 - 3(x^2 + 2x - 5) + 3x^2$
$= 5 - 3x^2 - 6x + 15 + 3x^2$
$= 20 - 6x$

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Algebra and functions
Exercise A, Question 18

Question:

Simplify this expression:

$(r^2 + 3t^2 + 9) - (2r^2 + 3t^2 - 4)$

Solution:

$(r^2 + 3t^2 + 9) - (2r^2 + 3t^2 - 4)$
$= r^2 + 3t^2 + 9 - 2r^2 - 3t^2 + 4$
$= 13 - r^2$

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Algebra and functions
Exercise B, Question 1

Question:

Simplify this expression:

$x^3 \times x^4$

Solution:

$= x^{3 + 4}$
$= x^7$

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Algebra and functions
Exercise B, Question 2

Question:

Simplify this expression:

$2x^3 \times 3x^2$

Solution:

$= 2 \times 3 \times x^{3+2}$
$= 6x^5$

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Algebra and functions
Exercise B, Question 3

Question:

Simplify this expression:

$4p^3 \div 2p$

Solution:

$= 4 \div 2 \times p^3 \div p$
$= 2 \times p^{3-1}$
$= 2p^2$

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Algebra and functions
Exercise B, Question 4

Question:

Simplify this expression:

$3x^{-4} \div x^{-2}$

Solution:

$= 3x^{-4 - -2}$
$= 3x^{-2}$

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Algebra and functions
Exercise B, Question 5

Question:

Simplify this expression:

$k^3 \div k^{-2}$

Solution:

$= k^{3 - -2}$
$= k^5$

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Algebra and functions
Exercise B, Question 6

Question:

Simplify this expression:

$(y^2)^5$

Solution:

$= y^{2 \times 5}$
$= y^{10}$

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Algebra and functions
Exercise B, Question 7

Question:

Simplify this expression:

$10x^5 \div 2x^{-3}$

Solution:

$= 5x^{5 - -3}$
$= 5x^8$

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Algebra and functions
Exercise B, Question 8

Question:

Simplify this expression:

$(p^3)^2 \div p^4$

Solution:

$= p^6 \div p^4$
$= p^{6-4}$
$= p^2$

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Algebra and functions
Exercise B, Question 9

Question:

Simplify this expression:

$(2a^3)^2 \div 2a^3$

Solution:

$= 4a^6 \div 2a^3$
$= 2a^{6-3}$
$= 2a^3$

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Algebra and functions
Exercise B, Question 10

Question:

Simplify this expression:

$8p^{-4} \div 4p^{3}$

Solution:

$= 2p^{-4-3}$
$= 2p^{-7}$

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Algebra and functions
Exercise B, Question 11

Question:

Simplify this expression:

$2a^{-4} \times 3a^{-5}$

Solution:

$= 6a^{-4 + -5}$
$= 6a^{-9}$

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Algebra and functions
Exercise B, Question 12

Question:

Simplify this expression:

$21a^3b^2 \div 7ab^4$

Solution:

$= 3a^{3-1}b^{2-4}$
$= 3a^2b^{-2}$

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Algebra and functions
Exercise B, Question 13

Question:

Simplify this expression:

$9x^2 \times 3 (x^2)^3$

Solution:

$= 27x^2 \times x^{2 \times 3}$
$= 27x^{2 + 6}$
$= 27x^8$

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Algebra and functions
Exercise B, Question 14

Question:

Simplify this expression:

$3x^3 \times 2x^2 \times 4x^6$

Solution:

$= 24 \times x^{3+2+6}$
$= 24x^{11}$

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Algebra and functions
Exercise B, Question 15

Question:

Simplify this expression:

$7a^4 \times (3a^4)^2$

Solution:

$= 7a^4 \times 9a^8$
$= 63a^{12}$

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Algebra and functions
Exercise B, Question 16

Question:

Simplify this expression:

$(4y^3)^3 \div 2y^3$

Solution:

$= 64y^9 \div 2y^3$
$= 32y^6$

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Algebra and functions
Exercise B, Question 17

Question:

Simplify this expression:

$2a^3 \div 3a^2 \times 6a^5$

Solution:

$= 4a^{3 - 2 + 5}$
$= 4a^6$

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Algebra and functions
Exercise B, Question 18

Question:

Simplify this expression:

$3a^4 \times 2a^5 \times a^3$

Solution:

$= 6a^{4+5+3}$
$= 6a^{12}$

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Algebra and functions
Exercise C, Question 1

Question:

Expand and simplify if possible:

$9 (x - 2)$

Solution:

$= 9x - 18$

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Algebra and functions
Exercise C, Question 2

Question:

Expand and simplify if possible:

$x(x+9)$

Solution:

$= x^2 + 9x$

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Algebra and functions
Exercise C, Question 3

Question:

Expand and simplify if possible:

$-3y(4-3y)$

Solution:

$= -12y + 9y^2$

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Algebra and functions
Exercise C, Question 4

Question:

Expand and simplify if possible:

$x (y + 5)$

Solution:

$= xy + 5x$

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Algebra and functions
Exercise C, Question 5

Question:

Expand and simplify if possible:

$-x(3x+5)$

Solution:

$= -3x^2 - 5x$

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Algebra and functions
Exercise C, Question 6

Question:

Expand and simplify if possible:

$-5x(4x+1)$

Solution:

$= -20x^2 - 5x$

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Algebra and functions
Exercise C, Question 7

Question:

Expand and simplify if possible:

$(4x + 5) x$

Solution:

$= 4x^2 + 5x$

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Algebra and functions
Exercise C, Question 8

Question:

Expand and simplify if possible:

$-3y(5-2y^2)$

Solution:

$= -15y+6y^3$

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Algebra and functions
Exercise C, Question 9

Question:

Expand and simplify if possible:

$-2x(5x-4)$

Solution:

$= -10x^2 + 8x$

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Algebra and functions
Exercise C, Question 10

Question:

Expand and simplify if possible:

$(3x - 5)x^2$

Solution:

$= 3x^3 - 5x^2$

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Algebra and functions
Exercise C, Question 11

Question:

Expand and simplify if possible:

$3(x+2)+(x-7)$

Solution:

$= 3x + 6 + x - 7$
$= 4x - 1$

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Algebra and functions
Exercise C, Question 12

Question:

Expand and simplify if possible:

$5x - 6 - (3x - 2)$

Solution:

$= 5x - 6 - 3x + 2$
$= 2x - 4$

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Algebra and functions
Exercise C, Question 13

Question:

Expand and simplify if possible:

$x (3x^2 - 2x + 5)$

Solution:

$= 3x^3 - 2x^2 + 5x$

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Algebra and functions
Exercise C, Question 14

Question:

Expand and simplify if possible:

$7y^2 (2 - 5y + 3y^2)$

Solution:

$= 14y^2 - 35y^3 + 21y^4$

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Algebra and functions
Exercise C, Question 15

Question:

Expand and simplify if possible:

$-2y^2(5-7y+3y^2)$

Solution:

$=-10y^2+14y^3-6y^4$

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Algebra and functions
Exercise C, Question 16

Question:

Expand and simplify if possible:

$7(x-2)+3(x+4)-6(x-2)$

Solution:

$= 7x - 14 + 3x + 12 - 6x + 12$
$= 4x + 10$

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Algebra and functions
Exercise C, Question 17

Question:

Expand and simplify if possible:

$5x - 3(4 - 2x) + 6$

Solution:

$= 5x - 12 + 6x + 6$
$= 11x - 6$

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Algebra and functions
Exercise C, Question 18

Question:

Expand and simplify if possible:

$3x^2 - x (3 - 4x) + 7$

Solution:

$= 3x^2 - 3x + 4x^2 + 7$
$= 7x^2 - 3x + 7$

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Algebra and functions
Exercise C, Question 19

Question:

Expand and simplify if possible:

$4x(x+3)-2x(3x-7)$

Solution:

$= 4x^2 + 12x - 6x^2 + 14x$
$= 26x - 2x^2$

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Algebra and functions
Exercise C, Question 20

Question:

Expand and simplify if possible:

$3x^2 (2x + 1) - 5x^2 (3x - 4)$

Solution:

$= 6x^3 + 3x^2 - 15x^3 + 20x^2$
$= 23x^2 - 9x^3$

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Algebra and functions
Exercise D, Question 1

Question:

Factorise this expression completely:

$4x + 8$

Solution:

$= 4 (x + 2)$

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Algebra and functions
Exercise D, Question 2

Question:

Factorise this expression completely:

$6x - 24$

Solution:

$= 6 (x - 4)$

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Algebra and functions
Exercise D, Question 3

Question:

Factorise this expression completely:

$20x + 15$

Solution:

$= 5 (4x + 3)$

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Algebra and functions
Exercise D, Question 4

Question:

Factorise this expression completely:

$2x^2 + 4$

Solution:

$= 2 (x^2 + 2)$

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Algebra and functions
Exercise D, Question 5

Question:

Factorise this expression completely:

$4x^2 + 20$

Solution:

$= 4 (x^2 + 5)$

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Algebra and functions
Exercise D, Question 6

Question:

Factorise this expression completely:

$6x^2 - 18x$

Solution:

$= 6x (x - 3)$

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Algebra and functions
Exercise D, Question 7

Question:

Factorise this expression completely:

$x^2 - 7x$

Solution:

$= x (x - 7)$

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Algebra and functions

Exercise D, Question 8

Question:

Factorise this expression completely:

$2x^2 + 4x$

Solution:

$= 2x (x + 2)$

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Algebra and functions
Exercise D, Question 9

Question:

Factorise this expression completely:

$3x^2 - x$

Solution:

$= x (3x - 1)$

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Algebra and functions
Exercise D, Question 10

Question:

Factorise this expression completely:

$6x^2 - 2x$

Solution:

$= 2x (3x - 1)$

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Algebra and functions
Exercise D, Question 11

Question:

Factorise this expression completely:

$10y^2 - 5y$

Solution:

$= 5y (2y - 1)$

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Algebra and functions
Exercise D, Question 12

Question:

Factorise this expression completely:

$35x^2 - 28x$

Solution:

$= 7x (5x - 4)$

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Algebra and functions
Exercise D, Question 13

Question:

Factorise this expression completely:

$x^2 + 2x$

Solution:

$= x (x + 2)$

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Algebra and functions
Exercise D, Question 14

Question:

Factorise this expression completely:

$3y^2 + 2y$

Solution:

$= y (3y + 2)$

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Algebra and functions
Exercise D, Question 15

Question:

Factorise this expression completely:

$4x^2 + 12x$

Solution:

$= 4x (x + 3)$

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Algebra and functions
Exercise D, Question 16

Question:

Factorise this expression completely:

$5y^2 - 20y$

Solution:

$= 5y (y - 4)$

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Algebra and functions
Exercise D, Question 17

Question:

Factorise this expression completely:

$9xy^2 + 12x^2y$

Solution:

$= 3xy\,(3y + 4x)$

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Algebra and functions
Exercise D, Question 18

Question:

Factorise this expression completely:

$6ab - 2ab^2$

Solution:

$= 2ab (3 - b)$

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Algebra and functions
Exercise D, Question 19

Question:

Factorise this expression completely:

$5x^2 - 25xy$

Solution:

$= 5x (x - 5y)$

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Algebra and functions
Exercise D, Question 20

Question:

Factorise this expression completely:

$12x^2y + 8xy^2$

Solution:

$= 4xy\ (\ 3x + 2y\ )$

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Algebra and functions
Exercise D, Question 21

Question:

Factorise this expression completely:

$15y - 20yz^2$

Solution:

$= 5y (3 - 4z^2)$

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Algebra and functions
Exercise D, Question 22

Question:

Factorise this expression completely:

$12x^2 - 30$

Solution:

$= 6 (2x^2 - 5)$

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Algebra and functions
Exercise D, Question 23

Question:

Factorise this expression completely:

$xy^2 - x^2y$

Solution:

$= xy(y - x)$

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Algebra and functions
Exercise D, Question 24

Question:

Factorise this expression completely:

$12y^2 - 4yx$

Solution:

$= 4y (3y - x)$

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Algebra and functions
Exercise E, Question 1

Question:

Factorise:

$x^2 + 4x$

Solution:

$= x (x + 4)$

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Algebra and functions
Exercise E, Question 2

Question:

Factorise:

$2x^2 + 6x$

Solution:

$= 2x (x + 3)$

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Algebra and functions
Exercise E, Question 3

Question:

Factorise:

$x^2 + 11x + 24$

Solution:

$= x^2 + 8x + 3x + 24$
$= x (x + 8) + 3 (x + 8)$
$= (x + 8) (x + 3)$

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Algebra and functions
Exercise E, Question 4

Question:

Factorise:

$x^2 + 8x + 12$

Solution:

$= x^2 + 2x + 6x + 12$
$= x (x + 2) + 6 (x + 2)$
$= (x + 2) (x + 6)$

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Algebra and functions
Exercise E, Question 5

Question:

Factorise:

$x^2 + 3x - 40$

Solution:

$= x^2 + 8x - 5x - 40$
$= x(x + 8) - 5(x + 8)$
$= (x + 8)(x - 5)$

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Algebra and functions
Exercise E, Question 6

Question:

Factorise:

$x^2 - 8x + 12$

Solution:

$= x^2 - 2x - 6x + 12$
$= x(x - 2) - 6(x - 2)$
$= (x - 2)(x - 6)$

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Algebra and functions
Exercise E, Question 7

Question:

Factorise:

$x^2 + 5x + 6$

Solution:

$= x^2 + 3x + 2x + 6$
$= x (x + 3) + 2 (x + 3)$
$= (x + 3) (x + 2)$

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Algebra and functions
Exercise E, Question 8

Question:

Factorise:

$x^2 - 2x - 24$

Solution:

$= x^2 - 6x + 4x - 24$
$= x(x - 6) + 4(x - 6)$
$= (x - 6)(x + 4)$

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Algebra and functions
Exercise E, Question 9

Question:

Factorise:

$x^2 - 3x - 10$

Solution:

$= x^2 - 5x + 2x - 10$
$= x(x - 5) + 2(x - 5)$
$= (x - 5)(x + 2)$

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Algebra and functions
Exercise E, Question 10

Question:

Factorise:

$x^2 + x - 20$

Solution:

$= x^2 - 4x + 5x - 20$
$= x (x - 4) + 5 (x - 4)$
$= (x - 4) (x + 5)$

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Algebra and functions
Exercise E, Question 11

Question:

Factorise:

$2x^2 + 5x + 2$

Solution:

$= 2x^2 + x + 4x + 2$
$= x(2x + 1) + 2(2x + 1)$
$= (2x + 1)(x + 2)$

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Edexcel Modular Mathematics for AS and A-Level

Algebra and functions
Exercise E, Question 12

Question:

Factorise:

$3x^2 + 10x - 8$

Solution:

$= 3x^2 - 2x + 12x - 8$
$= x(3x - 2) + 4(3x - 2)$
$= (3x - 2)(x + 4)$

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Algebra and functions
Exercise E, Question 13

Question:

Factorise:

$5x^2 - 16x + 3$

Solution:

$= 5x^2 - 15x - x + 3$
$= 5x(x - 3) - (x - 3)$
$= (x - 3)(5x - 1)$

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Algebra and functions
Exercise E, Question 14

Question:

Factorise:

$6x^2 - 8x - 8$

Solution:

$= 6x^2 - 12x + 4x - 8$
$= 6x (x - 2) + 4 (x - 2)$
$= (x - 2) (6x + 4) = 2 (x - 2) (3x + 2)$

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Algebra and functions
Exercise E, Question 15

Question:

Factorise:

$2x^2 + 7x - 15$

Solution:

$= 2x^2 + 10x - 3x - 15$
$= 2x (x + 5) - 3 (x + 5)$
$= (x + 5) (2x - 3)$

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Algebra and functions
Exercise E, Question 16

Question:

Factorise:

$2x^4 + 14x^2 + 24$

Solution:

$= 2y^2 + 14y + 24$
$= 2y^2 + 6y + 8y + 24$
$= 2y(y + 3) + 8(y + 3)$
$= (y + 3)(2y + 8)$
$= (x^2 + 3)(2x^2 + 8) = 2(x^2 + 3)(x^2 + 4)$

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Algebra and functions
Exercise E, Question 17

Question:

Factorise:

$x^2 - 4$

Solution:

$= x^2 - 2^2$
$= (x + 2) (x - 2)$

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Algebra and functions
Exercise E, Question 18

Question:

Factorise:

$x^2 - 49$

Solution:

$= x^2 - 7^2$
$= (x + 7) (x - 7)$

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Algebra and functions
Exercise E, Question 19

Question:

Factorise:

$4x^2 - 25$

Solution:

$= (2x)^2 - 5^2$
$= (2x + 5) (2x - 5)$

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Algebra and functions
Exercise E, Question 20

Question:

Factorise:

$9x^2 - 25y^2$

Solution:

$= (3x) ^2 - (5y) ^2$
$= (3x + 5y) (3x - 5y)$

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Algebra and functions
Exercise E, Question 21

Question:

Factorise:

$36x^2 - 4$

Solution:

$= 4 (9x^2 - 1)$
$= 4 [(3x) ^2 - 1]$
$= 4 (3x + 1) (3x - 1)$

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Algebra and functions
Exercise E, Question 22

Question:

Factorise:

$2x^2 - 50$

Solution:

$= 2 (x^2 - 25)$
$= 2 (x^2 - 5^2)$
$= 2 (x + 5) (x - 5)$

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Algebra and functions
Exercise E, Question 23

Question:

Factorise:

$6x^2 - 10x + 4$

Solution:

$= 2 (3x^2 - 5x + 2)$
$= 2 (3x^2 - 3x - 2x + 2)$
$= 2 [3x (x - 1) - 2 (x - 1)]$
$= 2 (x - 1) (3x - 2)$

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Algebra and functions
Exercise E, Question 24

Question:

Factorise:

$15x^2 + 42x - 9$

Solution:

$= 3 (5x^2 + 14x - 3)$
$= 3 (5x^2 - x + 15x - 3)$
$= 3 [x (5x - 1) + 3 (5x - 1)]$
$= 3 (5x - 1) (x + 3)$

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Algebra and functions
Exercise F, Question 1

Question:

Factorise:

Simplify:

(a) $x^3 \div x^{-2}$

(b) $x^5 \div x^7$

(c) $x^{\frac{3}{2}} \times x^{\frac{5}{2}}$

(d) $(x^2)^{\frac{3}{2}}$

(e) $(x^3)^{\frac{5}{3}}$

(f) $3x^{0.5} \times 4x^{-0.5}$

(g) $9x^{\frac{2}{3}} \div 3x^{\frac{1}{6}}$

(h) $5x^{1\frac{2}{5}} \div x^{\frac{2}{5}}$

(i) $3x^4 \times 2x^{-5}$

Solution:

(a) $= x^{3 - -2}$
$= x^5$

(b) $= x^{5-7}$
$= x^{-2}$

(c) $= x^{\frac{3}{2} + \frac{5}{2}}$
$= x^4$

(d) $= x^{2 \times \frac{3}{2}}$
$= x^3$

(e) $= x^{3 \times \frac{5}{3}}$
$= x^5$

(f) $= 12x^{0.5 + -0.5}$
$= 12x^0$

$= 12$

(g) $= 3x^{\frac{2}{3} - \frac{1}{6}}$

$= 3x^{\frac{1}{2}}$

(h) $= 5x^{1\frac{2}{5} - \frac{2}{5}}$

$= 5x$

(i) $= 6x^{4 + -5}$

$= 6x^{-1}$

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Algebra and functions
Exercise F, Question 2

Question:

Factorise:

Evaluate:

(a) $25^{\frac{1}{2}}$

(b) $81^{\frac{1}{2}}$

(c) $27^{\frac{1}{3}}$

(d) 4^{-2}

(e) $9^{-\frac{1}{2}}$

(f) $(-5)^{-3}$

(g) $\left(\frac{3}{4}\right)^{0}$

(h) $1296^{\frac{1}{4}}$

(i) $\left(1\frac{9}{16}\right)^{\frac{3}{2}}$

(j) $\left(\frac{27}{8}\right)^{\frac{2}{3}}$

(k) $\left(\frac{6}{5}\right)^{-1}$

(l) $\left(\frac{343}{512}\right)^{-\frac{2}{3}}$

Solution:

(a) $= \sqrt{25}$
$= \pm 5$

(b) $= \sqrt{81}$

$= \pm 9$

(c) $= \sqrt[3]{27}$
$= 3$

(d) $= \frac{1}{4^2}$

$= \frac{1}{16}$

(e) $= \frac{1}{9^{\frac{1}{2}}}$

$= \frac{1}{\sqrt{9}}$

$= \pm \frac{1}{3}$

(f) $= \frac{1}{(-5)^3}$

$= \frac{1}{-125}$

(g) $= 1$

(h) $= \sqrt[4]{1296}$
$= \pm 6$

(i) $= \left(\frac{25}{16} \right)^{\frac{3}{2}}$

$= \frac{(\sqrt{25})^3}{(\sqrt{16})^3}$

$= \frac{5^3}{4^3}$

$= \frac{125}{64}$

(j) $= \frac{(\sqrt[3]{27})^2}{(\sqrt[3]{8})^2}$

$= \frac{(3)^2}{(2)^2}$

$= \frac{9}{4}$

(k) $= \left(\frac{5}{6} \right)^1$

$= \frac{5}{6}$

(l) $\frac{(\sqrt[3]{512})^2}{(\sqrt[3]{343})^2}$

$= \frac{(8)^2}{(7)^2}$

$= \frac{64}{49}$

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Algebra and functions
Exercise G, Question 1

Question:

Simplify:

$\sqrt{28}$

Solution:

$= \sqrt{4} \times \sqrt{7}$
$= 2\sqrt{7}$

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Algebra and functions
Exercise G, Question 2

Question:

Simplify:

$\sqrt{72}$

Solution:

$= \sqrt{8} \times \sqrt{9}$
$= \sqrt{2} \times \sqrt{4} \times \sqrt{9}$
$= \sqrt{2} \times 2 \times 3$
$= 6\sqrt{2}$

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Algebra and functions
Exercise G, Question 3

Question:

Simplify:

$\sqrt{50}$

Solution:

$= \sqrt{25} \times \sqrt{2}$
$= 5\sqrt{2}$

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Algebra and functions
Exercise G, Question 4

Question:

Simplify:

$\sqrt{32}$

Solution:

$= \sqrt{16} \times \sqrt{2}$
$= 4\sqrt{2}$

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Algebra and functions
Exercise G, Question 5

Question:

Simplify:

$\sqrt{90}$

Solution:

$= \sqrt{9} \times \sqrt{10}$
$= 3\sqrt{10}$

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Algebra and functions
Exercise G, Question 6

Question:

Simplify:

$$\frac{\sqrt{12}}{2}$$

Solution:

$$= \frac{\sqrt{4} \times \sqrt{3}}{2}$$

$$= \frac{2 \times \sqrt{3}}{2}$$

$$= \sqrt{3}$$

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Algebra and functions
Exercise G, Question 7

Question:

Simplify:

$$\frac{\sqrt{27}}{3}$$

Solution:

$$= \frac{\sqrt{9} \times \sqrt{3}}{3}$$

$$= \frac{3 \times \sqrt{3}}{3}$$

$$= \sqrt{3}$$

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Algebra and functions
Exercise G, Question 8

Question:

Simplify:

$\sqrt{20} + \sqrt{80}$

Solution:

$= \sqrt{4}\sqrt{5} + \sqrt{16}\sqrt{5}$
$= 2\sqrt{5} + 4\sqrt{5}$
$= 6\sqrt{5}$

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Algebra and functions
Exercise G, Question 9

Question:

Simplify:

$\sqrt{200} + \sqrt{18} - \sqrt{72}$

Solution:

$= \sqrt{100}\sqrt{2} + \sqrt{9}\sqrt{2} - \sqrt{9}\sqrt{4}\sqrt{2}$
$= 10\sqrt{2} + 3\sqrt{2} - 6\sqrt{2}$
$= 7\sqrt{2}$

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Algebra and functions
Exercise G, Question 10

Question:

Simplify:

$\sqrt{175} + \sqrt{63} + 2\sqrt{28}$

Solution:

$= \sqrt{25} \times \sqrt{7} + \sqrt{9} \times \sqrt{7} + 2 \times \sqrt{4} \times \sqrt{7}$
$= 5\sqrt{7} + 3\sqrt{7} + 4\sqrt{7}$
$= 12\sqrt{7}$

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Algebra and functions
Exercise G, Question 11

Question:

Simplify:

$1\sqrt{28} - 2\sqrt{63} + \sqrt{7}$

Solution:

$= \sqrt{4}\sqrt{7} - 2\sqrt{9}\sqrt{7} + \sqrt{7}$
$= 2\sqrt{7} - 6\sqrt{7} + \sqrt{7}$
$= -3\sqrt{7}$

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Algebra and functions
Exercise G, Question 12

Question:

Simplify:

$\sqrt{80} - 2\sqrt{20} + 3\sqrt{45}$

Solution:

$= \sqrt{16}\sqrt{5} - 2\sqrt{4}\sqrt{5} + 3\sqrt{9}\sqrt{5}$
$= 4\sqrt{5} - 4\sqrt{5} + 9\sqrt{5}$
$= 9\sqrt{5}$

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Algebra and functions
Exercise G, Question 13

Question:

Simplify:

$3\sqrt{80} - 2\sqrt{20} + 5\sqrt{45}$

Solution:

$= 3\sqrt{16}\sqrt{5} - 2\sqrt{4}\sqrt{5} + 5\sqrt{9}\sqrt{5}$
$= 12\sqrt{5} - 4\sqrt{5} + 15\sqrt{5}$
$= 23\sqrt{5}$

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Algebra and functions
Exercise G, Question 14

Question:

Simplify:

$$\frac{\sqrt{44}}{\sqrt{11}}$$

Solution:

$$= \frac{\sqrt{4}\sqrt{11}}{\sqrt{11}}$$

$$= 2$$

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Algebra and functions
Exercise G, Question 15

Question:

Simplify:

$\sqrt{12} + 3\sqrt{48} + \sqrt{75}$

Solution:

$= \sqrt{4}\sqrt{3} + 3\sqrt{16}\sqrt{3} + \sqrt{25}\sqrt{3}$
$= 2\sqrt{3} + 12\sqrt{3} + 5\sqrt{3}$
$= 19\sqrt{3}$

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Algebra and functions
Exercise H, Question 1

Question:

Rationalise the denominator:

$\frac{1}{\sqrt{5}}$

Solution:

$= \frac{1 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}}$

$= \frac{\sqrt{5}}{5}$

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Algebra and functions
Exercise H, Question 2

Question:

Rationalise the denominator:

$$\frac{1}{\sqrt{11}}$$

Solution:

$$= \frac{1 \times \sqrt{11}}{\sqrt{11} \times \sqrt{11}}$$

$$= \frac{\sqrt{11}}{11}$$

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Algebra and functions
Exercise H, Question 3

Question:

Rationalise the denominator:

$\frac{1}{\sqrt{2}}$

Solution:

$= \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}}$

$= \frac{\sqrt{2}}{2}$

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Algebra and functions
Exercise H, Question 4

Question:

Rationalise the denominator:

$$\frac{\sqrt{3}}{\sqrt{15}}$$

Solution:

$$= \frac{\sqrt{3} \times \sqrt{15}}{\sqrt{15} \times \sqrt{15}}$$

$$= \frac{\sqrt{3 \times 15}}{15}$$

$$= \frac{\sqrt{45}}{15}$$

$$= \frac{\sqrt{9 \times 5}}{15}$$

$$= \frac{\sqrt{9} \times \sqrt{5}}{15}$$

$$= \frac{3 \times \sqrt{5}}{15}$$

$$= \frac{\sqrt{5}}{5}$$

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Algebra and functions
Exercise H, Question 5

Question:

Rationalise the denominator:

$$\frac{\sqrt{12}}{\sqrt{48}}$$

Solution:

$$= \frac{\sqrt{12}}{\sqrt{12} \times \sqrt{4}}$$

$$= \frac{1}{\sqrt{4}}$$

$$= \frac{1}{2}$$

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Algebra and functions
Exercise H, Question 6

Question:

Rationalise the denominator:

$$\frac{\sqrt{5}}{\sqrt{80}}$$

Solution:

$$= \frac{\sqrt{5}}{\sqrt{5} \times \sqrt{16}}$$

$$= \frac{1}{\sqrt{16}}$$

$$= \frac{1}{4}$$

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Algebra and functions
Exercise H, Question 7

Question:

Rationalise the denominator:

$$\frac{\sqrt{12}}{\sqrt{156}}$$

Solution:

$$= \frac{\sqrt{12}}{\sqrt{12} \times \sqrt{13}}$$

$$= \frac{1}{\sqrt{13}}$$

$$= \frac{1 \times \sqrt{13}}{\sqrt{13} \times \sqrt{13}}$$

$$= \frac{\sqrt{13}}{13}$$

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Algebra and functions
Exercise H, Question 8

Question:

Rationalise the denominator:

$$\frac{\sqrt{7}}{\sqrt{63}}$$

Solution:

$$\frac{\sqrt{7}}{\sqrt{7} \times \sqrt{9}}$$

$$= \frac{1}{\sqrt{9}}$$

$$= \frac{1}{3}$$

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Algebra and functions
Exercise H, Question 9

Question:

Rationalise the denominator:

$$\frac{1}{1+\sqrt{3}}$$

Solution:

$$= \frac{1 \times (1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})}$$

$$= \frac{1-\sqrt{3}}{1+\sqrt{3}-\sqrt{3}-3}$$

$$= \frac{1-\sqrt{3}}{-2} \text{ or}$$

$$= \frac{-1+\sqrt{3}}{2}$$

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Algebra and functions
Exercise H, Question 10

Question:

Rationalise the denominator:

$$\frac{1}{2+\sqrt{5}}$$

Solution:

$$= \frac{1\times(2-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})}$$

$$= \frac{2-\sqrt{5}}{4-5}$$

$$= \frac{2-\sqrt{5}}{-1}$$

$$= -2+\sqrt{5}$$

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Algebra and functions
Exercise H, Question 11

Question:

Rationalise the denominator:

$$\frac{1}{3-\sqrt{7}}$$

Solution:

$$= \frac{3+\sqrt{7}}{(3-\sqrt{7})(3+\sqrt{7})}$$

$$= \frac{3+\sqrt{7}}{9-7}$$

$$= \frac{3+\sqrt{7}}{2}$$

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Algebra and functions
Exercise H, Question 12

Question:

Rationalise the denominator:

$$\frac{4}{3-\sqrt{5}}$$

Solution:

$$= \frac{4 \times (3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})}$$

$$= \frac{12+4\sqrt{5}}{9-5}$$

$$= \frac{12+4\sqrt{5}}{4}$$

$$= 3+\sqrt{5}$$

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Algebra and functions
Exercise H, Question 13

Question:

Rationalise the denominator:

$$\frac{1}{\sqrt{5}-\sqrt{3}}$$

Solution:

$$= \frac{\sqrt{5}+\sqrt{3}}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})}$$

$$= \frac{\sqrt{5}+\sqrt{3}}{5-3}$$

$$= \frac{\sqrt{5}+\sqrt{3}}{2}$$

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Algebra and functions
Exercise H, Question 14

Question:

Rationalise the denominator:

$$\frac{3-\sqrt{2}}{4-\sqrt{5}}$$

Solution:

$$= \frac{(3-\sqrt{2})(4+\sqrt{5})}{(4-\sqrt{5})(4+\sqrt{5})}$$

$$= \frac{(3-\sqrt{2})(4+\sqrt{5})}{16-5}$$

$$= \frac{(3-\sqrt{2})(4+\sqrt{5})}{11}$$

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Algebra and functions
Exercise H, Question 15

Question:

Rationalise the denominator:

$$\frac{5}{2+\sqrt{5}}$$

Solution:

$$= \frac{5 \times (2-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})}$$

$$= \frac{5(2-\sqrt{5})}{4-5}$$

$$= \frac{5(2-\sqrt{5})}{-1}$$

$$= 5(\sqrt{5}-2)$$

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Algebra and functions
Exercise H, Question 16

Question:

Rationalise the denominator:

$$\frac{5\sqrt{2}}{\sqrt{8}-\sqrt{7}}$$

Solution:

$$= \frac{5\sqrt{2}(\sqrt{8}+\sqrt{7})}{(\sqrt{8}-\sqrt{7})(\sqrt{8}+\sqrt{7})}$$

$$= \frac{5(\sqrt{8\times 2}+\sqrt{2}\sqrt{7})}{8-7}$$

$$= \frac{5(\sqrt{16}+\sqrt{14})}{1}$$

$$= 5(4+\sqrt{14})$$

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Algebra and functions
Exercise H, Question 17

Question:

Rationalise the denominator:

$$\frac{11}{3+\sqrt{11}}$$

Solution:

$$= \frac{11(3-\sqrt{11})}{(3+\sqrt{11})(3-\sqrt{11})}$$

$$= \frac{11(3-\sqrt{11})}{9-11}$$

$$= \frac{11(3-\sqrt{11})}{-2}$$

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Algebra and functions
Exercise H, Question 18

Question:

Rationalise the denominator:

$$\frac{\sqrt{3}-\sqrt{7}}{\sqrt{3}+\sqrt{7}}$$

Solution:

$$= \frac{(\sqrt{3}-\sqrt{7})(\sqrt{3}-\sqrt{7})}{(\sqrt{3}+\sqrt{7})(\sqrt{3}-\sqrt{7})}$$

$$= \frac{3-\sqrt{21}-\sqrt{21}+7}{3-7}$$

$$= \frac{10-2\sqrt{21}}{-4}$$

$$= \frac{5-\sqrt{21}}{-2}$$

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Algebra and functions
Exercise H, Question 19

Question:

Rationalise the denominator:

$$\frac{\sqrt{17}-\sqrt{11}}{\sqrt{17}+\sqrt{11}}$$

Solution:

$$= \frac{(\sqrt{17}-\sqrt{11})(\sqrt{17}-\sqrt{11})}{(\sqrt{17}+\sqrt{11})(\sqrt{17}-\sqrt{11})}$$

$$= \frac{17-\sqrt{187}-\sqrt{187}+11}{17-11}$$

$$= \frac{28-2\sqrt{187}}{6}$$

$$= \frac{14-\sqrt{187}}{3}$$

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Algebra and functions
Exercise H, Question 20

Question:

Rationalise the denominator:

$$\frac{\sqrt{41}+\sqrt{29}}{\sqrt{41}-\sqrt{29}}$$

Solution:

$$= \frac{(\sqrt{41}+\sqrt{29})(\sqrt{41}+\sqrt{29})}{(\sqrt{41}-\sqrt{29})(\sqrt{41}+\sqrt{29})}$$

$$= \frac{41+2\sqrt{41}\sqrt{29}+29}{41-29}$$

$$= \frac{70+2\sqrt{1189}}{12}$$

$$= \frac{35+\sqrt{1189}}{6}$$

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Algebra and functions
Exercise H, Question 21

Question:

Rationalise the denominator:

$$\frac{\sqrt{2}-\sqrt{3}}{\sqrt{3}-\sqrt{2}}$$

Solution:

$$= \frac{(\sqrt{2}-\sqrt{3})(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}$$

$$= \frac{\sqrt{6}-3+2-\sqrt{6}}{3-2}$$

$$= \frac{-1}{1}$$

$$= -1$$

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Algebra and functions
Exercise I, Question 1

Question:

Simplify:

(a) $y^3 \times y^5$

(b) $3x^2 \times 2x^5$

(c) $(4x^2)^3 \div 2x^5$

(d) $4b^2 \times 3b^3 \times b^4$

Solution:

(a) $= y^{3+5}$
$= y^8$

(b) $= 3 \times 2 \times x^{2+5}$
$= 6x^7$

(c) $= 4^3 x^{2 \times 3} \div 2x^5$
$= 64x^6 \div 2x^5$
$= 32x^{6-5}$
$= 32x$

(d) $= 4 \times 3 \times b^{2+3+4}$
$= 12b^9$

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Algebra and functions
Exercise I, Question 2

Question:

Expand the brackets:

(a) $3(5y + 4)$

(b) $5x^2(3 - 5x + 2x^2)$

(c) $5x(2x + 3) - 2x(1 - 3x)$

(d) $3x^2(1 + 3x) - 2x(3x - 2)$

Solution:

(a) $= 15y + 12$

(b) $= 15x^2 - 25x^3 + 10x^4$

(c) $= 10x^2 + 15x - 2x + 6x^2$
$= 16x^2 + 13x$

(d) $= 3x^2 + 9x^3 - 6x^2 + 4x$
$= 9x^3 - 3x^2 + 4x$

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Algebra and functions
Exercise I, Question 3

Question:

Factorise these expressions completely:

(a) $3x^2 + 4x$

(b) $4y^2 + 10y$

(c) $x^2 + xy + xy^2$

(d) $8xy^2 + 10x^2y$

Solution:

(a) $= x (3x + 4)$

(b) $= 2y (2y + 5)$

(c) $= x (x + y + y^2)$

(d) $= 2xy (4y + 5x)$

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Algebra and functions
Exercise I, Question 4

Question:

Factorise:

(a) $x^2 + 3x + 2$

(b) $3x^2 + 6x$

(c) $x^2 - 2x - 35$

(d) $2x^2 - x - 3$

(e) $5x^2 - 13x - 6$

(f) $6 - 5x - x^2$

Solution:

(a) $= x^2 + x + 2x + 2$
$= x(x + 1) + 2(x + 1)$
$= (x + 1)(x + 2)$

(b) $= 3x(x + 2)$

(c) $= x^2 - 7x + 5x - 35$
$= x(x - 7) + 5(x - 7)$
$= (x - 7)(x + 5)$

(d) $= 2x^2 - 3x + 2x - 3$
$= x(2x - 3) + (2x - 3)$
$= (2x - 3)(x + 1)$

(e) $= 5x^2 + 2x - 15x - 6$
$= x(5x + 2) - 3(5x + 2)$
$= (5x + 2)(x - 3)$

(f) $= 6 + x - 6x - x^2$
$= (6 + x) - x(6 + x)$
$= (1 - x)(6 + x)$

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Algebra and functions
Exercise I, Question 5

Question:

Simplify:

(a) $9x^3 \div 3x^{-3}$

(b) $\left(4^{\frac{3}{2}}\right)^{\frac{1}{3}}$

(c) $3x^{-2} \times 2x^4$

(d) $3x^{\frac{1}{3}} \div 6x^{\frac{2}{3}}$

Solution:

(a) $= 3x^{3 - -3}$
$= 3x^6$

(b) $\left[\left(\sqrt{4} \right)^3 \right]^{\frac{1}{3}}$

$= \left(\sqrt{4} \right)^{3 \times \frac{1}{3}}$
$= \sqrt{4}$
$= \pm 2$

(c) $= 6x^{-2+4}$
$= 6x^2$

(d) $= \frac{1}{2}x^{\frac{1}{3} - \frac{2}{3}}$

$= \frac{1}{2}x^{-\frac{1}{3}}$ or

$= \frac{1}{2\left(\sqrt[3]{x}\right)}$

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise I, Question 6

Question:

Evaluate:

(a) $\left(\frac{8}{27} \right)^{\frac{2}{3}}$

(b) $\left(\frac{225}{289} \right)^{\frac{3}{2}}$

Solution:

(a) $= \left(\frac{\sqrt[3]{8}}{\sqrt[3]{27}} \right)^{2}$

$= \left(\frac{2}{3} \right)^{2}$

$= \frac{4}{9}$

(b) $= \left(\frac{\sqrt{225}}{\sqrt{289}} \right)^{3}$

$= \frac{15^3}{17^3}$

$= \frac{3375}{4913}$

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Algebra and functions
Exercise I, Question 7

Question:

Simplify:

(a) $\frac{3}{\sqrt{63}}$

(b) $\sqrt{20} + 2\sqrt{45} - \sqrt{80}$

Solution:

(a) $= \frac{3}{\sqrt{9} \times \sqrt{7}}$

$= \frac{3}{3\sqrt{7}}$

$= \frac{1}{\sqrt{7}}$

$= \frac{\sqrt{7}}{7}$ (If you rationalise)

(b) $= 2\sqrt{5} + 2 \times 3\sqrt{5} - 4\sqrt{5}$
$= 4\sqrt{5}$

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Algebra and functions
Exercise I, Question 8

Question:

Rationalise:

(a) $\frac{1}{\sqrt{3}}$

(b) $\frac{1}{\sqrt{2}-1}$

(c) $\frac{3}{\sqrt{3}-2}$

(d) $\frac{\sqrt{23}-\sqrt{37}}{\sqrt{23}+\sqrt{37}}$

Solution:

(a) $= \frac{1 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$

$= \frac{\sqrt{3}}{3}$

(b) $= \frac{\sqrt{2}+1}{(\sqrt{2}-1)(\sqrt{2}+1)}$

$= \frac{\sqrt{2}+1}{2-1}$

$= \sqrt{2}+1$

(c) $= \frac{3(\sqrt{3}+2)}{(\sqrt{3}-2)(\sqrt{3}+2)}$

$= \frac{3\sqrt{3}+6}{3-4}$

$= -3\sqrt{3}-6$

(d) $= \frac{(\sqrt{23}-\sqrt{37})(\sqrt{23}-\sqrt{37})}{(\sqrt{23}+\sqrt{37})(\sqrt{23}-\sqrt{37})}$

$= \frac{23-2\sqrt{23}\sqrt{37}+37}{23-37}$

$= \frac{60-2\sqrt{851}}{-14}$

$= \frac{30-\sqrt{851}}{-7}$

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Quadratic Equations
Exercise A, Question 1

Question:

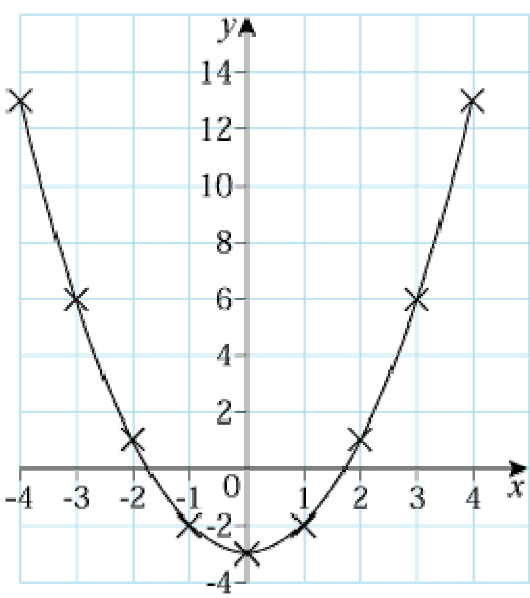
Draw a graph with the following equation, taking values of x from -4 to $+4$. For each graph write down the equation of the line of symmetry.

$y = x^2 - 3$

Solution:

$y = x^2 - 3$.

x	-4	-33	-2	-1	0	1	2	3	4
$x^2 - 3$	$16 - 3$	$9 - 3$	$4 - 3$	$1 - 3$	$0 - 3$	$1 - 3$	$4 - 3$	$9 - 3$	$16 - 3$
y	13	6	1	-2	-3	-2	1	6	13



Equation of line of symmetry is $x = 0$.

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Quadratic Equations
Exercise A, Question 2

Question:

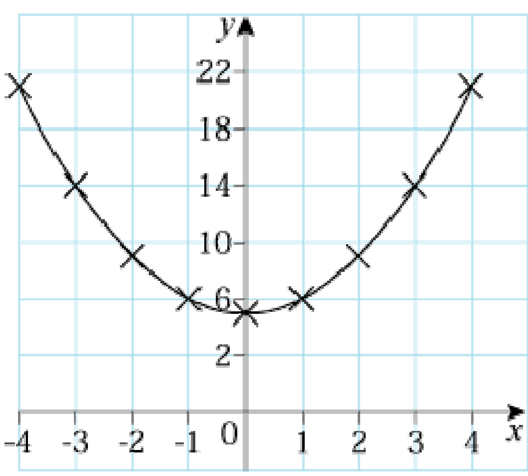
Draw a graph with the following equation, taking values of x from -4 to $+4$. For each graph write down the equation of the line of symmetry.

$y = x^2 + 5$

Solution:

$y = x^2 + 5$.

x	− 4	− 33	− 2	− 1	0	1	2	3	4
$x^2 + 5$	16 + 5	9 + 5	4 + 5	1 + 5	0 + 5	1 + 5	4 + 5	9 + 5	16 + 5
y	21	14	9	6	5	6	9	14	21



Equation of line of symmetry is $x = 0$.

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Quadratic Equations
Exercise A, Question 3

Question:

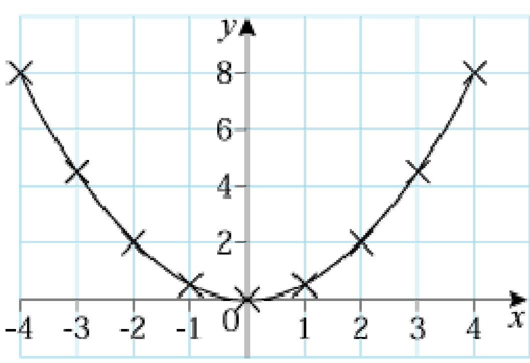
Draw a graph with the following equation, taking values of x from -4 to $+4$. For each graph write down the equation of the line of symmetry.

$y = \frac{1}{2}x^2$

Solution:

$y = \frac{1}{2}x^2$

x	-4	-33	-2	-1	0	1	2	3	4
$\frac{1}{2}x^2$	8	$4\frac{1}{2}$	2	$\frac{1}{2}$	0	$\frac{1}{2}$	2	$4\frac{1}{2}$	8
y	8	$4\frac{1}{2}$	2	$\frac{1}{2}$	0	$\frac{1}{2}$	2	$4\frac{1}{2}$	8



Equation of line of symmetry is $x = 0$.

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Quadratic Equations
Exercise A, Question 4

Question:

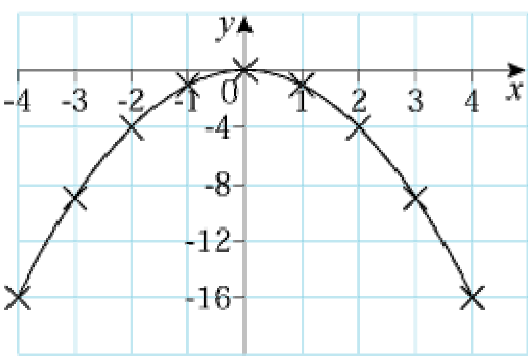
Draw a graph with the following equation, taking values of x from -4 to $+4$. For each graph write down the equation of the line of symmetry.

$y = -x^2$

Solution:

$y = -x^2$

x	-4	-33	-2	-1	0	1	2	3	4
$-x^2$	-16	-9	-4	-1	0	-1	-4	-9	-16
y	-16	-9	-4	-1	0	-1	-4	-9	-16



Equation of line of symmetry is $x = 0$.

Solutionbank C1

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Quadratic Equations

Exercise A, Question 5

Question:

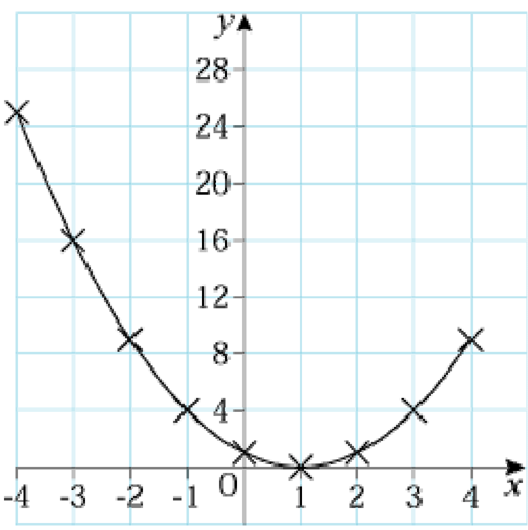
Draw a graph with the following equation, taking values of x from -4 to $+4$. For each graph write down the equation of the line of symmetry.

$y = (x - 1)^2$

Solution:

$y = (x - 1)^2$

x	-4	-33	-2	-1	0	1	2	3	4
$(x-1)^2$	25	16	9	4	1	0	1	4	9
y	25	16	9	4	1	0	1	4	9



Equation of line of symmetry is $x = 1$.

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Quadratic Equations
Exercise A, Question 6

Question:

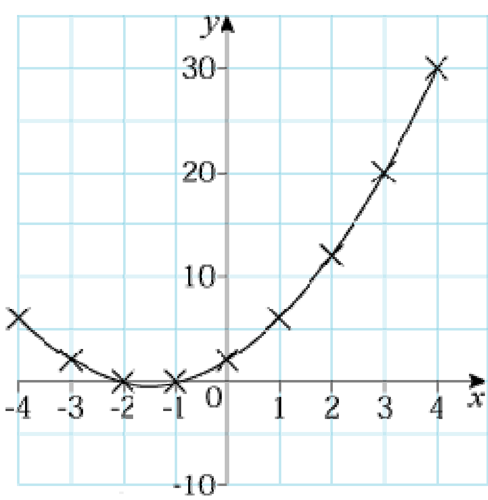
Draw a graph with the following equation, taking values of x from -4 to $+4$. For each graph write down the equation of the line of symmetry.

$y = x^2 + 3x + 2$

Solution:

$y = x^2 + 3x + 2$

x	-4	-33	-2	-1	0	1	2	3	4
$x^2 + 3x + 2$	$16 - 12 + 2$	$9 - 9 + 2$	$4 - 6 + 2$	$1 - 3 + 2$	$0 + 0 + 2$	$1 + 3 + 2$	$4 + 6 + 2$	$9 + 9 + 2$	$16 + 12 + 2$
y	6	2	0	0	2	6	12	20	30



Equation of line of symmetry is $x = -1\frac{1}{2}$.

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Quadratic Equations
Exercise A, Question 7

Question:

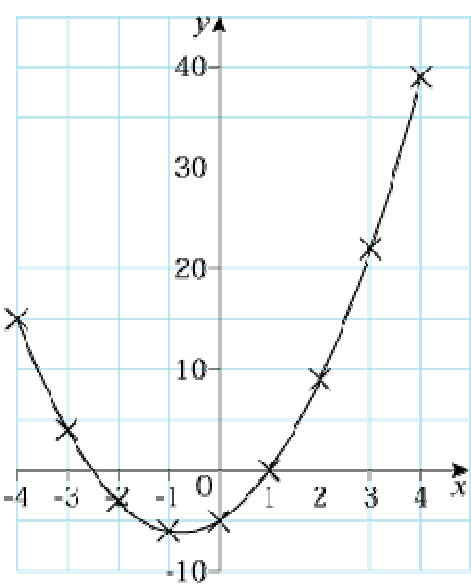
Draw a graph with the following equation, taking values of x from -4 to $+4$. For each graph write down the equation of the line of symmetry.

$y = 2x^2 + 3x - 5$

Solution:

$y = 2x^2 + 3x - 5$

x	-4	-33	-2	-1	0	1	2	3	4
$2x^2 + 3x - 5$	$32 - 12 - 5$	$18 - 9 - 5$	$8 - 6 - 5$	$2 - 3 - 5$	$0 + 0 - 5$	$2 + 3 - 5$	$8 + 6 - 5$	$18 + 9 - 5$	$32 + 12 - 5$
y	15	4	-3	-6	-5	0	9	22	39



Equation of line of symmetry is $x = -\frac{3}{4}$.

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Quadratic Equations
Exercise A, Question 8

Question:

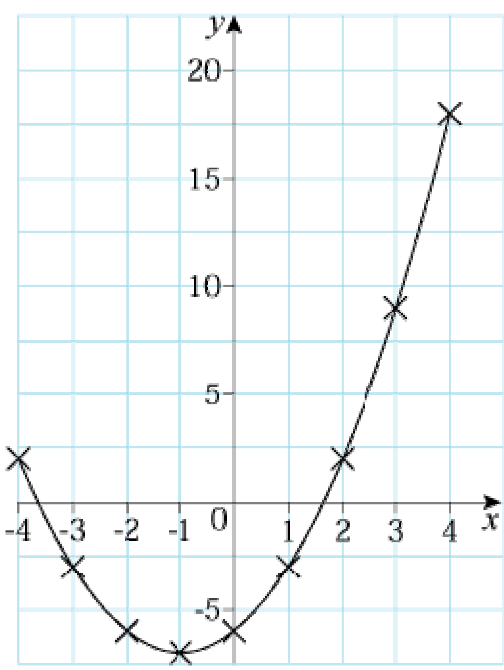
Draw a graph with the following equation, taking values of x from -4 to $+4$. For each graph write down the equation of the line of symmetry.

$y = x^2 + 2x - 6$

Solution:

$y = x^2 + 2x - 6$

x	-4	-33	-2	-1	0	1	2	3	4
$x^2 + 2x - 6$	$16 - 8 - 6$	$9 - 6 - 6$	$4 - 4 - 6$	$1 - 2 - 6$	$0 + 0 - 6$	$1 + 2 - 6$	$4 + 4 - 6$	$9 + 6 - 6$	$16 + 8 - 6$
y	2	-3	-6	-7	-6	-3	2	9	18



Equation of line of symmetry is $x = -1$.

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Quadratic Equations
Exercise A, Question 9

Question:

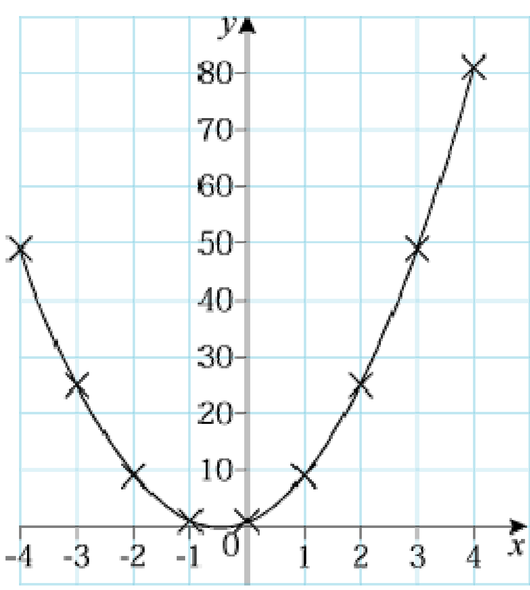
Draw a graph with the following equation, taking values of x from -4 to $+4$. For each graph write down the equation of the line of symmetry.

$y = (2x + 1)^2$

Solution:

$y = (2x + 1)^2$

x	-4	-33	-2	-1	0	1	2	3	4
$2x + 1$	$-8+1$	$-6+1$	$-4+1$	$-2+1$	$0+1$	$2+1$	$4+1$	$6+1$	$8+1$
$(2x + 1)$	-7	-5	-3	-1	1	3	5	7	9
$y = (2x + 1)^2$	49	25	9	1	1	9	25	49	81



Equation of line of symmetry is $x = -\frac{1}{2}$.

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Quadratic Equations
Exercise B, Question 1

Question:

Solve the following equation:

$x^2 = 4x$

Solution:

$x^2 - 4x = 0$
$x (x - 4) = 0$
$x = 0$ or $x - 4 = 0$
So $x = 0$ or $x = 4$

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Quadratic Equations
Exercise B, Question 2

Question:

Solve the following equation:

$x^2 = 25x$

Solution:

$x^2 - 25x = 0$
$x (x - 25) = 0$
$x = 0$ or $x - 25 = 0$
So $x = 0$ or $x = 25$

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Quadratic Equations
Exercise B, Question 3

Question:

Solve the following equation:

$3x^2 = 6x$

Solution:

$3x^2 - 6x = 0$
$3x (x - 2) = 0$
$x = 0$ or $x - 2 = 0$
So $x = 0$ or $x = 2$

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Quadratic Equations
Exercise B, Question 4

Question:

Solve the following equation:

$5x^2 = 30x$

Solution:

$5x^2 - 30x = 0$
$5x (x - 6) = 0$
$x = 0$ or $x - 6 = 0$
So $x = 0$ or $x = 6$

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Quadratic Equations
Exercise B, Question 5

Question:

Solve the following equation:

$x^2 + 3x + 2 = 0$

Solution:

$(x + 1) (x + 2) = 0$
$x + 1 = 0$ or $x + 2 = 0$
So $x = -1$ or $x = -2$

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Quadratic Equations
Exercise B, Question 6

Question:

Solve the following equation:

$x^2 + 5x + 4 = 0$

Solution:

$(x + 1)(x + 4) = 0$
$x + 1 = 0$ or $x + 4 = 0$
So $x = -1$ or $x = -4$

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Quadratic Equations
Exercise B, Question 7

Question:

Solve the following equation:

$x^2 + 7x + 10 = 0$

Solution:

$(x + 2)(x + 5) = 0$
$x + 2 = 0$ or $x + 5 = 0$
$x = -2$ or $x = -5$

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Quadratic Equations
Exercise B, Question 8

Question:

Solve the following equation:

$x^2 - x - 6 = 0$

Solution:

$(x - 3) (x + 2) = 0$
$x - 3 = 0$ or $x + 2 = 0$
So $x = 3$ or $x = -2$

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Quadratic Equations
Exercise B, Question 9

Question:

Solve the following equation:

$x^2 - 8x + 15 = 0$

Solution:

$(x - 3)(x - 5) = 0$
$x - 3 = 0$ or $x - 5 = 0$
So $x = 3$ or $x = 5$

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Quadratic Equations
Exercise B, Question 10

Question:

Solve the following equation:

$x^2 - 9x + 20 = 0$

Solution:

$(x - 4)(x - 5) = 0$
$x - 4 = 0$ or $x - 5 = 0$
So $x = 4$ or $x = 5$

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Quadratic Equations
Exercise B, Question 11

Question:

Solve the following equation:

$x^2 - 5x - 6 = 0$

Solution:

$(x - 6)(x + 1) = 0$
$x - 6 = 0$ or $x + 1 = 0$
So $x = 6$ or $x = -1$

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Quadratic Equations

Exercise B, Question 12

Question:

Solve the following equation:

$x^2 - 4x - 12 = 0$

Solution:

$(x - 6)(x + 2) = 0$
$x - 6 = 0$ or $x + 2 = 0$
So $x = 6$ or $x = -2$

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Quadratic Equations

Exercise B, Question 13

Question:

Solve the following equation:

$2x^2 + 7x + 3 = 0$

Solution:

$(2x + 1)(x + 3) = 0$
$2x + 1 = 0$ or $x + 3 = 0$
$2x = -1$ or $x = -3$
So $x = -\frac{1}{2}$ or $x = -3$

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Quadratic Equations
Exercise B, Question 14

Question:

Solve the following equation:

$6x^2 - 7x - 3 = 0$

Solution:

$(3x + 1)(2x - 3) = 0$
$3x + 1 = 0$ or $2x - 3 = 0$
So $x = -\frac{1}{3}$ or $x = \frac{3}{2}$

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Quadratic Equations
Exercise B, Question 15

Question:

Solve the following equation:

$6x^2 - 5x - 6 = 0$

Solution:

$(3x + 2)(2x - 3) = 0$
$3x + 2 = 0$ or $2x - 3 = 0$
So $x = -\frac{2}{3}$ or $x = \frac{3}{2}$

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Quadratic Equations
Exercise B, Question 16

Question:

Solve the following equation:

$4x^2 - 16x + 15 = 0$

Solution:

$(2x - 3)(2x - 5) = 0$
$2x - 3 = 0$ or $2x - 5 = 0$
So $x = \frac{3}{2}$ or $x = \frac{5}{2}$

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Quadratic Equations
Exercise B, Question 17

Question:

Solve the following equation:

$3x^2 + 5x = 2$

Solution:

$3x^2 + 5x - 2 = 0$
$(3x - 1)(x + 2) = 0$
$3x - 1 = 0$ or $x + 2 = 0$
So $x = \frac{1}{3}$ or $x = -2$

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Quadratic Equations
Exercise B, Question 18

Question:

Solve the following equation:

$(2x - 3)^2 = 9$

Solution:

$2x - 3 = \pm 3$
$2x = \pm 3 + 3$
$x = \frac{\pm 3 + 3}{2}$

So $x = 3$ or $x = 0$

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Quadratic Equations
Exercise B, Question 19

Question:

Solve the following equation:

$(x - 7)^2 = 36$

Solution:

$x - 7 = \pm 6$
$x = \pm 6 + 7$
So $x = 1$ or $x = 13$

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Quadratic Equations
Exercise B, Question 20

Question:

Solve the following equation:

$2x^2 = 8$

Solution:

$x^2 = 4$
$x = \pm 2$
So $x = 2$ or $x = -2$

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Quadratic Equations
Exercise B, Question 21

Question:

Solve the following equation:

$3x^2 = 5$

Solution:

$x^2 = \frac{5}{3}$

$x = \pm\sqrt{\frac{5}{3}}$

So $x = \sqrt{\frac{5}{3}}$ or $x = -\sqrt{\frac{5}{3}}$

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Quadratic Equations
Exercise B, Question 22

Question:

Solve the following equation:

$(x - 3) ^2 = 13$

Solution:

$x - 3 = \pm \sqrt{13}$
$x = 3 \pm \sqrt{13}$
So $x = 3 + \sqrt{13}$ or $x = 3 - \sqrt{13}$

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Quadratic Equations
Exercise B, Question 23

Question:

Solve the following equation:

$$(3x-1)^2 = 11$$

Solution:

$$3x - 1 = \pm \sqrt{11}$$
$$3x = 1 \pm \sqrt{11}$$
$$x = \frac{1 \pm \sqrt{11}}{3}$$

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Quadratic Equations
Exercise B, Question 24

Question:

Solve the following equation:

$5x^2 - 10x^2 = -7 + x + x^2$

Solution:

$-6x^2 - x + 7 = 0$
$(1 - x)(7 + 6x) = 0$
$x = 1$ or $6x = -7$

So $x = 1$ or $x = -\frac{7}{6}$

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Quadratic Equations
Exercise B, Question 25

Question:

Solve the following equation:

$6x^2 - 7 = 11x$

Solution:

$6x^2 - 11x - 7 = 0$
$(3x - 7)(2x + 1) = 0$
$3x - 7 = 0$ or $2x + 1 = 0$
So $x = \frac{7}{3}$ or $x = -\frac{1}{2}$

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Quadratic Equations
Exercise B, Question 26

Question:

Solve the following equation:

$4x^2 + 17x = 6x - 2x^2$

Solution:

$6x^2 + 11x = 0$
$x(6x + 11) = 0$
$x = 0$ or $6x + 11 = 0$
So $x = 0$ or $x = -\frac{11}{6}$

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Quadratic Equations
Exercise C, Question 1

Question:

Complete the square for the expression:

$x^2 + 4x$

Solution:

$= (x + 2)^2 - 4$

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Quadratic Equations
Exercise C, Question 2

Question:

Complete the square for the expression:

$x^2 - 6x$

Solution:

$= (x - 3) ^2 - 9$

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Quadratic Equations
Exercise C, Question 3

Question:

Complete the square for the expression:

$x^2 - 16x$

Solution:

$= (x - 8)^2 - 64$

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Quadratic Equations
Exercise C, Question 4

Question:

Complete the square for the expression:

$x^2 + x$

Solution:

$$= \left(x + \frac{1}{2} \right)^2 - \frac{1}{4}$$

Solutionbank C1
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Quadratic Equations
Exercise C, Question 5

Question:

Complete the square for the expression:

$x^2 - 14x$

Solution:

$= (x - 7)^2 - 49$

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Quadratic Equations
Exercise C, Question 6

Question:

Complete the square for the expression:

$2x^2 + 16x$

Solution:

$= 2 (x^2 + 8x)$
$= 2 [(x + 4) ^2 - 16]$
$= 2 (x + 4) ^2 - 32$

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Quadratic Equations
Exercise C, Question 7

Question:

Complete the square for the expression:

$3x^2 - 24x$

Solution:

$= 3 (x^2 - 8x)$
$= 3 [(x - 4)^2 - 16]$
$= 3 (x - 4)^2 - 48$

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Quadratic Equations
Exercise C, Question 8

Question:

Complete the square for the expression:

$2x^2 - 4x$

Solution:

$= 2 (x^2 - 2x)$
$= 2 [(x - 1)^2 - 1]$
$= 2 (x - 1)^2 - 2$

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Quadratic Equations
Exercise C, Question 9

Question:

Complete the square for the expression:

$5x^2 + 20x$

Solution:

$= 5 (x^2 + 4x)$
$= 5 [(x + 2)^2 - 4]$
$= 5 (x + 2)^2 - 20$

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Quadratic Equations
Exercise C, Question 10

Question:

Complete the square for the expression:

$2x^2 - 5x$

Solution:

$$= 2\left(x^2 - \frac{5}{2}x\right)$$

$$= 2\left[\left(x - \frac{5}{4}\right)^2 - \frac{25}{16}\right]$$

$$= 2\left(x - \frac{5}{4}\right)^2 - \frac{25}{8}$$

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Quadratic Equations
Exercise C, Question 11

Question:

Complete the square for the expression:

$3x^2 + 9x$

Solution:

$= 3 \left(x^2 + 3x \right)$

$= 3 \left[\left(x + \frac{3}{2} \right)^2 - \frac{9}{4} \right]$

$= 3 \left(x + \frac{3}{2} \right)^2 - \frac{27}{4}$

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Quadratic Equations
Exercise C, Question 12

Question:

Complete the square for the expression:

$3x^2 - x$

Solution:

$$= 3\left(x^2 - \frac{1}{3}x\right)$$

$$= 3\left[\left(x - \frac{1}{6}\right)^2 - \frac{1}{36}\right]$$

$$= 3\left(x - \frac{1}{6}\right)^2 - \frac{3}{36}$$

$$= 3\left(x - \frac{1}{6}\right)^2 - \frac{1}{12}$$

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Quadratic Equations

Exercise D, Question 1

Question:

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

$x^2 + 6x + 1 = 0$

Solution:

$x^2 + 6x = -1$
$(x + 3)^2 - 9 = -1$
$(x + 3)^2 = -1 + 9$
$(x + 3)^2 = 8$
$x + 3 = \pm \sqrt{8}$
$x = -3 \pm \sqrt{8}$
$x = -3 \pm \sqrt{2}\sqrt{4}$
$x = -3 \pm 2\sqrt{2}$
So $x = -3 + 2\sqrt{2}$ or $x = -3 - 2\sqrt{2}$

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Quadratic Equations
Exercise D, Question 2

Question:

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

$x^2 + 12x + 3 = 0$

Solution:

$x^2 + 12x = -3$
$(x + 6)^2 - 36 = -3$
$(x + 6)^2 = 33$
$x + 6 = \pm\sqrt{33}$
$x = -6 \pm \sqrt{33}$
So $x = -6 + \sqrt{33}$ or $x = -6 - \sqrt{33}$

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Quadratic Equations
Exercise D, Question 3

Question:

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

$x^2 - 10x = 5$

Solution:

$(x - 5)^2 - 25 = 5$
$(x - 5)^2 = 5 + 25$
$(x - 5)^2 = 30$
$x - 5 = \pm \sqrt{30}$
$x = 5 \pm \sqrt{30}$
So $x = 5 + \sqrt{30}$ or $x = 5 - \sqrt{30}$

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Quadratic Equations
Exercise D, Question 4

Question:

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

$x^2 + 4x - 2 = 0$

Solution:

$x^2 + 4x = 2$
$(x + 2)^2 - 4 = 2$
$(x + 2)^2 = 6$
$x + 2 = \pm \sqrt{6}$
So $x = -2 + \sqrt{6}$ or $x = -2 - \sqrt{6}$

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Quadratic Equations
Exercise D, Question 5

Question:

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

$x^2 - 3x - 5 = 0$

Solution:

$x^2 - 3x = 5$

$\left(x - \frac{3}{2} \right)^2 - \frac{9}{4} = 5$

$\left(x - \frac{3}{2} \right)^2 = 5 + \frac{9}{4}$

$\left(x - \frac{3}{2} \right)^2 = \frac{29}{4}$

$x - \frac{3}{2} = \pm \frac{\sqrt{29}}{2}$

$x = \frac{3}{2} \pm \frac{\sqrt{29}}{2}$

So $x = \frac{3 + \sqrt{29}}{2}$ or $x = \frac{3 - \sqrt{29}}{2}$

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Quadratic Equations

Exercise D, Question 6

Question:

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

$2x^2 - 7 = 4x$

Solution:

$2x^2 - 4x = 7$

$x^2 - 2x = \frac{7}{2}$

$(x - 1)^2 - 1 = \frac{7}{2}$

$(x - 1)^2 = \frac{9}{2}$

$x - 1 = \pm \frac{3}{\sqrt{2}}$

$x = 1 \pm \frac{3}{\sqrt{2}}$

$x = 1 \pm \frac{3\sqrt{2}}{2}$

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Quadratic Equations
Exercise D, Question 7

Question:

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

$4x^2 - x = 8$

Solution:

$x^2 - \frac{1}{4}x = 2$

$\left(x - \frac{1}{8} \right)^2 - \frac{1}{64} = 2$

$\left(x - \frac{1}{8} \right)^2 = 2 + \frac{1}{64}$

$\left(x - \frac{1}{8} \right)^2 = \frac{129}{64}$

$x - \frac{1}{8} = \pm \frac{\sqrt{129}}{8}$

$x = \frac{1}{8} \pm \frac{\sqrt{129}}{8}$

So $x = \frac{1 + \sqrt{129}}{8}$ or $x = \frac{1 - \sqrt{129}}{8}$

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Quadratic Equations
Exercise D, Question 8

Question:

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

$10 = 3x - x^2$

Solution:

$x^2 - 3x = -10$

$\left(x - \frac{3}{2} \right)^2 - \frac{9}{4} = -10$

$\left(x - \frac{3}{2} \right)^2 = -\frac{31}{4}$

No real roots as RHS is negative.

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Quadratic Equations
Exercise D, Question 9

Question:

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

$15 - 6x - 2x^2 = 0$

Solution:

$2x^2 + 6x = 15$

$x^2 + 3x = \frac{15}{2}$

$\left(x + \frac{3}{2} \right)^2 - \frac{9}{4} = \frac{15}{2}$

$\left(x + \frac{3}{2} \right)^2 = \frac{39}{4}$

$x + \frac{3}{2} = \pm \frac{\sqrt{39}}{2}$

$x = - \frac{3}{2} \pm \frac{\sqrt{39}}{2}$

So $x = - \frac{3}{2} + \frac{\sqrt{39}}{2}$ or $x = - \frac{3}{2} - \frac{\sqrt{39}}{2}$

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Quadratic Equations
Exercise D, Question 10

Question:

Solve the quadratic equation by completing the square (remember to leave your answer in surd form):

$5x^2 + 8x - 2 = 0$

Solution:

$x^2 + \frac{8}{5}x = \frac{2}{5}$

$\left(x + \frac{4}{5} \right)^2 - \frac{16}{25} = \frac{2}{5}$

$\left(x + \frac{4}{5} \right)^2 = \frac{26}{25}$

$x + \frac{4}{5} = \pm \frac{\sqrt{26}}{5}$

$x = - \frac{4}{5} \pm \frac{\sqrt{26}}{5}$

So $x = \frac{-4 + \sqrt{26}}{5}$ or $x = \frac{-4 - \sqrt{26}}{5}$

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Quadratic Equations
Exercise E, Question 1

Question:

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

$x^2 + 3x + 1 = 0$

Solution:

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(1)}}{2 \times 1}$$

$$x = \frac{-3 \pm \sqrt{9 - 4}}{2}$$

$$x = \frac{-3 \pm \sqrt{5}}{2}$$

Then $x = \frac{-3 + \sqrt{5}}{2}$ or $x = \frac{-3 - \sqrt{5}}{2}$

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Quadratic Equations
Exercise E, Question 2

Question:

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

$x^2 - 3x - 2 = 0$

Solution:

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-2)}}{2 \times 1}$$

$$x = \frac{+3 \pm \sqrt{9 + 8}}{2}$$

$$x = \frac{3 \pm \sqrt{17}}{2}$$

Then $x = \frac{3 + \sqrt{17}}{2}$ or $x = \frac{3 - \sqrt{17}}{2}$

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Quadratic Equations
Exercise E, Question 3

Question:

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

$x^2 + 6x + 6 = 0$

Solution:

$x = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(6)}}{2 \times 1}$

$x = \frac{-6 \pm \sqrt{36 - 24}}{2}$

$x = \frac{-6 \pm \sqrt{12}}{2}$

$x = \frac{-6 \pm 2\sqrt{3}}{2}$

$x = -3 \pm \sqrt{3}$

Then $x = -3 + \sqrt{3}$ or $x = -3 - \sqrt{3}$

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Quadratic Equations
Exercise E, Question 4

Question:

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

$x^2 - 5x - 2 = 0$

Solution:

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-2)}}{2 \times 1}$$

$$x = \frac{+5 \pm \sqrt{25 + 8}}{2}$$

$$x = \frac{5 \pm \sqrt{33}}{2}$$

Then $x = \frac{5 + \sqrt{33}}{2}$ or $x = \frac{5 - \sqrt{33}}{2}$

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Quadratic Equations

Exercise E, Question 5

Question:

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

$3x^2 + 10x - 2 = 0$

Solution:

$$x = \frac{-10 \pm \sqrt{10^2 - 4(3)(-2)}}{2 \times 3}$$

$$x = \frac{-10 \pm \sqrt{100 + 24}}{6}$$

$$x = \frac{-10 \pm \sqrt{124}}{6}$$

$$x = \frac{-10 \pm 2\sqrt{31}}{6}$$

Then $x = \frac{-5 + \sqrt{31}}{3}$ or $x = \frac{-5 - \sqrt{31}}{3}$

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Quadratic Equations
Exercise E, Question 6

Question:

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

$4x^2 - 4x - 1 = 0$

Solution:

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(-1)}}{2 \times 4}$$

$$x = \frac{+4 \pm \sqrt{16 + 16}}{8}$$

$$x = \frac{4 \pm \sqrt{32}}{8}$$

$$x = \frac{4 \pm 4\sqrt{2}}{8}$$

Then $x = \frac{1 + \sqrt{2}}{2}$ or $x = \frac{1 - \sqrt{2}}{2}$

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Quadratic Equations
Exercise E, Question 7

Question:

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

$7x^2 + 9x + 1 = 0$

Solution:

$$x = \frac{-9 \pm \sqrt{9^2 - 4(7)(1)}}{2 \times 7}$$

$$x = \frac{-9 \pm \sqrt{81 - 28}}{14}$$

$$x = \frac{-9 \pm \sqrt{53}}{14}$$

Then $x = \frac{-9 + \sqrt{53}}{14}$ or $x = \frac{-9 - \sqrt{53}}{14}$

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Quadratic Equations
Exercise E, Question 8

Question:

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

$5x^2 + 4x - 3 = 0$

Solution:

$$x = \frac{-4 \pm \sqrt{4^2 - 4(5)(-3)}}{2 \times 5}$$

$$x = \frac{-4 \pm \sqrt{16 + 60}}{10}$$

$$x = \frac{-4 \pm \sqrt{76}}{10}$$

$$x = \frac{-4 \pm 2\sqrt{19}}{10}$$

Then $x = \frac{-2 + \sqrt{19}}{5}$ or $x = \frac{-2 - \sqrt{19}}{5}$

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Quadratic Equations
Exercise E, Question 9

Question:

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

$4x^2 - 7x = 2$

Solution:

$4x^2 - 7x - 2 = 0$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(4)(-2)}}{2 \times 4}$$

$$x = \frac{+7 \pm \sqrt{49 + 32}}{8}$$

$$x = \frac{7 \pm \sqrt{81}}{8}$$

$$x = \frac{7 \pm 9}{8}$$

Then $x = 2$ or $x = -\frac{1}{4}$

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Quadratic Equations
Exercise E, Question 10

Question:

Solve the following quadratic equation by using the formula, giving the solution in surd form. Simplify your answer:

$11x^2 + 2x - 7 = 0$

Solution:

$$x = \frac{-2 \pm \sqrt{2^2 - 4(11)(-7)}}{2 \times 11}$$

$$x = \frac{-2 \pm \sqrt{4 + 308}}{22}$$

$$x = \frac{-2 \pm \sqrt{312}}{22}$$

$$x = \frac{-2 \pm 2\sqrt{78}}{22}$$

$$x = \frac{-1 \pm \sqrt{78}}{11}$$

Then $x = \frac{-1 + \sqrt{78}}{11}$ or $x = \frac{-1 - \sqrt{78}}{11}$

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Quadratic Equations
Exercise F, Question 1

Question:

Sketch the graphs of the following equations:

(a) $y = x^2 + 3x + 2$

(b) $y = x^2 - 3x + 10$

(c) $y = x^2 + 2x - 15$

(d) $y = 2x^2 + 7x + 3$

(e) $y = 2x^2 + x - 3$

(f) $y = 6x^2 - 19x + 10$

(g) $y = 3x^2 - 2x - 5$

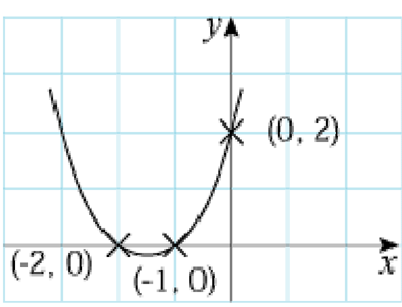
(h) $y = 3x^2 - 13x$

(i) $y = -x^2 + 6x + 7$

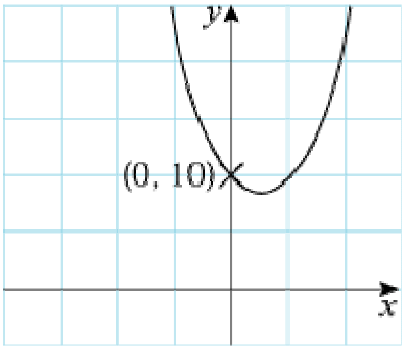
(j) $y = 4 - 7x - 2x^2$

Solution:

(a) $a > 0$ so graph is a $\cup$ shape.
$b^2 = 9$, $4ac = 8$
$b^2 > 4ac$, so there are two different roots of the equation $y = 0$.
When $y = 0$,
$(x + 2) (x + 1) = 0$
$x = -2$ or $x = -1$
So crossing points are $(-2 , 0)$ and $(-1 , 0)$.
When $x = 0$, $y = 2$, so $(0 , 2)$ is a crossing point.



(b) $a > 0$ so graph is a $\cup$ shape.
$b^2 = 9$, $4ac = 40$
$b^2 < 4ac$, so there are no real roots of the equation $y = 0$.
So there are no crossing points at $y = 0$.
When $x = 0$, $y = 10$, so crossing point is $(0 , 10)$.



(c) $a > 0$ so graph is a $\cup$ shape.

$b^2 = 4$, $4ac = -60$

$b^2 > 4ac$, so two different roots of $y = 0$.

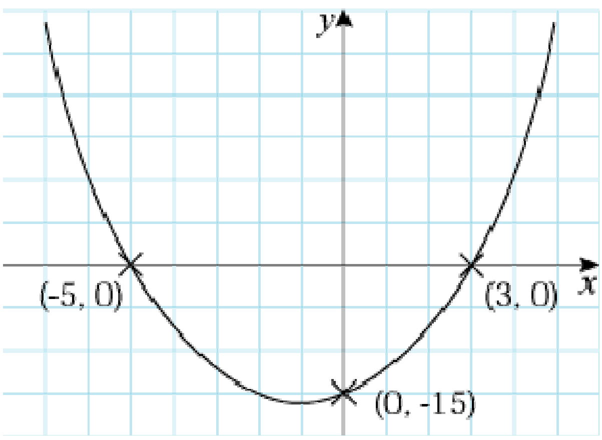
When $y = 0$,

$0 = (x + 5)(x - 3)$

$x = -5$ or $x = 3$

So crossing points are $(-5, 0)$ and $(3, 0)$.

When $x = 0$, $y = -15$, so crossing point is $(0, -15)$.



(d) $a > 0$ so graph is a $\cup$ shape.

$b^2 = 49$, $4ac = 24$

$b^2 > 4ac$, so two different roots of $y = 0$.

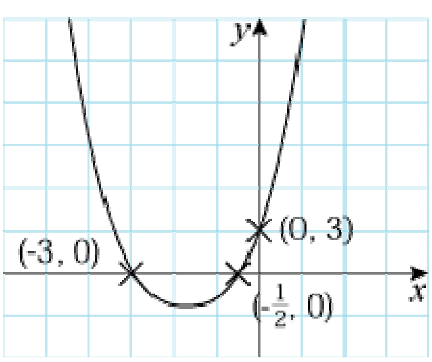
When $y = 0$,

$0 = (2x + 1)(x + 3)$

$x = -\frac{1}{2}$ or $x = -3$

So crossing points are $\left(-\frac{1}{2}, 0\right)$ and $(-3, 0)$.

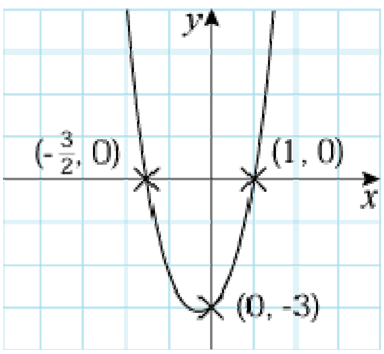
When $x = 0$, $y = 3$, so crossing point is $(0, 3)$.



(e) $a > 0$ so graph is a $\cup$ shape.
$b^2 = 1$, $4ac = -24$
$b^2 > 4ac$, so two different roots of $y = 0$.
When $y = 0$,
$0 = (2x + 3)(x - 1)$
$x = -\frac{3}{2}$ or $x = 1$

So crossing points are $\left(-\frac{3}{2}, 0\right)$ and $(1, 0)$.

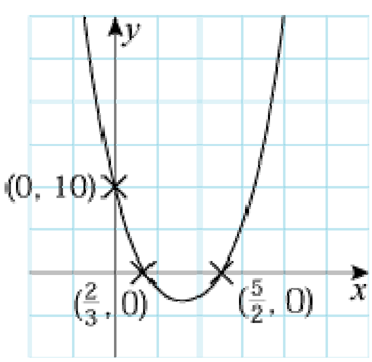
When $x = 0$, $y = -3$, so crossing point is $(0, -3)$.



(f) $a > 0$ so graph is a $\cup$ shape.
$b^2 = 361$, $4ac = 240$
$b^2 > 4ac$, so two different roots of $y = 0$.
When $y = 0$,
$0 = (3x - 2)(2x - 5)$
$x = \frac{2}{3}$ or $x = \frac{5}{2}$

So crossing points are $\left(\frac{2}{3}, 0\right)$ and $\left(\frac{5}{2}, 0\right)$.

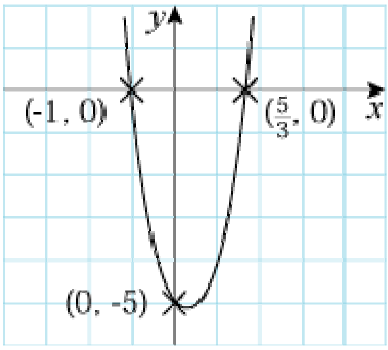
When $x = 0$, $y = 10$, so crossing point is $(0, 10)$.



(g) $a >$ so graph is a $\cup$ shape.
$b^2 = 4$, $4ac = -60$
$b^2 > 4ac$, so two different roots of $y = 0$.
When $y = 0$,
$0 = (3x - 5)(x + 1)$
$x = \frac{5}{3}$ or $x = -1$

So crossing points are $\left(\frac{5}{3} , 0 \right)$ and $(- 1 , 0)$.

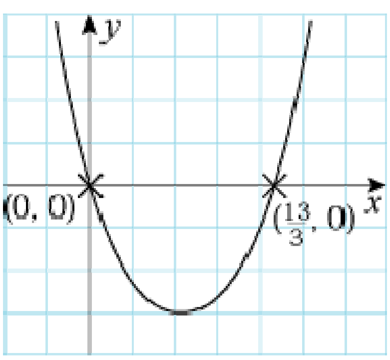
When $x = 0$, $y = - 5$, so crossing point is $(0 , - 5)$.



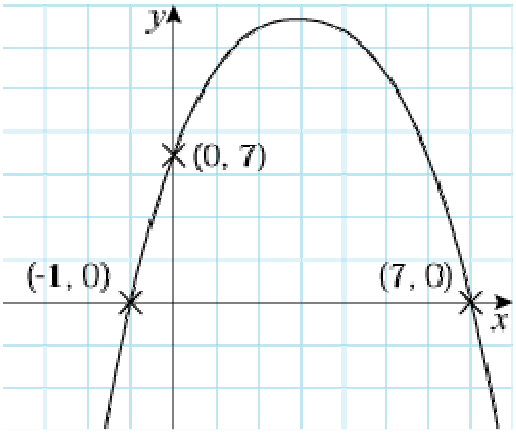
(h) $a > 0$ so graph is a $\cup$ shape.
$b^2 = 169$, $4ac = 0$
$b^2 > 4ac$, so two different roots of $y = 0$.
When $y = 0$,
$0 = x (3x - 13)$
$x = 0$ or $x = \frac{13}{3}$

So crossing points are $(0 , 0)$ and $\left(\frac{13}{3} , 0 \right)$.

When $x = 0$, $y = 0$, so crossing point is $(0 , 0)$.



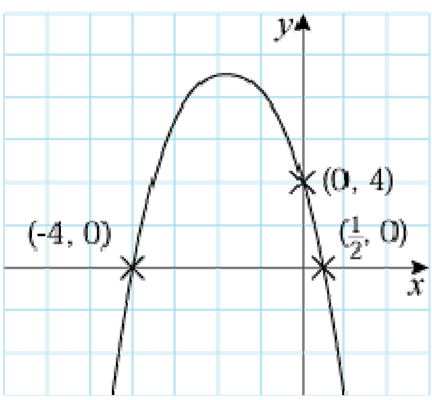
(i) $a < 0$ so graph is a $\cap$ shape.
$b^2 = 36$, $4ac = - 28$
$b^2 > 4ac$, so two different roots of $y = 0$.
When $y = 0$,
$0 = (7 - x) (1 + x)$
$x = 7$ or $x = - 1$
So crossing points are $(7 , 0)$ and $(- 1 , 0)$.
When $x = 0$, $y = 7$, so crossing point is $(0 , 7)$.



(j) $a < 0$ so graph is a $\cap$ shape.
$b^2 = 49$, $4ac = -32$
$b^2 > 4ac$, so two different roots of $y = 0$.
When $y = 0$,
$0 = (1 - 2x)(4 + x)$
$x = \frac{1}{2}$ or $x = -4$

So crossing points are $\left(\frac{1}{2}, 0\right)$ and $(-4, 0)$.

When $x = 0$, $y = 4$, so crossing point is $(0, 4)$.



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Quadratic Equations
Exercise F, Question 2

Question:

Find the values of k for which $x^2 + kx + 4 = 0$ has equal roots.

Solution:

$x^2 + kx + 4 = 0$ has equal roots if
$b^2 = 4ac$
i.e.
$k^2 = 4 \times 1 \times 4 = 16 \quad \Rightarrow \quad k = \pm 4$

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Quadratic Equations
Exercise F, Question 3

Question:

Find the values of k for which $kx^2 + 8x + k = 0$ has equal roots.

Solution:

$kx^2 + 8x + k = 0$ has equal roots if
$b^2 = 4ac$
i.e.
$8^2 = 4 \times k \times k = 4k^2$
So $k^2 = \frac{64}{4} = 16 \quad \Rightarrow \quad k = \pm 4$

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Quadratic Equations
Exercise G, Question 1

Question:

Draw the graphs with the following equations, choosing appropriate values for x. For each graph write down the equation of the line of symmetry.

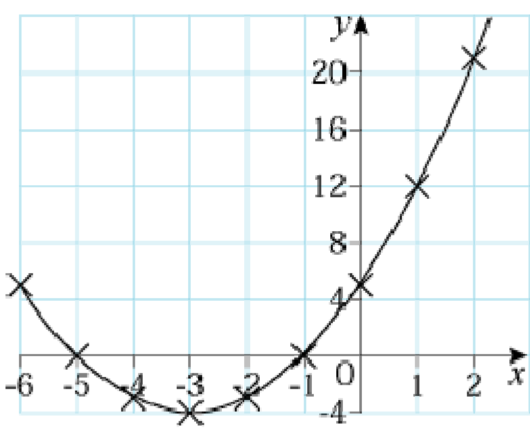
(a) $y = x^2 + 6x + 5$

(b) $y = 2x^2 - 3x - 4$

Solution:

(a)

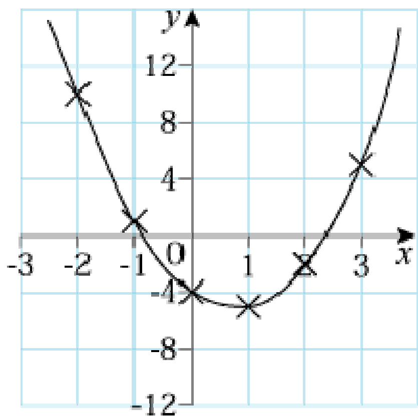
x	-6	-5	-4	-3	-2	-1	0	1	2
x^2	36	25	16	9	4	1	0	1	4
$+6x$	-36	-30	-24	-18	-12	-6	0	$+6$	$+12$
$+5$	$+5$	$+5$	$+5$	$+5$	$+5$	$+5$	$+5$	$+5$	$+5$
y	5	0		-3	-4	-3	0	5	12 21



$x = -3$ is line of symmetry.

(b)

x	-2	-1	0	1	2	3
$2x^2$	8	2	0	2	8	18
$-3x$	$+6$	$+3$	0	-3	-6	-9
-4	-4	-4	-4	-4	-4	-4
y	10	1		-4	-5	-2 5



$x = \frac{3}{4}$ is line of symmetry.

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Quadratic Equations
Exercise G, Question 2

Question:

Solve the following equations:

(a) $y^2 + 3y + 2 = 0$

(b) $3x^2 + 13x - 10 = 0$

(c) $5x^2 - 10x = 4x + 3$

(d) $(2x - 5)^2 = 7$

Solution:

(a) $(y + 1)(y + 2) = 0$
$y = -1$ or $y = -2$

(b) $(3x - 2)(x + 5) = 0$
$x = \frac{2}{3}$ or $x = -5$

(c) $5x^2 - 14x - 3 = 0$
$(5x + 1)(x - 3) = 0$
$x = -\frac{1}{5}$ or $x = 3$

(d) $2x - 5 = \pm\sqrt{7}$
$2x = \pm\sqrt{7} + 5$
$x = \frac{5 \pm \sqrt{7}}{2}$

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Quadratic Equations
Exercise G, Question 3

Question:

Solve the following equations by:

(i) Completing the square.

(ii) Using the formula.

(a) $x^2 + 5x + 2 = 0$

(b) $x^2 - 4x - 3 = 0$

(c) $5x^2 + 3x - 1 = 0$

(d) $3x^2 - 5x = 4$

Solution:

(a) (i) $x^2 + 5x = -2$

$$\left(x + \frac{5}{2} \right)^2 - \frac{25}{4} = -2$$

$$\left(x + \frac{5}{2} \right)^2 = \frac{17}{4}$$

$$x + \frac{5}{2} = \pm \frac{\sqrt{17}}{2}$$

$$x = \frac{-5 \pm \sqrt{17}}{2}$$

(ii) $x = \dfrac{-5 \pm \sqrt{5^2 - 4(1)(2)}}{2}$

$$x = \frac{-5 \pm \sqrt{25 - 8}}{2}$$

$$x = \frac{-5 \pm \sqrt{17}}{2}$$

(b)(i) $x^2 - 4x = 3$

$(x - 2)^2 - 4 = 3$

$(x - 2)^2 = 7$

$x - 2 = \pm \sqrt{7}$

$x = 2 \pm \sqrt{7}$

(ii) $x = \dfrac{-(-4) \pm \sqrt{16 - 4(1)(-3)}}{2}$

$$x = \frac{+4 \pm \sqrt{16 + 12}}{2}$$

$$x = \frac{4 \pm \sqrt{4 \times 7}}{2}$$

$x = \frac{4 \pm 2\sqrt{7}}{2}$

$x = 2 \pm \sqrt{7}$

(c) (i) $5x^2 + 3x = 1$

$5\left(x^2 + \frac{3}{5}x\right) = 1$

$5\left[\left(x + \frac{3}{10}\right)^2 - \frac{9}{100}\right] = 1$

$\left(x + \frac{3}{10}\right)^2 - \frac{9}{100} = \frac{1}{5}$

$\left(x + \frac{3}{10}\right)^2 = \frac{29}{100}$

$x + \frac{3}{10} = \pm \frac{\sqrt{29}}{10}$

$x = \frac{-3 \pm \sqrt{29}}{10}$

(ii) $x = \frac{-3 \pm \sqrt{9 - 4(5)(-1)}}{10}$

$x = \frac{-3 \pm \sqrt{29}}{10}$

(d)(i) $3\left(x^2 - \frac{5}{3}x\right) = 4$

$3\left[\left(x - \frac{5}{6}\right)^2 - \frac{25}{36}\right] = 4$

$\left(x - \frac{5}{6}\right)^2 - \frac{25}{36} = \frac{4}{3}$

$\left(x - \frac{5}{6}\right)^2 = \frac{73}{36}$

$x - \frac{5}{6} = \pm \frac{\sqrt{73}}{6}$

$x = \frac{5 \pm \sqrt{73}}{6}$

(ii) $x = \frac{-(-5) \pm \sqrt{25 - 4(3)(-4)}}{6}$

$x = \frac{+5 \pm \sqrt{25 + 48}}{6}$

$x = \frac{5 \pm \sqrt{73}}{6}$

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Quadratic Equations
Exercise G, Question 4

Question:

Sketch graphs of the following equations:

(a) $y = x^2 + 5x + 4$

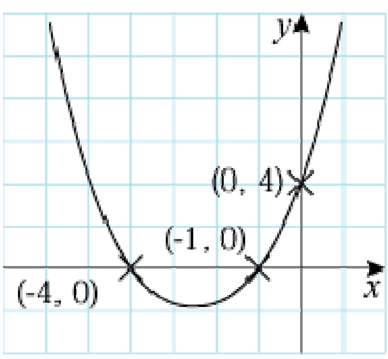
(b) $y = 2x^2 + x - 3$

(c) $y = 6 - 10x - 4x^2$

(d) $y = 15x - 2x^2$

Solution:

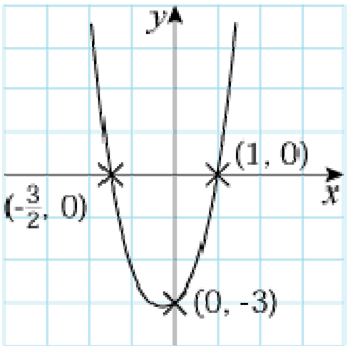
(a) $a > 0$ so $\cup$ shape
$b^2 = 25$, $4ac = 16$
$b^2 > 4ac$, so two different roots of $y = 0$.
$y = 0 \Rightarrow 0 = (x + 1) (x + 4)$
$x = -1$ or $x = -4$
So x-axis crossing points are $(-1 , 0)$ and $(-4 , 0)$.
$x = 0 \Rightarrow y = 4$ So y-axis crossing point is $(0 , 4)$.



(b) $a > 0$ So $\cup$ shape
$b^2 = 1$, $4ac = -24$
$b^2 > 4ac$, so two different roots of $y = 0$.
$y = 0 \Rightarrow 0 = (2x + 3) (x - 1)$
$x = -\frac{3}{2}$ or $x = 1$

So x-axis crossing points are $\left(-\frac{3}{2} , 0 \right)$ and $(1 , 0)$.

$x = 0 \Rightarrow y = -3$ so y-axis crossing point in $(0 , -3)$.

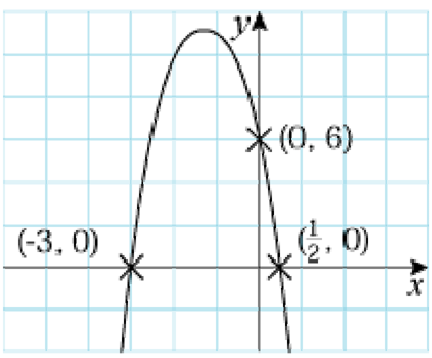


(c) $a < 0$ So $\cap$ shape
$b^2 = 100$, $4ac = -96$
$b^2 > 4ac$, so two different roots of $y = 0$.
$y = 0 \Rightarrow 0 = (1 - 2x)(6 + 2x)$

$x = \frac{1}{2}$ or $x = -3$

So x-axis crossing points are $\left(\frac{1}{2}, 0\right)$ and $(-3, 0)$.

$x = 0 \Rightarrow y = 6$ so y-axis crossing point is $(0, 6)$.

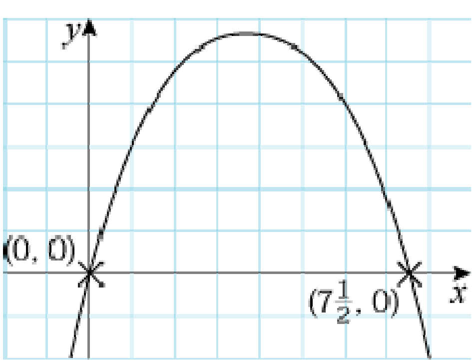


(d) $a < 0$ so $\cap$ shape
$b^2 = 225$, $4ac = 0$
$b^2 > 4ac$, so two different roots of $y = 0$.
$y = 0 \Rightarrow 0 = x(15 - 2x)$

$x = 0$ or $x = 7\frac{1}{2}$

So x-axis crossing points are $(0, 0)$ and $\left(7\frac{1}{2}, 0\right)$.

$x = 0 \Rightarrow y = 0$ So y-axis crossing point is $(0, 0)$.



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Quadratic Equations
Exercise G, Question 5

Question:

Given that for all values of x :

$3x^2 + 12x + 5 = p(x + q)^2 + r$

(a) Find the values of p, q and r.

(b) Solve the equation $3x^2 + 12x + 5 = 0$. **[E]**

Solution:

(a) $3x^2 + 12x + 5 = p(x^2 + 2qx + q^2) + r$
$3x^2 + 12x + 5 = px^2 + 2pqx + pq^2 + r$
Comparing x^2 : $p = 3$ ①
Comparing x : $2pq = 12$ ②
Comparing constants : $pq^2 + r = 5$ ③
Substitute ① into ②:
$2 \times 3q = 12$
$q = 2$
Substitute $p = 3$ and $q = 2$ into ③:
$3 \times 2^2 + r = 5$
$12 + r = 5$
$r = -7$
So $p = 3$, $q = 2$, $r = -7$

(b) $3x^2 + 12x + 5 = 0$
$\Rightarrow \quad 3(x + 2)^2 - 7 = 0$
$\Rightarrow \quad 3(x + 2)^2 = 7$
$\Rightarrow \quad (x + 2)^2 = \frac{7}{3}$
$\Rightarrow \quad x + 2 = \pm \sqrt{\frac{7}{3}}$
So $x = -2 \pm \sqrt{\frac{7}{3}}$

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Quadratic Equations
Exercise G, Question 6

Question:

Find, as surds, the roots of the equation

$2(x+1)(x-4)-(x-2)^2=0$

Solution:

$2(x^2-3x-4)-(x^2-4x+4)=0$

$2x^2-6x-8-x^2+4x-4=0$

$x^2-2x-12=0$

$x=\dfrac{-(-2)\pm\sqrt{4-4(1)(-12)}}{2}$

$x=\dfrac{+2\pm\sqrt{52}}{2}$

$x=\dfrac{2\pm\sqrt{4\times 13}}{2}$

$x=\dfrac{2\pm 2\sqrt{13}}{2}$

$x=1\pm\sqrt{13}$

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Quadratic Equations
Exercise G, Question 7

Question:

Use algebra to solve $(x-1)(x+2)=18$. **[E]**

Solution:

$x^2+x-2=18$
$x^2+x-20=0$
$(x+5)(x-4)=0$
$x=-5$ or $x=4$

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Equations and inequalities
Exercise A, Question 1

Question:

Solve these simultaneous equations by elimination:

$2x - y = 6$
$4x + 3y = 22$

Solution:

$6x - 3y = 18$
$4x + 3y = 22$
Add:
$10x = 40$
$x = 4$
Substitute into $2x - y = 6$:
$8 - y = 6$
$y = 2$
So solution is $x = 4$, $y = 2$

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Equations and inequalities
Exercise A, Question 2

Question:

Solve these simultaneous equations by elimination:

$7x + 3y = 16$
$2x + 9y = 29$

Solution:

$21x + 9y = 48$
$2x + 9y = 29$
Subtract:
$19x = 19$
$x = 1$
Substitute into $7x + 3y = 16$:
$7 + 3y = 16$
$3y = 9$
$y = 3$
So solution is $x = 1$, $y = 3$

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Equations and inequalities

Exercise A, Question 3

Question:

Solve these simultaneous equations by elimination:

$5x + 2y = 6$
$3x - 10y = 26$

Solution:

$25x + 10y = 30$
$3x - 10y = 26$
Add:
$28x = 56$
$x = 2$
Substitute into $5x + 2y = 6$:
$10 + 2y = 6$
$2y = -4$
$y = -2$
So solution is $x = 2$, $y = -2$

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Equations and inequalities
Exercise A, Question 4

Question:

Solve these simultaneous equations by elimination:

$2x - y = 12$
$6x + 2y = 21$

Solution:

$4x - 2y = 24$
$6x + 2y = 21$
Add:
$10x = 45$
$x = 4\frac{1}{2}$

Substitute into $2x - y = 12$:
$9 - y = 12$
$-y = 3$
$y = -3$

So solution is $x = 4\frac{1}{2}$, $y = -3$

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Equations and inequalities
Exercise A, Question 5

Question:

Solve these simultaneous equations by elimination:

$3x - 2y = -6$
$6x + 3y = 2$

Solution:

$6x - 4y = -12$
$6x + 3y = 2$
Subtract:
$-7y = -14$
$y = 2$
Substitute into $3x - 2y = -6$:
$3x - 4 = -6$
$3x = -2$
$x = -\frac{2}{3}$

So solution is $x = -\frac{2}{3}$, $y = 2$

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Equations and inequalities
Exercise A, Question 6

Question:

Solve these simultaneous equations by elimination:

$3x + 8y = 33$
$6x = 3 + 5y$

Solution:

$6x + 16y = 66$
$6x = 3 + 5y$
$6x + 16y = 66$
$6x - 5y = 3$
Subtract:
$21y = 63$
$y = 3$
Substitute into $3x + 8y = 33$:
$3x + 24 = 33$
$3x = 9$
$x = 3$
So solution is $x = 3$, $y = 3$

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Equations and inequalities
Exercise B, Question 1

Question:

Solve these simultaneous equations by substitution:

$x + 3y = 11$
$4x - 7y = 6$

Solution:

$x = 11 - 3y$
Substitute into $4x - 7y = 6$:
$4 (11 - 3y) - 7y = 6$
$44 - 12y - 7y = 6$
$-19y = -38$
$y = 2$
Substitute into $x = 11 - 3y$:
$x = 11 - 6$
$x = 5$
So solution is $x = 5$, $y = 2$

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Equations and inequalities
Exercise B, Question 2

Question:

Solve these simultaneous equations by substitution:

$4x - 3y = 40$
$2x + y = 5$

Solution:

$y = 5 - 2x$
Substitute into $4x - 3y = 40$:
$4x - 3(5 - 2x) = 40$
$4x - 15 + 6x = 40$
$10x = 55$
$x = 5\frac{1}{2}$

Substitute into $y = 5 - 2x$:
$y = 5 - 11$
$y = -6$

So solution is $x = 5\frac{1}{2}$, $y = -6$

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Equations and inequalities
Exercise B, Question 3

Question:

Solve these simultaneous equations by substitution:

$3x - y = 7$
$10x + 3y = -2$

Solution:

$-y = 7 - 3x$
$y = 3x - 7$
Substitute into $10x + 3y = -2$:
$10x + 3 (3x - 7) = -2$
$10x + 9x - 21 = -2$
$19x = 19$
$x = 1$
Substitute into $y = 3x - 7$:
$y = 3 - 7$
$y = -4$
So solution is $x = 1$, $y = -4$

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Equations and inequalities
Exercise B, Question 4

Question:

Solve these simultaneous equations by substitution:

$2y = 2x - 3$
$3y = x - 1$

Solution:

$x = 3y + 1$
Substitute into $2y = 2x - 3$:
$2y = 2 (3y + 1) - 3$
$2y = 6y + 2 - 3$
$-4y = -1$
$y = \frac{1}{4}$
Substitute into $x = 3y + 1$:
$x = \frac{3}{4} + 1$

$x = 1 \frac{3}{4}$

So solution is $x = 1 \frac{3}{4}, y = \frac{1}{4}$

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Equations and inequalities
Exercise C, Question 1

Question:

Solve the simultaneous equations:

(a) $x + y = 11$
$xy = 30$

(b) $2x + y = 1$
$x^2 + y^2 = 1$

(c) $y = 3x$
$2y^2 - xy = 15$

(d) $x + y = 9$
$x^2 - 3xy + 2y^2 = 0$

(e) $3a + b = 8$
$3a^2 + b^2 = 28$

(f) $2u + v = 7$
$uv = 6$

Solution:

(a) $y = 11 - x$
Substitute into $xy = 30$:
$x(11 - x) = 30$
$11x - x^2 = 30$
$0 = x^2 - 11x + 30$
$0 = (x - 5)(x - 6)$
$x = 5$ or $x = 6$
Substitute into $y = 11 - x$:
when $x = 5$, $y = 11 - 5 = 6$
when $x = 6$, $y = 11 - 6 = 5$
Solutions are $x = 5$, $y = 6$ and $x = 6$, $y = 5$

(b) $y = 1 - 2x$
Substitute into $x^2 + y^2 = 1$:
$x^2 + (1 - 2x)^2 = 1$
$x^2 + 1 - 4x + 4x^2 = 1$
$5x^2 - 4x = 0$
$x(5x - 4) = 0$
$x = 0$ or $x = \frac{4}{5}$
Substitute into $y = 1 - 2x$:
when $x = 0$, $y = 1$
when $x = \frac{4}{5}$, $y = 1 - \frac{8}{5} = -\frac{3}{5}$
Solutions are $x = 0$, $y = 1$ and $x = \frac{4}{5}$, $y = -\frac{3}{5}$

(c) $y = 3x$
Substitute into $2y^2 - xy = 15$:

$2(3x)^2 - x(3x) = 15$
$18x^2 - 3x^2 = 15$
$15x^2 = 15$
$x^2 = 1$
$x = -1$ or $x = 1$
Substitute into $y = 3x$:
when $x = -1$, $y = -3$
when $x = 1$, $y = 3$
Solutions are $x = -1$, $y = -3$ and $x = 1$, $y = 3$

(d) $x = 9 - y$
Substitute into $x^2 - 3xy + 2y^2 = 0$:
$(9 - y)^2 - 3y(9 - y) + 2y^2 = 0$
$81 - 18y + y^2 - 27y + 3y^2 + 2y^2 = 0$
$6y^2 - 45y + 81 = 0$
Divide by 3:
$2y^2 - 15y + 27 = 0$
$(2y - 9)(y - 3) = 0$
$y = \frac{9}{2}$ or $y = 3$

Substitute into $x = 9 - y$:
when $y = \frac{9}{2}$, $x = 9 - \frac{9}{2} = \frac{9}{2}$

when $y = 3$, $x = 9 - 3 = 6$

Solutions are $x = 4\frac{1}{2}$, $y = 4\frac{1}{2}$ and $x = 6$, $y = 3$

(e) $b = 8 - 3a$
Substitute into $3a^2 + b^2 = 28$:
$3a^2 + (8 - 3a)^2 = 28$
$3a^2 + 64 - 48a + 9a^2 = 28$
$12a^2 - 48a + 36 = 0$
Divide by 12:
$a^2 - 4a + 3 = 0$
$(a - 1)(a - 3) = 0$
$a = 1$ or $a = 3$
Substitute into $b = 8 - 3a$:
when $a = 1$, $b = 8 - 3 = 5$
when $a = 3$, $b = 8 - 9 = -1$
Solutions are $a = 1$, $b = 5$ and $a = 3$, $b = -1$

(f) $v = 7 - 2u$
Substitute into $uv = 6$:
$u(7 - 2u) = 6$
$7u - 2u^2 = 6$
$0 = 2u^2 - 7u + 6$
$0 = (2u - 3)(u - 2)$
$u = \frac{3}{2}$ or $u = 2$

Substitute into $v = 7 - 2u$:
when $u = \frac{3}{2}$, $v = 7 - 3 = 4$

when $u = 2$, $v = 7 - 4 = 3$

Solutions are $u = \frac{3}{2}$, $v = 4$ and $u = 2$, $v = 3$

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Equations and inequalities
Exercise C, Question 2

Question:

Find the coordinates of the points at which the line with equation $y = x - 4$ intersects the curve with equation $y^2 = 2x^2 - 17$.

Solution:

$y = x - 4$
Substitute into $y^2 = 2x^2 - 17$:
$(x - 4) ^2 = 2x^2 - 17$
$x^2 - 8x + 16 = 2x^2 - 17$
$0 = x^2 + 8x - 33$
$0 = (x + 11) (x - 3)$
$x = - 11$ or $x = 3$
Substitute into $y = x - 4$:
when $x = - 11$, $y = - 11 - 4 = - 15$
when $x = 3$, $y = 3 - 4 = - 1$
Intersection points: $(- 11 , - 15)$ and $(3 , - 1)$

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Equations and inequalities
Exercise C, Question 3

Question:

Find the coordinates of the points at which the line with equation $y = 3x - 1$ intersects the curve with equation $y^2 - xy = 15$.

Solution:

$y = 3x - 1$
Substitute into $y^2 - xy = 15$:
$(3x - 1)^2 - x(3x - 1) = 15$
$9x^2 - 6x + 1 - 3x^2 + x = 15$
$6x^2 - 5x - 14 = 0$
$(6x + 7)(x - 2) = 0$
$x = -\frac{7}{6}$ or $x = 2$

Substitute into $y = 3x - 1$:
when $x = -\frac{7}{6}$, $y = -\frac{21}{6} - 1 = -\frac{9}{2}$
when $x = 2$, $y = 6 - 1 = 5$
Intersection points: $\left(-1\frac{1}{6}, -4\frac{1}{2}\right)$ and $(2, 5)$

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Equations and inequalities
Exercise C, Question 4

Question:

Solve the simultaneous equations:

(a) $3x + 2y = 7$
$x^2 + y = 8$

(b) $2x + 2y = 7$
$x^2 - 4y^2 = 8$

Solution:

(a) $2y = 7 - 3x$

$y = \frac{1}{2}(7 - 3x)$

Substitute into $x^2 + y = 8$:

$x^2 + \frac{1}{2}(7 - 3x) = 8$

Multiply by 2:
$2x^2 + (7 - 3x) = 16$
$2x^2 - 3x - 9 = 0$
$(2x + 3)(x - 3) = 0$

$x = -\frac{3}{2}$ or $x = 3$

Substitute into $y = \frac{1}{2}\left(7 - 3x\right)$:

when $x = -\frac{3}{2}$, $y = \frac{1}{2}\left(7 + \frac{9}{2}\right) = \frac{23}{4}$

when $x = 3$, $y = \frac{1}{2}\left(7 - 9\right) = -1$

Solutions are $x = -1\frac{1}{2}$, $y = 5\frac{3}{4}$ and $x = 3$, $y = -1$

(b) $2x = 7 - 2y$

$x = \frac{1}{2}\left(7 - 2y\right)$

Substitute into $x^2 - 4y^2 = 8$:

$\left[\frac{1}{2}\left(7 - 2y\right)\right]^2 - 4y^2 = 8$

$\frac{1}{4}(7 - 2y)^2 - 4y^2 = 8$

Multiply by 4:
$(7 - 2y)^2 - 16y^2 = 32$
$49 - 28y + 4y^2 - 16y^2 = 32$
$0 = 12y^2 + 28y - 17$
$0 = (6y + 17)(2y - 1)$

$y = -\frac{17}{6}$ or $y = \frac{1}{2}$

Substitute into $x = \frac{1}{2}\left(7 - 2y\right)$:

when $y = -\frac{17}{6}$, $x = \frac{1}{2}\left(7 + \frac{17}{3}\right) = \frac{19}{3}$

when $y = \frac{1}{2}$, $x = \frac{1}{2}\left(7 - 1\right) = 3$

Solutions are $x = 6\frac{1}{3}$, $y = -2\frac{5}{6}$ and $x = 3$, $y = \frac{1}{2}$

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Equations and inequalities
Exercise C, Question 5

Question:

Solve the simultaneous equations, giving your answers in their simplest surd form:

(a) $x - y = 6$
$xy = 4$

(b) $2x + 3y = 13$
$x^2 + y^2 = 78$

Solution:

(a) $x = 6 + y$
Substitute into $xy = 4$:
$y(6 + y) = 4$
$6y + y^2 = 4$
$y^2 + 6y - 4 = 0$
$a = 1, b = 6, c = -4$

$$y = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a} = \frac{-6 \pm \sqrt{(36 + 16)}}{2} = \frac{-6 \pm \sqrt{52}}{2}$$

$$\sqrt{52} = \sqrt{(4 \times 13)} = \sqrt{4}\sqrt{13} = 2\sqrt{13}$$

$$y = \frac{-6 \pm 2\sqrt{13}}{2} = -3 \pm \sqrt{13}$$

Substitute into $x = 6 + y$:
when $y = -3 - \sqrt{13}$, $x = 6 - 3 - \sqrt{13} = 3 - \sqrt{13}$
when $y = -3 + \sqrt{13}$, $x = 6 - 3 + \sqrt{13} = 3 + \sqrt{13}$
Solutions are $x = 3 - \sqrt{13}$, $y = -3 - \sqrt{13}$ and $x = 3 + \sqrt{13}$, $y = -3 + \sqrt{13}$

(b) $2x = 13 - 3y$

$$x = \frac{1}{2}\left(13 - 3y\right)$$

Substitute into $x^2 + y^2 = 78$:

$$\left[\frac{1}{2}\left(13 - 3y\right)\right]^2 + y^2 = 78$$

$$\frac{1}{4}(13 - 3y)^2 + y^2 = 78$$

Multiply by 4:
$(13 - 3y)^2 + 4y^2 = 312$
$169 - 78y + 9y^2 + 4y^2 = 312$
$13y^2 - 78y - 143 = 0$
Divide by 13:
$y^2 - 6y - 11 = 0$
$a = 1, b = -6, c = -11$

$$y = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a} = \frac{6 \pm \sqrt{(36 + 44)}}{2} = \frac{6 \pm \sqrt{80}}{2}$$

$$\sqrt{80} = \sqrt{(16 \times 5)} = \sqrt{16}\sqrt{5} = 4\sqrt{5}$$

$$y = \frac{6 \pm 4\sqrt{5}}{2} = 3 \pm 2\sqrt{5}$$

Substitute into $x = \frac{1}{2}\left(13 - 3y\right)$:

when $y = 3 - 2\sqrt{5}$, $x = \frac{1}{2}\left[13 - 3(3 - 2\sqrt{5})\right] = \frac{1}{2}\left[13 - 9 + 6\sqrt{5}\right] = 2 + 3\sqrt{5}$

when $y = 3 + 2\sqrt{5}$, $x = \frac{1}{2}\left[13 - 3(3 + 2\sqrt{5})\right] = \frac{1}{2}\left[13 - 9 - 6\sqrt{5}\right] = 2 - 3\sqrt{5}$

Solutions are $x = 2 - 3\sqrt{5}$, $y = 3 + 2\sqrt{5}$ and $x = 2 + 3\sqrt{5}$, $y = 3 - 2\sqrt{5}$

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Equations and inequalities
Exercise D, Question 1

Question:

Find the set of values of x for which:

(a) $2x - 3 < 5$

(b) $5x + 4 \geq 39$

(c) $6x - 3 > 2x + 7$

(d) $5x + 6 \leq -12 - x$

(e) $15 - x > 4$

(f) $21 - 2x > 8 + 3x$

(g) $1 + x < 25 + 3x$

(h) $7x - 7 < 7 - 7x$

(i) $5 - 0.5x \geq 1$

(j) $5x + 4 > 12 - 2x$

Solution:

(a) $2x < 5 + 3$
$2x < 8$
$x < 4$

(b) $5x \geq 39 - 4$
$5x \geq 35$
$x \geq 7$

(c) $6x - 2x > 7 + 3$
$4x > 10$
$x > 2\frac{1}{2}$

(d) $5x + x \leq -12 - 6$
$6x \leq -18$
$x \leq -3$

(e) $-x > 4 - 15$
$-x > -11$
$x < 11$

(f) $21 - 8 > 3x + 2x$
$13 > 5x$
$5x < 13$
$x < 2\frac{3}{5}$

(g) $x - 3x < 25 - 1$
$-2x < 24$
$x > -12$

(h) $7x + 7x < 7 + 7$
$14x < 14$
$x < 1$

(i) $-0.5x \geq 1 - 5$
$-0.5x \geq -4$
$x \leq 8$

(j) $5x + 2x > 12 - 4$
$7x > 8$
$x > 1\frac{1}{7}$

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Equations and inequalities
Exercise D, Question 2

Question:

Find the set of values of x for which:

(a) $2(x-3) \geq 0$

(b) $8(1-x) > x-1$

(c) $3(x+7) \leq 8-x$

(d) $2(x-3)-(x+12) < 0$

(e) $1+11(2-x) < 10(x-4)$

(f) $2(x-5) \geq 3(4-x)$

(g) $12x-3(x-3) < 45$

(h) $x-2(5+2x) < 11$

(i) $x(x-4) \geq x^2+2$

(j) $x(5-x) \geq 3+x-x^2$

Solution:

(a) $2x-6 \geq 0$
$2x \geq 6$
$x \geq 3$

(b) $8-8x > x-1$
$8+1 > x+8x$
$9 > 9x$
$1 > x$
$x < 1$

(c) $3x+21 \leq 8-x$
$3x+x \leq 8-21$
$4x \leq -13$
$x \leq -3\frac{1}{4}$

(d) $2x-6-x-12 < 0$
$2x-x < 6+12$
$x < 18$

(e) $1+22-11x < 10x-40$
$1+22+40 < 10x+11x$
$63 < 21x$
$3 < x$
$x > 3$

(f) $2x-10 \geq 12-3x$

$2x + 3x \geq 12 + 10$
$5x \geq 22$
$x \geq 4\frac{2}{5}$

(g) $12x - 3x + 9 < 45$
$12x - 3x < 45 - 9$
$9x < 36$
$x < 4$

(h) $x - 10 - 4x < 11$
$x - 4x < 11 + 10$
$-3x < 21$
$x > -7$

(i) $x^2 - 4x \geq x^2 + 2$
$x^2 - x^2 - 4x \geq 2$
$-4x \geq 2$
$x \leq -\frac{1}{2}$

(j) $5x - x^2 \geq 3 + x - x^2$
$5x - x - x^2 + x^2 \geq 3$
$4x \geq 3$
$x \geq \frac{3}{4}$

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Edexcel Modular Mathematics for AS and A-Level

Equations and inequalities
Exercise D, Question 3

Question:

Find the set of values of x for which:

(a) $3(x-2) > x-4$ and $4x+12 > 2x+17$

(b) $2x-5 < x-1$ and $7(x+1) > 23-x$

(c) $2x-3 > 2$ and $3(x+2) < 12+x$

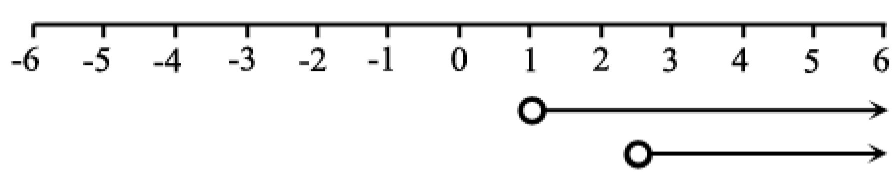
(d) $15-x < 2(11-x)$ and $5(3x-1) > 12x+19$

(e) $3x+8 \leq 20$ and $2(3x-7) \geq x+6$

Solution:

(a) $3x-6 > x-4$
$3x-x > -4+6$
$2x > 2$
$x > 1$

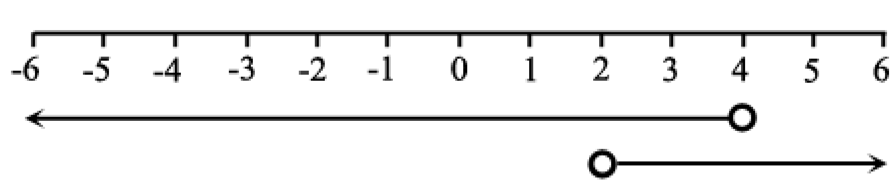
$4x+12 > 2x+17$
$4x-2x > 17-12$
$2x > 5$
$x > 2\frac{1}{2}$



So the required set of values is $x > 2\frac{1}{2}$

(b) $2x-x < -1+5$
$x < 4$

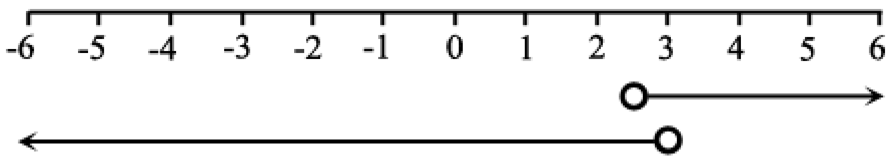
$7(x+1) > 23-x$
$7x+7 > 23-x$
$7x+x > 23-7$
$8x > 16$
$x > 2$



So the required set of values is $2 < x < 4$

(c) $2x > 2 + 3$
$2x > 5$
$x > 2\frac{1}{2}$

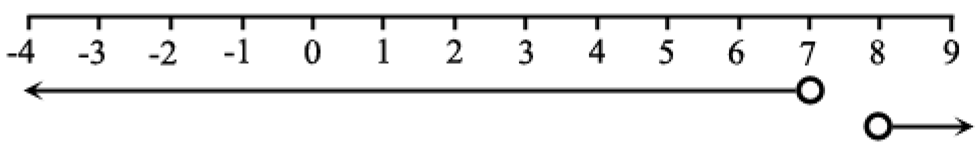
$3(x + 2) < 12 + x$
$3x + 6 < 12 + x$
$3x - x < 12 - 6$
$2x < 6$
$x < 3$



So the required set of values is $2\frac{1}{2} < x < 3$

(d) $15 - x < 22 - 2x$
$-x + 2x < 22 - 15$
$x < 7$

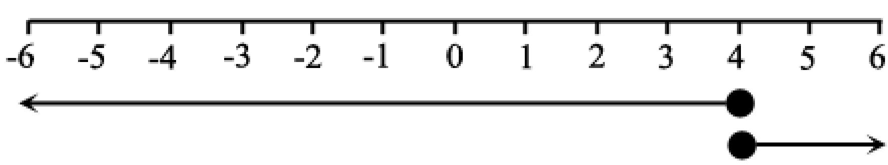
$5(3x - 1) > 12x + 19$
$15x - 5 > 12x + 19$
$15x - 12x > 19 + 5$
$3x > 24$
$x > 8$



There are no values satisfying both inequalities.

(e) $3x \leq 20 - 8$
$3x \leq 12$
$x \leq 4$

$2(3x - 7) \geq x + 6$
$6x - 14 \geq x + 6$
$6x - x \geq 6 + 14$
$5x \geq 20$
$x \geq 4$



There is just one value, $x = 4$.

Solutionbank C1
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Equations and inequalities
Exercise E, Question 1

Question:

Find the set of values of x for which:

(a) $x^2 - 11x + 24 < 0$

(b) $12 - x - x^2 > 0$

(c) $x^2 - 3x - 10 > 0$

(d) $x^2 + 7x + 12 \geq 0$

(e) $7 + 13x - 2x^2 > 0$

(f) $10 + x - 2x^2 < 0$

(g) $4x^2 - 8x + 3 \leq 0$

(h) $-2 + 7x - 3x^2 < 0$

(i) $x^2 - 9 < 0$

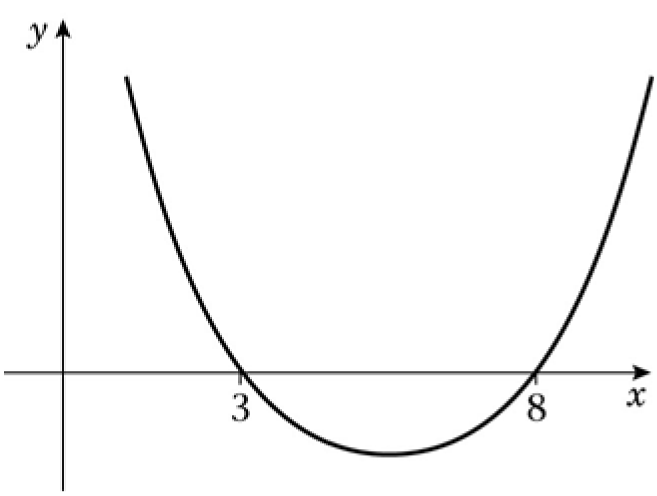
(j) $6x^2 + 11x - 10 > 0$

(k) $x^2 - 5x > 0$

(l) $2x^2 + 3x \leq 0$

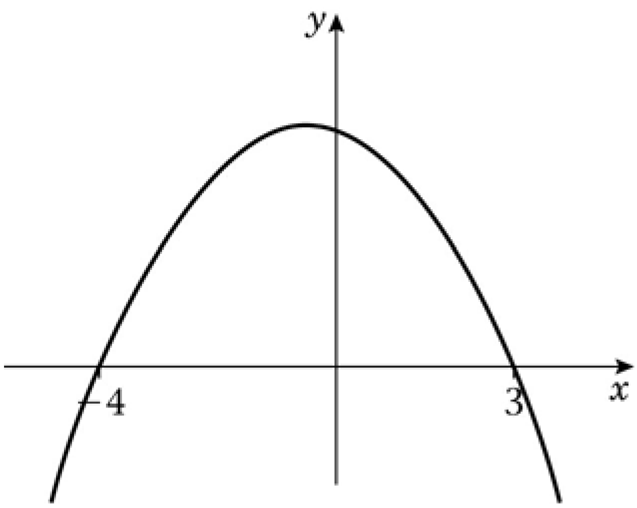
Solution:

(a) $x^2 - 11x + 24 = 0$
$(x - 3) (x - 8) = 0$
$x = 3, x = 8$
Sketch of $y = x^2 - 11x + 24$:



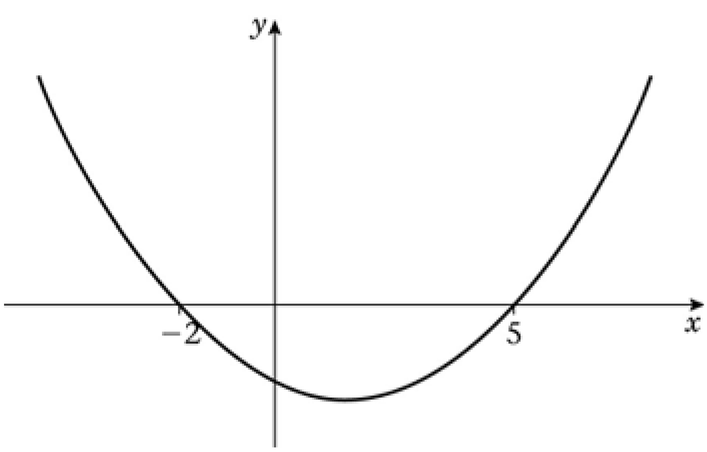
$x^2 - 11x + 24 < 0$ when $3 < x < 8$

(b) $12 - x - x^2 = 0$
$0 = x^2 + x - 12$
$0 = (x + 4)(x - 3)$
$x = -4, x = 3$
Sketch of $y = 12 - x - x^2$:



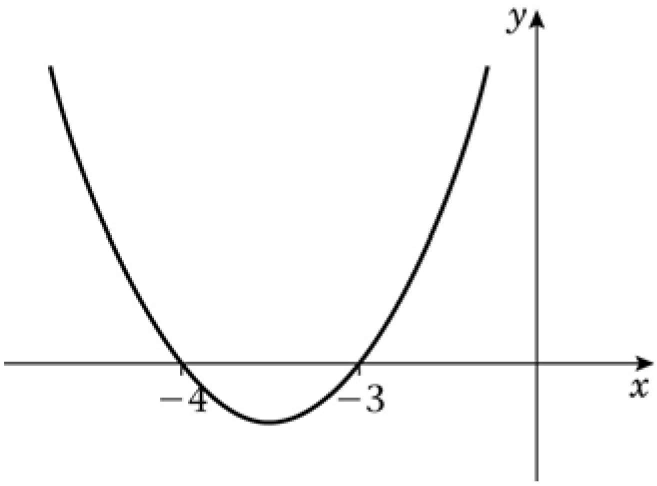
$12 - x - x^2 > 0$ when $-4 < x < 3$

(c) $x^2 - 3x - 10 = 0$
$(x + 2)(x - 5) = 0$
$x = -2, x = 5$
Sketch of $y = x^2 - 3x - 10$:



$x^2 - 3x - 10 > 0$ when $x < -2$ or $x > 5$

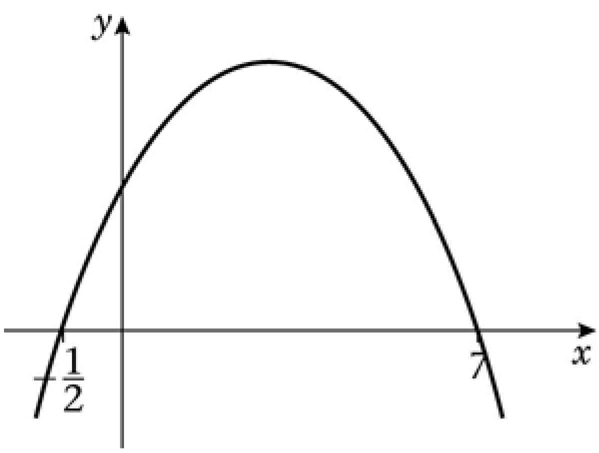
(d) $x^2 + 7x + 12 = 0$
$(x + 4)(x + 3) = 0$
$x = -4, x = -3$
Sketch of $y = x^2 + 7x + 12$:



$x^2 + 7x + 12 \geq 0$ when $x \leq -4$ or $x \geq -3$

(e) $7 + 13x - 2x^2 = 0$
$2x^2 - 13x - 7 = 0$
$(2x + 1)(x - 7) = 0$
$x = -\frac{1}{2}, x = 7$

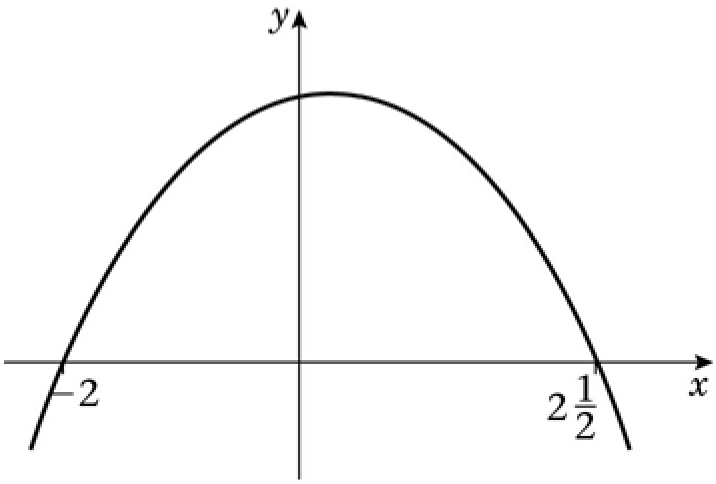
Sketch of $y = 7 + 13x - 2x^2$:



$7 + 13x - 2x^2 > 0$ when $-\frac{1}{2} < x < 7$

(f) $10 + x - 2x^2 = 0$
$2x^2 - x - 10 = 0$
$(2x - 5)(x + 2) = 0$
$x = 2\frac{1}{2}, x = -2$

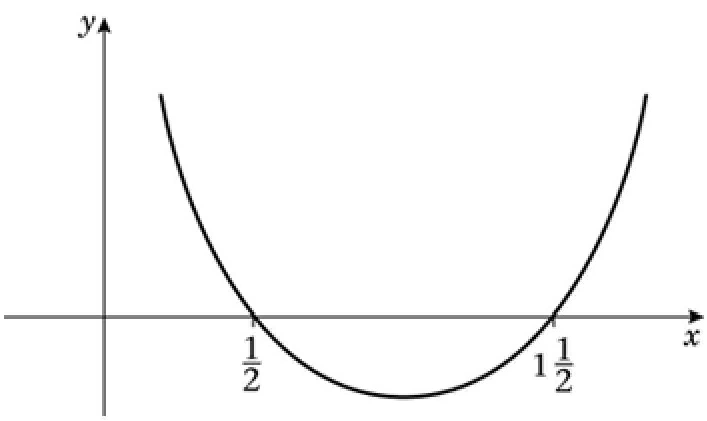
Sketch of $y = 10 + x - 2x^2$:



$10 + x - 2x^2 < 0$ when $x < -2$ or $x > 2\frac{1}{2}$

(g) $4x^2 - 8x + 3 = 0$
$(2x - 1)(2x - 3) = 0$
$x = \frac{1}{2}, x = 1\frac{1}{2}$

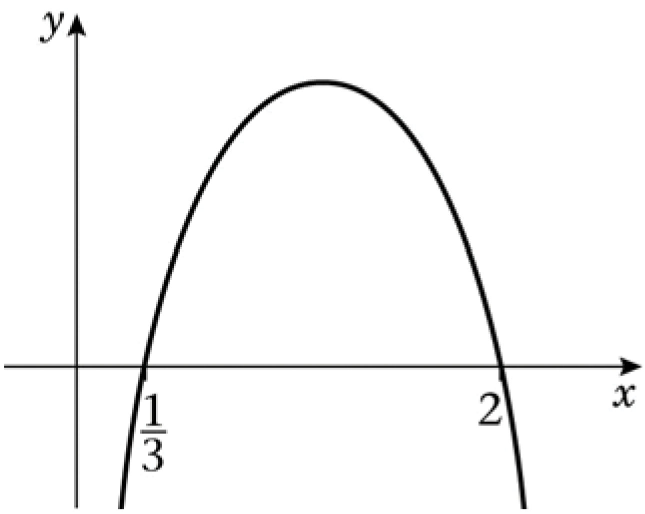
Sketch of $y = 4x^2 - 8x + 3$:



$4x^2 - 8x + 3 \leq 0$ when $\frac{1}{2} \leq x \leq 1\frac{1}{2}$

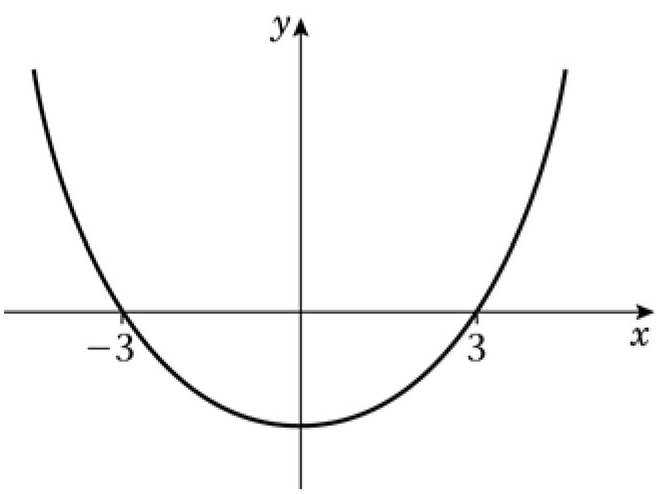
(h) $-2 + 7x - 3x^2 = 0$
$3x^2 - 7x + 2 = 0$
$(3x - 1)(x - 2) = 0$
$x = \frac{1}{3}, x = 2$

Sketch of $y = -2 + 7x - 3x^2$:



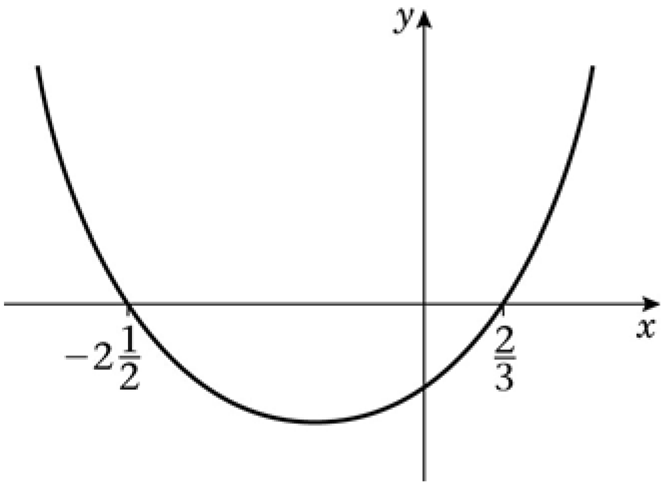
$-2 + 7x - 3x^2 < 0$ when $x < \frac{1}{3}$ or $x > 2$

(i) $x^2 - 9 = 0$
$(x + 3)(x - 3) = 0$
$x = -3, x = 3$
Sketch of $y = x^2 - 9$:



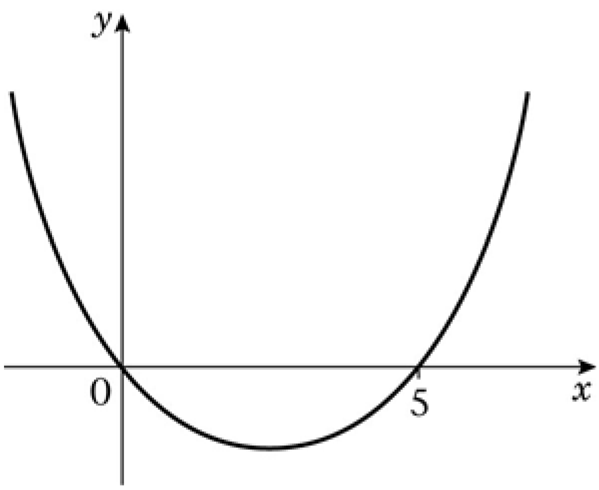
$x^2 - 9 < 0$ when $-3 < x < 3$

(j) $6x^2 + 11x - 10 = 0$
$(3x - 2)(2x + 5) = 0$
$x = \frac{2}{3}, x = -2\frac{1}{2}$
Sketch of $y = 6x^2 + 11x - 10$:



$6x^2 + 11x - 10 > 0$ when $x < -2\frac{1}{2}$ or $x > \frac{2}{3}$

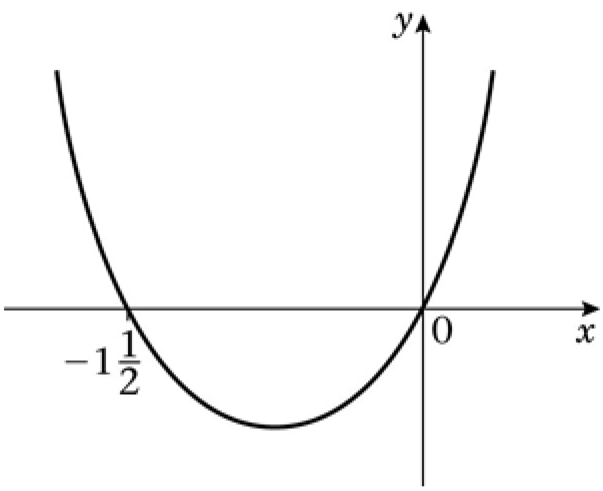
(k) $x^2 - 5x = 0$
$x(x - 5) = 0$
$x = 0, x = 5$
Sketch of $y = x^2 - 5x$:



$x^2 - 5x > 0$ when $x < 0$ or $x > 5$

(l) $2x^2 + 3x = 0$
$x(2x + 3) = 0$
$x = 0, x = -1\frac{1}{2}$

Sketch of $y = 2x^2 + 3x$:



$2x^2 + 3x \leq 0$ when $-1\frac{1}{2} \leq x \leq 0$

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Equations and inequalities
Exercise E, Question 2

Question:

Find the set of values of x for which:

(a) $x^2 < 10 - 3x$

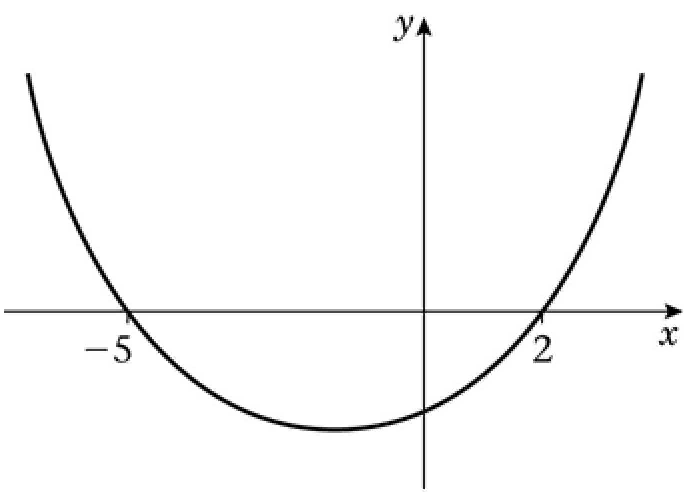
(b) $11 < x^2 + 10$

(c) $x (3 - 2x) > 1$

(d) $x (x + 11) < 3 (1 - x^2)$

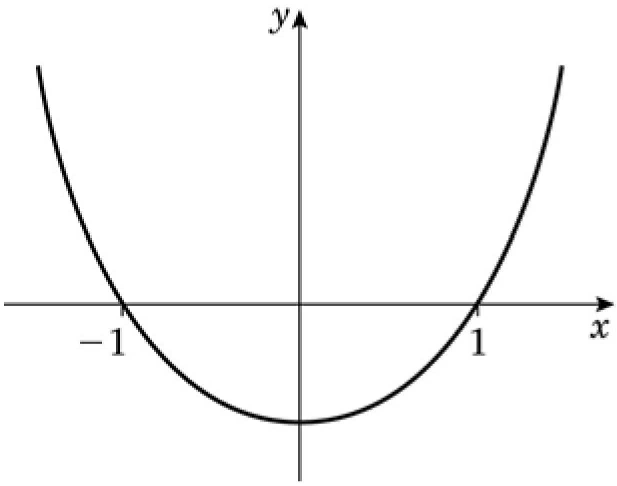
Solution:

(a) $x^2 = 10 - 3x$
$x^2 + 3x - 10 = 0$
$(x + 5) (x - 2) = 0$
$x = -5, x = 2$
$x^2 < 10 - 3x \Rightarrow x^2 + 3x - 10 < 0$
Sketch of $y = x^2 + 3x - 10$:



$x^2 + 3x - 10 < 0$ when $-5 < x < 2$

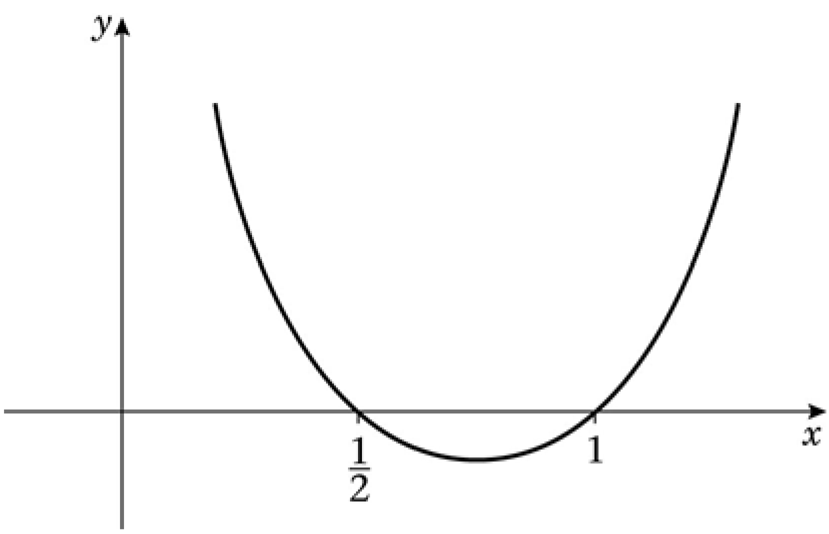
(b) $11 = x^2 + 10$
$x^2 = 1$
$x = -1, x = 1$
$11 < x^2 + 10 \quad \Rightarrow \quad 0 < x^2 + 10 - 11 \quad \Rightarrow \quad x^2 - 1 > 0$
Sketch of $y = x^2 - 1$:



$x^2 - 1 > 0$ when $x < -1$ or $x > 1$

(c) $x(3 - 2x) = 1$
$3x - 2x^2 = 1$
$0 = 2x^2 - 3x + 1$
$0 = (2x - 1)(x - 1)$
$x = \frac{1}{2}, x = 1$

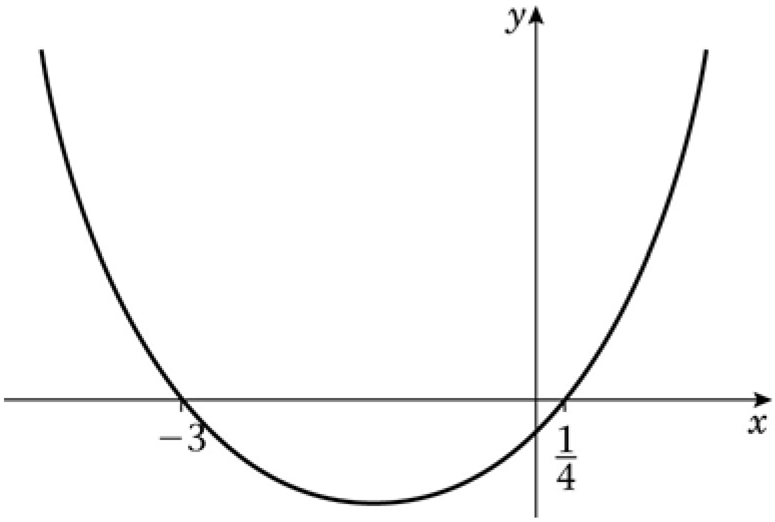
$x(3 - 2x) > 1 \Rightarrow -2x^2 + 3x - 1 > 0 \Rightarrow 2x^2 - 3x + 1 < 0$
Sketch of $y = 2x^2 - 3x + 1$:



$2x^2 - 3x + 1 < 0$ when $\frac{1}{2} < x < 1$

(d) $x(x + 11) = 3(1 - x^2)$
$x^2 + 11x = 3 - 3x^2$
$x^2 + 3x^2 + 11x - 3 = 0$
$4x^2 + 11x - 3 = 0$
$(4x - 1)(x + 3) = 0$
$x = \frac{1}{4}, x = -3$

$x(x + 11) < 3(1 - x^2) \Rightarrow 4x^2 + 11x - 3 < 0$
Sketch of $y = 4x^2 + 11x - 3$:



$4x^2 + 11x - 3 < 0$ when $-3 < x < \frac{1}{4}$

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Equations and inequalities
Exercise E, Question 3

Question:

Find the set of values of x for which:

(a) $x^2 - 7x + 10 < 0$ and $3x + 5 < 17$

(b) $x^2 - x - 6 > 0$ and $10 - 2x < 5$

(c) $4x^2 - 3x - 1 < 0$ and $4(x + 2) < 15 - (x + 7)$

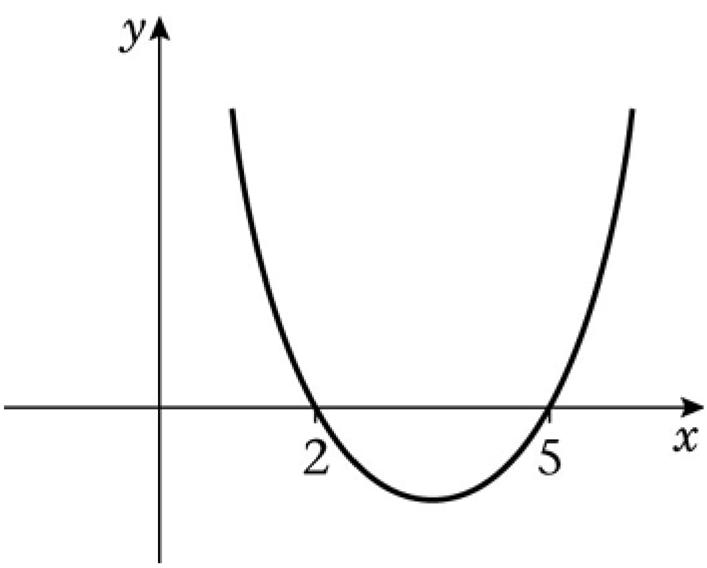
(d) $2x^2 - x - 1 < 0$ and $14 < 3x - 2$

(e) $x^2 - x - 12 > 0$ and $3x + 17 > 2$

(f) $x^2 - 2x - 3 < 0$ and $x^2 - 3x + 2 > 0$

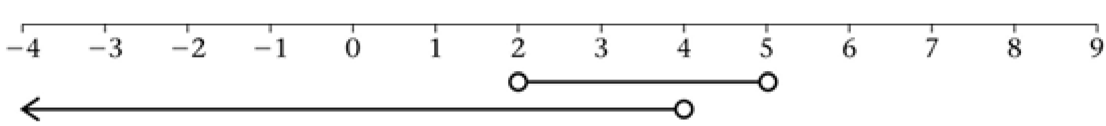
Solution:

(a) $x^2 - 7x + 10 = 0$
$(x - 2)(x - 5) = 0$
$x = 2, x = 5$
Sketch of $y = x^2 - 7x + 10$:



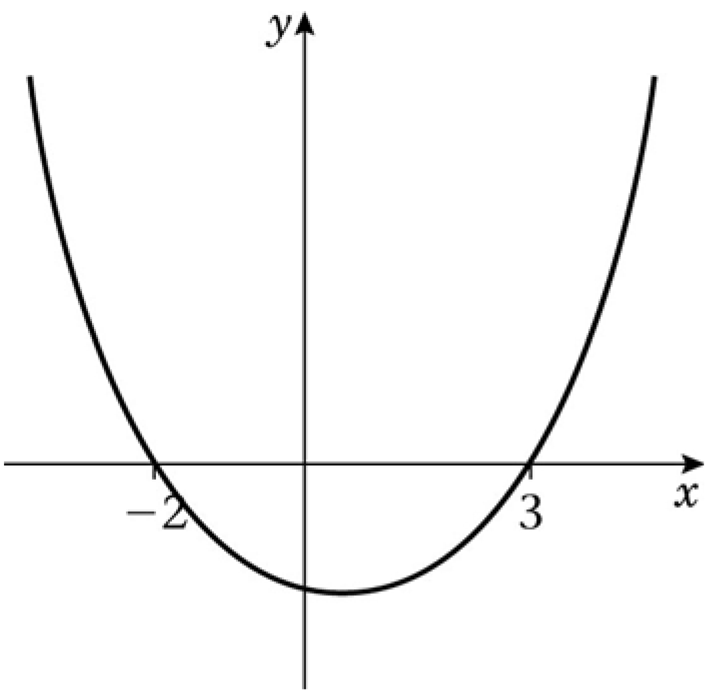
$x^2 - 7x + 10 < 0$ when $2 < x < 5$.

$3x + 5 < 17$
$3x < 17 - 5$
$3x < 12$
$x < 4$



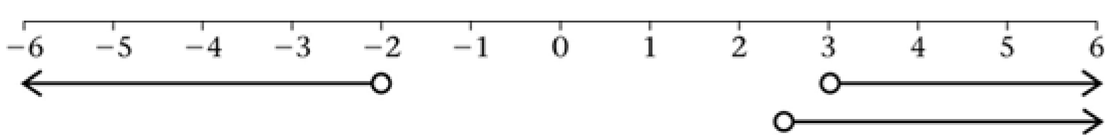
Intersection is $2 < x < 4$.

(b) $x^2 - x - 6 = 0$
$(x + 2)(x - 3) = 0$
$x = -2, x = 3$
Sketch of $y = x^2 - x - 6$:



$x^2 - x - 6 > 0$ when $x < -2$ or $x > 3$

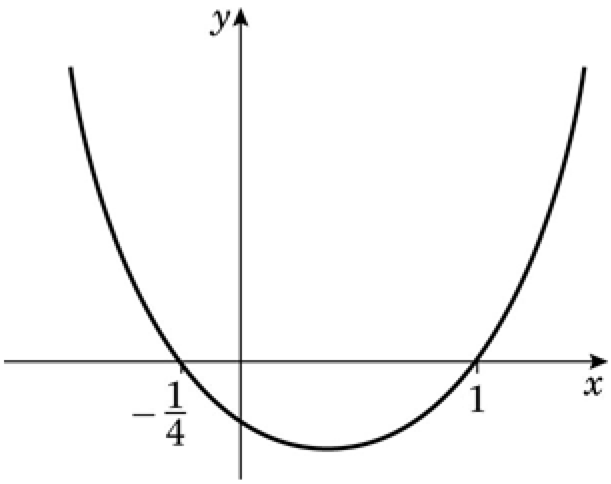
$10 - 2x < 5$
$-2x < 5 - 10$
$-2x < -5$
$x > 2\frac{1}{2}$



Intersection is $x > 3$.

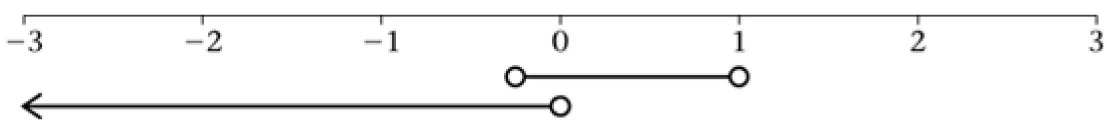
(c) $4x^2 - 3x - 1 = 0$
$(4x + 1)(x - 1) = 0$
$x = -\frac{1}{4}, x = 1$

Sketch of $y = 4x^2 - 3x - 1$:



$4x^2 - 3x - 1 < 0$ when $-\frac{1}{4} < x < 1$

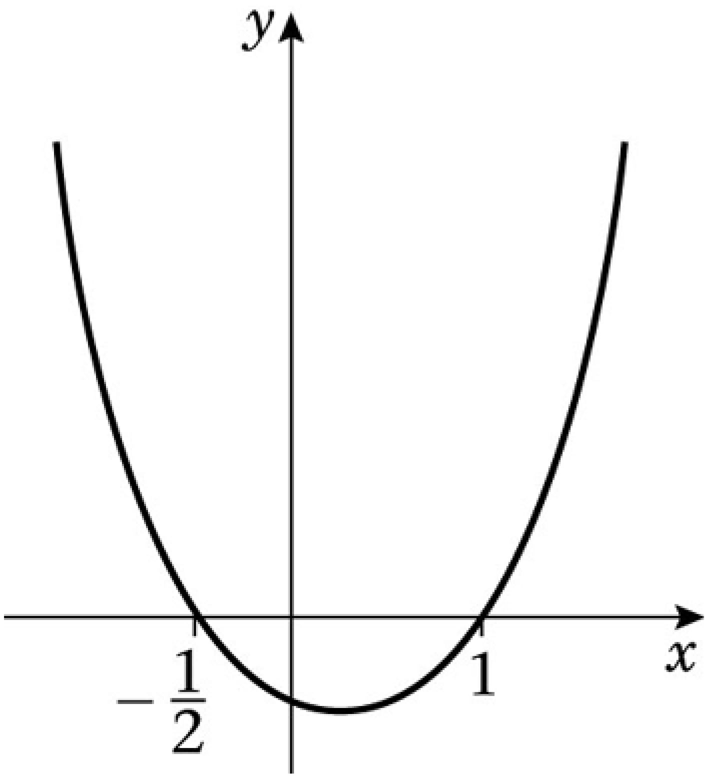
$4(x + 2) < 15 - (x + 7)$
$4x + 8 < 15 - x - 7$
$4x + 8 < 8 - x$
$4x + x < 8 - 8$
$5x < 0$
$x < 0$



Intersection is $-\frac{1}{4} < x < 0$

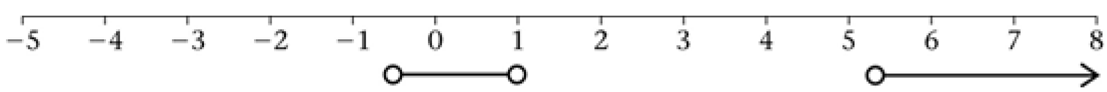
(d) $2x^2 - x - 1 = 0$
$(2x + 1)(x - 1) = 0$
$x = -\frac{1}{2}, x = 1$

Sketch of $y = 2x^2 - x - 1$:



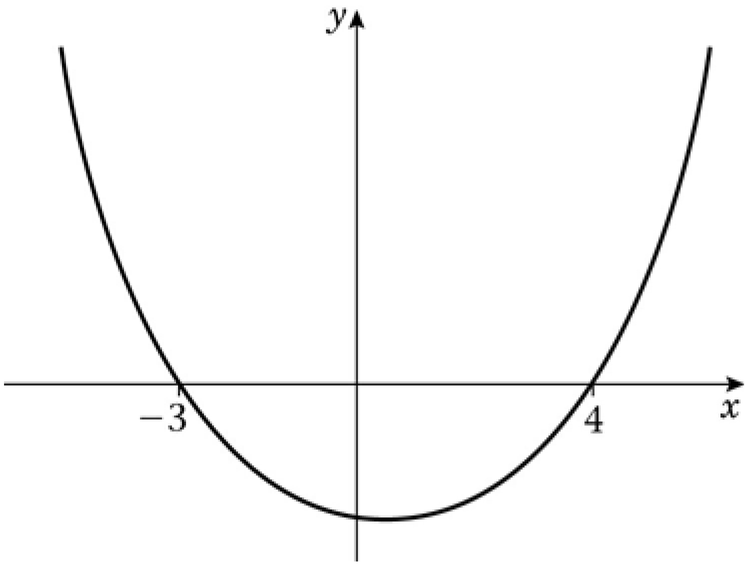
$2x^2 - x - 1 < 0$ when $-\frac{1}{2} < x < 1$

$14 < 3x - 2$
$14 + 2 < 3x$
$3x > 16$
$x > 5\frac{1}{3}$



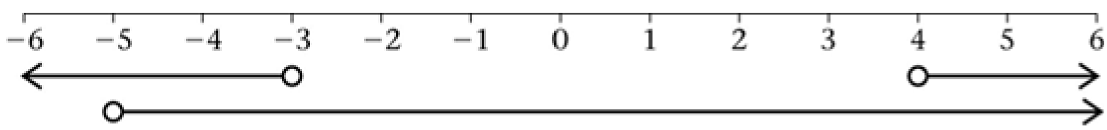
No intersection.
There are no values of x for which both inequalities are true.

(e) $x^2 - x - 12 = 0$
$(x + 3)(x - 4) = 0$
$x = -3, x = 4$
Sketch of $y = x^2 - x - 12$:



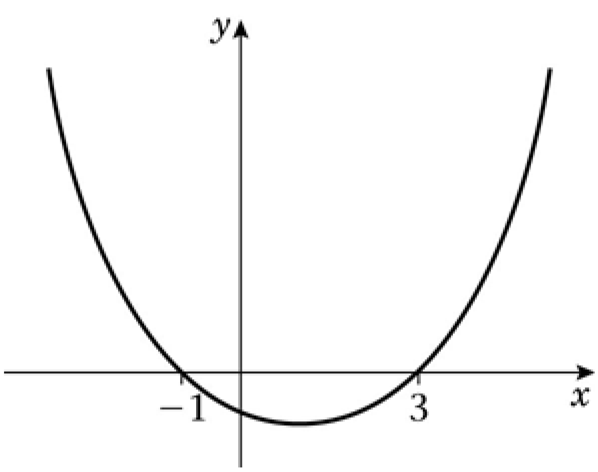
$x^2 - x - 12 > 0$ when $x < -3$ or $x > 4$

$3x + 17 > 2$
$3x > 2 - 17$
$3x > -15$
$x > -5$

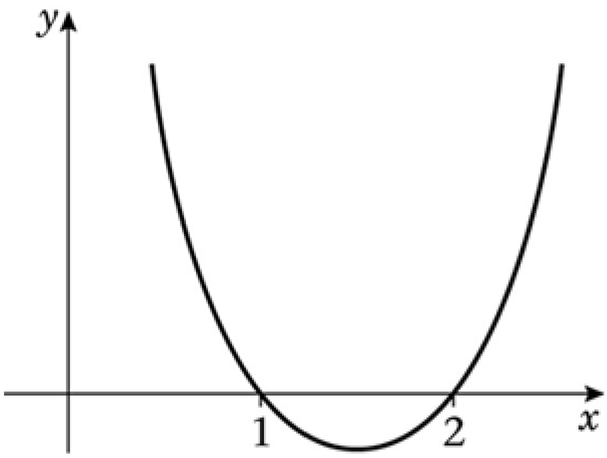


Intersection is $-5 < x < -3$, $x > 4$.

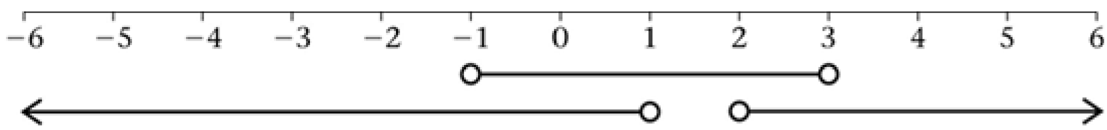
(f) $x^2 - 2x - 3 = 0$
$(x + 1)(x - 3) = 0$
$x = -1$, $x = 3$
Sketch of $y = x^2 - 2x - 3$:



$x^2 - 2x - 3 < 0$ when $-1 < x < 3$
$x^2 - 3x + 2 = 0$
$(x - 1)(x - 2) = 0$
$x = 1$, $x = 2$
Sketch of $y = x^2 - 3x + 2$:



$x^2 - 3x + 2 > 0$ when $x < 1$ or $x > 2$



Intersection is $-1 < x < 1, 2 < x < 3$.

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Equations and inequalities
Exercise E, Question 4

Question:

(a) Find the range of values of k for which the equation $x^2 - kx + (k + 3) = 0$ has no real roots.

(b) Find the range of values of p for which the roots of the equation $px^2 + px - 2 = 0$ are real.

Solution:

(a) $a = 1$, $b = - k$, $c = k + 3$
$b^2 - 4ac < 0$ for no real roots, so
$k^2 - 4 (k + 3) < 0$
$k^2 - 4k - 12 < 0$
$(k - 6) (k + 2) < 0$
$- 2 < k < 6$

(b) $a = p$, $b = p$, $c = - 2$
$b^2 - 4ac < 0$ for no real roots, so
$p^2 + 8p < 0$
$p (p + 8) < 0$
$- 8 < p < 0$

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Equations and inequalities
Exercise F, Question 1

Question:

Solve the simultaneous equations:

$x + 2y = 3$
$x^2 - 4y^2 = -33$ **[E]**

Solution:

$x = 3 - 2y$
Substitute into $x^2 - 4y^2 = -33$:
$(3 - 2y)^2 - 4y^2 = -33$
$9 - 12y + 4y^2 - 4y^2 = -33$
$-12y = -33 - 9$
$-12y = -42$
$y = 3\frac{1}{2}$
Substitute into $x = 3 - 2y$:
$x = 3 - 7 = -4$
So solution is $x = -4$, $y = 3\frac{1}{2}$

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Equations and inequalities
Exercise F, Question 2

Question:

Show that the elimination of x from the simultaneous equations:

$x - 2y = 1$
$3xy - y^2 = 8$
produces the equation
$5y^2 + 3y - 8 = 0$.
Solve this quadratic equation and hence find the pairs (x, y) for which the simultaneous equations are satisfied. **[E]**

Solution:

$x = 1 + 2y$
Substitute into $3xy - y^2 = 8$:
$3y(1 + 2y) - y^2 = 8$
$3y + 6y^2 - y^2 = 8$
$5y^2 + 3y - 8 = 0$
$(5y + 8)(y - 1) = 0$
$y = -\frac{8}{5}$ or $y = 1$

Substitute into $x = 1 + 2y$:
when $y = -\frac{8}{5}$, $x = 1 - \frac{16}{5} = -\frac{11}{5}$
when $y = 1$, $x = 1 + 2 = 3$
Solutions are $\left(-2\frac{1}{5}, -1\frac{3}{5}\right)$ and $(3, 1)$

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Equations and inequalities
Exercise F, Question 3

Question:

(a) Given that $3^x = 9^{y-1}$, show that $x = 2y - 2$.

(b) Solve the simultaneous equations:
$x = 2y - 2$
$x^2 = y^2 + 7$ **[E]**

Solution:

(a) $9 = 3^2$, so $3^x = (3^2)^{y-1} \Rightarrow 3^x = 3^{2(y-1)}$

Equate powers: $x = 2(y-1) \Rightarrow x = 2y - 2$

(b) $x = 2y - 2$
Substitute into $x^2 = y^2 + 7$:
$(2y-2)^2 = y^2 + 7$
$4y^2 - 8y + 4 = y^2 + 7$
$4y^2 - y^2 - 8y + 4 - 7 = 0$
$3y^2 - 8y - 3 = 0$
$(3y+1)(y-3) = 0$
$y = -\frac{1}{3}$ or $y = 3$

Substitute into $x = 2y - 2$:
when $y = -\frac{1}{3}$, $x = -\frac{2}{3} - 2 = -2\frac{2}{3}$
when $y = 3$, $x = 6 - 2 = 4$
Solutions are $x = -2\frac{2}{3}$, $y = -\frac{1}{3}$ and $x = 4$, $y = 3$

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Equations and inequalities
Exercise F, Question 4

Question:

Solve the simultaneous equations:

$x + 2y = 3$
$x^2 - 2y + 4y^2 = 18$ **[E]**

Solution:

$x = 3 - 2y$
Substitute into $x^2 - 2y + 4y^2 = 18$:
$(3 - 2y)^2 - 2y + 4y^2 = 18$
$9 - 12y + 4y^2 - 2y + 4y^2 = 18$
$8y^2 - 14y + 9 - 18 = 0$
$8y^2 - 14y - 9 = 0$
$(4y - 9)(2y + 1) = 0$
$y = \frac{9}{4}$ or $y = -\frac{1}{2}$

Substitute into $x = 3 - 2y$:
when $y = \frac{9}{4}$, $x = 3 - \frac{9}{2} = -\frac{3}{2}$

when $y = -\frac{1}{2}$, $x = 3 + 1 = 4$

Solutions are $x = -1\frac{1}{2}$, $y = 2\frac{1}{4}$ and $x = 4$, $y = -\frac{1}{2}$

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Equations and inequalities

Exercise F, Question 5

Question:

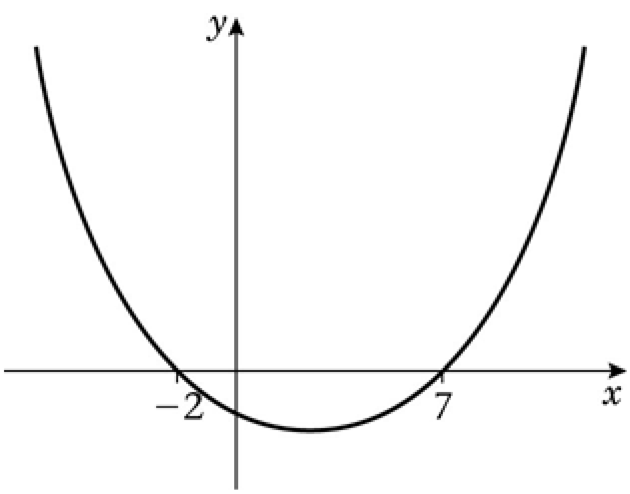
(a) Solve the inequality $3x - 8 > x + 13$.

(b) Solve the inequality $x^2 - 5x - 14 > 0$. **[E]**

Solution:

(a) $3x - x > 13 + 8$
$2x > 21$
$x > 10\frac{1}{2}$

(b) $x^2 - 5x - 14 = 0$
$(x + 2)(x - 7) = 0$
$x = -2$ or $x = 7$
Sketch of $y = x^2 - 5x - 14$:



$x^2 - 5x - 14 > 0$ when $x < -2$ or $x > 7$

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Equations and inequalities
Exercise F, Question 6

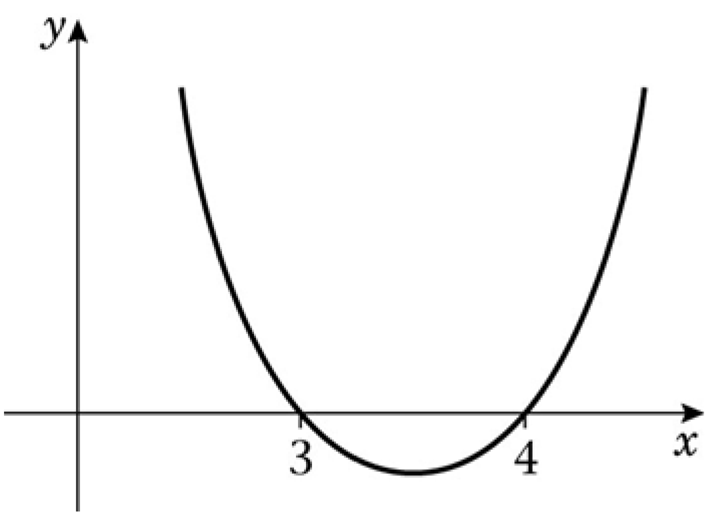
Question:

Find the set of values of x for which $(x-1)(x-4) < 2(x-4)$. **[E]**

Solution:

$x^2 - 5x + 4 < 2x - 8$
$x^2 - 5x - 2x + 4 + 8 < 0$
$x^2 - 7x + 12 < 0$

$x^2 - 7x + 12 = 0$
$(x-3)(x-4) = 0$
$x = 3$ or $x = 4$
Sketch of $y = x^2 - 7x + 12$:



$x^2 - 7x + 12 < 0$ when $3 < x < 4$.

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Equations and inequalities
Exercise F, Question 7

Question:

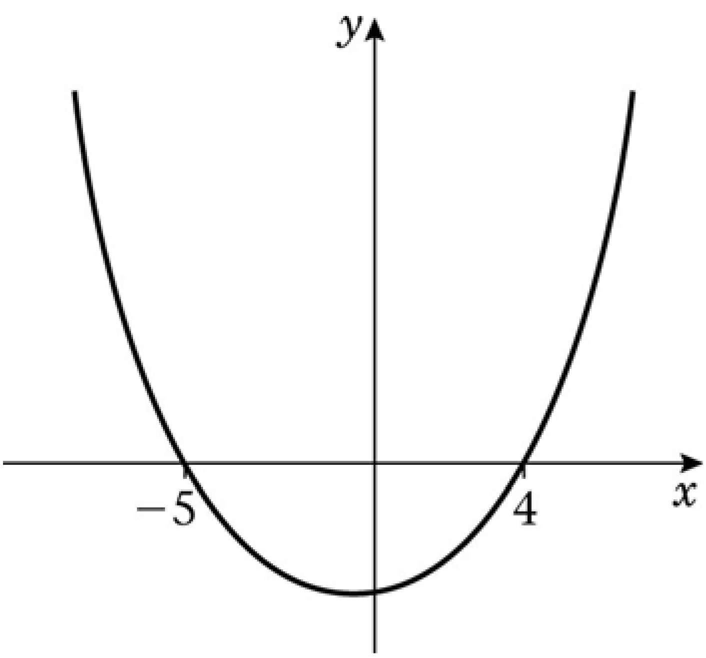
(a) Use algebra to solve $(x-1)(x+2)=18$.

(b) Hence, or otherwise, find the set of values of x for which $(x-1)(x+2)>18$. **[E]**

Solution:

(a) $x^2+x-2=18$
$x^2+x-20=0$
$(x+5)(x-4)=0$
$x=-5$ or $x=4$

(b) $(x-1)(x+2)>18 \quad \Rightarrow \quad x^2+x-20>0$
Sketch of $y=x^2+x-20$:



$x^2+x-20>0$ when $x<-5$ or $x>4$

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Equations and inequalities
Exercise F, Question 8

Question:

Find the set of values of x for which:

(a) $6x - 7 < 2x + 3$

(b) $2x^2 - 11x + 5 < 0$

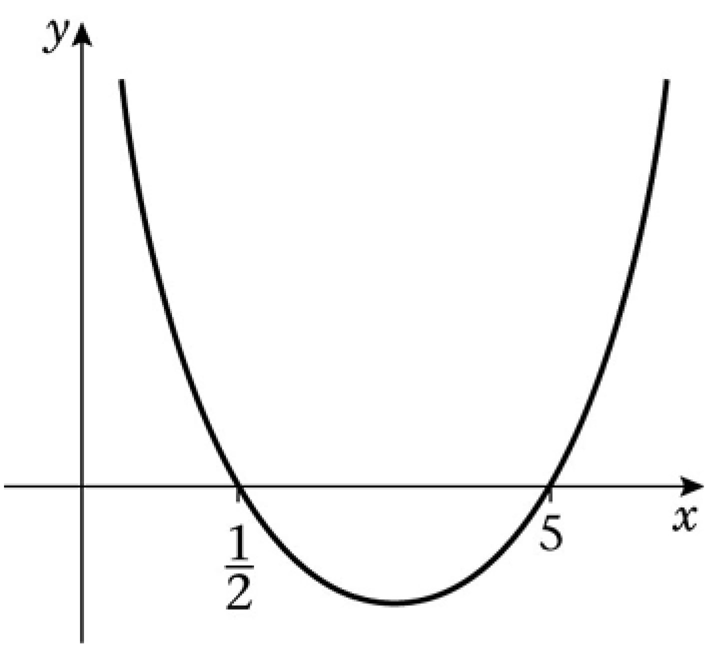
(c) both $6x - 7 < 2x + 3$ and $2x^2 - 11x + 5 < 0$. **[E]**

Solution:

(a) $6x - 2x < 3 + 7$
$4x < 10$
$x < 2\frac{1}{2}$

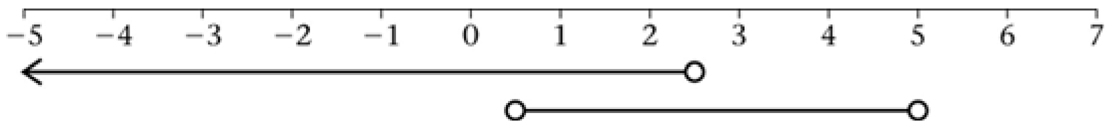
(b) $2x^2 - 11x + 5 = 0$
$(2x - 1)(x - 5) = 0$
$x = \frac{1}{2}$ or $x = 5$

Sketch of $y = 2x^2 - 11x + 5$:



$2x^2 - 11x + 5 < 0$ when $\frac{1}{2} < x < 5$

(c)



Intersection is $\frac{1}{2} < x < 2\frac{1}{2}$.

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Equations and inequalities
Exercise F, Question 9

Question:

Find the values of k for which $kx^2 + 8x + 5 = 0$ has real roots.

Solution:

$a = k$, $b = 8$, $c = 5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$b^2 - 4ac \geq 0$ for real roots. So

$8^2 - 4k \times 5 \geq 0$

$64 - 20k \geq 0$

$64 \geq 20k$

$\frac{64}{20} \geq k$

$k \leq 3\frac{1}{5}$

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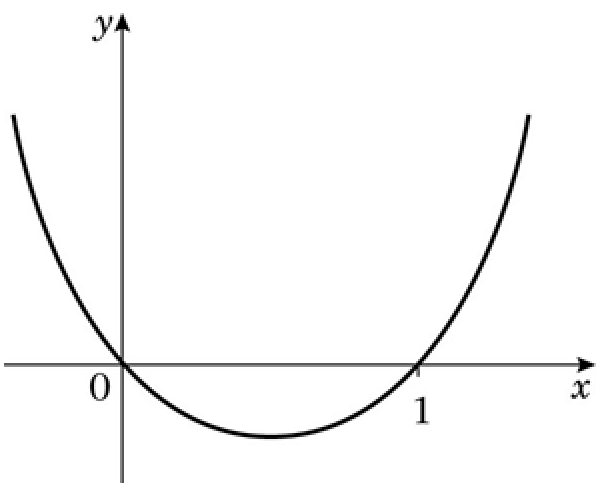
Equations and inequalities
Exercise F, Question 10

Question:

Find algebraically the set of values of x for which $(2x - 3)(x + 2) > 3(x - 2)$. **[E]**

Solution:

$2x^2 + x - 6 > 3x - 6$
$2x^2 + x - 3x - 6 + 6 > 0$
$2x^2 - 2x > 0$
$2x(x - 1) > 0$
Solve the equation:
$2x(x - 1) = 0$
$x = 0$ or $x = 1$
Sketch of $y = 2x^2 - 2x$:



$2x^2 - 2x > 0$ when $x < 0$ or $x > 1$

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Equations and inequalities
Exercise F, Question 11

Question:

(a) Find, as surds, the roots of the equation $2(x+1)(x-4)-(x-2)^2=0$.

(b) Hence find the set of values of x for which $2(x+1)(x-4)-(x-2)^2>0$. **[E]**

Solution:

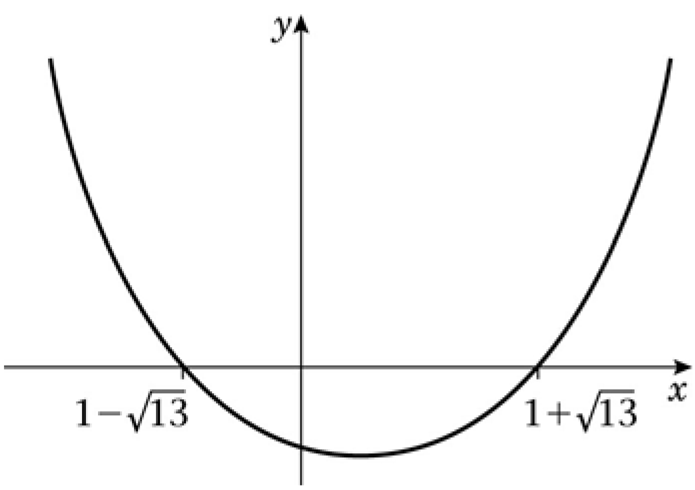
(a) $2(x^2-3x-4)-(x^2-4x+4)=0$
$2x^2-6x-8-x^2+4x-4=0$
$x^2-2x-12=0$
$a=1, b=-2, c=-12$

$$x=\frac{-b\pm\sqrt{(b^2-4ac)}}{2a}:$$

$$x=\frac{2\pm\sqrt{(-2)^2+48}}{2}=\frac{2\pm\sqrt{52}}{2}$$

$\sqrt{52}=\sqrt{4}\sqrt{13}=2\sqrt{13}$
$x=1+\sqrt{13}$ or $x=1-\sqrt{13}$

(b) $2(x+1)(x-4)-(x-2)^2>0 \Rightarrow x^2-2x-12>0$
Sketch of $y=x^2-2x-12$:



$x^2-2x-12>0$ when $x<1-\sqrt{13}$ or $x>1+\sqrt{13}$

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Equations and inequalities
Exercise F, Question 12

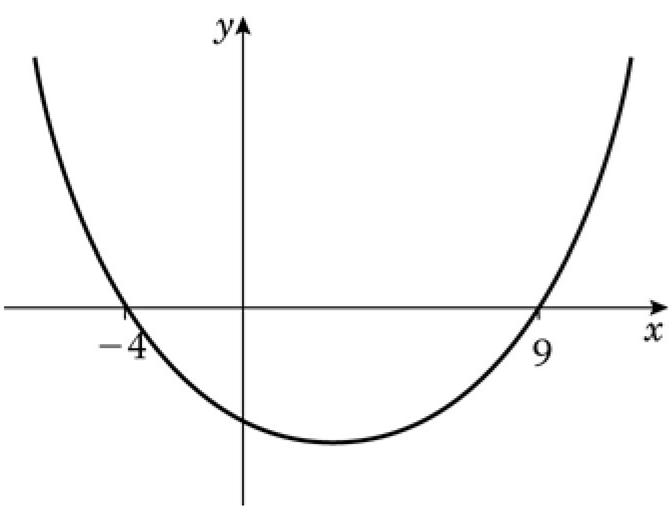
Question:

(a) Use algebra to find the set of values of x for which $x(x-5) > 36$.

(b) Using your answer to part (a), find the set of values of y for which $y^2(y^2-5) > 36$.

Solution:

(a) $x^2 - 5x > 36$
$x^2 - 5x - 36 > 0$
Solve the equation:
$x^2 - 5x - 36 = 0$
$(x+4)(x-9) = 0$
$x = -4$ or $x = 9$
Sketch of $y = x^2 - 5x - 36$:



$x^2 - 5x - 36 > 0$ when $x < -4$ or $x > 9$

(b) Either $y^2 < -4$ or $y^2 > 9$
$y^2 < -4$ is not possible. No values.
$y^2 > 9 \quad \Rightarrow \quad y > 3$ or $y < -3$

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Equations and inequalities
Exercise F, Question 13

Question:

The specification for a rectangular car park states that the length x m is to be 5 m more than the breadth. The perimeter of the car park is to be greater than 32 m.

(a) Form a linear inequality in x.
The area of the car park is to be less than 104m^2.

(b) Form a quadratic inequality in x.

(c) By solving your inequalities, determine the set of possible values of x. **[E]**

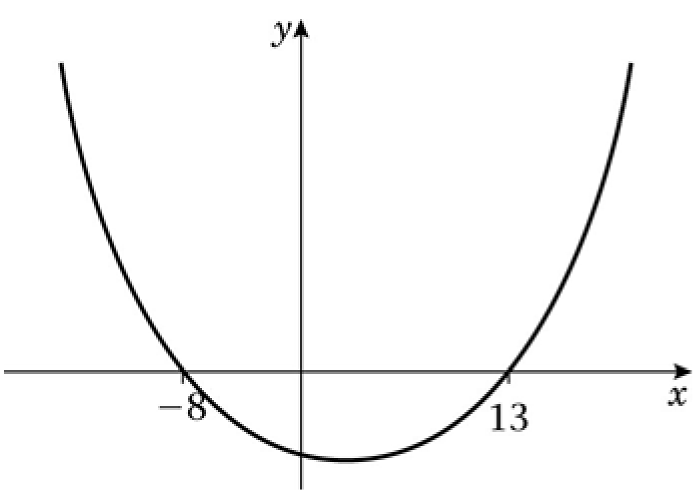
Solution:

(a) Length is x metres, breadth is $(x - 5)$ metres.
Perimeter is $x + x + (x - 5) + (x - 5) = (4x - 10)$ metres
So $4x - 10 > 32$

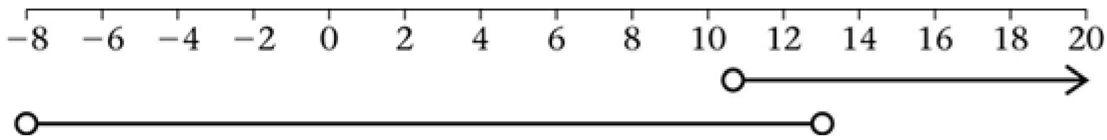
(b) Area is $x (x - 5)$ m^2.
So $x (x - 5) < 104$

(c) Linear:
$4x - 10 > 32$
$4x > 32 + 10$
$4x > 42$
$x > 10 \frac{1}{2}$

Quadratic:
$x^2 - 5x < 104$
$x^2 - 5x - 104 < 0$
Solve the equation:
$x^2 - 5x - 104 = 0$
$(x + 8) (x - 13) = 0$
$x = - 8$ or $x = 13$
Sketch of $y = x^2 - 5x - 104$:



$x^2 - 5x - 104 < 0$ when $- 8 < x < 13$



Intersection is $10\frac{1}{2} < x < 13$.

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Sketching curves
Exercise A, Question 1

Question:

Sketch the following curves and indicate clearly the points of intersection with the axes:

(a) $y = (x - 3)(x - 2)(x + 1)$

(b) $y = (x - 1)(x + 2)(x + 3)$

(c) $y = (x + 1)(x + 2)(x + 3)$

(d) $y = (x + 1)(1 - x)(x + 3)$

(e) $y = (x - 2)(x - 3)(4 - x)$

(f) $y = x(x - 2)(x + 1)$

(g) $y = x(x + 1)(x - 1)$

(h) $y = x(x + 1)(1 - x)$

(i) $y = (x - 2)(2x - 1)(2x + 1)$

(j) $y = x(2x - 1)(x + 3)$

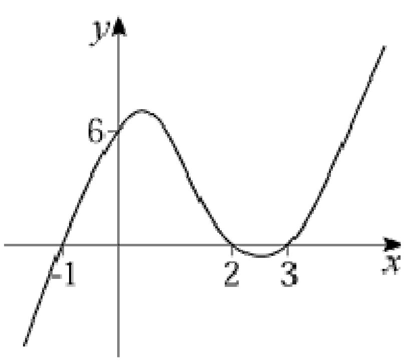
Solution:

(a) $y = 0 \quad \Rightarrow \quad x = -1, 2, 3$

$x = 0 \quad \Rightarrow \quad y = 6$

$x \to \infty, y \to \infty$

$x \to -\infty, y \to -\infty$

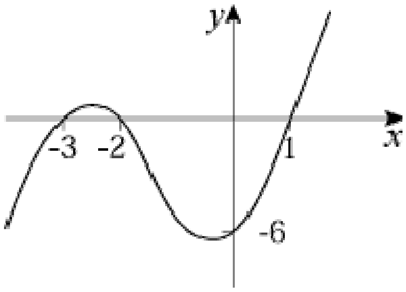


(b) $y = 0 \quad \Rightarrow \quad x = 1, -2, -3$

$x = 0 \quad \Rightarrow \quad y = -6$

$x \to \infty, y \to \infty$

$x \to -\infty, y \to -\infty$

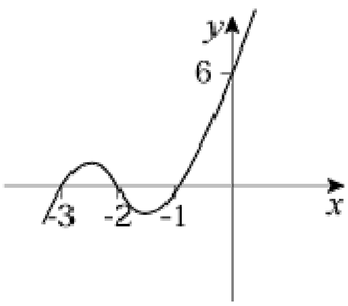


(c) $y = 0 \quad \Rightarrow \quad x = -1, -2, -3$

$x = 0 \quad \Rightarrow \quad y = 6$

$x \to \infty, y \to \infty$

$x \to -\infty, y \to -\infty$

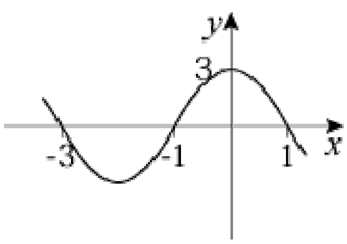


(d) $y = 0 \quad \Rightarrow \quad x = -1, 1, -3$

$x = 0 \quad \Rightarrow \quad y = 3$

$x \to \infty, y \to -\infty$

$x \to -\infty, y \to \infty$

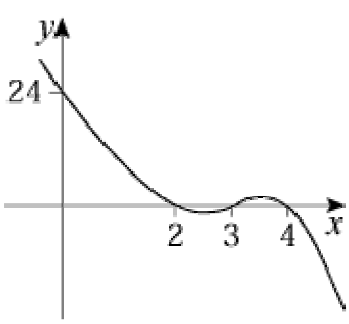


(e) $y = 0 \quad \Rightarrow \quad x = 2, 3, 4$

$x = 0 \quad \Rightarrow \quad y = 24$

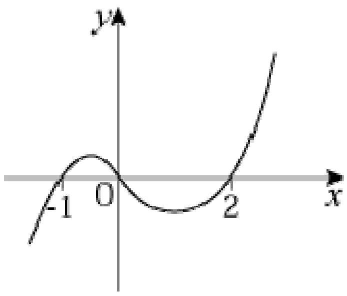
$x \to \infty, y \to -\infty$

$x \to -\infty, y \to \infty$

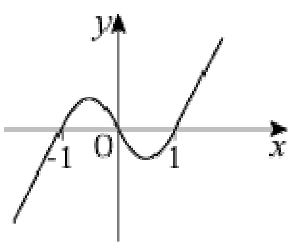


(f) $y = 0 \quad \Rightarrow \quad x = 0, -1, 2$

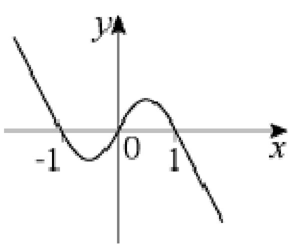
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



(g) $y = 0 \quad \Rightarrow \quad x = 0, -1, 1$
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$

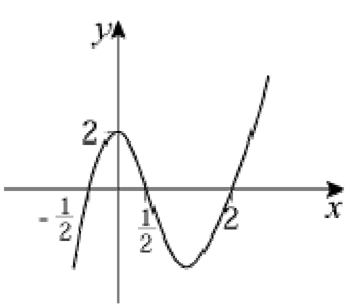


(h) $y = 0 \quad \Rightarrow \quad x = 0, -1, 1$
$x \to \infty, y \to -\infty$
$x \to -\infty, y \to \infty$



(i) $y = 0 \quad \Rightarrow \quad x = 2, \frac{1}{2}, -\frac{1}{2}$

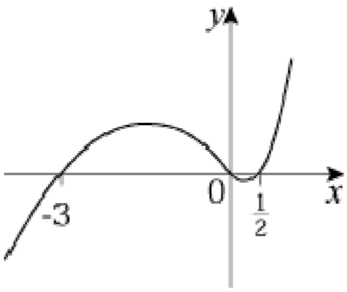
$x = 0 \quad \Rightarrow \quad y = 2$
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



(j) $y = 0 \quad \Rightarrow \quad x = 0, \frac{1}{2}, -3$

$x \to \infty, y \to \infty$

$x \to -\infty, y \to -\infty$



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Sketching curves
Exercise A, Question 2

Question:

Sketch the curves with the following equations:

(a) $y = (x + 1)^2(x - 1)$

(b) $y = (x + 2)(x - 1)^2$

(c) $y = (2 - x)(x + 1)^2$

(d) $y = (x - 2)(x + 1)^2$

(e) $y = x^2(x + 2)$

(f) $y = (x - 1)^2 x$

(g) $y = (1 - x)^2(3 + x)$

(h) $y = (x - 1)^2(3 - x)$

(i) $y = x^2(2 - x)$

(j) $y = x^2(x - 2)$

Solution:

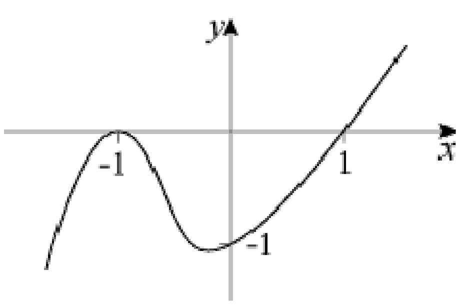
(a) $y = 0 \quad \Rightarrow \quad x = -1$ (twice), 1

$x = 0 \quad \Rightarrow \quad y = -1$

Turning point at $(-1, 0)$.

$x \to \infty, y \to \infty$

$x \to -\infty, y \to -\infty$



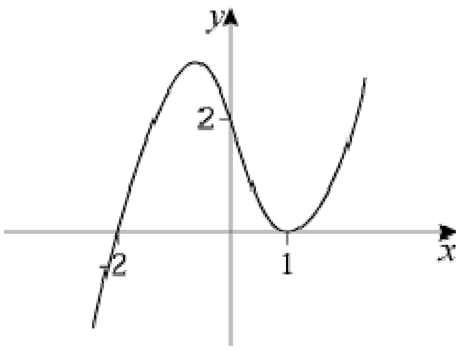
(b) $y = 0 \quad \Rightarrow \quad x = -2$, 1 (twice)

$x = 0 \quad \Rightarrow \quad y = 2$

Turning point at $(1, 0)$.

$x \to \infty, y \to \infty$

$x \to -\infty, y \to -\infty$



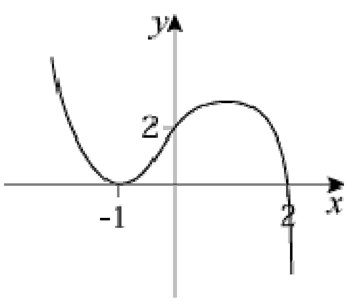
(c) $y = 0 \quad \Rightarrow \quad x = 2, \ -1$ (twice)

$x = 0 \quad \Rightarrow \quad y = 2$

Turning point at $(-1, 0)$.

$x \to \infty, y \to -\infty$

$x \to -\infty, y \to \infty$



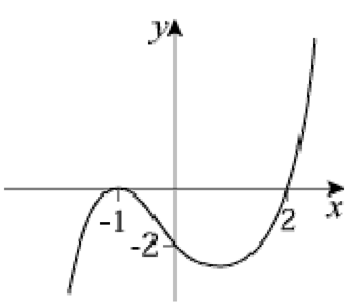
(d) $y = 0 \quad \Rightarrow \quad x = 2, \ -1$ (twice)

$x = 0 \quad \Rightarrow \quad y = -2$

Turning point at $(-1, 0)$.

$x \to \infty, y \to \infty$

$x \to -\infty, y \to -\infty$

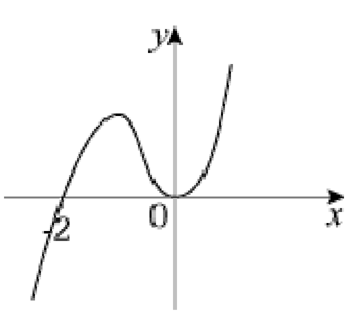


(e) $y = 0 \quad \Rightarrow \quad x = 0$ (twice), -2

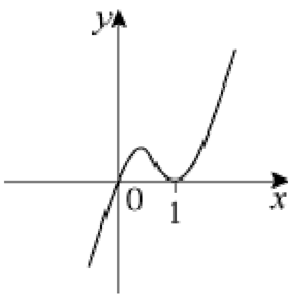
Turning point at $(0, 0)$.

$x \to \infty, y \to \infty$

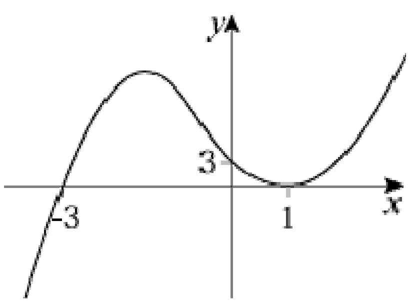
$x \to -\infty, y \to -\infty$



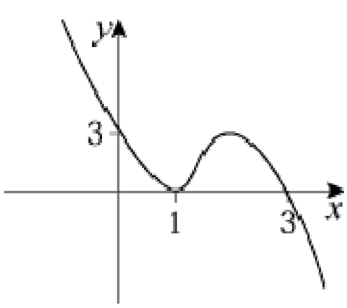
(f) $y = 0 \quad \Rightarrow \quad x = 0, 1$ (twice)
Turning point at $(1, 0)$.
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



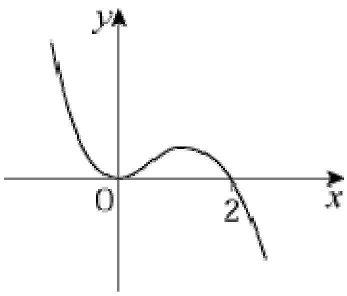
(g) $y = 0 \quad \Rightarrow \quad x = 1$ (twice), -3
$x = 0 \quad \Rightarrow \quad y = 3$
Turning point at $(1, 0)$.
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



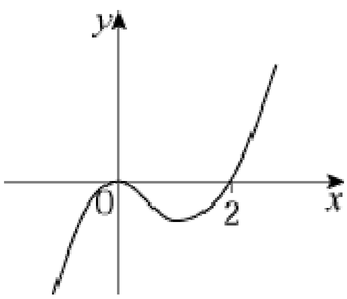
(h) $y = 0 \quad \Rightarrow \quad x = 1$ (twice), 3
$x = 0 \quad \Rightarrow \quad y = 3$
Turning point at $(1, 0)$.
$x \to \infty, y \to -\infty$
$x \to -\infty, y \to \infty$



(i) $y = 0 \quad \Rightarrow \quad x = 0$ (twice), 2
Turning point at $(0, 0)$.
$x \to \infty, y \to -\infty$
$x \to -\infty, y \to \infty$



(j) $y = 0 \quad \Rightarrow \quad x = 0$ (twice), 2
Turning point at (0 , 0).
$x \to \infty$, $y \to \infty$
$x \to -\infty$, $y \to -\infty$



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Edexcel Modular Mathematics for AS and A-Level

Sketching curves

Exercise A, Question 3

Question:

Factorise the following equations and then sketch the curves:

(a) $y = x^3 + x^2 - 2x$

(b) $y = x^3 + 5x^2 + 4x$

(c) $y = x^3 + 2x^2 + x$

(d) $y = 3x + 2x^2 - x^3$

(e) $y = x^3 - x^2$

(f) $y = x - x^3$

(g) $y = 12x^3 - 3x$

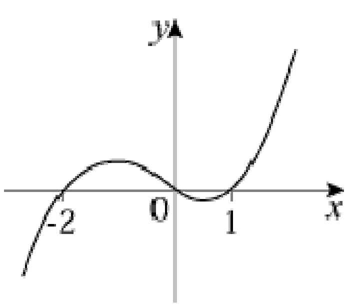
(h) $y = x^3 - x^2 - 2x$

(i) $y = x^3 - 9x$

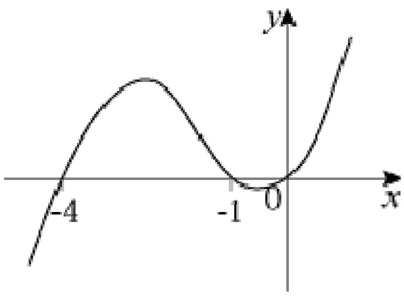
(j) $y = x^3 - 9x^2$

Solution:

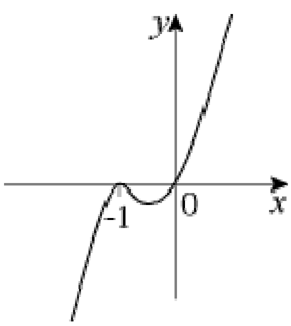
(a) $y = x^3 + x^2 - 2x = x(x^2 + x - 2)$
So $y = x(x + 2)(x - 1)$
$y = 0 \quad \Rightarrow \quad x = 0, 1, -2$
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



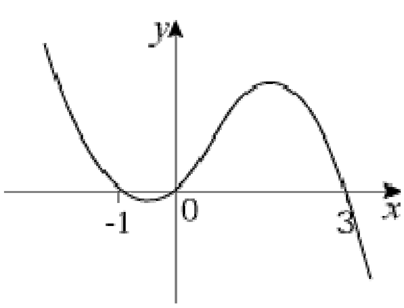
(b) $y = x^3 + 5x^2 + 4x = x(x^2 + 5x + 4)$
So $y = x(x + 4)(x + 1)$
$y = 0 \quad \Rightarrow \quad x = 0, -4, -1$
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



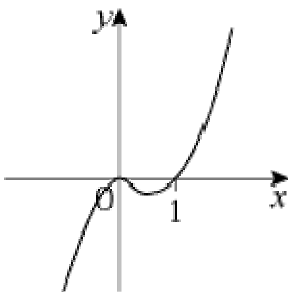
(c) $y = x^3 + 2x^2 + x = x(x^2 + 2x + 1)$
So $y = x(x+1)^2$
$y = 0 \quad \Rightarrow \quad x = 0, \ -1$ (twice)
Turning point at $(-1, 0)$.
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



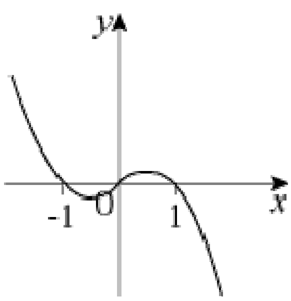
(d) $y = 3x + 2x^2 - x^3 = x(3 + 2x - x^2)$
So $y = x(3-x)(1+x)$
$y = 0 \quad \Rightarrow \quad x = 0, 3, \ -1$
$x \to \infty, y \to -\infty$
$x \to -\infty, y \to \infty$



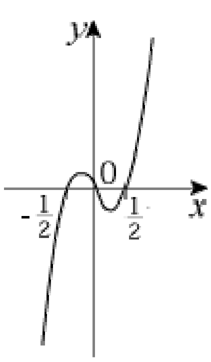
(e) $y = x^3 - x^2 = x^2(x-1)$
$y = 0 \quad \Rightarrow \quad x = 0$ (twice), 1
Turning point at $(0, 0)$.
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



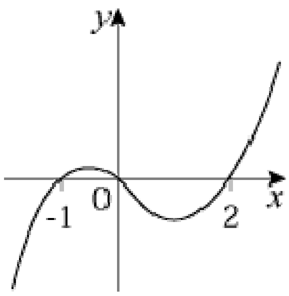
(f) $y = x - x^3 = x(1 - x^2)$
So $y = x(1 - x)(1 + x)$
$y = 0 \quad \Rightarrow \quad x = 0, 1, -1$
$x \to \infty, y \to -\infty$
$x \to -\infty, y \to \infty$



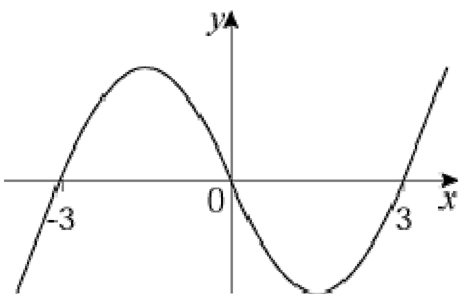
(g) $y = 12x^3 - 3x = 3x(4x^2 - 1)$
So $y = 3x(2x - 1)(2x + 1)$
$y = 0 \quad \Rightarrow \quad x = 0, \frac{1}{2}, -\frac{1}{2}$
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



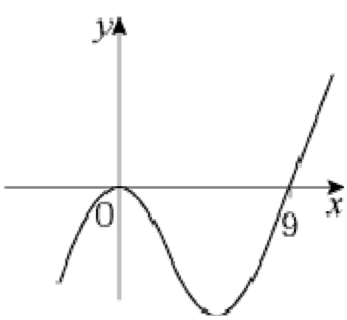
(h) $y = x^3 - x^2 - 2x = x(x^2 - x - 2)$
So $y = x(x + 1)(x - 2)$
$y = 0 \quad \Rightarrow \quad x = 0, -1, 2$
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



(i) $y = x^3 - 9x = x(x^2 - 9)$
So $y = x(x - 3)(x + 3)$
$y = 0 \Rightarrow x = 0, 3, -3$
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



(j) $y = x^3 - 9x^2 = x^2(x - 9)$
$y = 0 \Rightarrow x = 0$ (twice), 9
Turning point at (0,0).
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



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Edexcel Modular Mathematics for AS and A-Level

Sketching curves
Exercise B, Question 1

Question:

Sketch the following curves and show their positions relative to the curve $y = x^3$:

(a) $y = (x - 2)^3$

(b) $y = (2 - x)^3$

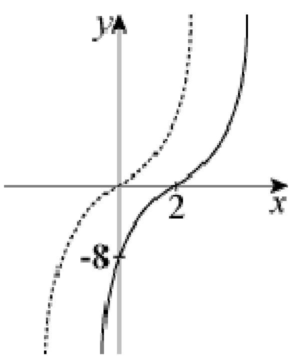
(c) $y = (x - 1)^3$

(d) $y = (x + 2)^3$

(e) $y = - (x + 2)^3$

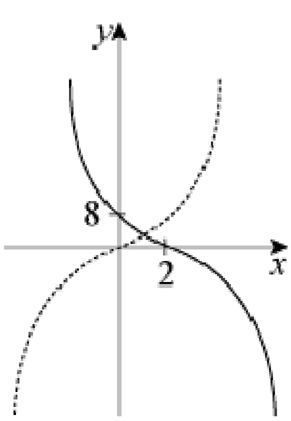
Solution:

(a) $y = 0 \Rightarrow x = 2$, so curve crosses x -axis at $(2 , 0)$
$x = 0 \Rightarrow y = - 8$, so curve crosses y-axis at $(0 , - 8)$



Curve is a translation of $+ 2$ in x direction of the curve $y = x^3$.

(b) $y = 0 \Rightarrow x = 2$, so curve crosses x-axis at $(2 , 0)$
$x = 0 \Rightarrow y = 8$, so curve crosses y-axis at $(0 , 8)$
$y = (2 - x)^3 = - (x - 2)^3$, so shape is like $y = - x^3$

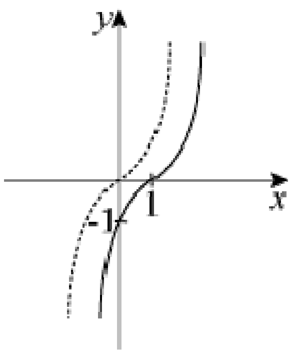


This is a horizontal translation of $+2$ of the curve $y = -x^3$.

(c) $y = 0 \Rightarrow x = 1$, so curve crosses x-axis at $(1, 0)$

$x = 0 \Rightarrow y = -1$, so curve crosses y-axis at $(0, -1)$

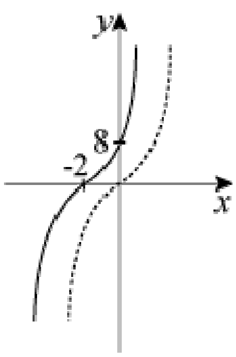
$y = (x - 1)^3$ is a horizontal translation of $+1$ of $y = x^3$.



(d) $y = 0 \Rightarrow x = -2$, so curve crosses x-axis at $(-2, 0)$

$x = 0 \Rightarrow y = 8$, so curve crosses y-axis at $(0, 8)$

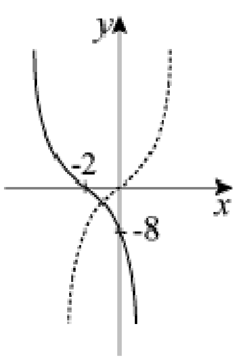
$y = (x + 2)^3$ is same shape as $y = x^3$ but translated horizontally by -2.



(e) $y = 0 \Rightarrow x = -2$, so curve crosses x-axis at $(-2, 0)$

$x = 0 \Rightarrow y = -8$, so curve crosses y-axis at $(0, -8)$

$y = -(x + 2)^3$ is a reflection in x-axis of $y = -(x + 2)^3$.



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Sketching curves
Exercise B, Question 2

Question:

Sketch the following and indicate the coordinates of the points where the curves cross the axes:

(a) $y = (x + 3)^3$

(b) $y = (x - 3)^3$

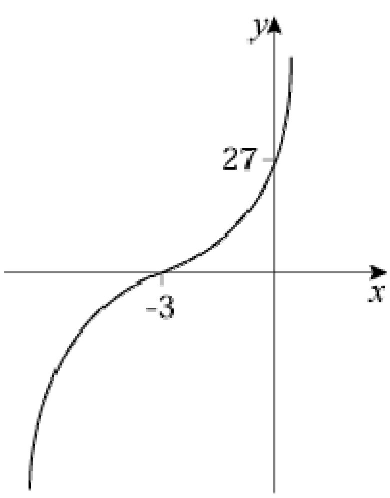
(c) $y = (1 - x)^3$

(d) $y = -(x - 2)^3$

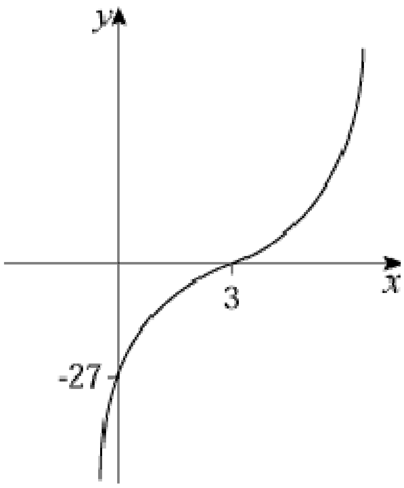
(e) $y = -(x - \frac{1}{2})^3$

Solution:

(a) $y = 0 \Rightarrow x = -3$, so curve crosses x-axis at $(-3, 0)$
$x = 0 \Rightarrow y = 27$, so curve crosses y-axis at $(0, 27)$
$y = (x + 3)^3$ is a translation of -3 in x-direction of $y = x^3$.



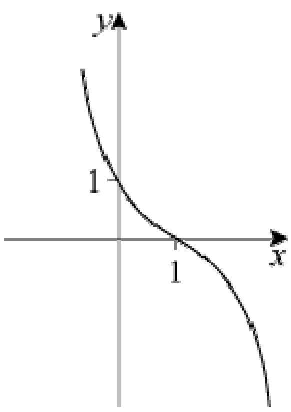
(b) $y = 0 \Rightarrow x = 3$, so curve crosses x-axis at $(3, 0)$
$x = 0 \Rightarrow y = -27$, so curve crosses y-axis at $(0, -27)$
$y = (x - 3)^3$ is a horizontal translation of $+3$ of $y = x^3$.



(c) $y = 0 \quad \Rightarrow \quad x = 1$, so curve crosses x-axis at $(1\ ,\ 0)$

$x = 0 \quad \Rightarrow \quad y = 1$, so curve crosses y-axis at $(0\ ,\ 1)$

$y = (1 - x)^3$ is a horizontal translation of $y = -x^3$.

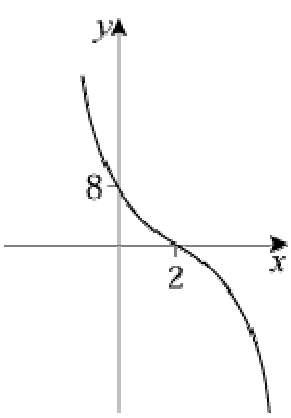


Horizontal translation $+1$ of $y = -x^3$.

(d) $y = 0 \quad \Rightarrow \quad x = 2$, so curve crosses x-axis at $(2\ ,\ 0)$

$x = 0 \quad \Rightarrow \quad y = 8$, so curve crosses y-axis at $(0\ ,\ 8)$

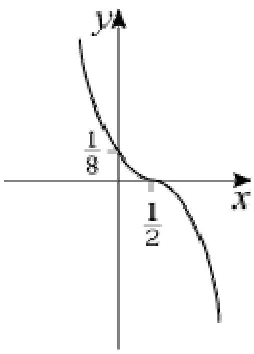
$y = -(x - 2)^3$ is a translation ($+2$ in x-direction) of $y = -x^3$.



(e) $y = 0 \quad \Rightarrow \quad x = \frac{1}{2}$, so curve crosses x-axis at $(\frac{1}{2}\ ,\ 0)$

$x = 0 \quad \Rightarrow \quad y = \frac{1}{8}$, so curve crosses y-axis at $(0\ ,\ \frac{1}{8})$

$y = -(x - \frac{1}{2})^3$ is a horizontal translation $(+\ \frac{1}{2})$ of $y = -x^3$.



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Sketching curves
Exercise C, Question 1

Question:

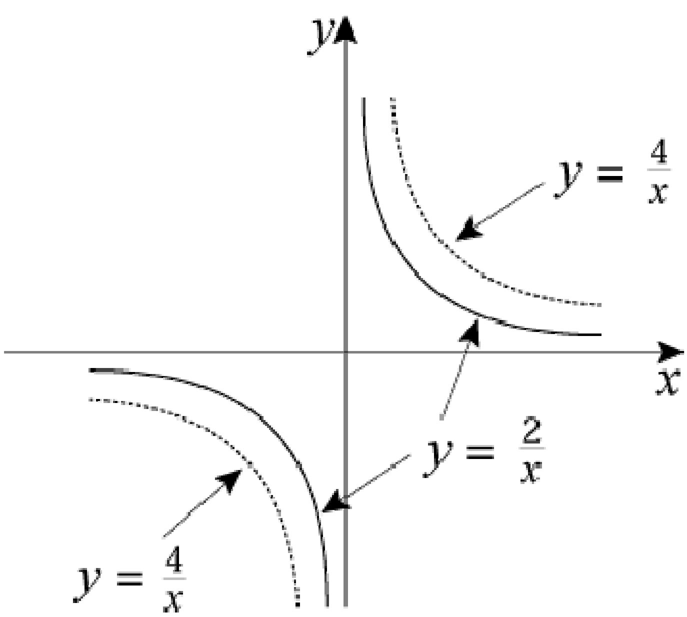
Sketch on the same diagram

$y = \frac{2}{x}$ and $y = \frac{4}{x}$

Solution:

For $x > 0$, $\frac{4}{x} > \frac{2}{x}$ (since $4 > 2$)

So $\frac{4}{x}$ is ‘on top’ of $\frac{2}{x}$ in 1st quadrant and ‘below’ in 3rd quadrant



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Sketching curves

Exercise C, Question 2

Question:

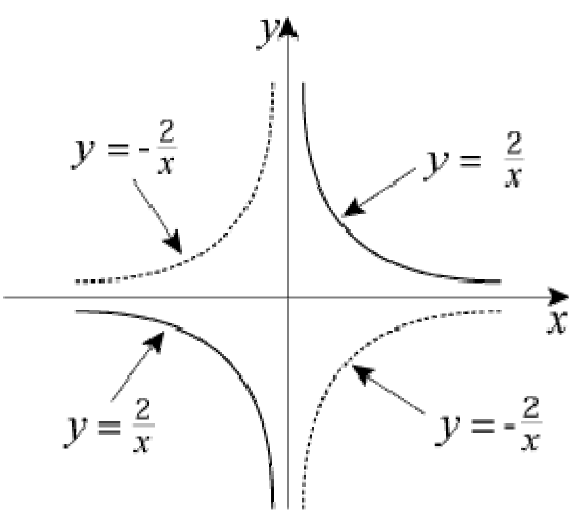
Sketch on the same diagram

$y = \frac{2}{x}$ and $y = - \frac{2}{x}$

Solution:

$y = \frac{2}{x} > 0$ for $x > 0$

$y = - \frac{2}{x} < 0$ for $x > 0$



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Sketching curves
Exercise C, Question 3

Question:

Sketch on the same diagram

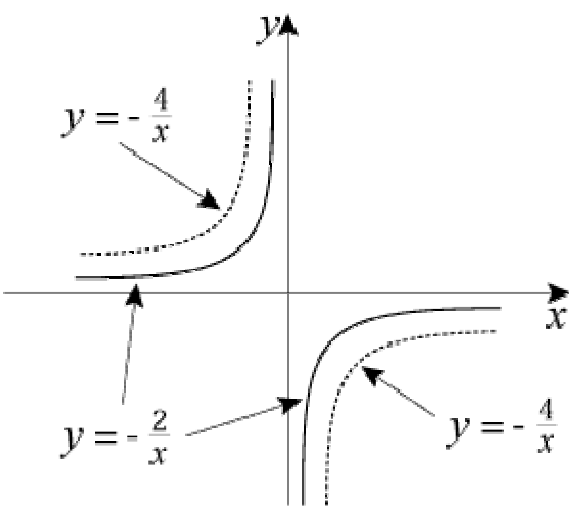
$y = -\frac{4}{x}$ and $y = -\frac{2}{x}$

Solution:

Graphs are like $y = -\frac{1}{x}$ and so exist in 2nd and 4th quadrants.

For $x < 0$, $-\frac{4}{x} > -\frac{2}{x}$

So $-\frac{4}{x}$ is 'on top' of $-\frac{2}{x}$ in 2nd quadrant and 'below' in 4th quadrant.



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Sketching curves
Exercise C, Question 4

Question:

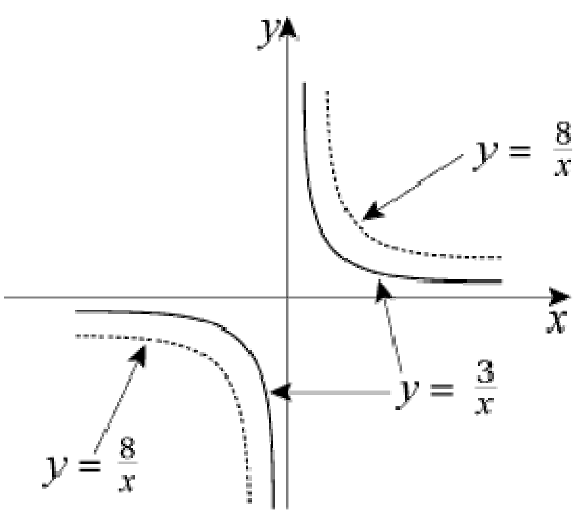
Sketch on the same diagram

$y = \frac{3}{x}$ and $y = \frac{8}{x}$

Solution:

For $x > 0$, $\frac{8}{x} > \frac{3}{x}$

So $y = \frac{8}{x}$ is 'on top' of $y = \frac{3}{x}$ in 1st quadrant and 'below' in 3rd quadrant.



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Sketching curves
Exercise C, Question 5

Question:

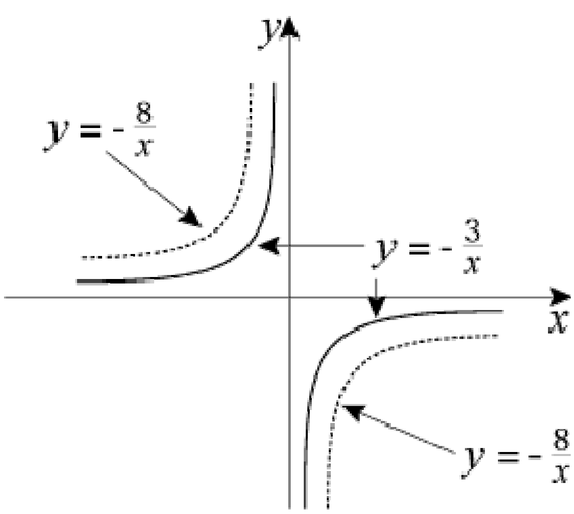
Sketch on the same diagram

$y = -\frac{3}{x}$ and $y = -\frac{8}{x}$

Solution:

For $x < 0$, $-\frac{8}{x} > -\frac{3}{x}$

So $y = -\frac{8}{x}$ is ‘on top’ of $y = -\frac{3}{x}$ in 2nd quadrant and ‘below’ in 4th quadrant.



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Sketching curves
Exercise D, Question 1

Question:

In each case:

(i) sketch the two curves on the same axes

(ii) state the number of points of intersection

(iii) write down a suitable equation which would give the x-coordinates of these points. (You are not required to solve this equation.)

(a) $y = x^2$, $y = x(x^2 - 1)$

(b) $y = x(x + 2)$, $y = -\frac{3}{x}$

(c) $y = x^2$, $y = (x + 1)(x - 1)^2$

(d) $y = x^2(1 - x)$, $y = -\frac{2}{x}$

(e) $y = x(x - 4)$, $y = \frac{1}{x}$

(f) $y = x(x - 4)$, $y = -\frac{1}{x}$

(g) $y = x(x - 4)$, $y = (x - 2)^3$

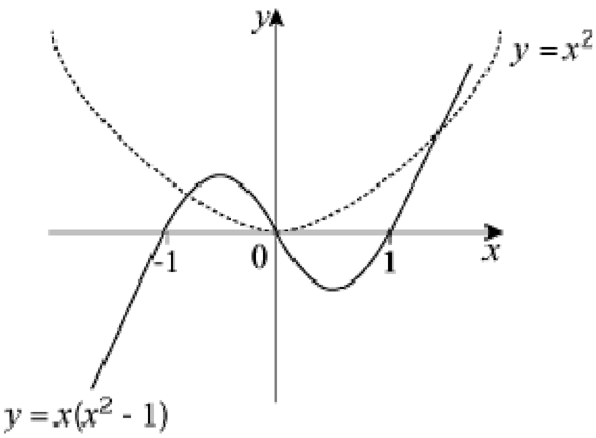
(h) $y = -x^3$, $y = -\frac{2}{x}$

(i) $y = -x^3$, $y = x^2$

(j) $y = -x^3$, $y = -x(x + 2)$

Solution:

(a) (i) $y = x^2$ is standard
$y = x(x^2 - 1) = x(x - 1)(x + 1)$
$y = 0 \quad \Rightarrow \quad x = 0, 1, -1$
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



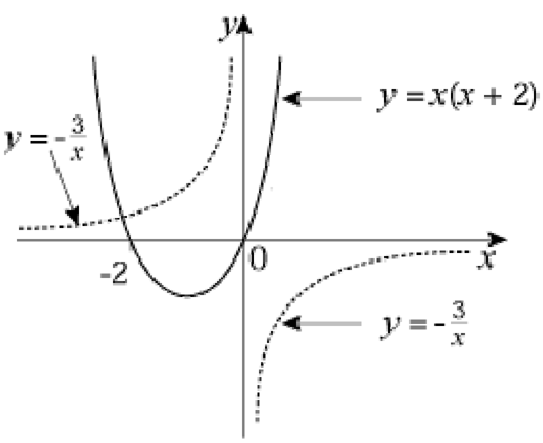
(ii) $y = x^2$ cuts $y = x(x^2 - 1)$ in 3 places.

(iii) Solutions given by $x^2 = x(x^2 - 1)$

(b) (i) $y = x(x + 2)$ is a $\cup$ -shaped curve

$y = 0 \Rightarrow x = 0, -2$

$y = -\frac{3}{x}$ is like $y = -\frac{1}{x}$



(ii) Curves cross at only 1 point.

(iii) Equation: $-\frac{3}{x} = x(x + 2)$

(c) (i) $y = x^2$ is standard

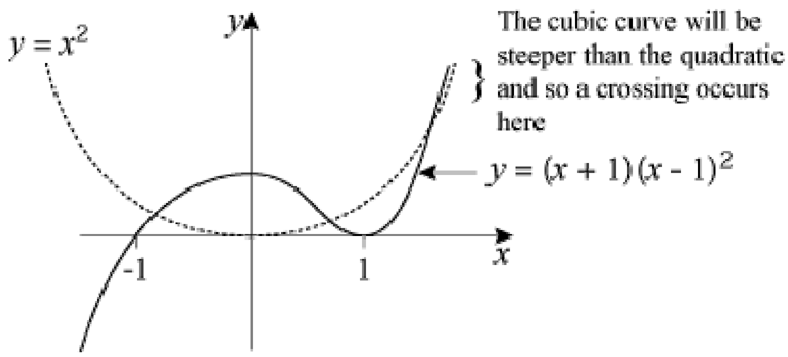
$y = (x + 1)(x - 1)^2$

$y = 0 \Rightarrow x = -1, 1$ (twice)

Turning point at (1 , 0)

$x \to \infty, y \to +\infty$

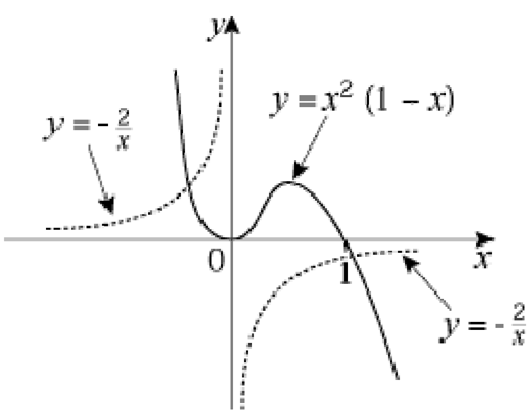
$x \to -\infty, y \to -\infty$



(ii) 3 points of intersection

(iii) Equation: $x^2 = (x + 1)(x - 1)^2$

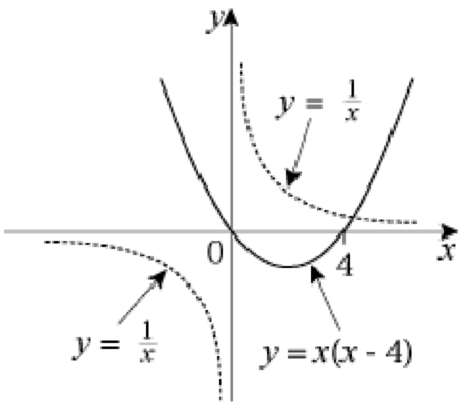
(d) (i) $y = x^2(1 - x)$
$y = 0 \Rightarrow x = 0$ (twice), 1
Turning point at (0 , 0)
$x \to \infty$, $y \to -\infty$
$x \to -\infty$, $y \to \infty$
$y = -\frac{2}{x}$ is like $y = -\frac{1}{x}$ and in 2nd and 4th quadrants



(ii) 2 points of intersection

(iii) Equation: $-\frac{2}{x} = x^2(1 - x)$

(e) (i) $y = x(x - 4)$ is a $\cup$ -shaped curve
$y = 0 \Rightarrow x = 0, 4$
$y = \frac{1}{x}$ is standard



(ii) 1 point of intersection

(iii) Equation: $\frac{1}{x}$ = $x(x - 4)$

(f) (i) $y = x(x - 4)$ is a $\cup$ -shaped curve

$y = 0 \quad \Rightarrow \quad x = 0, 4$

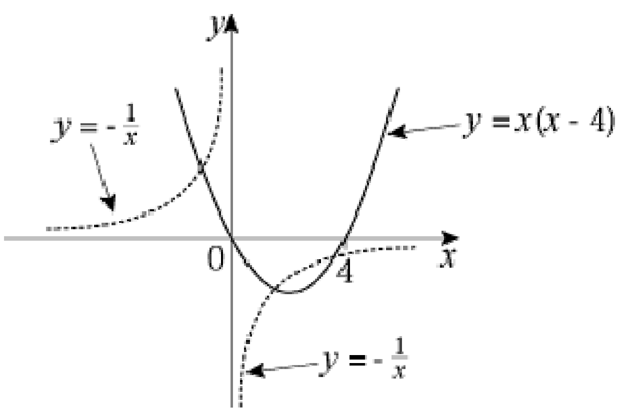
$y = -\frac{1}{x}$ is standard and in 2nd and 4th quadrants

At $x = 2$,

$y = -\frac{1}{x}$ gives $y = -\frac{1}{2}$

$y = x(x - 4)$ gives $y = 2(-2) = -4$

So $y = -\frac{1}{x}$ cuts $y = x(x - 4)$ in 4th quadrant.



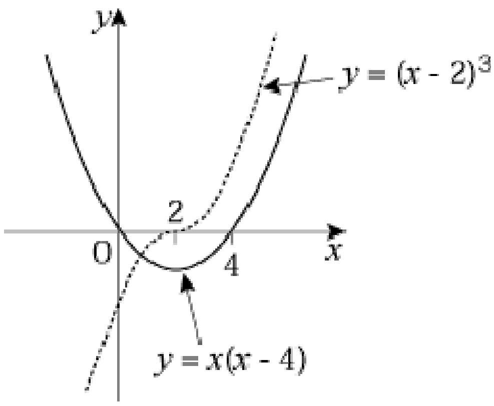
(ii) 3 points of intersection

(iii) Equation: $-\frac{1}{x} = x(x - 4)$

(g) (i) $y = x(x - 4)$ is a $\cup$ -shaped curve

$y = 0 \quad \Rightarrow \quad x = 0, 4$

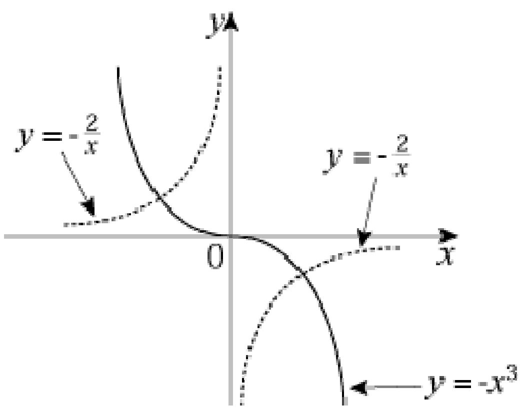
$y = (x - 2)^3$ is a translation of + 2 in the x-direction of $y = x^3$.



(ii) 1 point of intersection

(iii) $x(x-4)=(x-2)^3$

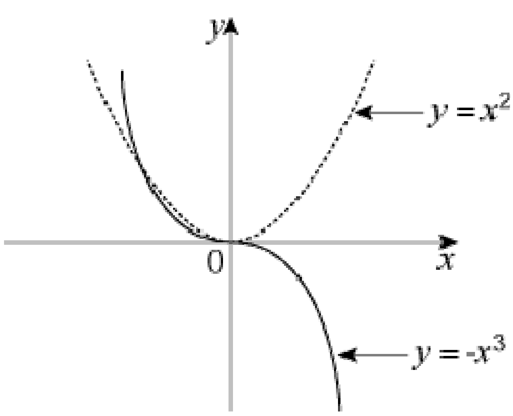
(h) (i) $y=-x^3$ is standard
$y=-\frac{2}{x}$ is like $y=-\frac{1}{x}$ and in 2nd and 4th quadrants.



(ii) 2 points of intersection

(iii) $-x^3=-\frac{2}{x}$ or $x^3=\frac{2}{x}$

(i) (i) $y=-x^3$ is standard
$y=x^2$ is standard

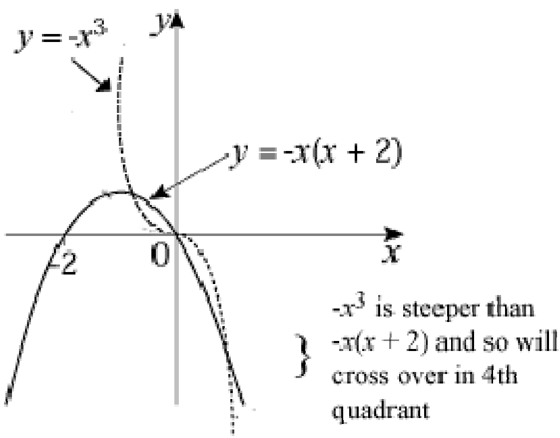


(ii) 2 points of intersection

[At (0,0) the curves actually touch. They intersect in the second quadrant.]

(iii) $x^2 = -x^3$

(j) (i) $y = -x^3$ is standard
$y = -x(x + 2)$ is $\cap$ shaped
$y = 0 \Rightarrow x = 0, -2$



(ii) 3 points of intersection

(iii) $-x^3 = -x(x + 2)$ or $x^3 = x(x + 2)$

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Sketching curves
Exercise D, Question 2

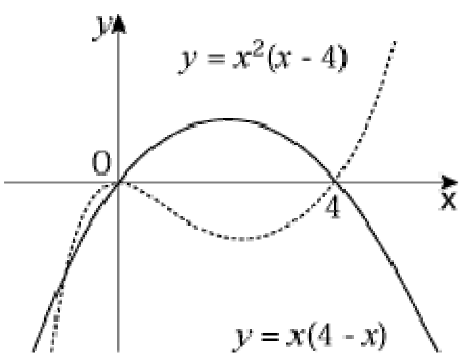
Question:

(a) On the same axes sketch the curves given by $y = x^2(x - 4)$ and $y = x(4 - x)$.

(b) Find the coordinates of the points of intersection.

Solution:

(a) $y = x^2(x - 4)$
$y = 0 \quad \Rightarrow \quad x = 0$ (twice), 4
Turning point at (0 , 0)
$y = x(4 - x)$ is $\cap$ shaped
$y = 0 \quad \Rightarrow \quad x = 0, 4$



(b) $x(4 - x) = x^2(x - 4)$
$\Rightarrow \quad 0 = x^2(x - 4) - x(4 - x)$
Factorise: $0 = x(x - 4)(x + 1)$
So intersections at $x = 0, -1, 4$
So points are [using $y = x(4 - x)$] (0, 0) ; (− 1, − 5) ; (4, 0)

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Sketching curves
Exercise D, Question 3

Question:

(a) On the same axes sketch the curves given by $y = x(2x + 5)$ and $y = x(1 + x)^2$

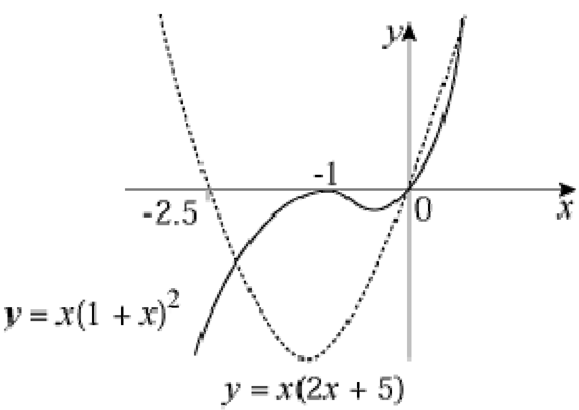
(b) Find the coordinates of the points of intersection.

Solution:

(a) $y = x(2x + 5)$ is $\cup$ shaped

$y = 0 \quad \Rightarrow \quad x = 0, \; -\frac{5}{2}$

$y = x(1 + x)^2$
$y = 0 \quad \Rightarrow \quad x = 0, \; -1$ (twice)
Turning point at $(-1, 0)$
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



(b) $x(1 + x)^2 = x(2x + 5)$
$\Rightarrow \quad x[x^2 + 2x + 1 - (2x + 5)] = 0$
$\Rightarrow \quad x(x^2 - 4) = 0$
$\Rightarrow \quad x(x - 2)(x + 2) = 0$
$\Rightarrow \quad x = 0, 2, -2$
So points are [using $y = x(2x + 5)$]: $(0, 0)$; $(2, 18)$; $(-2, -2)$

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Sketching curves
Exercise D, Question 4

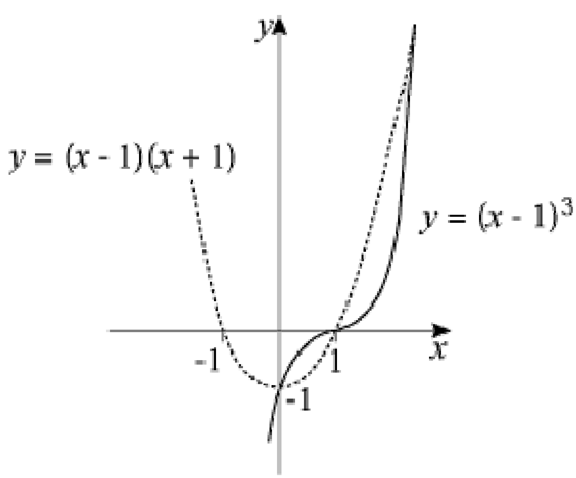
Question:

(a) On the same axes sketch the curves given by $y = (x - 1)^3$ and $y = (x - 1)(1 + x)$.

(b) Find the coordinates of the points of intersection.

Solution:

(a) $y = (x - 1)^3$ is like $y = x^3$ with crossing points at $(1, 0)$ and $(0, -1)$
$y = (x - 1)(1 + x)$ is a $\cup$ -shaped curve.
$y = 0 \Rightarrow x = 1, -1$



(b) Intersect when $(x - 1)^3 = (x - 1)(x + 1)$
i.e. $(x - 1)^3 - (x - 1)(x + 1) = 0$
$\Rightarrow \quad (x - 1)[x^2 - 2x + 1 - (x + 1)] = 0$
$\Rightarrow \quad (x - 1)(x^2 - 3x) = 0$
$\Rightarrow \quad (x - 1)(x - 3)x = 0$
$\Rightarrow \quad x = 0, 1, 3$
So intersections at $(0, -1)$; $(1, 0)$; $(3, 8)$

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Sketching curves
Exercise D, Question 5

Question:

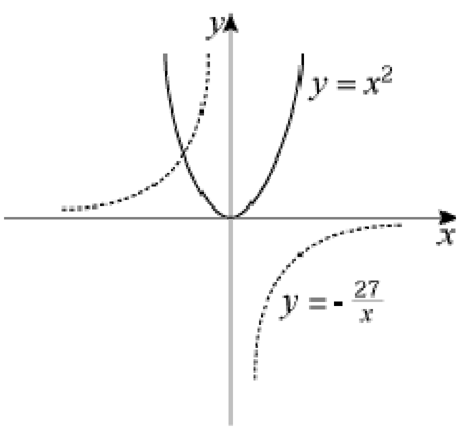
(a) On the same axes sketch the curves given by $y = x^2$ and $y = -\frac{27}{x}$.

(b) Find the coordinates of the point of intersection.

Solution:

(a) $y = -\frac{27}{x}$ is like $y = -\frac{1}{x}$ and in 2nd and 4th quadrants.

$y = x^2$ is standard



(b) $-\frac{27}{x} = x^2$

$\Rightarrow \quad -27 = x^3$

$\Rightarrow \quad x = -3$

Substitute in $y = -\frac{27}{x} \quad \Rightarrow \quad y = 9$

So intersection at $(-3 , 9)$

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Sketching curves
Exercise D, Question 6

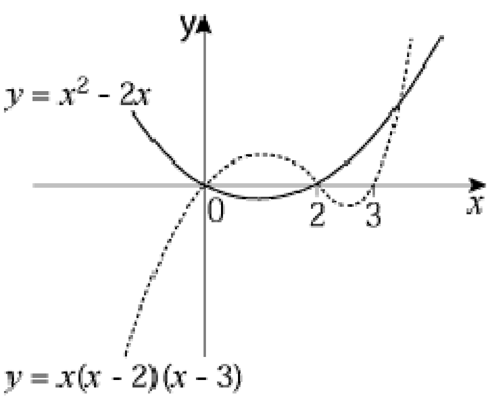
Question:

(a) On the same axes sketch the curves given by $y = x^2 - 2x$ and $y = x(x - 2)(x - 3)$.

(b) Find the coordinates of the point of intersection.

Solution:

(a) $y = x(x - 2)(x - 3)$
$y = 0 \quad \Rightarrow \quad x = 0, 2, 3$
$y = x^2 - 2x = x(x - 2)$ is $\cup$ shaped
$y = 0 \quad \Rightarrow \quad x = 0, 2$



(b) $x(x - 2) = x(x - 2)(x - 3)$
$\Rightarrow \quad 0 = x(x - 2)(x - 3 - 1)$
$\Rightarrow \quad 0 = x(x - 2)(x - 4)$
$\Rightarrow \quad x = 0, 2, 4$
Substitute in $y = x(x - 2) \quad \Rightarrow \quad y = 0, 0, 8$
So intersections at (0 , 0); (2 , 0); (4 , 8)

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Sketching curves
Exercise D, Question 7

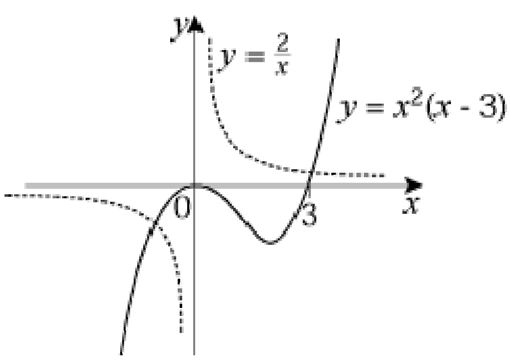
Question:

(a) On the same axes sketch the curves given by $y = x^2(x - 3)$ and $y = \frac{2}{x}$.

(b) Explain how your sketch shows that there are only two solutions to the equation $x^3(x - 3) = 2$.

Solution:

(a) $y = x^2(x - 3)$
$y = 0 \quad \Rightarrow \quad x = 0$ (twice), 3
Turning point at (0 , 0)
$y = \frac{2}{x}$ is like $y = \frac{1}{x}$



(b) Curves only cross at two points. So two solutions to

$\frac{2}{x} = x^2(x - 3)$

$2 = x^3(x - 3)$

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Sketching curves
Exercise D, Question 8

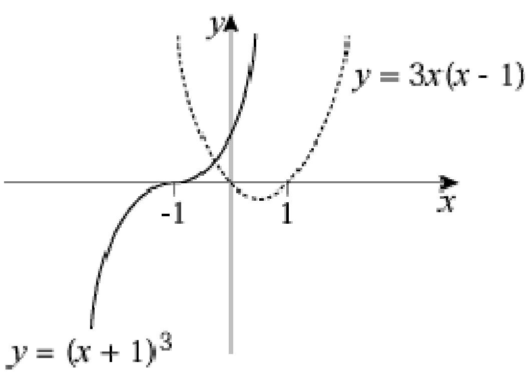
Question:

(a) On the same axes sketch the curves given by $y = (x + 1)^3$ and $y = 3x(x - 1)$.

(b) Explain how your sketch shows that there is only one solution to the equation $x^3 + 6x + 1 = 0$.

Solution:

(a) $y = (x + 1)^3$ is like $y = x^3$ and crosses at $(- 1 , 0)$ and $(0, 1)$.
$y = 3x(x - 1)$ is $\cup$ shaped
$y = 0 \Rightarrow x = 0, 1$



(b) Curves only cross once. So only one solution to
$(x + 1)^3 = 3x(x - 1)$
$x^3 + \cancel{3x^2} + 3x + 1 = \cancel{3x^2} - 3x$
$x^3 + 6x + 1 = 0$

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Sketching curves
Exercise D, Question 9

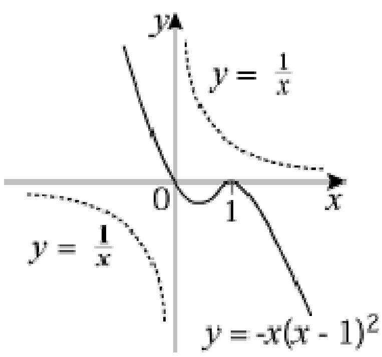
Question:

(a) On the same axes sketch the curves given by $y = \frac{1}{x}$ and $y = -x(x-1)^2$.

(b) Explain how your sketch shows that there are no solutions to the equation $1 + x^2(x-1)^2 = 0$.

Solution:

(a) $y = -x(x-1)^2$
$y = 0 \Rightarrow x = 0, 1$ (twice)
Turning point at (1 , 0)
$x \to \infty$, $y \to -\infty$
$x \to -\infty$, $y \to \infty$



(b) Curves do not cross, so no solutions to
$\frac{1}{x} = -x(x-1)^2$
$1 = -x^2(x-1)^2$
$1 + x^2(x-1)^2 = 0$

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Sketching curves
Exercise D, Question 10

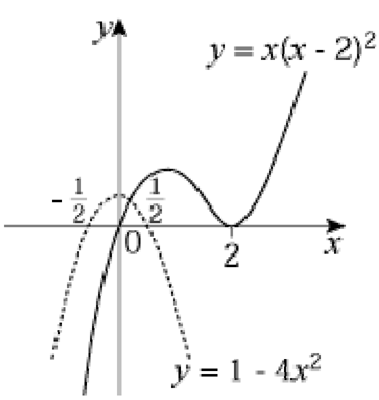
Question:

(a) On the same axes sketch the curves given by $y = 1 - 4x^2$ and $y = x(x - 2)^2$.

(b) State, with a reason, the number of solutions to the equation $x^3 + 4x - 1 = 0$.

Solution:

(a) $y = x(x - 2)^2$
$y = 0 \quad \Rightarrow \quad x = 0, 2$ (twice)
Turning point at (2 , 0)
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$
$y = 1 - 4x^2 = (1 - 2x)(1 + 2x)$ is $\cap$ shaped
$y = 0 \quad \Rightarrow \quad x = \frac{1}{2}, -\frac{1}{2}$



(b) Curves cross once. So one solution to
$1 - 4x^2 = x(x - 2)^2$
$1 - 4x^2 = x(x^2 - 4x + 4)$
$1 - 4x^2 = x^3 - 4x^2 + 4x$
$0 = x^3 + 4x - 1$

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Sketching curves
Exercise D, Question 11

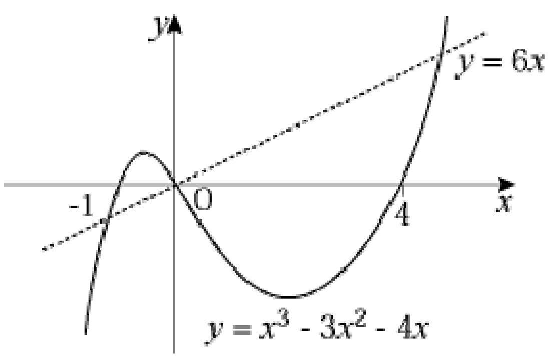
Question:

(a) On the same axes sketch the curve $y = x^3 - 3x^2 - 4x$ and the line $y = 6x$.

(b) Find the coordinates of the points of intersection.

Solution:

(a) $y = x^3 - 3x^2 - 4x = x(x^2 - 3x - 4)$
So $y = x(x - 4)(x + 1)$
$y = 0 \quad \Rightarrow \quad x = 0, \ -1, 4$
$x \to \infty , y \to \infty$
$x \to -\infty , y \to -\infty$
$y = 6x$ is a straight line through $(0 , 0)$



(b) $x^3 - 3x^2 - 4x = 6x$
$\Rightarrow \quad x^3 - 3x^2 - 10x = 0$
$\Rightarrow \quad x(x^2 - 3x - 10) = 0$
$\Rightarrow \quad x(x - 5)(x + 2) = 0$
$\Rightarrow \quad x = 0, 5, \ -2$
So (using $y = 6x$) the points of intersection are: $(0 , 0)$; $(5 , 30)$; $(-2 , -12)$

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Sketching curves

Exercise D, Question 12

Question:

(a) On the same axes sketch the curve $y = (x^2 - 1)(x - 2)$ and the line $y = 14x + 2$.

(b) Find the coordinates of the points of intersection.

Solution:

(a) $y = (x^2 - 1)(x - 2) = (x - 1)(x + 1)(x - 2)$

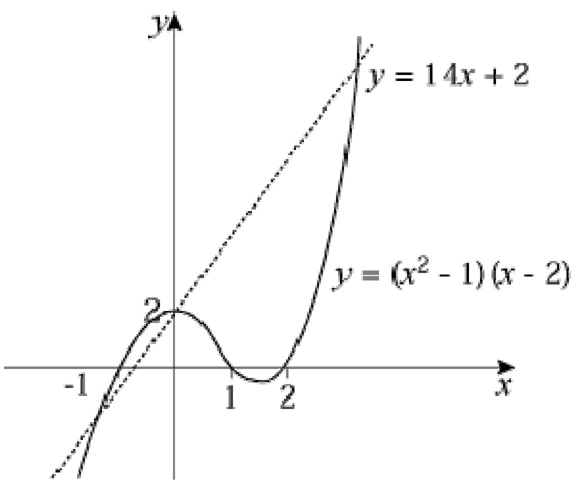
$y = 0 \quad \Rightarrow \quad x = 1, \ -1, 2$

$x = 0 \quad \Rightarrow \quad y = 2$

$x \to \infty , y \to \infty$

$x \to -\infty , y \to -\infty$

$y = 14x + 2$ is a straight line passing through $(0 , 2)$ and $\left(-\frac{1}{7} , 0\right)$.



(b) Intersection when $14x + 2 = (x^2 - 1)(x - 2)$

$\Rightarrow \quad 14x + 2 = x^3 - 2x^2 - x + 2$

$\Rightarrow \quad 0 = x^3 - 2x^2 - 15x$

$\Rightarrow \quad 0 = x(x^2 - 2x - 15)$

$\Rightarrow \quad 0 = x(x - 5)(x + 3)$

$\Rightarrow \quad x = 0, 5, \ -3$

So (using $y = 14x + 2$) the points of intersection are: $(0 , 2)$; $(5 , 72)$; $(-3 , \ -40)$

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Sketching curves
Exercise D, Question 13

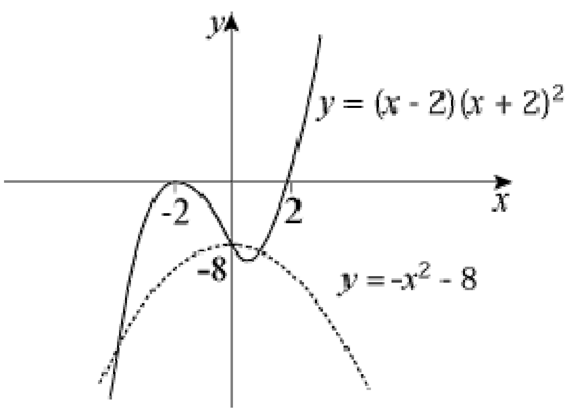
Question:

(a) On the same axes sketch the curves with equations $y = (x-2)(x+2)^2$ and $y = -x^2 - 8$.

(b) Find the coordinates of the points of intersection.

Solution:

(a) $y = (x-2)(x+2)^2$
$y = 0 \quad \Rightarrow \quad x = -2$ (twice), 2
$x = 0 \quad \Rightarrow \quad y = -8$
Turning point at $(-2, 0)$
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$
$y = -x^2 - 8$ is $\cap$ shaped with a maximum at $(0, -8)$



(b) Intersections when $-x^2 - 8 = (x+2)^2(x-2)$
$\Rightarrow \quad -x^2 - 8 = (x^2 + 4x + 4)(x - 2)$
$\Rightarrow \quad -x^2 - 8 = x^3 + 4x^2 + 4x - 2x^2 - 8x - 8$
$\Rightarrow \quad 0 = x^3 + 3x^2 - 4x$
$\Rightarrow \quad 0 = x(x^2 + 3x - 4)$
$\Rightarrow \quad 0 = x(x+4)(x-1)$
$\Rightarrow \quad x = 0, 1, -4$
So (using $y = -x^2 - 8$) points of intersection are: $(0, -8)$; $(1, -9)$; $(-4, -24)$

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Sketching curves
Exercise E, Question 1

Question:

Apply the following transformations to the curves with equations $y = f(x)$ where:

(i) $f(x) = x^2$

(ii) $f(x) = x^3$

(iii) $f(x) = \frac{1}{x}$

In each case state the coordinates of points where the curves cross the axes and in (iii) state the equations of any asymptotes.

(a) $f(x + 2)$

(b) $f(x) + 2$

(c) $f(x - 1)$

(d) $f(x) - 1$

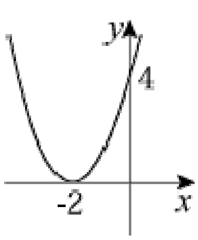
(e) $f(x) - 3$

(f) $f(x - 3)$

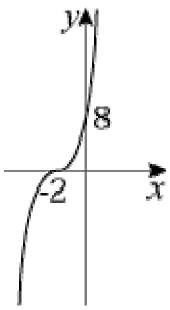
Solution:

(a) $f(x + 2)$ is a horizontal translation of -2.

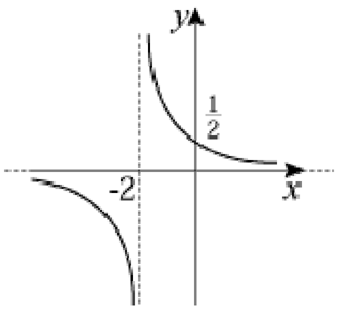
(i) $y = x^2 \rightarrow y = (x + 2)^2$



(ii) $y = x^3 \rightarrow y = (x + 2)^3$



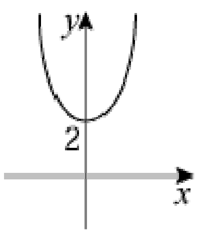
(iii) $y = \frac{1}{x} \rightarrow y = \frac{1}{x+2}$



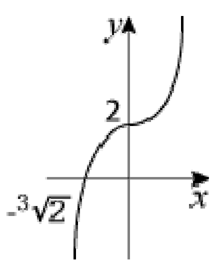
Asymptotes: $x = -2$ and $y = 0$

(b) f(x) + 2 is a vertical translation of + 2.

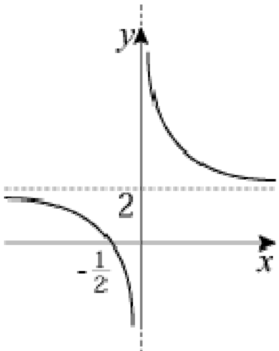
(i) $y = x^2 \rightarrow y = x^2 + 2$



(ii) $y = x^3 \rightarrow y = x^3 + 2$



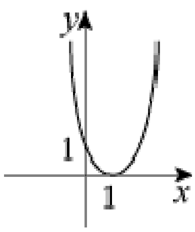
(iii) $y = \frac{1}{x} \rightarrow y = \frac{1}{x} + 2$



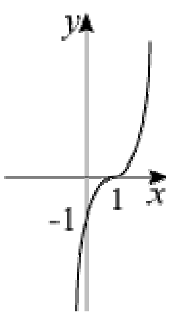
Asymptotes: $x = 0$ and $y = 2$

(c) f(x − 1) is a horizontal translation of + 1.

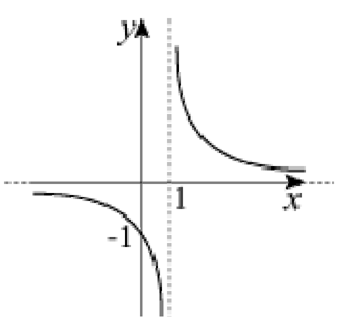
(i) $y = x^2 \rightarrow y = (x - 1)^2$



(ii) $y = x^3 \rightarrow y = (x - 1)^3$



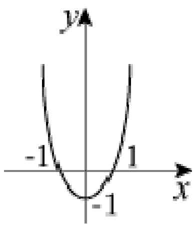
(iii) $y = \frac{1}{x} \rightarrow y = \frac{1}{x-1}$



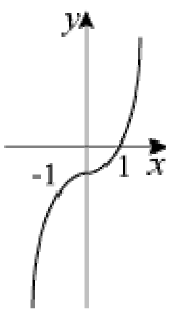
Asymptotes: $x = 1$, $y = 0$

(d) f(x) − 1 is a vertical translation of − 1.

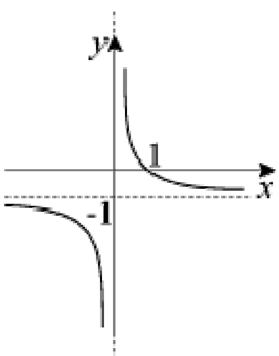
(i) $y = x^2 \rightarrow y = x^2 - 1$



(ii) $y = x^3 \rightarrow y = x^3 - 1$



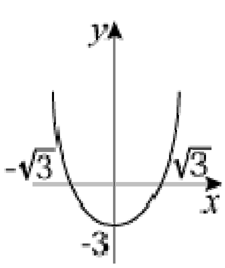
(iii) $y = \frac{1}{x} \rightarrow y = \frac{1}{x} - 1$



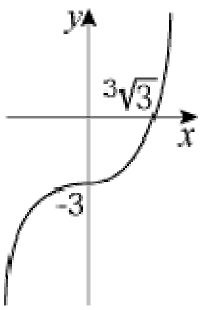
Asymptotes: $x = 0$, $y = -1$

(e) $f(x) - 3$ is a vertical translation of -3.

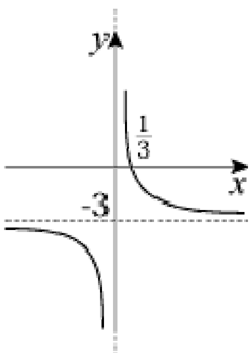
(i) $y = x^2 \rightarrow y = x^2 - 3$



(ii) $y = x^3 \rightarrow y = x^3 - 3$



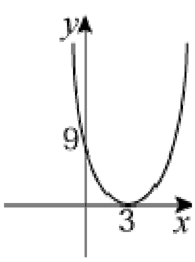
(iii) $y = \frac{1}{x} \to y = \frac{1}{x} - 3$



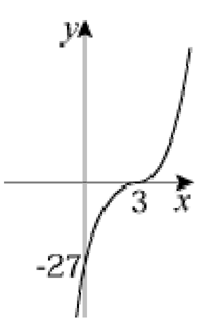
Asymptotes: $x = 0$, $y = -3$

(f) $f(x - 3)$ is a horizontal translation of $+3$.

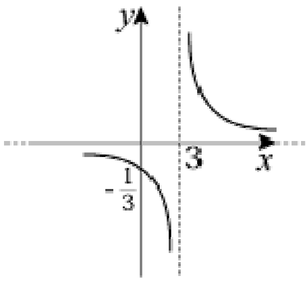
(i) $y = x^2 \to y = (x - 3)^2$



(ii) $y = x^3 \to y = (x - 3)^3$



(iii) $y = \frac{1}{x} \to y = \frac{1}{x - 3}$



Asymptotes: $x = 3$, $y = 0$

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Sketching curves
Exercise E, Question 2

Question:

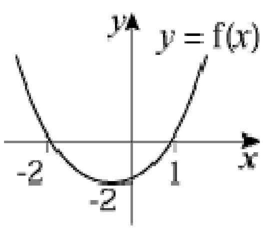
(a) Sketch the curve $y = f(x)$ where $f(x) = (x - 1)(x + 2)$.

(b) On separate diagrams sketch the graphs of (i) $y = f(x + 2)$ (ii) $y = f(x) + 2$.

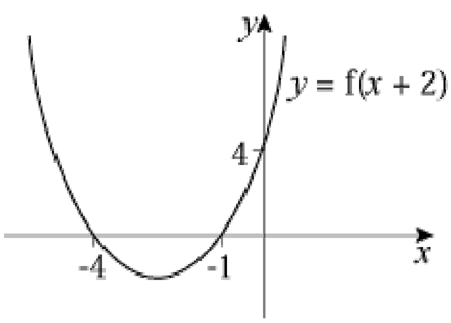
(c) Find the equations of the curves $y = f(x + 2)$ and $y = f(x) + 2$, in terms of x, and use these equations to find the coordinates of the points where your graphs in part (b) cross the y-axis.

Solution:

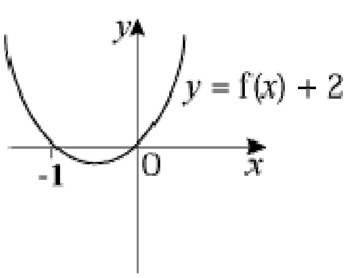
(a) $f(x) = 0 \quad \Rightarrow \quad x = 1, - 2$



(b)(i) $f(x + 2)$ is a horizontal translation of $- 2$.



(ii) $f(x) + 2$ is a vertical translation of $+ 2$



Since axis of symmetry of $f(x)$ is at $x = - \frac{1}{2}$, the same axis of symmetry applies to $f(x) + 2$. Since one root is at $x = 0$, the other must be symmetric at $x = - 1$.

(c) $y = f(x + 2)$ is $y = (x + 1)(x + 4)$. So $x = 0 \quad \Rightarrow \quad y = 4$

$y = \mathrm{f}(x) + 2$ is $y = x^2 + x = x(x + 1)$. So $x = 0 \quad \Rightarrow \quad y = 0$

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sketching curves
Exercise E, Question 3

Question:

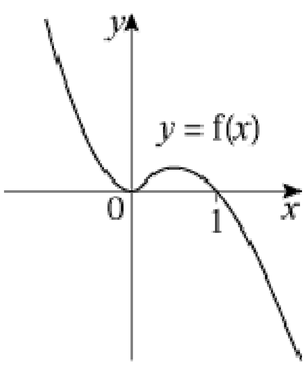
(a) Sketch the graph of $y = \mathrm{f}(x)$ where $\mathrm{f}(x) = x^2(1 - x)$.

(b) Sketch the curve with equation $y = \mathrm{f}(x + 1)$.

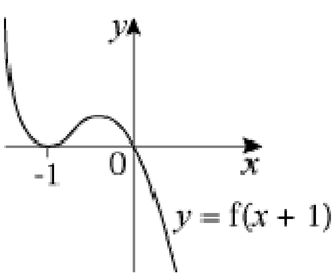
(c) By finding the equation $\mathrm{f}(x + 1)$ in terms of x, find the coordinates of the point in part (b) where the curve crosses the y-axis.

Solution:

(a) $y = x^2(1 - x)$
$y = 0 \quad \Rightarrow \quad x = 0$ (twice), 1
Turning point at (0 , 0)
$x \to \infty$, $y \to -\infty$
$x \to -\infty$, $y \to \infty$



(b) $\mathrm{f}(x + 1)$ is a horizontal translation of -1.



(c) $\mathrm{f}(x + 1) = (x + 1)^2 [1 - (x + 1)] = -(x + 1)^2 x$
So $y = 0 \quad \Rightarrow \quad x = 0$, i.e. curve passes through (0 , 0).

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Sketching curves

Exercise E, Question 4

Question:

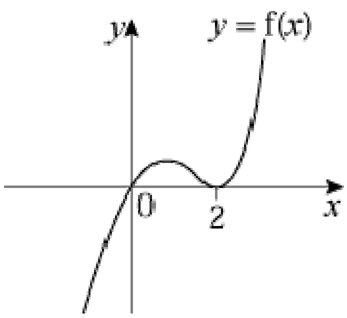
(a) Sketch the graph of $y = \mathrm{f}(x)$ where $\mathrm{f}(x) = x(x - 2)^2$.

(b) Sketch the curves with equations $y = \mathrm{f}(x) + 2$ and $y = \mathrm{f}(x + 2)$.

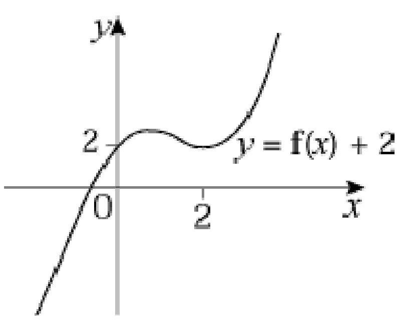
(c) Find the coordinates of the points where the graph of $y = \mathrm{f}(x + 2)$ crosses the axes.

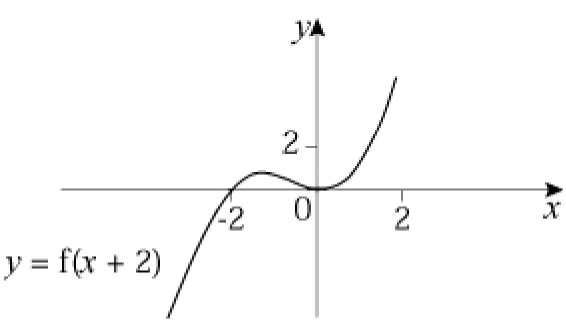
Solution:

(a) $y = x(x - 2)^2$
$y = 0 \quad \Rightarrow \quad x = 0, 2$ (twice)
Turning point at $(2, 0)$
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



(b)





(c) $\mathrm{f}(x + 2) = 0$ at points where $(x + 2)[(x + 2) - 2]^2 = 0$

$\Rightarrow \quad (x + 2)(x)^2 = 0$

$\Rightarrow \quad x = 0$ and $x = -2$

So graph crosses axes at $(0 , 0)$; $(-2 , 0)$.

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Sketching curves
Exercise E, Question 5

Question:

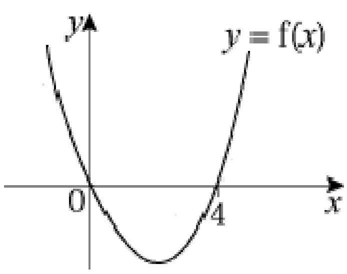
(a) Sketch the graph of $y = f(x)$ where $f(x) = x(x - 4)$.

(b) Sketch the curves with equations $y = f(x + 2)$ and $y = f(x) + 4$.

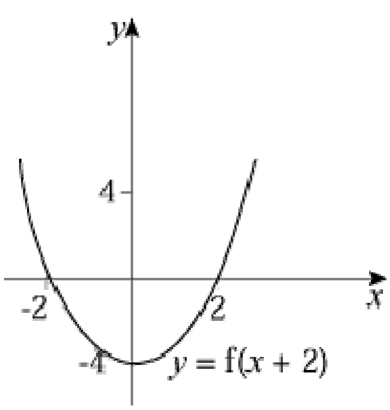
(c) Find the equations of the curves in part (b) in terms of x and hence find the coordinates of the points where the curves cross the axes.

Solution:

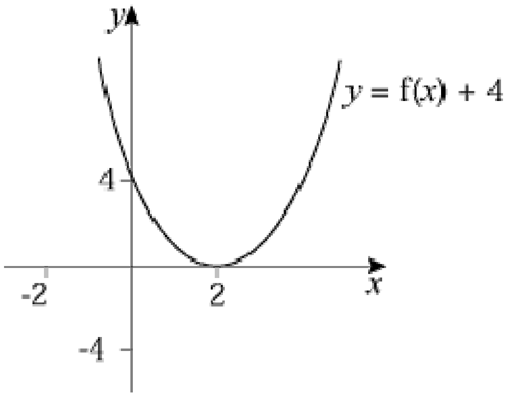
(a) $y = x(x - 4)$ is $\cup$ shaped and passes though (0 , 0) and (4 , 0).



(b) $f(x + 2)$ is a horizontal translation of $- 2$.



$f(x) + 4$ is a vertical translation of $+ 4$.



(c) $f(x + 2) = (x + 2)[(x + 2) - 4] = (x + 2)(x - 2)$

$y = 0 \quad \Rightarrow \quad x = -2, 2$

$f(x) + 4 = x(x - 4) + 4 = x^2 - 4x + 4 = (x - 2)^2$

$y = 0 \quad \Rightarrow \quad x = 2$

The minimum point on $y = f(x)$ is when $x = 2$ (by symmetry) and then $f(2) = -4$.
So $y = f(x + 2)$ crosses y-axis at $(0, -4)$
and $y = f(x) + 4$ touches x-axis at $(2, 0)$.

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Sketching curves
Exercise F, Question 1

Question:

Apply the following transformations to the curves with equations $y = \mathrm{f}(x)$ where:

(i) $\mathrm{f}(x) = x^2$

(ii) $\mathrm{f}(x) = x^3$

(iii) $\mathrm{f}(x) = \frac{1}{x}$

In each case show both $\mathrm{f}(x)$ and the transformation on the same diagram.

(a) $\mathrm{f}(2x)$

(b) $\mathrm{f}(-x)$

(c) $\mathrm{f}(\frac{1}{2}x)$

(d) $\mathrm{f}(4x)$

(e) $\mathrm{f}(\frac{1}{4}x)$

(f) $2\mathrm{f}(x)$

(g) $-\mathrm{f}(x)$

(h) $4\mathrm{f}(x)$

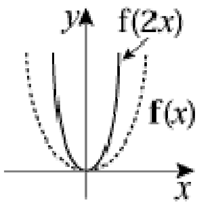
(i) $\frac{1}{2}\mathrm{f}(x)$

(j) $\frac{1}{4}\mathrm{f}(x)$

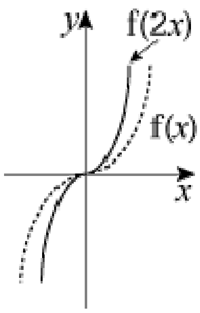
Solution:

(a) $\mathrm{f}(2x)$ means multiply x-coordinates by $\frac{1}{2}$.

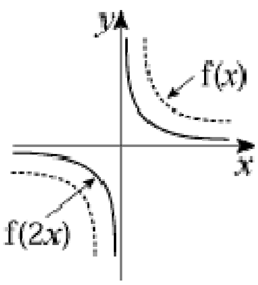
(i) $y = x^2 \rightarrow y = (2x)^2 = 4x^2$



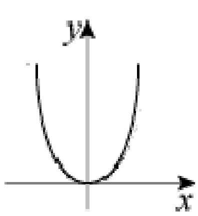
(ii) $y = x^3 \rightarrow y = (2x)^3 = 8x^3$



(iii) $y = \frac{1}{x} \rightarrow y = \frac{1}{2x} = \frac{1}{2} \times \frac{1}{x}$

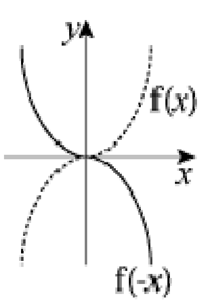


(b) (i) $y = x^2 \rightarrow y = (- x)^2 = x^2$

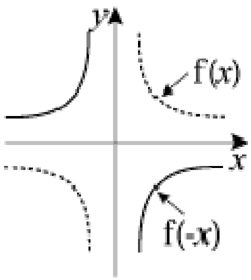


$f(x) = f(-x)$

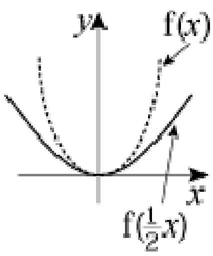
(ii) $y = x^3 \rightarrow y = (- x)^3 = - x^3$



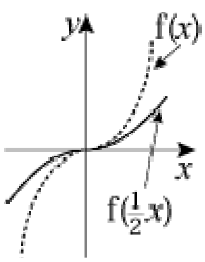
(iii) $y = \frac{1}{x} \rightarrow y = \frac{1}{-x} = -\frac{1}{x}$



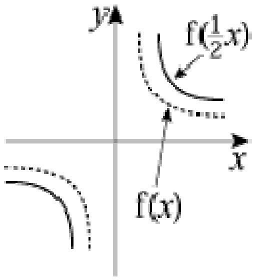
(c) (i) $y = x^3 \rightarrow y = (\frac{1}{2}x)^2 = \frac{x^2}{4}$



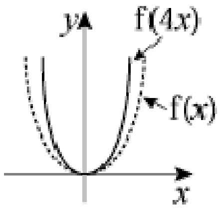
(ii) $y = x^3 \rightarrow y = (\frac{1}{2}x)^3 = \frac{x^3}{8}$



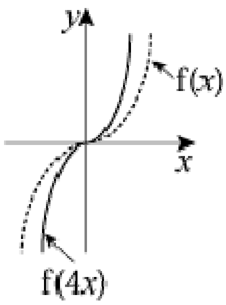
(iii) $y = \frac{1}{x} \rightarrow y = \frac{1}{\frac{1}{2}x} = \frac{2}{x}$



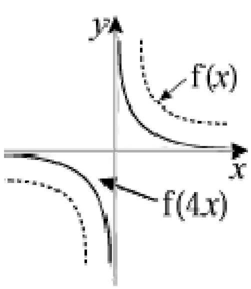
(d) (i) $y = x^2 \rightarrow y = (4x)^2 = 16x^2$



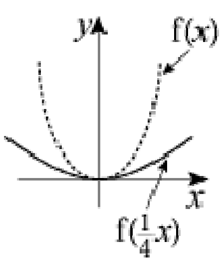
(ii) $y = x^3 \rightarrow y = (4x)^3 = 64x^3$



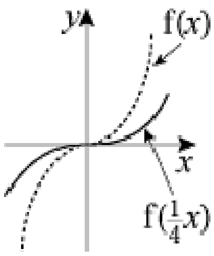
(iii) $y = \frac{1}{x} \rightarrow y = \frac{1}{4x} = \frac{1}{4} \times \frac{1}{x}$



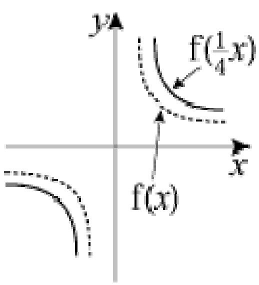
(e) (i) $y = x^2 \rightarrow y = (\frac{1}{4}x)^2 = \frac{x^2}{16}$



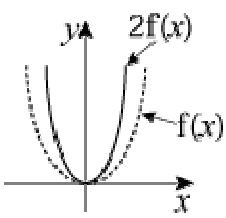
(ii) $y = x^3 \rightarrow y = (\frac{1}{4}x)^3 = \frac{x^3}{64}$



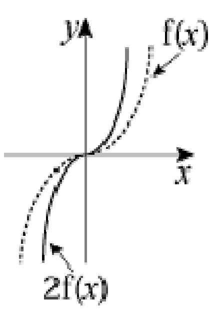
(iii) $y = \frac{1}{x} \rightarrow y = \frac{1}{\frac{1}{4}x} = \frac{4}{x}$



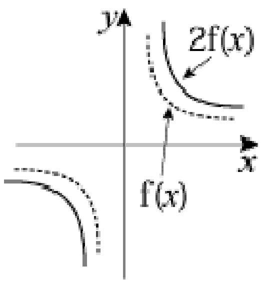
(f) (i) $y = x^2 \rightarrow y = 2x^2$



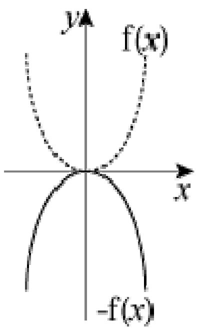
(ii) $y = x^3 \rightarrow y = 2x^3$



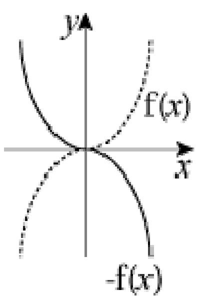
(iii) $y = \frac{1}{x} \rightarrow y = 2 \times \frac{1}{x} = \frac{2}{x}$



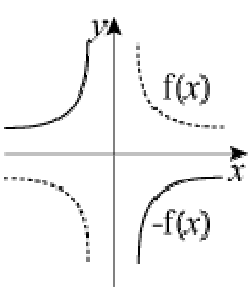
(g) (i) $y = x^2 \rightarrow y = -x^2$



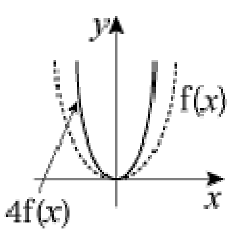
(ii) $y = x^3 \rightarrow y = -x^3$



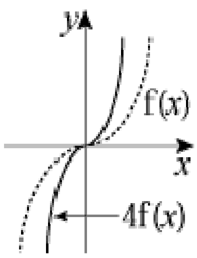
(iii) $y = \frac{1}{x} \rightarrow y = -\frac{1}{x}$



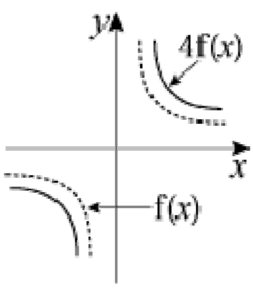
(h) (i) $y = x^2 \rightarrow y = 4x^2$



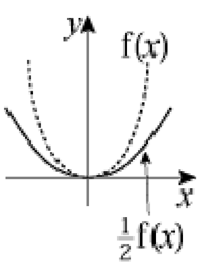
(ii) $y = x^3 \rightarrow y = 4x^3$



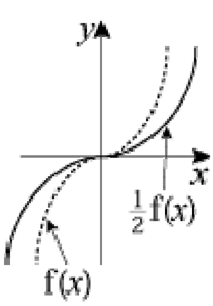
(iii) $y = \frac{1}{x} \rightarrow y = \frac{4}{x}$



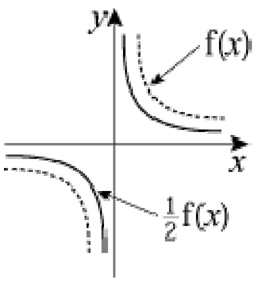
(i) (i) $y = x^2 \rightarrow y = \frac{1}{2}x^2$



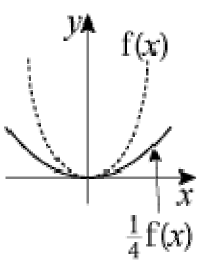
(ii) $y = x^3 \rightarrow y = \frac{1}{2}x^3$



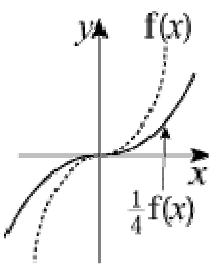
(iii) $y = \frac{1}{x} \rightarrow y = \frac{1}{2} \times \frac{1}{x}$



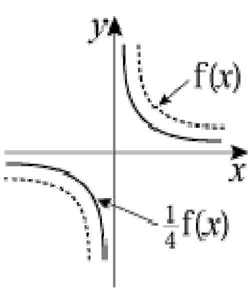
(j) (i) $y = x^2 \rightarrow y = \frac{1}{4}x^2$



(ii) $y = x^3 \rightarrow y = \frac{1}{4}x^3$



(iii) $y = \frac{1}{x} \rightarrow y = \frac{1}{4} \times \frac{1}{x}$



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Sketching curves
Exercise F, Question 2

Question:

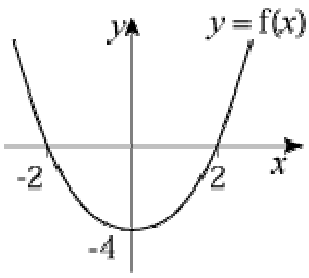
(a) Sketch the curve with equation $y = \mathrm{f}(x)$ where $\mathrm{f}(x) = x^2 - 4$.

(b) Sketch the graphs of $y = \mathrm{f}(4x)$, $y = 3\mathrm{f}(x)$, $y = \mathrm{f}(-x)$ and $y = -\mathrm{f}(x)$.

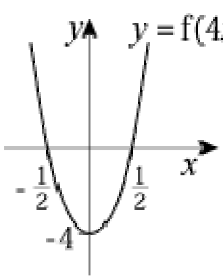
Solution:

(a) $y = x^2 - 4 = (x - 2)(x + 2)$ and is $\cup$ shaped

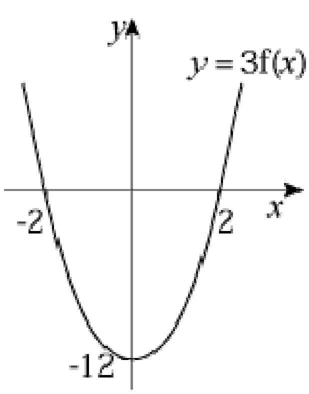
$y = 0 \quad \Rightarrow \quad x = 2, \ -2$



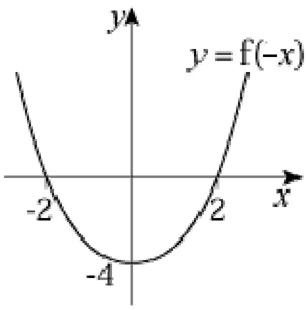
(b) $\mathrm{f}(4x)$ is a stretch $\times \frac{1}{4}$ horizontally



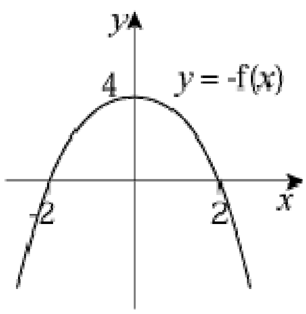
$3\mathrm{f}(x)$ is a stretch $\times 3$ vertically



$\mathrm{f}(-x)$ is a reflection in y-axis



− f(x) is a reflection in x-axis



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Sketching curves
Exercise F, Question 3

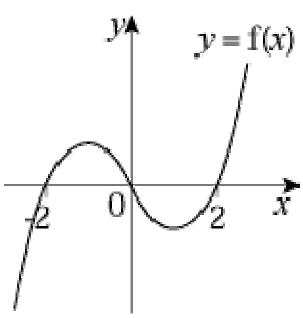
Question:

(a) Sketch the curve with equation $y = f(x)$ where $f(x) = (x - 2)(x + 2)x$.

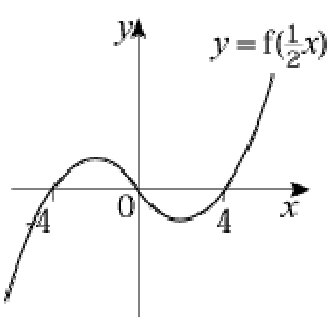
(b) Sketch the graphs of $y = f(\frac{1}{2}x)$, $y = f(2x)$ and $y = -f(x)$.

Solution:

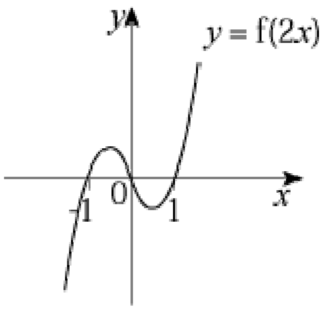
(a) $y = (x - 2)(x + 2)x$
$y = 0 \quad \Rightarrow \quad x = 2, -2, 0$
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



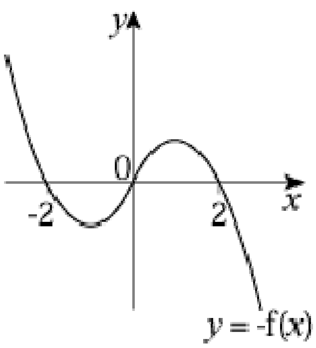
(b) $f(\frac{1}{2}x)$ is a stretch $\times 2$ horizontally



$f(2x)$ is a stretch $\times \frac{1}{2}$ horizontally



$-$ f(x) is a reflection in x-axis



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Sketching curves
Exercise F, Question 4

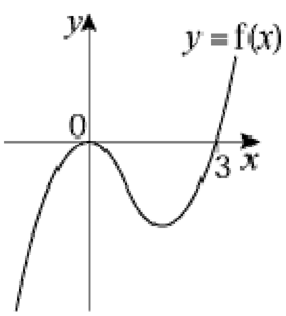
Question:

(a) Sketch the curve with equation $y = \mathrm{f}(x)$ where $\mathrm{f}(x) = x^2(x - 3)$.

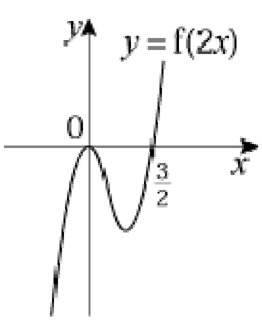
(b) Sketch the curves with equations $y = \mathrm{f}(2x)$, $y = -\mathrm{f}(x)$ and $y = \mathrm{f}(-x)$.

Solution:

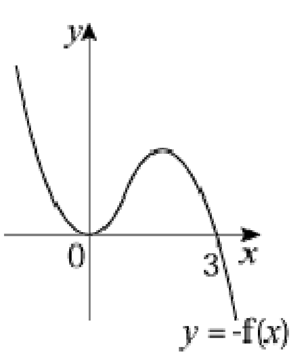
(a) $y = x^2(x - 3)$
$y = 0 \Rightarrow x = 0$ (twice), 3
Turning point at (0 , 0)
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



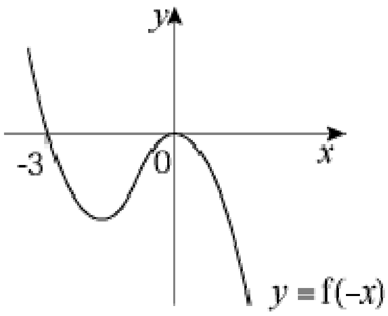
(b) f(2x) is a stretch $\times \frac{1}{2}$ horizontally



$-\mathrm{f}(x)$ is a reflection in x-axis



f(− x) is a reflection in y-axis



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Sketching curves
Exercise F, Question 5

Question:

(a) Sketch the curve with equation $y = \mathrm{f}(x)$ where $\mathrm{f}(x) = (x - 2)(x - 1)(x + 2)$.

(b) Sketch the curves with equations $y = \mathrm{f}(2x)$ and $\mathrm{f}(\frac{1}{2}x)$.

Solution:

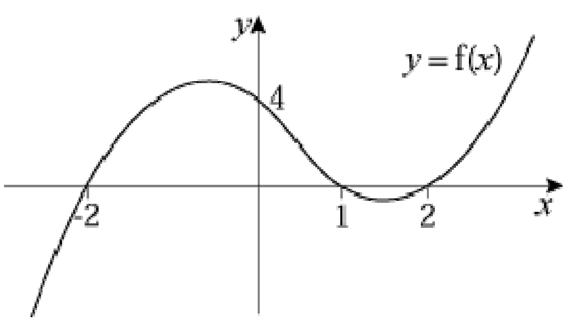
(a) $y = (x - 2)(x - 1)(x + 2)$

$y = 0 \quad \Rightarrow \quad x = 2, 1, -2$

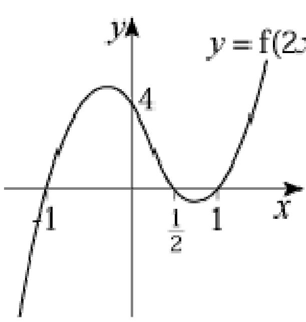
$x = 0 \quad \Rightarrow \quad y = 4$

$x \to \infty , y \to \infty$

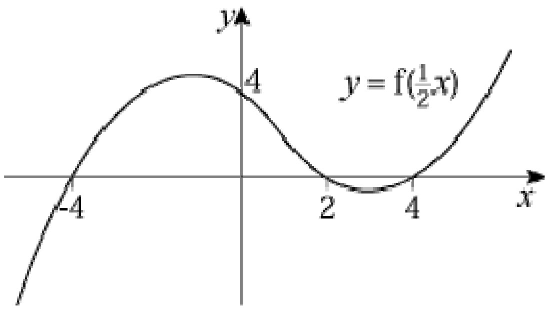
$x \to -\infty , y \to -\infty$



(b) $\mathrm{f}(2x)$ is a stretch $\times \frac{1}{2}$ horizontally



$\mathrm{f}(\frac{1}{2}x)$ is a stretch $\times 2$ horizontally


y
4
$y = f(\frac{1}{2}x)$
-4
2
4
x

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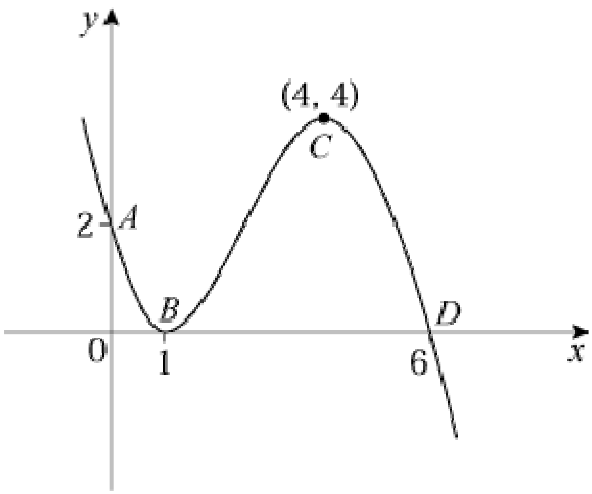
Sketching curves
Exercise G, Question 1

Question:

The following diagram shows a sketch of the curve with equation y = f(x). The points A(0, 2), B(1, 0), C(4, 4) and D(6, 0) lie on the curve.



Sketch the following graphs and give the coordinates of the points A, B, C and D after each transformation:

(a) f(x + 1)

(b) f(x) − 4

(c) f(x + 4)

(d) f(2x)

(e) 3f(x)

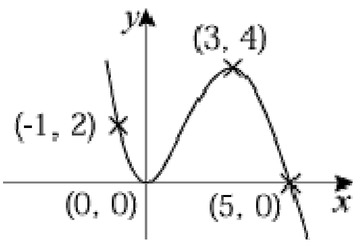
(f) f($\frac{1}{2}x$)

(g) $\frac{1}{2}$f(x)

(h) f(− x)

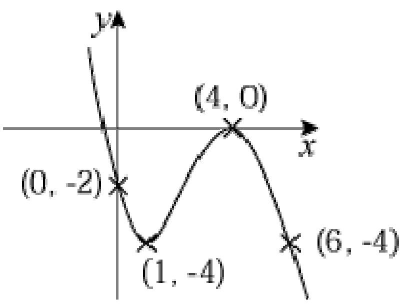
Solution:

(a) f(x + 1) is a translation of − 1 horizontally.



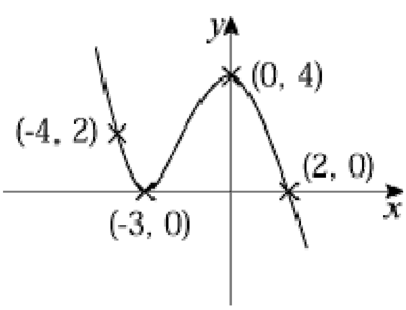
$A'(-1, 2)$; $B'(0, 0)$; $C'(3, 4)$; $D'(5, 0)$

(b) $f(x) - 4$ is a vertical translation of -4.



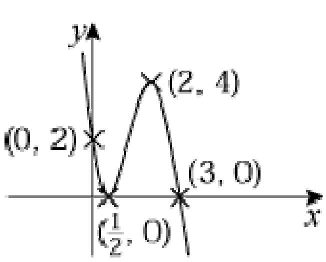
$A'(0, -2)$; $B'(1, -4)$; $C'(4, 0)$; $D'(6, -4)$

(c) $f(x + 4)$ is a translation of -4 horizontally.



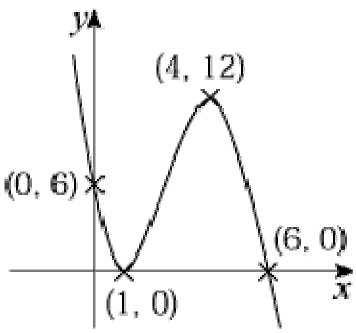
$A'(-4, 2)$; $B'(-3, 0)$; $C'(0, 4)$; $D'(2, 0)$

(d) $f(2x)$ is a stretch of $\frac{1}{2}$ horizontally.



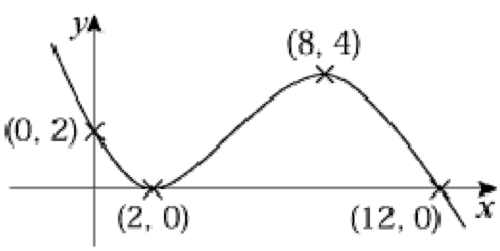
$A'(0, 2)$; $B'(\frac{1}{2}, 0)$; $C'(2, 4)$; $D'(3, 0)$

(e) $3f(x)$ is a stretch of 3 vertically.



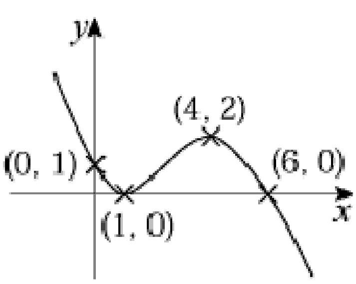
$A'(0\ ,\ 6)$; $B'(1\ ,\ 0)$; $C'(4\ ,\ 12)$; $D'(6\ ,\ 0\ )$

(f) $f(\frac{1}{2}x)$ is a stretch of 2 horizontally.



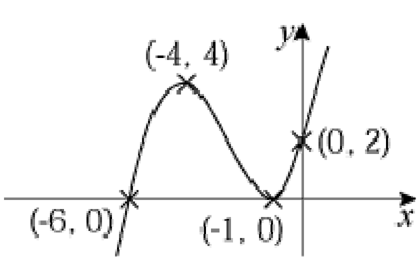
$A'(0\ ,\ 2)$; $B'(2\ ,\ 0)$; $C'(8\ ,\ 4)$; $D'(12\ ,\ 0)$

(g) $\frac{1}{2}f(x)$ is a stretch of $\frac{1}{2}$ vertically.



$A'(0\ ,\ 1)$; $B'(1\ ,\ 0)$; $C'(4\ ,\ 2)$; $D'(6\ ,\ 0)$

(h) $f(-x)$ is a reflection is the y-axis.



$A'(0\ ,\ 2)$; $B'(-1\ ,\ 0)$; $C'(-4\ ,\ 4)$; $D'(-6\ ,\ 0)$

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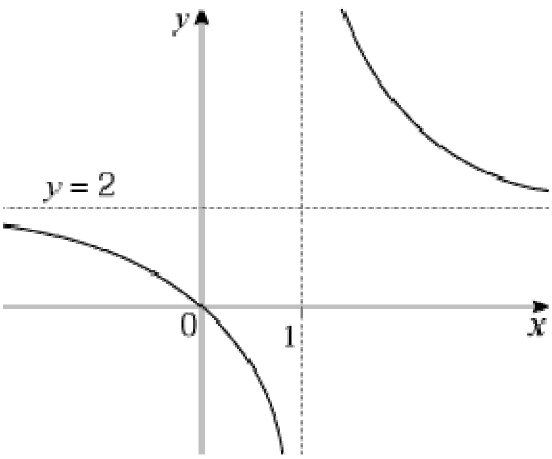
Sketching curves

Exercise G, Question 2

Question:

The curve $y = \mathrm{f}(x)$ passes through the origin and has horizontal asymptote $y = 2$ and vertical asymptote $x = 1$, as shown in the diagram.



Sketch the following graphs and give the equations of any asymptotes and, for all graphs except (a), give coordinates of intersections with the axes after each transformation.

(a) $\mathrm{f}(x) + 2$

(b) $\mathrm{f}(x + 1)$

(c) $2\mathrm{f}(x)$

(d) $\mathrm{f}(x) - 2$

(e) $\mathrm{f}(2x)$

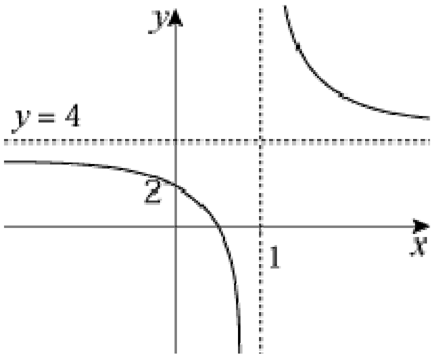
(f) $\mathrm{f}\left(\frac{1}{2}x\right)$

(g) $\frac{1}{2}\mathrm{f}(x)$

(h) $-\mathrm{f}(x)$

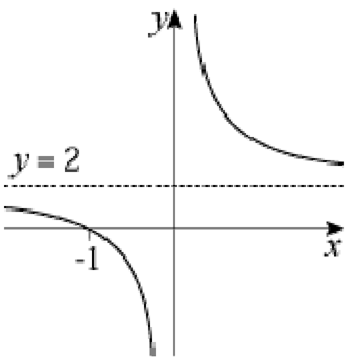
Solution:

(a) $\mathrm{f}(x) + 2$ is a translation of $+2$ in a vertical direction.



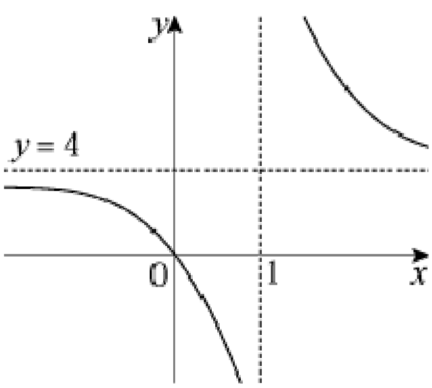
Asymptotes: $x = 1$, $y = 4$. Intersections: $(0 , 2)$ and $(a , 0)$, where $0 < a < 1$.

(b) $f(x + 1)$ is a horizontal translation of -1.



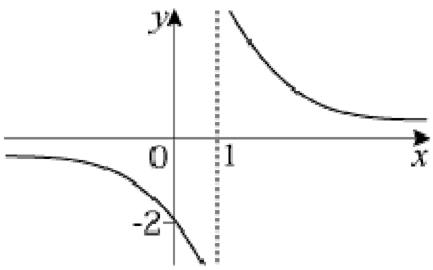
Asymptotes: $x = 0$, $y = 2$. Intersections: $(-1 , 0)$

(c) $2f(x)$ is a stretch of 2 in a vertical direction.



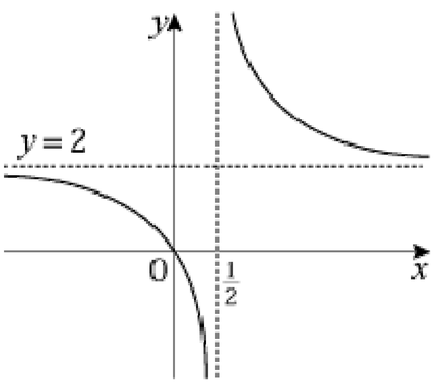
Asymptotes: $x = 1$, $y = 4$. Intersections: $(0 , 0)$

(d) $f(x) - 2$ is a vertical translation of -2.



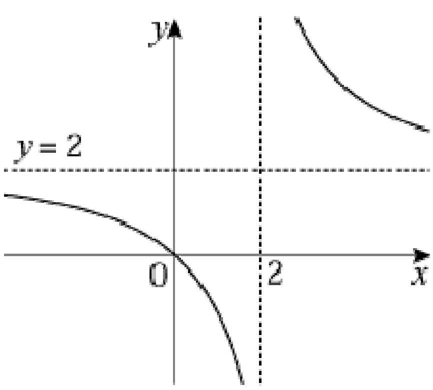
Asymptotes: x = 1, y = 0. Intersections: (0 , − 2)

(e) f(2x) is a stretch of $\frac{1}{2}$ in a horizontal direction.



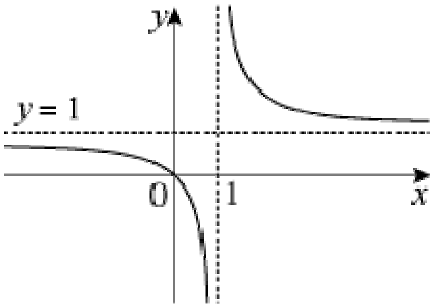
Asymptotes: x = $\frac{1}{2}$, y = 2. Intersections: (0 , 0)

(f) f($\frac{1}{2}x$) is a stretch of 2 in a horizontal direction.



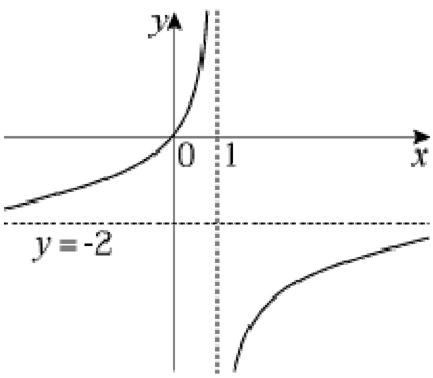
Asymptotes: x = 2, y = 2. Intersections: (0 , 0)

(g) $\frac{1}{2}$ f(x) is a stretch of $\frac{1}{2}$ in a vertical direction.



Asymptotes: $x = 1$, $y = 1$. Intesections: (0 , 0)

(h) $-\mathrm{f}(x)$ is a reflection in the x-axis.



Asymptotes: $x = 1$, $y = -2$. Intersections: (0 , 0)

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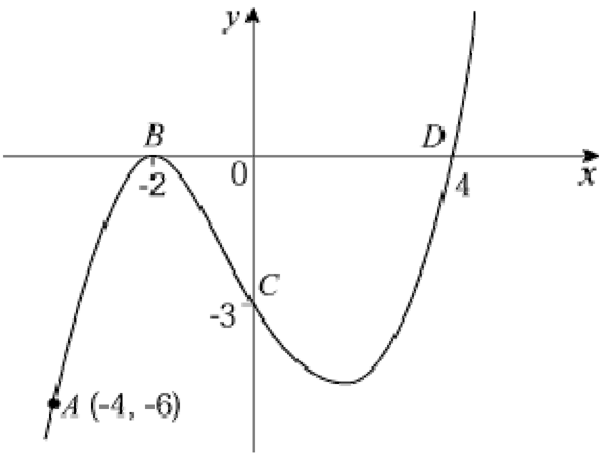
Sketching curves
Exercise G, Question 3

Question:

The curve with equation $y = f(x)$ passes through the points $A(-4, -6)$, $B(-2, 0)$, $C(0, -3)$ and $D(4, 0)$ as shown in the diagram.



Sketch the following and give the coordinates of the points *A*, *B*, *C* and *D* after each transformation.

(a) $f(x-2)$

(b) $f(x)+6$

(c) $f(2x)$

(d) $f(x+4)$

(e) $f(x)+3$

(f) $3f(x)$

(g) $\frac{1}{3}f(x)$

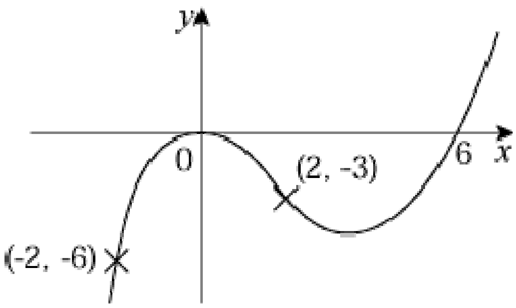
(h) $f(\frac{1}{4}x)$

(i) $-f(x)$

(j) $f(-x)$

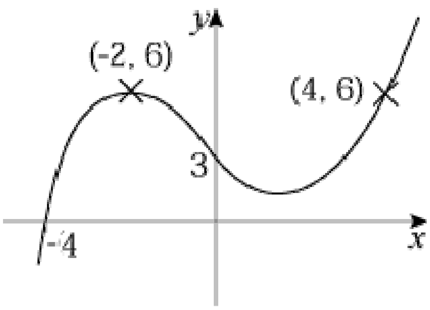
Solution:

(a) $f(x-2)$ is a horizontal translation of $+2$.



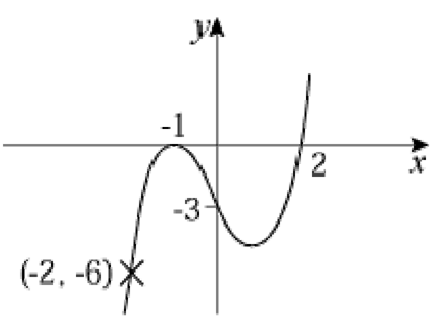
$A'(-2, -6)$; $B'(0, 0)$; $C'(2, -3)$; $D'(6, 0)$

(b) f(x) + 6 is a vertical translation of + 6.



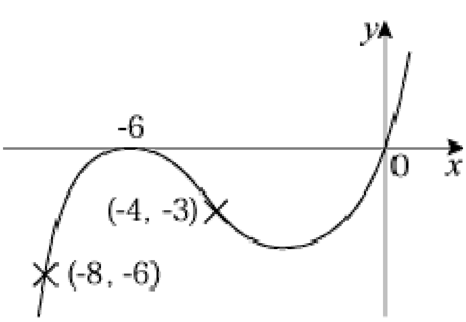
$A'(-4, 0)$; $B'(-2, 6)$; $C'(0, 3)$; $D'(4, 6)$

(c) f(2x) is a horizontal stretch of $\frac{1}{2}$.



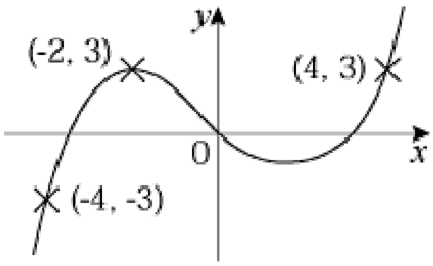
$A'(-2, -6)$; $B'(-1, 0)$; $C'(0, -3)$; $D'(2, 0)$

(d) f(x + 4) is a horizontal translation of − 4.



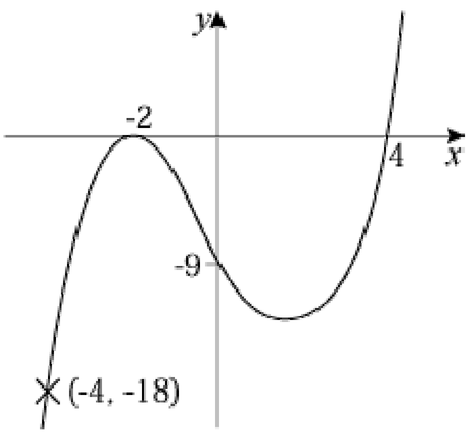
$A'(-8, -6)$; $B'(-6, 0)$; $C'(-4, -3)$; $D'(0, 0)$

(e) f(x) + 3 is a vertical translation of + 3.



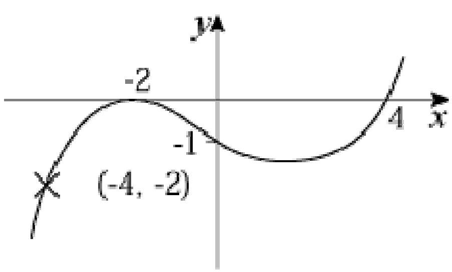
$A'(-4\ ,\ -3)$; $B'(-2\ ,\ 3)$; $C'(0\ ,\ 0)$; $D'(4\ ,\ 3)$

(f) $3\mathrm{f}(x)$ is a vertical stretch of 3.



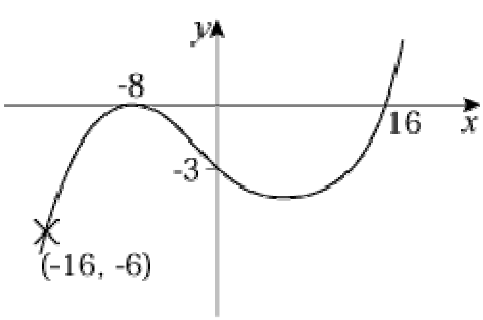
$A'(-4\ ,\ -18)$; $B'(-2\ ,\ 0)$; $C'(0\ ,\ -9)$; $D'(4\ ,\ 0)$

(g) $\frac{1}{3}\mathrm{f}(x)$ is a vertical stretch of $\frac{1}{3}$.



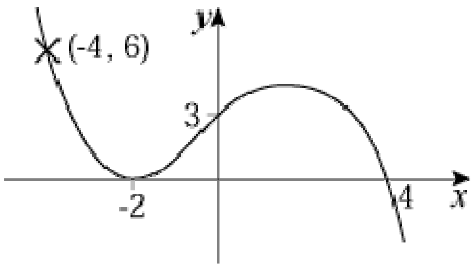
$A'(-4\ ,\ -2)$; $B'(-2\ ,\ 0)$; $C'(0\ ,\ -1)$; $D'(4\ ,\ 0)$

(h) $\mathrm{f}(\frac{1}{4}x)$ is a horizontal stretch of 4.



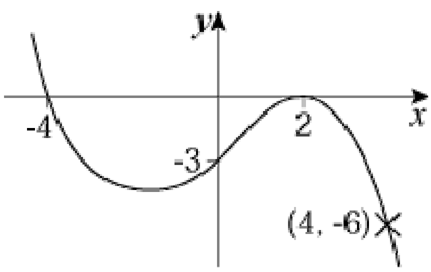
$A'(-16, -6)$; $B'(-8, 0)$; $C'(0, -3)$; $D'(16, 0)$

(i) $-f(x)$ is a reflection in the x-axis.



$A'(-4, 6)$; $B'(-2, 0)$; $C'(0, 3)$; $D'(4, 0)$

(j) $f(-x)$ is a reflection in the y-axis.



$A'(4, -6)$; $B'(2, 0)$; $C'(0, -3)$; $D'(-4, 0)$

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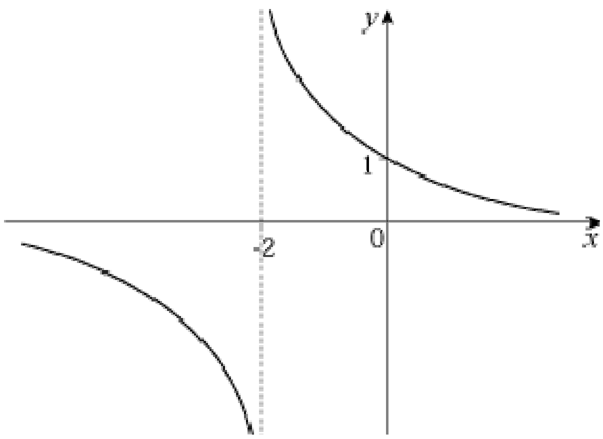
Sketching curves
Exercise G, Question 4

Question:

A sketch of the curve $y = \text{f}(x)$ is shown in the diagram. The curve has vertical asymptote $x = -2$ and a horizontal asymptote with equation $y = 0$. The curve crosses the y-axis at $(0, 1)$.



(a) Sketch, on separate diagrams, the graphs of:

(i) $2\text{f}(x)$

(ii) $\text{f}(2x)$

(iii) $\text{f}(x - 2)$

(iv) $\text{f}(x) - 1$

(v) $\text{f}(-x)$

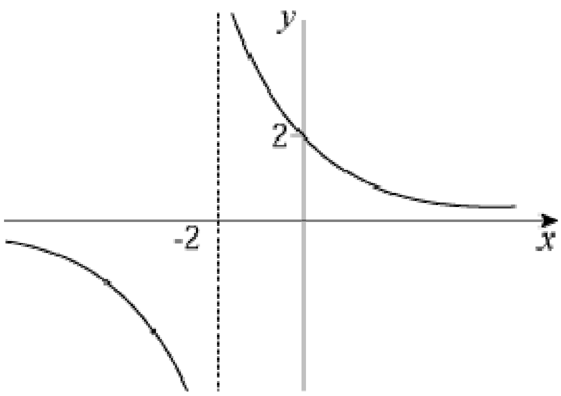
(vi) $-\text{f}(x)$

In each case state the equations of any asymptotes and, if possible, points where the curve cuts the axes.

(b) Suggest a possible equation for $\text{f}(x)$.

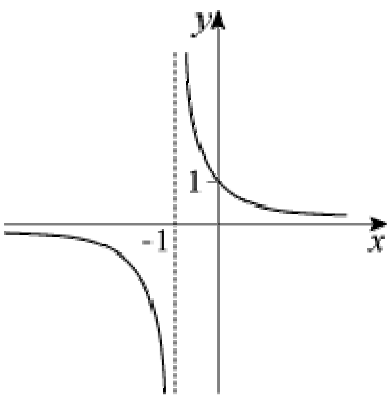
Solution:

(a) (i) $2\text{f}(x)$ is a vertical stretch of 2.



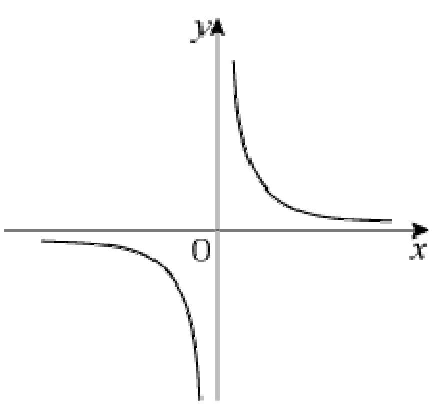
Asymptotes: $x = -2$, $y = 0$. Intersections: (0 , 2)

(ii) f(2x) is a horizontal stretch of $\frac{1}{2}$.



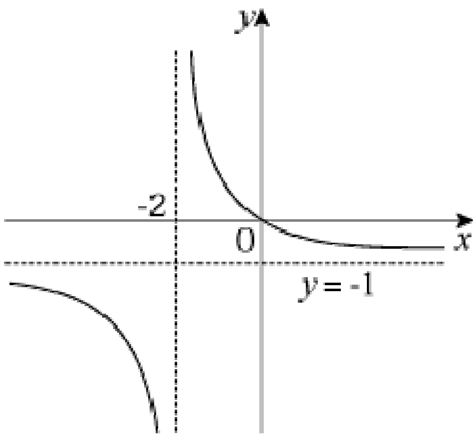
Asymptotes: $x = -1$, $y = 0$. Intersections: (0 , 1)

(iii) f($x - 2$) is a translation of $+2$ in the x-direction.



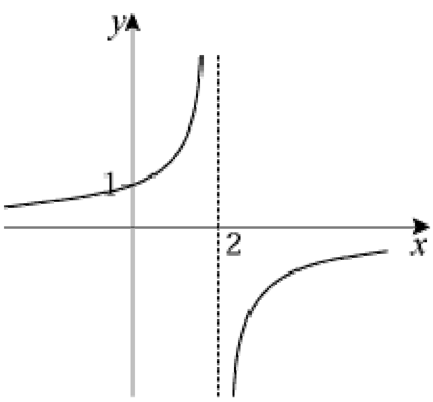
Asymptotes: $x = 0$, $y = 0$. No intersections with axes.

(iv) f(x) − 1 is a translation of -1 in the y-direction.



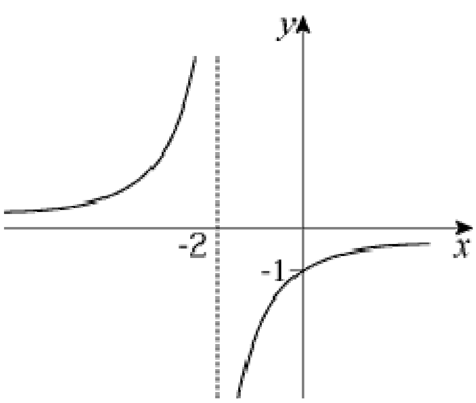
Asymptotes: $x = -2$, $y = -1$. Intersections: $(0, 0)$

(v) $f(-x)$ is a reflection in the y-axis.



Asymptotes: $x = 2$, $y = 0$. Intersections: $(0, 1)$

(vi) $-f(x)$ is a reflection in the x-axis.



Asymptotes: $x = -2$, $y = 0$. Intersections: $(0, -1)$

(b) The shape of the curve is like $y = \frac{k}{x}$, $k > 0$.

$x = -2$ asymptote suggests denominator is zero when $x = -2$, so denominator is $x + 2$.
Also, $f(0) = 1$ means 2 required on numerator.

$f(x) = \frac{2}{x+2}$

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Sketching curves
Exercise H, Question 1

Question:

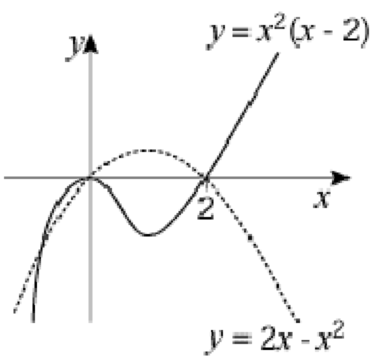
(a) On the same axes sketch the graphs of $y = x^2(x - 2)$ and $y = 2x - x^2$.

(b) By solving a suitable equation find the points of intersection of the two graphs.

Solution:

(a) $y = x^2(x - 2)$
$y = 0 \quad \Rightarrow \quad x = 0$ (twice), 2
Turning point at (0 , 0).
$x \to \infty , y \to \infty$
$x \to -\infty , y \to -\infty$

$y = 2x - x^2 = x(2 - x)$ is $\cap$ shaped
$y = 0 \quad \Rightarrow \quad x = 0, 2$



(b) $x^2(x - 2) = x(2 - x)$
$\Rightarrow \quad x^2(x - 2) - x(2 - x) = 0$
$\Rightarrow \quad x(x - 2)(x + 1) = 0$
$\Rightarrow \quad x = 0, 2, -1$
Using $y = x(2 - x)$ the points of intersection are:
(0 , 0); (2 , 0); (− 1 , − 3)

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Sketching curves
Exercise H, Question 2

Question:

(a) On the same axes sketch the curves with equations $y = \frac{6}{x}$ and $y = 1 + x$.

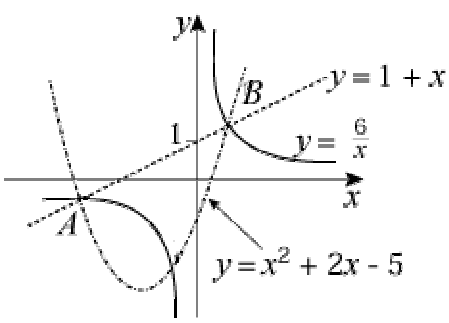
(b) The curves intersect at the points A and B. Find the coordinates of A and B.

(c) The curve C with equation $y = x^2 + px + q$, where p and q are integers, passes through A and B. Find the values of p and q.

(d) Add C to your sketch.

Solution:

(a) $y = \frac{6}{x}$ is like $y = \frac{1}{x}$ and $y = 1 + x$ is a straight line.



(b) $\frac{6}{x} = 1 + x$

$\Rightarrow \quad 6 = x + x^2$

$\Rightarrow \quad 0 = x^2 + x - 6$

$\Rightarrow \quad 0 = (x + 3)(x - 2)$

$\Rightarrow \quad x = 2, \ -3$

So A is $(-3 , \ -2)$; B is $(2 , 3)$

(c) Substitute the points A and B into $y = x^2 + px + q$:

$A \quad \Rightarrow \quad -2 = 9 - 3p + q$ ①

$B \quad \Rightarrow \quad 3 = 4 + 2p + q$ ②

① − ②: $-5 = 5 - 5p$

$\Rightarrow \quad p = 2$

$\Rightarrow \quad q = -5$

(d) $y = x^2 + 2x - 5 = (x + 1)^2 - 6 \quad \Rightarrow \quad$ minimum at $(-1 , \ -6)$

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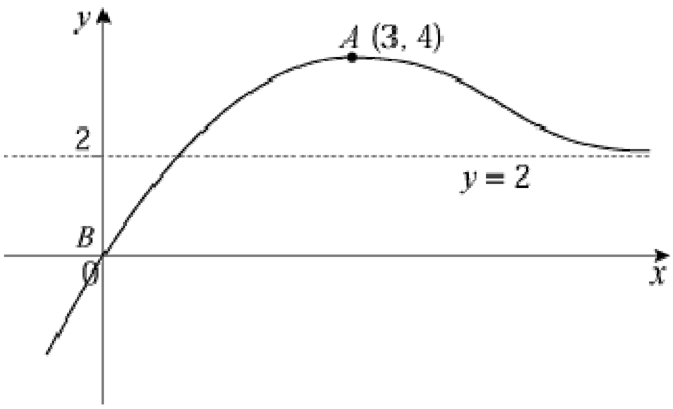
Sketching curves
Exercise H, Question 3

Question:

The diagram shows a sketch of the curve $y = f(x)$. The point $B(0 , 0)$ lies on the curve and the point $A(3 , 4)$ is a maximum point. The line $y = 2$ is an asymptote.



Sketch the following and in each case give the coordinates of the new positions of A and B and state the equation of the asymptote:

(a) $f(2x)$

(b) $\frac{1}{2}f(x)$

(c) $f(x) - 2$

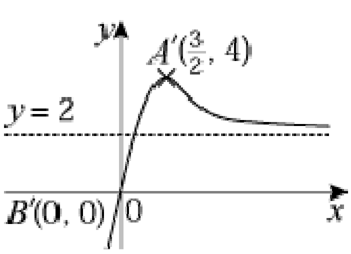
(d) $f(x + 3)$

(e) $f(x - 3)$

(f) $f(x) + 1$

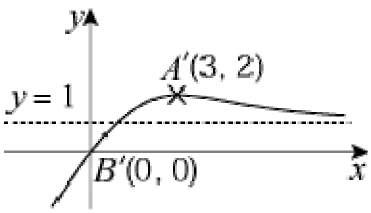
Solution:

(a) $f(2x)$ is a horizontal stretch of $\frac{1}{2}$.



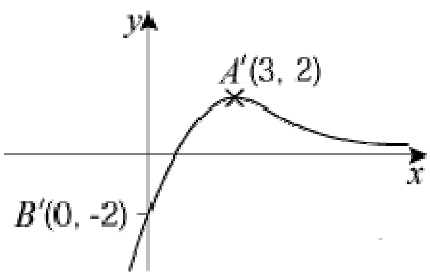
$A'(\frac{3}{2} , 4)$; $B'(0 , 0)$. Asymptote: $y = 2$.

(b) $\frac{1}{2}$f(x) is a vertical stretch of $\frac{1}{2}$.



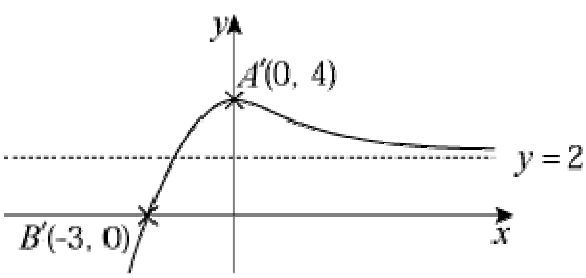
A'(3 , 2); B'(0 , 0). Asymptote: y = 1.

(c) f(x) − 2 is a vertical translation of − 2.



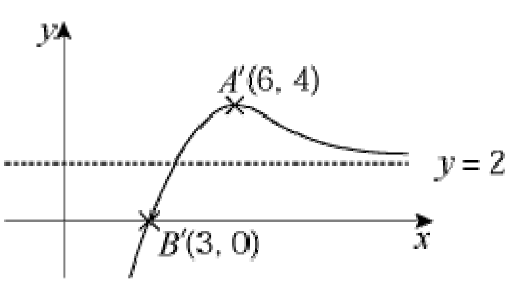
A'(3 , 2); B'(0 , − 2). Asymptote: y = 0.

(d) f(x + 3) is a horizontal translation of − 3.



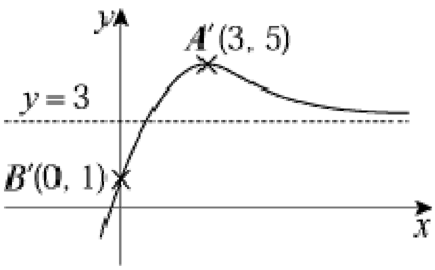
A'(0 , 4); B'(− 3 , 0). Asymptote: y = 2.

(e) f(x − 3) is a horizontal translation of + 3.



A'(6 , 4); B'(3 , 0). Asymptote: y = 2.

(f) f(x) + 1 is a vertical translation of + 1.



$A'(3, 5)$; $B'(0, 1)$. Asymptote: $y = 3$.

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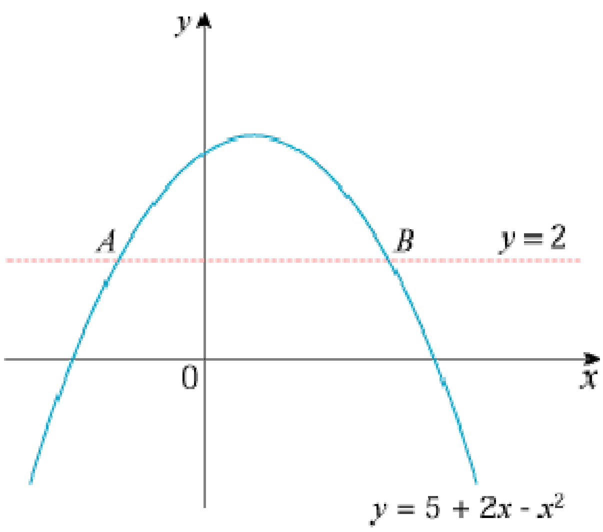
Sketching curves
Exercise H, Question 4

Question:

The diagram shows the curve with equation $y = 5 + 2x - x^2$ and the line with equation $y = 2$. The curve and the line intersect at the points A and B.



Find the x-coordinates of A and B. **[E]**

Solution:

$2 = 5 + 2x - x^2$
$x^2 - 2x - 3 = 0$
$(x - 3)(x + 1) = 0$
$x = -1, 3$

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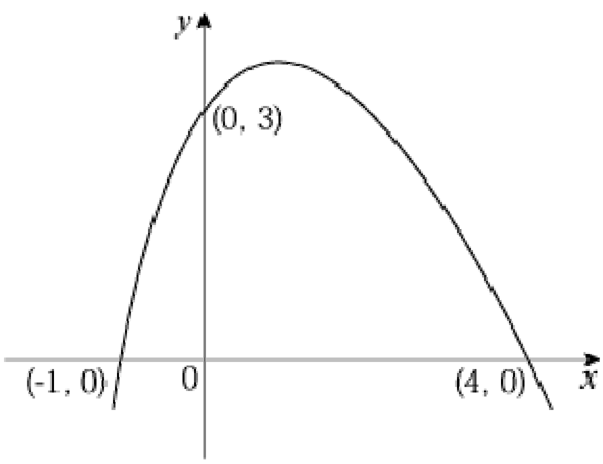
Sketching curves
Exercise H, Question 5

Question:

The curve with equation $y = f(x)$ meets the coordinate axes at the points $(-1, 0)$, $(4, 0)$ and $(0, 3)$, as shown in the diagram.



Using a separate diagram for each, sketch the curve with equation

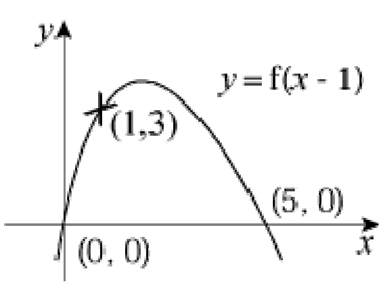
(a) $y = f(x - 1)$

(b) $y = -f(x)$

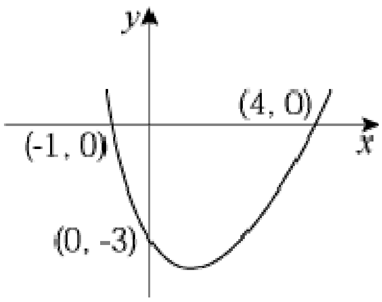
On each sketch, write in the coordinates of the points at which the curve meets the coordinate axes. **[E]**

Solution:

(a) $f(x - 1)$ is a translation of $+1$ in the x-direction.



(b) $-f(x)$ is a reflection in the x-axis.


y
(4, 0)
x
(-1, 0)
(0, -3)

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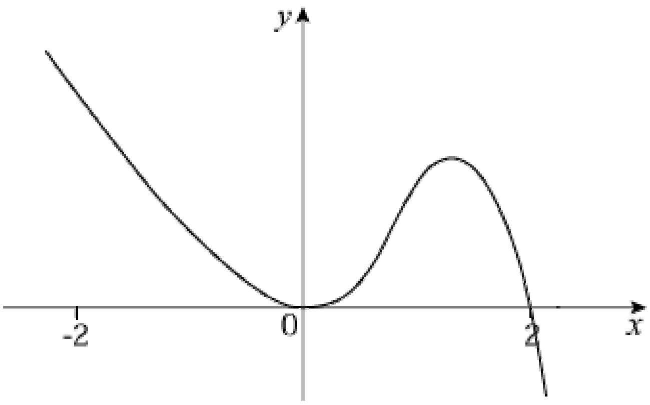
Sketching curves
Exercise H, Question 6

Question:

The figure shows a sketch of the curve with equation $y = f(x)$.



In separate diagrams show, for $-2 \leq x \leq 2$, sketches of the curves with equation:

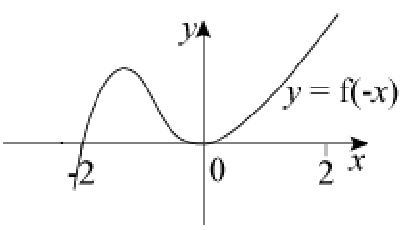
(a) $y = f(-x)$

(b) $y = -f(x)$

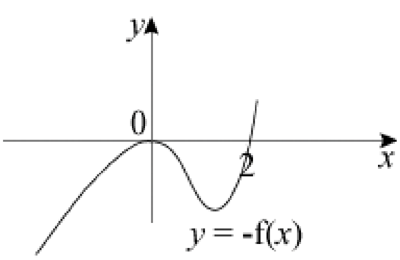
Mark on each sketch the x-coordinate of any point, or points, where a curve touches or crosses the x-axis. **[E]**

Solution:

(a) $f(-x)$ is a reflection in the y-axis.



(b) $-f(x)$ is a reflection in the x-axis.



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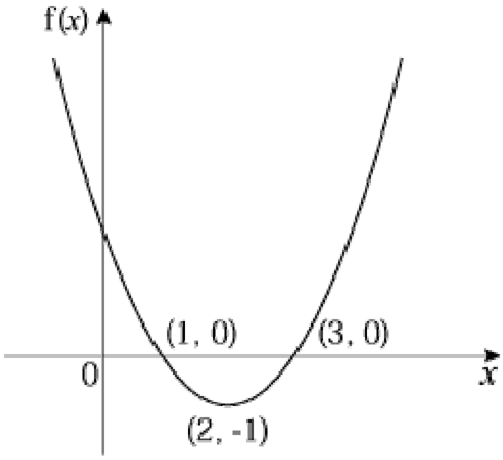
Sketching curves
Exercise H, Question 7

Question:

The diagram shows the graph of the quadratic function f. The graph meets the x-axis at (1 , 0) and (3 , 0) and the minimum point is (2 , − 1).



(a) Find the equation of the graph in the form $y = \mathrm{f}(x)$.

(b) On separate axes, sketch the graphs of

(i) $y = \mathrm{f}(x + 2)$

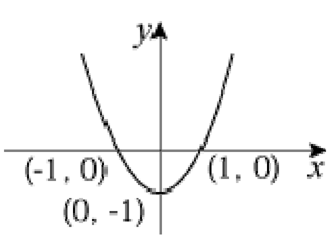
(ii) $y = \mathrm{f}(2x)$

(c) On each graph write in the coordinates of the points at which the graph meets the x-axis and write in the coordinates of the minimum point. **[E]**

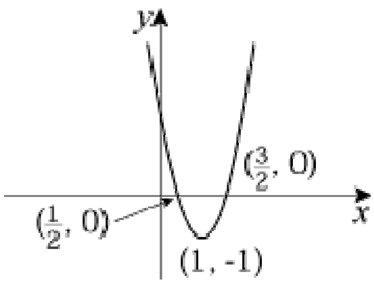
Solution:

(a) Let $y = a(x - p)(x - q)$
Since (1 , 0) and (3 , 0) are on the curve then $p = 1$, $q = 3$
So $y = a(x - 1)(x - 3)$
Using $(2 , - 1) \Rightarrow - 1 = a(1)(- 1) \Rightarrow a = 1$
So $y = (x - 1)(x - 3) = x^2 - 4x + 3$

(b) (i) $\mathrm{f}(x + 2) = (x + 1)(x - 1)$, or translation of − 2 in the x-direction.



(ii) $\mathrm{f}(2x) = (2x - 1)(2x - 3)$, or horizontal stretch of $\frac{1}{2}$.


y
x
($\frac{3}{2}$, 0)
($\frac{1}{2}$, 0)
(1, -1)

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Coordinate geometry in the (x, y) plane
Exercise A, Question 1

Question:

Work out the gradients of these lines:

(a) $y = -2x + 5$

(b) $y = -x + 7$

(c) $y = 4 + 3x$

(d) $y = \frac{1}{3}x - 2$

(e) $y = -\frac{2}{3}x$

(f) $y = \frac{5}{4}x + \frac{2}{3}$

(g) $2x - 4y + 5 = 0$

(h) $10x - 5y + 1 = 0$

(i) $-x + 2y - 4 = 0$

(j) $-3x + 6y + 7 = 0$

(k) $4x + 2y - 9 = 0$

(l) $9x + 6y + 2 = 0$

Solution:

(a) Gradient $= -2$

(b) Gradient $= -1$

(c) Gradient $= 3$

(d) Gradient $= \frac{1}{3}$

(e) Gradient $= -\frac{2}{3}$

(f) Gradient $= \frac{5}{4}$

(g) $2x - 4y + 5 = 0$
$2x + 5 = 4y$

$4y = 2x + 5$

$y = \frac{2}{4}x + \frac{5}{4}$

$y = \frac{1}{2}x + \frac{5}{4}$

Gradient $= \frac{1}{2}$

(h) $10x - 5y + 1 = 0$
$10x + 1 = 5y$
$5y = 10x + 1$

$y = \frac{10}{5}x + \frac{1}{5}$

$y = 2x + \frac{1}{5}$

Gradient $= 2$

(i) $-x + 2y - 4 = 0$
$2y - 4 = x$
$2y = x + 4$

$y = \frac{1}{2}x + 2$

Gradient $= \frac{1}{2}$

(j) $-3x + 6y + 7 = 0$
$6y + 7 = 3x$
$6y = 3x - 7$

$y = \frac{3}{6}x - \frac{7}{6}$

$y = \frac{1}{2}x - \frac{7}{6}$

Gradient $= \frac{1}{2}$

(k) $4x + 2y - 9 = 0$
$2y - 9 = -4x$
$2y = -4x + 9$

$y = -\frac{4}{2}x + \frac{9}{2}$

$y = -2x + \frac{9}{2}$

Gradient $= -2$

(l) $9x + 6y + 2 = 0$
$6y + 2 = -9x$
$6y = -9x - 2$

$y = -\frac{9}{6}x - \frac{2}{6}$

$y = -\frac{3}{2}x - \frac{1}{3}$

Gradient $= -\frac{3}{2}$

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Coordinate geometry in the (x, y) plane
Exercise A, Question 2

Question:

These lines intercept the y-axis at $(0 , c)$. Work out the value of c in each case.

(a) $y = - x + 4$

(b) $y = 2x - 5$

(c) $y = \frac{1}{2}x - \frac{2}{3}$

(d) $y = - 3x$

(e) $y = \frac{6}{7}x + \frac{7}{5}$

(f) $y = 2 - 7x$

(g) $3x - 4y + 8 = 0$

(h) $4x - 5y - 10 = 0$

(i) $- 2x + y - 9 = 0$

(j) $7x + 4y + 12 = 0$

(k) $7x - 2y + 3 = 0$

(l) $- 5x + 4y + 2 = 0$

Solution:

(a) $c = 4$

(b) $c = - 5$

(c) $c = - \frac{2}{3}$

(d) $y = - 3x$
$y = - 3x + 0$
$c = 0$

(e) $c = \frac{7}{5}$

(f) $y = 2 - 7x$
$y = - 7x + 2$
$c = 2$

(g) $3x - 4y + 8 = 0$

$3x + 8 = 4y$
$4y = 3x + 8$

$y = \frac{3}{4}x + \frac{8}{4}$

$y = \frac{3}{4}x + 2$

$c = 2$

(h) $4x - 5y - 10 = 0$
$4x - 10 = 5y$
$5y = 4x - 10$

$y = \frac{4}{5}x - \frac{10}{5}$

$y = \frac{4}{5}x - 2$

$c = -2$

(i) $-2x + y - 9 = 0$
$y - 9 = 2x$
$y = 2x + 9$
$c = 9$

(j) $7x + 4y + 12 = 0$
$4y + 12 = -7x$
$4y = -7x - 12$

$y = -\frac{7}{4}x - \frac{12}{4}$

$y = -\frac{7}{4}x - 3$

$c = -3$

(k) $7x - 2y + 3 = 0$
$7x + 3 = 2y$
$2y = 7x + 3$

$y = \frac{7}{2}x + \frac{3}{2}$

$c = \frac{3}{2}$

(l) $-5x + 4y + 2 = 0$
$4y + 2 = 5x$
$4y = 5x - 2$

$y = \frac{5}{4}x - \frac{2}{4}$

$y = \frac{5}{4}x - \frac{1}{2}$

$c = -\frac{1}{2}$

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Coordinate geometry in the (x, y) plane
Exercise A, Question 3

Question:

Write these lines in the form $ax + by + c = 0$.

(a) $y = 4x + 3$

(b) $y = 3x - 2$

(c) $y = -6x + 7$

(d) $y = \frac{4}{5}x - 6$

(e) $y = \frac{5}{3}x + 2$

(f) $y = \frac{7}{3}x$

(g) $y = 2x - \frac{4}{7}$

(h) $y = -3x + \frac{2}{9}$

(i) $y = -6x - \frac{2}{3}$

(j) $y = -\frac{1}{3}x + \frac{1}{2}$

(k) $y = \frac{2}{3}x + \frac{5}{6}$

(l) $y = \frac{3}{5}x + \frac{1}{2}$

Solution:

(a) $y = 4x + 3$
$0 = 4x + 3 - y$
$4x + 3 - y = 0$
$4x - y + 3 = 0$

(b) $y = 3x - 2$
$0 = 3x - 2 - y$
$3x - 2 - y = 0$
$3x - y - 2 = 0$

(c) $y = -6x + 7$
$6x + y = 7$
$6x + y - 7 = 0$

(d) $y = \frac{4}{5}x - 6$

Multiply each term by 5:
$5y = 4x - 30$
$0 = 4x - 30 - 5y$
$4x - 30 - 5y = 0$
$4x - 5y - 30 = 0$

(e) $y = \frac{5}{3}x + 2$

Multiply each term by 3:
$3y = 5x + 6$
$0 = 5x + 6 - 3y$
$5x + 6 - 3y = 0$
$5x - 3y + 6 = 0$

(f) $y = \frac{7}{3}x$

Multiply each term by 3:
$3y = 7x$
$0 = 7x - 3y$
$7x - 3y = 0$

(g) $y = 2x - \frac{4}{7}$

Multiply each term by 7:
$7y = 14x - 4$
$0 = 14x - 4 - 7y$
$14x - 4 - 7y = 0$
$14x - 7y - 4 = 0$

(h) $y = -3x + \frac{2}{9}$

Multiply each term by 9:
$9y = -27x + 2$
$27x + 9y = 2$
$27x + 9y - 2 = 0$

(i) $y = -6x - \frac{2}{3}$

Multiply each term by 3:
$3y = -18x - 2$
$18x + 3y = -2$
$18x + 3y + 2 = 0$

(j) $y = -\frac{1}{3}x + \frac{1}{2}$

Multiply each term by 6 (6 is divisible by both 3 and 2):
$6y = -2x + 3$
$2x + 6y = 3$
$2x + 6y - 3 = 0$

(k) $y = \frac{2}{3}x + \frac{5}{6}$

Multiply each term by 6 (6 is divisible by both 3 and 6):
$6y = 4x + 5$

$0 = 4x + 5 - 6y$
$4x + 5 - 6y = 0$
$4x - 6y + 5 = 0$

(l) $y = \frac{3}{5}x + \frac{1}{2}$

Multiply each term by 10 (10 is divisible by both 5 and 2):
$10y = 6x + 5$
$0 = 6x + 5 - 10y$
$6x + 5 - 10y = 0$
$6x - 10y + 5 = 0$

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Coordinate geometry in the (x, y) plane
Exercise A, Question 4

Question:

A line is parallel to the line $y = 5x + 8$ and its intercept on the y-axis is $(0 , 3)$. Write down the equation of the line.

Solution:

The line is parallel to $y = 5x + 8$, so $m = 5$.
The line intercepts the y-axis at $(0 , 3)$, so $c = 3$.
Using $y = mx + c$, the equation of the line is $y = 5x + 3$.

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Coordinate geometry in the (x, y) plane
Exercise A, Question 5

Question:

A line is parallel to the line $y = -\frac{2}{5}x + 1$ and its intercept on the y-axis is $(0, -4)$. Work out the equation of the line. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers.

Solution:

The line is parallel to $y = -\frac{2}{5}x + 1$, so $m = -\frac{2}{5}$.

The line intercepts the y-axis at $(0, -4)$, so $c = -4$.
Using $y = mx + c$, the equation of the line is

$y = -\frac{2}{5}x - 4$

Multiply each term by 5:
$5y = -2x - 20$
$2x + 5y = -20$
$2x + 5y + 20 = 0$

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Coordinate geometry in the (x, y) plane
Exercise A, Question 6

Question:

A line is parallel to the line $3x + 6y + 11 = 0$ and its intercept on the y-axis is $(0, 7)$. Write down the equation of the line.

Solution:

$3x + 6y + 11 = 0$
$6y + 11 = -3x$
$6y = -3x - 11$
$y = -\frac{3}{6}x - \frac{11}{6}$

$y = -\frac{1}{2}x - \frac{11}{6}$

The line is parallel to $y = -\frac{1}{2}x - \frac{11}{6}$, so $m = -\frac{1}{2}$.

The line intercepts the y-axis at $(0, 7)$, so $c = 7$.

Using $y = mx + c$, the equation of the line is $y = -\frac{1}{2}x + 7$

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Coordinate geometry in the (x, y) plane
Exercise A, Question 7

Question:

A line is parallel to the line $2x - 3y - 1 = 0$ and it passes through the point $(0 , 0)$. Write down the equation of the line.

Solution:

$2x - 3y - 1 = 0$
$2x - 1 = 3y$
$3y = 2x - 1$

$y = \frac{2}{3}x - \frac{1}{3}$

The line is parallel to $y = \frac{2}{3}x - \frac{1}{3}$, so $m = \frac{2}{3}$.

The intercept on the y-axis is $(0 , 0)$, so $c = 0$.
Using $y = mx + c$:

$y = \frac{2}{3}x + 0$

$y = \frac{2}{3}x$

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Coordinate geometry in the (x, y) plane
Exercise A, Question 8

Question:

The line $y = 6x - 18$ meets the x-axis at the point P. Work out the coordinates of P.

Solution:

$y = 6x - 18$
Substitute $y = 0$:
$6x - 18 = 0$
$6x = 18$
$x = 3$
The line meets the x-axis at $P(3, 0)$.

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Coordinate geometry in the (x, y) plane
Exercise A, Question 9

Question:

The line $3x + 2y - 5 = 0$ meets the x-axis at the point R. Work out the coordinates of R.

Solution:

$3x + 2y - 5 = 0$
Substitute $y = 0$:
$3x + 2(0) - 5 = 0$
$3x - 5 = 0$
$3x = 5$
$x = \frac{5}{3}$

The line meets the x-axis at $R\left(\frac{5}{3}, 0\right)$.

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Coordinate geometry in the (x, y) plane
Exercise A, Question 10

Question:

The line $5x - 4y + 20 = 0$ meets the y-axis at the point A and the x-axis at the point B. Work out the coordinates of the points A and B.

Solution:

$5x - 4y + 20 = 0$
Substitute $x = 0$:
$5 (0) - 4y + 20 = 0$
$- 4y + 20 = 0$
$20 = 4y$
$4y = 20$
$y = 5$
The line meets the y-axis at $A (0 , 5)$.
Substitute $y = 0$:
$5x - 4 (0) + 20 = 0$
$5x + 20 = 0$
$5x = - 20$
$x = - 4$
The line meets the x-axis at $B (- 4 , 0)$.

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Coordinate geometry in the (x, y) plane
Exercise B, Question 1

Question:

Work out the gradient of the line joining these pairs of points:

(a) $(4, 2)$, $(6, 3)$

(b) $(-1, 3)$, $(5, 4)$

(c) $(-4, 5)$, $(1, 2)$

(d) $(2, -3)$, $(6, 5)$

(e) $(-3, 4)$, $(7, -6)$

(f) $(-12, 3)$, $(-2, 8)$

(g) $(-2, -4)$, $(10, 2)$

(h) $\left(\frac{1}{2}, 2\right)$, $\left(\frac{3}{4}, 4\right)$

(i) $\left(\frac{1}{4}, \frac{1}{2}\right)$, $\left(\frac{1}{2}, \frac{2}{3}\right)$

(j) $(-2.4, 9.6)$, $(0, 0)$

(k) $(1.3, -2.2)$, $(8.8, -4.7)$

(l) $(0, 5a)$, $(10a, 0)$

(m) $(3b, -2b)$, $(7b, 2b)$

(n) (p, p^2), (q, q^2)

Solution:

(a) $(x_1, y_1) = (4, 2)$, $(x_2, y_2) = (6, 3)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 2}{6 - 4} = \frac{1}{2}$$

(b) $(x_1, y_1) = (-1, 3)$, $(x_2, y_2) = (5, 4)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 3}{5 - (-1)} = \frac{1}{6}$$

(c) $(x_1, y_1) = (-4, 5)$, $(x_2, y_2) = (1, 2)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 5}{1 - (- 4)} = - \frac{3}{5}$$

(d) $(x_1 , y_1) = (2 , - 3) , (x_2 , y_2) = (6 , 5)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (- 3)}{6 - 2} = \frac{8}{4} = 2$$

(e) $(x_1 , y_1) = (- 3 , 4) , (x_2 , y_2) = (7 , - 6)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{- 6 - 4}{7 - (- 3)} = - \frac{10}{10} = - 1$$

(f) $(x_1 , y_1) = (- 12 , 3) , (x_2 , y_2) = (- 2 , 8)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 3}{- 2 - (- 12)} = \frac{5}{- 2 + 12} = \frac{5}{10} = \frac{1}{2}$$

(g) $(x_1 , y_1) = (- 2 , - 4) , (x_2 , y_2) = (10 , 2)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (- 4)}{10 - (- 2)} = \frac{6}{12} = \frac{1}{2}$$

(h) $\left(x_1 , y_1 \right) = \left(\frac{1}{2} , 2 \right) , \left(x_2 , y_2 \right) = \left(\frac{3}{4} , 4 \right)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 2}{\frac{3}{4} - \frac{1}{2}} = \frac{2}{\frac{1}{4}} = 8$$

(i) $\left(x_1 , y_1 \right) = \left(\frac{1}{4} , \frac{1}{2} \right) , \left(x_2 , y_2 \right) = \left(\frac{1}{2} , \frac{2}{3} \right)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{\frac{2}{3} - \frac{1}{2}}{\frac{1}{2} - \frac{1}{4}} = \frac{\frac{1}{6}}{\frac{1}{4}} = \frac{2}{3}$$

(j) $(x_1 , y_1) = (- 2.4 , 9.6) , (x_2 , y_2) = (0 , 0)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 9.6}{0 - (- 2.4)} = \frac{- 9.6}{2.4} = - 4$$

(k) $(x_1 , y_1) = (1.3 , - 2.2) , (x_2 , y_2) = (8.8 , - 4.7)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{- 4.7 - (- 2.2)}{8.8 - 1.3} = \frac{- 2.5}{7.5} = - \frac{1}{3}$$

(l) $(x_1 , y_1) = (0 , 5a) , (x_2 , y_2) = (10a , 0)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 5a}{10a - 0} = \frac{- 5a}{10a} = \frac{- 5}{10} = - \frac{1}{2}$$

(m) $(x_1 , y_1) = (3b , - 2b) , (x_2 , y_2) = (7b , 2b)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2b - (-2b)}{7b - 3b} = \frac{4b}{4b} = 1$$

(n) $(x_1, y_1) = (p, p^2)$, $(x_2, y_2) = (q, q^2)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{q^2 - p^2}{q - p} = \frac{(q - p)(q + p)}{q - p} = q + p$$

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Coordinate geometry in the (x, y) plane
Exercise B, Question 2

Question:

The line joining $(3, -5)$ to $(6, a)$ has gradient 4. Work out the value of a.

Solution:

$(x_1, y_1) = (3, -5), (x_2, y_2) = (6, a)$

$\frac{y_2 - y_1}{x_2 - x_1} = 4$

so $\frac{a - (-5)}{6 - 3} = 4$

$\Rightarrow \quad \frac{a + 5}{3} = 4$

$\Rightarrow \quad a + 5 = 12$

$\Rightarrow \quad a = 7$

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Coordinate geometry in the (x, y) plane
Exercise B, Question 3

Question:

The line joining $(5, b)$ to $(8, 3)$ has gradient -3. Work out the value of b.

Solution:

$(x_1, y_1) = (5, b)$, $(x_2, y_2) = (8, 3)$

$\frac{3-b}{8-5} = -3$

$\frac{3-b}{3} = -3$

$3 - b = -9$
$b = 12$

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Coordinate geometry in the (x, y) plane
Exercise B, Question 4

Question:

The line joining $(c, 4)$ to $(7, 6)$ has gradient $\frac{3}{4}$. Work out the value of c.

Solution:

$(x_1, y_1) = (c, 4)$, $(x_2, y_2) = (7, 6)$

$$\frac{6-4}{7-c} = \frac{3}{4}$$

$$\frac{2}{7-c} = \frac{3}{4}$$

$$2 = \frac{3}{4}\left(7-c\right)$$

$$8 = 3(7-c)$$
$$8 = 21 - 3c$$
$$-13 = -3c$$
$$c = \frac{-13}{-3} = \frac{13}{3} = 4\frac{1}{3}$$

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Coordinate geometry in the (x, y) plane
Exercise B, Question 5

Question:

The line joining $(-1, 2b)$ to $(1, 4)$ has gradient $-\frac{1}{4}$. Work out the value of d.

Solution:

$(x_1, y_1) = (-1, 2b), (x_2, y_2) = (1, 4)$

$\frac{4-2b}{1-(-1)} = -\frac{1}{4}$

$\frac{4-2b}{2} = -\frac{1}{4}$

$2 - b = -\frac{1}{4}$

$2\frac{1}{4} - b = 0$

$b = 2\frac{1}{4}$

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Coordinate geometry in the (x, y) plane
Exercise B, Question 6

Question:

The line joining $(-3, -2)$ to $(2e, 5)$ has gradient 2. Work out the value of e.

Solution:

$(x_1, y_1) = (-3, -2), (x_2, y_2) = (2e, 5)$

$\frac{5-(-2)}{2e-(-3)} = 2$

$\frac{7}{2e+3} = 2$

$7 = 2(2e+3)$
$7 = 4e+6$
$4e = 1$

$e = \frac{1}{4}$

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Coordinate geometry in the (x, y) plane
Exercise B, Question 7

Question:

The line joining $(7, 2)$ to $(f, 3f)$ has gradient 4. Work out the value of f.

Solution:

$(x_1, y_1) = (7, 2)$, $(x_2, y_2) = (f, 3f)$

$\frac{3f - 2}{f - 7} = 4$

$3f - 2 = 4(f - 7)$
$3f - 2 = 4f - 28$
$-2 = f - 28$
$28 - 2 = f$
$f = 26$

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Coordinate geometry in the (x, y) plane
Exercise B, Question 8

Question:

The line joining $(3 , -4)$ to $(-g , 2g)$ has gradient -3. Work out the value of g.

Solution:

$(x_1 , y_1) = (3 , -4) , (x_2 , y_2) = (-g , 2g)$

$\frac{2g - (-4)}{-g - 3} = -3$

$\frac{2g + 4}{-g - 3} = -3$

$2g + 4 = -3 (-g - 3)$
$2g + 4 = 3g + 9$
$4 = g + 9$
$g = -5$

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Coordinate geometry in the (x, y) plane
Exercise B, Question 9

Question:

Show that the points $A\left(2,3\right)$, $B\left(4,4\right)$, $C\left(10,7\right)$ can be joined by a straight line. (Hint: Find the gradient of the lines joining the points: **i** *A* and *B* and **ii** *A* and *C*.)

Solution:

The gradient of AB is $\frac{4-3}{4-2}=\frac{1}{2}$

The gradient of AC is $\frac{7-3}{10-2}=\frac{4}{8}=\frac{1}{2}$

The gradients are equal so the points can be joined by a straight line.

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Coordinate geometry in the (x, y) plane
Exercise B, Question 10

Question:

Show that the points $(-2a, 5a)$, $(0, 4a)$, $(6a, a)$ are collinear (i.e. on the same straight line).

Solution:

The gradient of the line joining $(-2a, 5a)$ and $(0, 4a)$ is

$$\frac{4a-5a}{0-(-2a)}=\frac{-a}{2a}=\frac{-1}{2}$$

The gradient of the line joining $(-2a, 5a)$ and $(6a, a)$ is

$$\frac{a-5a}{6a-(-2a)}=\frac{-4a}{8a}=\frac{-4}{8}=\frac{-1}{2}$$

The gradients are equal so the points can be joined by a straight line (i.e. they are collinear).

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Coordinate geometry in the (x, y) plane
Exercise C, Question 1

Question:

Find the equation of the line with gradient m that passes through the point (x_1 , y_1) when:

(a) $m = 2$ and $(x_1 , y_1) = (2 , 5)$

(b) $m = 3$ and $(x_1 , y_1) = (-2 , 1)$

(c) $m = -1$ and $(x_1 , y_1) = (3 , -6)$

(d) $m = -4$ and $(x_1 , y_1) = (-2 , -3)$

(e) $m = \frac{1}{2}$ and $(x_1 , y_1) = (-4 , 10)$

(f) $m = -\frac{2}{3}$ and $(x_1 , y_1) = (-6 , -1)$

(g) $m = 2$ and $(x_1 , y_1) = (a , 2a)$

(h) $m = -\frac{1}{2}$ and $(x_1 , y_1) = (-2b , 3b)$

Solution:

(a) $y - y_1 = m (x - x_1)$
$y - 5 = 2 (x - 2)$
$y - 5 = 2x - 4$
$y = 2x + 1$

(b) $y - y_1 = m (x - x_1)$
$y - 1 = 3 [x - (-2)]$
$y - 1 = 3 (x + 2)$
$y - 1 = 3x + 6$
$y = 3x + 7$

(c) $y - y_1 = m (x - x_1)$
$y - (-6) = -1 (x - 3)$
$y + 6 = -x + 3$
$y = -x - 3$

(d) $y - y_1 = m (x - x_1)$
$y - (-3) = -4 [x - (-2)]$
$y + 3 = -4 (x + 2)$
$y + 3 = -4x - 8$
$y = -4x - 11$

(e) $y - y_1 = m (x - x_1)$

$$y - 10 = \frac{1}{2}\left[x - \left(-4\right)\right]$$

$$y - 10 = \frac{1}{2}\left(x + 4\right)$$

$$y - 10 = \frac{1}{2}x + 2$$

$$y = \frac{1}{2}x + 12$$

(f) $y - y_1 = m\left(x - x_1\right)$

$$y - \left(-1\right) = -\frac{2}{3}\left[x - \left(-6\right)\right]$$

$$y + 1 = -\frac{2}{3}\left(x + 6\right)$$

$$y + 1 = -\frac{2}{3}x - 4$$

$$y = -\frac{2}{3}x - 5$$

(g) $y - y_1 = m\left(x - x_1\right)$
$y - 2a = 2\left(x - a\right)$
$y - 2a = 2x - 2a$
$y = 2x$

(h) $y - y_1 = m\left(x - x_1\right)$

$$y - 3b = -\frac{1}{2}\left[x - \left(-2b\right)\right]$$

$$y - 3b = -\frac{1}{2}\left(x + 2b\right)$$

$$y - 3b = -\frac{1}{2}x - b$$

$$y = -\frac{1}{2}x - b + 3b$$

$$y = -\frac{1}{2}x + 2b$$

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Coordinate geometry in the (x, y) plane
Exercise C, Question 2

Question:

The line $y = 4x - 8$ meets the x-axis at the point A. Find the equation of the line with gradient 3 that passes through the point A.

Solution:

$y = 4x - 8$
Substitute $y = 0$:
$4x - 8 = 0$
$4x = 8$
$x = 2$
So A has coordinates $(2 , 0)$.

$y - y_1 = m (x - x_1)$
$y - 0 = 3 (x - 2)$
$y = 3x - 6$
The equation of the line is $y = 3x - 6$.

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Coordinate geometry in the (x, y) plane
Exercise C, Question 3

Question:

The line $y = -2x + 8$ meets the y-axis at the point B. Find the equation of the line with gradient 2 that passes through the point B.

Solution:

$y = -2x + 8$
Substitute $x = 0$:
$y = -2(0) + 8$
$y = 8$
So B has coordinates $(0, 8)$.

$y - y_1 = m(x - x_1)$
$y - 8 = 2(x - 0)$
$y - 8 = 2x$
$y = 2x + 8$
The equation of the line is $y = 2x + 8$.

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Coordinate geometry in the (x, y) plane
Exercise C, Question 4

Question:

The line $y = \frac{1}{2}x + 6$ meets the x-axis at the point C. Find the equation of the line with gradient $\frac{2}{3}$ that passes through the point C. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers.

Solution:

$y = \frac{1}{2}x + 6$

Substitute $y = 0$:

$\frac{1}{2}x + 6 = 0$

$\frac{1}{2}x = -6$

$x = -12$

So C has coordinates $(-12, 0)$.

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{2}{3}\left[x - \left(-12\right)\right]$

$y = \frac{2}{3}\left(x + 12\right)$

$y = \frac{2}{3}x + 8$

Multiply each term by 3:

$3y = 2x + 24$

$0 = 2x + 24 - 3y$

$2x - 3y + 24 = 0$

The equation of the line is $2x - 3y + 24 = 0$.

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Coordinate geometry in the (x, y) plane
Exercise C, Question 5

Question:

The line $y = \frac{1}{4}x + 2$ meets the y-axis at the point B. The point C has coordinates $(-5 , 3)$. Find the gradient of the line joining the points B and C.

Solution:

$y = \frac{1}{4}x + 2$

Substitute $x = 0$:

$y = \frac{1}{4}\left(0 \right) + 2$

$y = 2$

So B has coordinates $(0 , 2)$.

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 2}{-5 - 0} = \frac{1}{-5} = -\frac{1}{5}$

The gradient of the line joining B and C is $-\frac{1}{5}$.

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Coordinate geometry in the (x, y) plane
Exercise C, Question 6

Question:

The lines $y = x$ and $y = 2x - 5$ intersect at the point A. Find the equation of the line with gradient $\frac{2}{5}$ that passes through the point A. (Hint: Solve $y = x$ and $y = 2x - 5$ simultaneously.)

Solution:

Substitute $y = x$:
$x = 2x - 5$
$0 = x - 5$
$x = 5$

$y = x$
Substitute $x = 5$:
$y = 5$
The coordinates of A are $(5 , 5)$.

$y - y_1 = m (x - x_1)$

$y - 5 = \frac{2}{5} \left(x - 5 \right)$

$y - 5 = \frac{2}{5}x - 2$

$y = \frac{2}{5}x + 3$

The equation of the line is $y = \frac{2}{5}x + 3$.

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Coordinate geometry in the (x, y) plane
Exercise C, Question 7

Question:

The lines $y = 4x - 10$ and $y = x - 1$ intersect at the point T. Find the equation of the line with gradient $-\frac{2}{3}$ that passes through the point T. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers.

Solution:

Substitute $y = x - 1$:
$x - 1 = 4x - 10$
$-1 = 3x - 10$
$9 = 3x$
$x = 3$

$y = x - 1$
Substitute $x = 3$:
$y = 3 - 1 = 2$
The coordinates of T are $(3 , 2)$.

$y - y_1 = m (x - x_1)$

$y - 2 = -\frac{2}{3}\left(x - 3 \right)$

$y - 2 = -\frac{2}{3}x + 2$

$\frac{2}{3}x + y - 2 = 2$

$\frac{2}{3}x + y - 4 = 0$

$2x + 3y - 12 = 0$
The equation of the line is $2x + 3y - 12 = 0$.

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Coordinate geometry in the (x, y) plane
Exercise C, Question 8

Question:

The line p has gradient $\frac{2}{3}$ and passes through the point $(6, -12)$. The line q has gradient -1 and passes through the point $(5, 5)$. The line p meets the y-axis at A and the line q meets the x-axis at B. Work out the gradient of the line joining the points A and B.

Solution:

The equation of p is

$$y - \left(-12 \right) = \frac{2}{3} \left(x - 6 \right)$$

$$y + 12 = \frac{2}{3}x - 4$$

$$y = \frac{2}{3}x - 16$$

The equation of q is
$y - 5 = -1(x - 5)$
$y - 5 = -x + 5$
$y = -x + 10$

For the coordinates of A substitute $x = 0$ into

$$y = \frac{2}{3}x - 16$$

$$y = \frac{2}{3}\left(0 \right) - 16$$

$y = -16$
Coordinates are $A(0, -16)$

For the coordinates of B substitute $y = 0$ into
$y = -x + 10$
$0 = -x + 10$
$x = 10$
Coordinates are $B(10, 0)$

Gradient of AB is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-16 - 0}{0 - 10} = \frac{-16}{-10} = \frac{8}{5}$$

The gradient of the line joining A and B is $\frac{8}{5}$.

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Coordinate geometry in the (x, y) plane
Exercise C, Question 9

Question:

The line $y = -2x + 6$ meets the x-axis at the point P. The line $y = \frac{3}{2}x - 4$ meets the y-axis at the point Q. Find the equation of the line joining the points P and Q. (Hint: First work out the gradient of the line joining the points P and Q.)

Solution:

$y = -2x + 6$
Substitute $y = 0$:
$0 = -2x + 6$
$2x = 6$
$x = 3$
P has coordinates $(3 , 0)$.

$y = \frac{3}{2}x - 4$

Substitute $x = 0$:

$y = \frac{3}{2}\left(0 \right) - 4$

$y = -4$
Q has coordinates $(0 , -4)$

Gradient of PQ is

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - (-4)}{3 - 0} = \frac{4}{3}$

Equation of PQ is
$y - y_1 = m (x - x_1)$
Substitute $(3 , 0)$:

$y - 0 = \frac{4}{3}\left(x - 3 \right)$

$y = \frac{4}{3}x - 4$

The equation of the line through P and Q is $y = \frac{4}{3}x - 4$.

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Coordinate geometry in the (x, y) plane
Exercise C, Question 10

Question:

The line $y = 3x - 5$ meets the x-axis at the point M. The line $y = -\frac{2}{3}x + \frac{2}{3}$ meets the y-axis at the point N. Find the equation of the line joining the points M and N. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers.

Solution:

$y = 3x - 5$
Substitute $y = 0$:
$3x - 5 = 0$
$3x = 5$
$x = \frac{5}{3}$

M has coordinates $\left(\frac{5}{3} , 0 \right)$.

$y = -\frac{2}{3}x + \frac{2}{3}$
Substitute $x = 0$:
$y = -\frac{2}{3}\left(0 \right) + \frac{2}{3} = \frac{2}{3}$

N has coordinates $\left(0 , \frac{2}{3} \right)$.

Gradient of MN is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - \frac{2}{3}}{\frac{5}{3} - 0} = \frac{-\frac{2}{3}}{\frac{5}{3}} = -\frac{2}{5}$$

Equation of MN is
$y - y_1 = m (x - x_1)$
Substitute $\left(\frac{5}{3} , 0 \right)$:

$y - 0 = -\frac{2}{5}\left(x - \frac{5}{3} \right)$

$y = -\frac{2}{5}x + \frac{2}{3}$
Multiply each term by 15:
$15y = -6x + 10$
$6x + 15y = 10$
$6x + 15y - 10 = 0$

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Coordinate geometry in the (x, y) plane
Exercise D, Question 1

Question:

Find the equation of the line that passes through these pairs of points:

(a) $(2 , 4)$ and $(3 , 8)$

(b) $(0 , 2)$ and $(3 , 5)$

(c) $(-2 , 0)$ and $(2 , 8)$

(d) $(5 , -3)$ and $(7 , 5)$

(e) $(3 , -1)$ and $(7 , 3)$

(f) $(-4 , -1)$ and $(6 , 4)$

(g) $(-1 , -5)$ and $(-3 , 3)$

(h) $(-4 , -1)$ and $(-3 , -9)$

(i) $\left(\frac{1}{3} , \frac{2}{5} \right)$ and $\left(\frac{2}{3} , \frac{4}{5} \right)$

(j) $\left(-\frac{3}{4} , \frac{1}{7} \right)$ and $\left(\frac{1}{4} , \frac{3}{7} \right)$

Solution:

(a) $(x_1 , y_1) = (2 , 4)$, $(x_2 , y_2) = (3 , 8)$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 4}{8 - 4} = \frac{x - 2}{3 - 2}$$

$$\frac{y - 4}{4} = \frac{x - 2}{1}$$

$$\frac{y - 4}{4} = x - 2$$

Multiply each side by 4:

$$4 \times \frac{y - 4}{4} = 4 \left(x - 2 \right)$$

$y - 4 = 4 (x - 2)$
$y - 4 = 4x - 8$
$y = 4x - 4$

(b) $(x_1 , y_1) = (0 , 2)$, $(x_2 , y_2) = (3 , 5)$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 2}{5 - 2} = \frac{x - 0}{3 - 0}$$

$$\frac{y - 2}{3} = \frac{x}{3}$$

Multiply each side by 3:

$$3 \times \frac{y - 2}{3} = 3 \times \frac{x}{3}$$

$y - 2 = x$
$y = x + 2$

(c) $(x_1 , y_1) = (- 2 , 0)$, $(x_2 , y_2) = (2 , 8)$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 0}{8 - 0} = \frac{x - (- 2)}{2 - (- 2)}$$

$$\frac{y}{8} = \frac{x + 2}{4}$$

Multiply each side by 8:

$$8 \times \frac{y}{8} = 8 \times \frac{x + 2}{4}$$

$y = 2 (x + 2)$
$y = 2x + 4$

(d) $(x_1 , y_1) = (5 , - 3)$, $(x_2 , y_2) = (7 , 5)$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - (- 3)}{5 - (- 3)} = \frac{x - 5}{7 - 5}$$

$$\frac{y + 3}{8} = \frac{x - 5}{2}$$

Multiply each side by 8:

$$8 \times \frac{y + 3}{8} = 8 \times \frac{x - 5}{2}$$

$y + 3 = 4 (x - 5)$
$y + 3 = 4x - 20$
$y = 4x - 23$

(e) $(x_1 , y_1) = (3 , - 1)$, $(x_2 , y_2) = (7 , 3)$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - (- 1)}{3 - (- 1)} = \frac{x - 3}{7 - 3}$$

$$\frac{y + 1}{4} = \frac{x - 3}{4}$$

Multiply each side by 4:
$y + 1 = x - 3$
$y = x - 4$

(f) $(x_1 , y_1) = (- 4 , - 1)$, $(x_2 , y_2) = (6 , 4)$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y-(-1)}{4-(-1)} = \frac{x-(-4)}{6-(-4)}$$

$$\frac{y+1}{5} = \frac{x+4}{10}$$

Multiply each side by 10:

$2(y+1) = x+4$

$2y+2 = x+4$

$2y = x+2$

Divide each term by 2:

$y = \frac{1}{2}x + 1$

(g) $(x_1, y_1) = (-1, -5)$, $(x_2, y_2) = (-3, 3)$

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y-(-5)}{3-(-5)} = \frac{x-(-1)}{-3-(-1)}$$

$$\frac{y+5}{8} = \frac{x+1}{-2}$$

Multiply each side by 8:

$y+5 = -4(x+1)$ (**Note:** $\frac{8}{-2} = -4$)

$y+5 = -4x-4$

$y = -4x-9$

(h) $(x_1, y_1) = (-4, -1)$, $(x_2, y_2) = (-3, -9)$

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y-(-1)}{-9-(-1)} = \frac{x-(-4)}{-3-(-4)}$$

$$\frac{y+1}{-8} = \frac{x+4}{1}$$

Multiply each side by -8:

$y+1 = -8(x+4)$

$y+1 = -8x-32$

$y = -8x-33$

(i) $\left(x_1, y_1\right) = \left(\frac{1}{3}, \frac{2}{5}\right)$, $\left(x_2, y_2\right) = \left(\frac{2}{3}, \frac{4}{5}\right)$

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y-\frac{2}{5}}{\frac{4}{5}-\frac{2}{5}} = \frac{x-\frac{1}{3}}{\frac{2}{3}-\frac{1}{3}}$$

$$\frac{y-\frac{2}{5}}{\frac{2}{5}} = \frac{x-\frac{1}{3}}{\frac{1}{3}}$$

$\frac{5}{2}\left(y - \frac{2}{5}\right) = 3\left(x - \frac{1}{3}\right)$ (**Note:** $\frac{1}{\frac{2}{5}} = \frac{5}{2}$ and $\frac{1}{\frac{1}{3}} = 3$)

$\frac{5}{2}y - 1 = 3x - 1$

$\frac{5}{2}y = 3x$

$5y = 6x$

$y = \frac{6}{5}x$

(j) $\left(x_1, y_1\right) = \left(\frac{-3}{4}, \frac{1}{7}\right), \left(x_2, y_2\right) = \left(\frac{1}{4}, \frac{3}{7}\right)$

$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$

$\frac{y - \frac{1}{7}}{\frac{3}{7} - \frac{1}{7}} = \frac{x - \left(-\frac{3}{4}\right)}{\frac{1}{4} - \left(-\frac{3}{4}\right)}$

$\frac{y - \frac{1}{7}}{\frac{2}{7}} = \frac{x + \frac{3}{4}}{1}$

Multiply each side by $\frac{2}{7}$:

$y - \frac{1}{7} = \frac{2}{7}\left(x + \frac{3}{4}\right)$

$y - \frac{1}{7} = \frac{2}{7}x + \frac{3}{14}$

$y = \frac{2}{7}x + \frac{3}{14} + \frac{1}{7}$

$y = \frac{2}{7}x + \frac{5}{14}$

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Coordinate geometry in the (x, y) plane
Exercise D, Question 2

Question:

The line that passes through the points $(2, -5)$ and $(-7, 4)$ meets the *x*-axis at the point *P*. Work out the coordinates of the point *P*.

Solution:

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$$

$$\frac{y-(-5)}{4-(-5)}=\frac{x-2}{-7-2}$$

$$\frac{y+5}{9}=\frac{x-2}{-9}$$

Multiply each side by 9:

$y+5=-1(x-2)$ (**Note:** $\frac{9}{-9}=-1$)

$y+5=-x+2$
$y=-x-3$
Substitute $y=0$:
$0=-x-3$
$x=-3$
So the line meets the *x*-axis at $P(-3, 0)$.

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Coordinate geometry in the (x, y) plane
Exercise D, Question 3

Question:

The line that passes through the points $(-3, -5)$ and $(4, 9)$ meets the *y*-axis at the point *G*. Work out the coordinates of the point *G*.

Solution:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - (-5)}{9 - (-5)} = \frac{x - (-3)}{4 - (-3)}$$

$$\frac{y + 5}{14} = \frac{x + 3}{7}$$

Multiply each side by 14:
$y + 5 = 2(x + 3)$
$y + 5 = 2x + 6$
$y = 2x + 1$
Substitute $x = 0$:
$y = 2(0) + 1 = 1$
The coordinates of *G* are $(0, 1)$.

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Coordinate geometry in the (x, y) plane
Exercise D, Question 4

Question:

The line that passes through the points $\left(3 , 2 \frac{1}{2} \right)$ and $\left(- 1 \frac{1}{2} , 4 \right)$ meets the y-axis at the point J. Work out the coordinates of the point J.

Solution:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 2 \frac{1}{2}}{4 - 2 \frac{1}{2}} = \frac{x - 3}{- 1 \frac{1}{2} - 3}$$

$$\frac{y - 2 \frac{1}{2}}{1 \frac{1}{2}} = \frac{x - 3}{- 4 \frac{1}{2}}$$

Multiply top and bottom of each fraction by 2:

$$\frac{2y - 5}{3} = \frac{2x - 6}{- 9}$$

Multiply each side by 9:

$3 (2y - 5) = - 1 (2x - 6)$ (**Note:** $\frac{9}{- 9} = - 1$)

$6y - 15 = - 2x + 6$
$6y = - 2x + 21$

$y = - \frac{2}{6}x + \frac{21}{6}$

$y = - \frac{1}{3}x + \frac{7}{2}$

Substitute $x = 0$:

$y = - \frac{1}{3} \left(0 \right) + \frac{7}{2} = \frac{7}{2}$

The coordinates of J are $\left(0 , \frac{7}{2} \right)$ or $\left(0 , 3 \frac{1}{2} \right)$.

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Coordinate geometry in the (x, y) plane
Exercise D, Question 5

Question:

The line $y = 2x - 10$ meets the x-axis at the point A. The line $y = -2x + 4$ meets the y-axis at the point B. Find the equation of the line joining the points A and B. (Hint: First work out the coordinates of the points A and B.)

Solution:

$y = 2x - 10$
Substitute $y = 0$:
$2x - 10 = 0$
$2x = 10$
$x = 5$
The coordinates of A are $(5, 0)$.

$y = -2x + 4$
Substitute $x = 0$:
$y = -2(0) + 4 = 4$
The coordinates of B are $(0, 4)$.

Equation of AB:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 0}{4 - 0} = \frac{x - 5}{0 - 5}$$

$$\frac{y}{4} = \frac{x - 5}{-5}$$

Multiply each side by 4:

$$y = 4\frac{(x - 5)}{-5} = \frac{4}{-5}\left(x - 5\right) = -\frac{4}{5}\left(x - 5\right) = -\frac{4}{5}x + 4$$

The equation of the line is $y = -\frac{4}{5}x + 4$.

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Coordinate geometry in the (x, y) plane
Exercise D, Question 6

Question:

The line $y = 4x + 5$ meets the y-axis at the point C. The line $y = -3x - 15$ meets the x-axis at the point D. Find the equation of the line joining the points C and D. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers.

Solution:

$y = 4x + 5$
Substitute $x = 0$:
$y = 4(0) + 5 = 5$
The coordinates of C are $(0, 5)$.

$y = -3x - 15$
Substitute $y = 0$:
$0 = -3x - 15$
$3x = -15$
$x = -5$
The coordinates of D are $(-5, 0)$.

Equation of CD:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 5}{0 - 5} = \frac{x - 0}{-5 - 0}$$

$$\frac{y - 5}{-5} = \frac{x}{-5}$$

Multiply each side by -5:
$y - 5 = x$
$-5 = x - y$
$0 = x - y + 5$
The equation of the line is $x - y + 5 = 0$.

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Coordinate geometry in the (x, y) plane
Exercise D, Question 7

Question:

The lines $y = x - 5$ and $y = 3x - 13$ intersect at the point S. The point T has coordinates $(- 4 , 2)$. Find the equation of the line that passes through the points S and T.

Solution:

$y = 3x - 13$
$y = x - 5$
So $3x - 13 = x - 5$

$\Rightarrow \quad 3x = x + 8$

$\Rightarrow \quad 2x = 8$

$\Rightarrow \quad x = 4$

when $x = 4$, $y = 4 - 5 = - 1$
The coordinates of S are $(4 , - 1)$.
Equation of ST:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - (- 1)}{2 - (- 1)} = \frac{x - 4}{- 4 - 4}$$

$$\frac{y + 1}{3} = \frac{x - 4}{- 8}$$

Multiply each side by 3:

$$y + 1 = 3 \times \frac{(x - 4)}{- 8}$$

$$y + 1 = \frac{3}{- 8} \times \left(x - 4 \right)$$

$$y + 1 = - \frac{3}{8} \left(x - 4 \right)$$

$$y + 1 = - \frac{3}{8}x + \frac{3}{2}$$

$$y = - \frac{3}{8}x + \frac{1}{2}$$

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Coordinate geometry in the (x, y) plane
Exercise D, Question 8

Question:

The lines $y = -2x + 1$ and $y = x + 7$ intersect at the point L. The point M has coordinates $(-3, 1)$. Find the equation of the line that passes through the points L and M.

Solution:

$y = x + 7$
$y = -2x + 1$
So $x + 7 = -2x + 1$

$\Rightarrow \quad 3x + 7 = 1$

$\Rightarrow \quad 3x = -6$

$\Rightarrow \quad x = -2$

when $x = -2$, $y = (-2) + 7 = 5$
The coordinates of L are $(-2, 5)$.
Equation of LM:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 5}{1 - 5} = \frac{x - (-2)}{-3 - (-2)}$$

$$\frac{y - 5}{-4} = \frac{x + 2}{-1}$$

Multiply each side by -4:

$y - 5 = 4(x + 2)$ (**Note:** $\frac{-4}{-1} = 4$)

$y - 5 = 4x + 8$
$y = 4x + 13$

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Coordinate geometry in the (x, y) plane
Exercise D, Question 9

Question:

The vertices of the triangle ABC have coordinates $A(3,5)$, $B(-2,0)$ and $C(4,-1)$. Find the equations of the sides of the triangle.

Solution:

(1) Equation of AB:
$(x_1, y_1) = (3, 5)$, $(x_2, y_2) = (-2, 0)$

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y-5}{0-5} = \frac{x-3}{-2-3}$$

$$\frac{y-5}{-5} = \frac{x-3}{-5}$$

Multiply each side by -5:
$y-5 = x-3$
$y = x+2$

(2) Equation of AC:
$(x_1, y_1) = (3, 5)$, $(x_2, y_2) = (4, -1)$

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y-5}{-1-5} = \frac{x-3}{4-3}$$

$$\frac{y-5}{-6} = \frac{x-3}{1}$$

Multiply each side by -6:
$y-5 = -6(x-3)$
$y-5 = -6x+18$
$y = -6x+23$

(3) Equation of BC:
$(x_1, y_1) = (-2, 0)$, $(x_2, y_2) = (4, -1)$

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y-0}{-1-0} = \frac{x-(-2)}{4-(-2)}$$

$$\frac{y}{-1} = \frac{x+2}{6}$$

Multiply each side by -1:

$$y = -1\frac{(x+2)}{6}$$

$$y = -\frac{1}{6}\left(x+2\right)$$

$$y = -\frac{1}{6}x - \frac{1}{3}$$

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Coordinate geometry in the (x, y) plane
Exercise D, Question 10

Question:

The line V passes through the points $(-5 , 3)$ and $(7 , -3)$ and the line W passes through the points $(2 , -4)$ and $(4 , 2)$. The lines V and W intersect at the point A. Work out the coordinates of the point A.

Solution:

(1) The equation of V:
$(x_1 , y_1) = (-5 , 3)$, $(x_2 , y_2) = (7 , -3)$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 3}{-3 - 3} = \frac{x - (-5)}{7 - (-5)}$$

$$\frac{y - 3}{-6} = \frac{x + 5}{12}$$

Multiply each side by -6:

$y - 3 = -\frac{1}{2}\left(x + 5 \right)$ (**Note:** $\frac{-6}{12} = -\frac{1}{2}$)

$$y - 3 = -\frac{1}{2}x - \frac{5}{2}$$

$$y = -\frac{1}{2}x + \frac{1}{2}$$

(2) The equation of W:
$(x_1 , y_1) = (2 , -4)$, $(x_2 , y_2) = (4 , 2)$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - (-4)}{2 - (-4)} = \frac{x - 2}{4 - 2}$$

$$\frac{y + 4}{6} = \frac{x - 2}{2}$$

Multiply each side by 6:

$y + 4 = 3 (x - 2)$ (**Note:** $\frac{6}{2} = 3$)

$y + 4 = 3x - 6$
$y = 3x - 10$

Solving simultaneously:

$$y = -\frac{1}{2}x + \frac{1}{2}$$

$$y = 3x - 10$$

So $3x - 10 = -\frac{1}{2}x + \frac{1}{2}$

$$\Rightarrow \quad \frac{7}{2}x - 10 = \frac{1}{2}$$

$$\Rightarrow \quad \frac{7}{2}x = \frac{21}{2}$$

$\Rightarrow \quad 7x = 21$

$\Rightarrow \quad x = 3$

When $x = 3$, $y = 3(3) - 10 = 9 - 10 = -1$
The lines intersect at $A(3, -1)$.

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Coordinate geometry in the (x, y) plane
Exercise E, Question 1

Question:

Work out if these pairs of lines are parallel, perpendicular or neither:

(a) $y = 4x + 2$
$y = -\frac{1}{4}x - 7$

(b) $y = \frac{2}{3}x - 1$

$y = \frac{2}{3}x - 11$

(c) $y = \frac{1}{5}x + 9$
$y = 5x + 9$

(d) $y = -3x + 2$
$y = \frac{1}{3}x - 7$

(e) $y = \frac{3}{5}x + 4$

$y = -\frac{5}{3}x - 1$

(f) $y = \frac{5}{7}x$

$y = \frac{5}{7}x - 3$

(g) $y = 5x - 3$
$5x - y + 4 = 0$

(h) $5x - y - 1 = 0$
$y = -\frac{1}{5}x$

(i) $y = -\frac{3}{2}x + 8$
$2x - 3y - 9 = 0$

(j) $4x - 5y + 1 = 0$
$8x - 10y - 2 = 0$

(k) $3x + 2y - 12 = 0$
$2x + 3y - 6 = 0$

(l) $5x - y + 2 = 0$

$2x + 10y - 4 = 0$

Solution:

(a) The gradients of the lines are 4 and $-\frac{1}{4}$.

$4 \times -\frac{1}{4} = -1$

The lines are **perpendicular**.

(b) The gradients of the lines are $\frac{2}{3}$ and $\frac{2}{3}$, i.e. they have the same gradient.

The lines are **parallel**.

(c) The gradients of the lines are $\frac{1}{5}$ and 5.

$\frac{1}{5} \times 5 = 1$

The lines are **neither** perpendicular nor parallel.

(d) The gradients of the lines are -3 and $\frac{1}{3}$.

$-3 \times \frac{1}{3} = -1$

The lines are **perpendicular**.

(e) The gradients of the lines are $\frac{3}{5}$ and $-\frac{5}{3}$.

$\frac{3}{5} \times -\frac{5}{3} = -1$

The lines are **perpendicular**.

(f) The gradients of the lines are $\frac{5}{7}$ and $\frac{5}{7}$, i.e. they have the same gradient.

The lines are **parallel**.

(g) The gradient of $y = 5x - 3$ is 5.
$5x - y + 4 = 0$
$5x + 4 = y$
$y = 5x + 4$
The gradient of $5x - y + 4 = 0$ is 5.
The lines have the same gradient.
The lines are **parallel**.

(h) $5x - y - 1 = 0$
$5x - 1 = y$
$y = 5x - 1$
The gradient of $5x - y - 1 = 0$ is 5.
The gradient of $y = -\frac{1}{5}x$ is $-\frac{1}{5}$.

The product of the gradients is $5 \times -\frac{1}{5} = -1$

So the lines are **perpendicular**.

(i) The gradient of $y = -\frac{3}{2}x + 8$ is $-\frac{3}{2}$.

$2x - 3y - 9 = 0$

$2x - 9 = 3y$
$3y = 2x - 9$
$y = \frac{2}{3}x - 3$

The gradient of $2x - 3y - 9 = 0$ is $\frac{2}{3}$.

The product of the gradients is $\frac{2}{3} \times - \frac{3}{2} = -1$

So the lines are **perpendicular**.

(j) $4x - 5y + 1 = 0$
$4x + 1 = 5y$
$5y = 4x + 1$
$y = \frac{4}{5}x + \frac{1}{5}$

The gradient of $4x - 5y + 1 = 0$ is $\frac{4}{5}$.

$8x - 10y - 2 = 0$
$8x - 2 = 10y$
$10y = 8x - 2$
$y = \frac{8}{10}x - \frac{2}{10}$

$y = \frac{4}{5}x - \frac{1}{5}$

The gradient of $8x - 10y - 2 = 0$ is $\frac{4}{5}$.

The lines have the same gradient, they are **parallel**.

(k) $3x + 2y - 12 = 0$
$3x + 2y = 12$
$2y = -3x + 12$
$y = -\frac{3}{2}x + 6$

The gradient of $3x + 2y - 12 = 0$ is $-\frac{3}{2}$.

$2x + 3y - 6 = 0$
$2x + 3y = 6$
$3y = -2x + 6$
$y = -\frac{2}{3}x + 2$

The gradient of $2x + 3y - 6 = 0$ is $-\frac{2}{3}$.

The product of the gradient is
$-\frac{3}{2} \times -\frac{2}{3} = 1$

So the lines are **neither** parallel nor perpendicular.

(l) $5x - y + 2 = 0$
$5x + 2 = y$
$y = 5x + 2$
The gradient of $5x - y + 2 = 0$ is 5.
$2x + 10y - 4 = 0$
$2x + 10y = 4$
$10y = -2x + 4$
$y = -\frac{2}{10}x + \frac{4}{10}$

$y = -\frac{1}{5}x + \frac{2}{5}$

The gradient of $2x + 10y - 4 = 0$ is $-\frac{1}{5}$.

The product of the gradients is

$5 \times -\frac{1}{5} = -1$

So the lines are **perpendicular**.

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane
Exercise E, Question 2

Question:

Find an equation of the line that passes through the point $(6 , - 2)$ and is perpendicular to the line $y = 3x + 5$.

Solution:

The gradient of $y = 3x + 5$ is 3.

The gradient of a line perpendicular to $y = 3x + 5$ is $-\frac{1}{3}$.

$$y - y_1 = m (x - x_1)$$

$$y - \left(- 2 \right) = - \frac{1}{3} \left(x - 6 \right)$$

$$y + 2 = - \frac{1}{3}x + 2$$

$$y = - \frac{1}{3}x$$

The equation of the line is $y = - \frac{1}{3}x$.

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane
Exercise E, Question 3

Question:

Find an equation of the line that passes through the point $(-2 , 7)$ and is parallel to the line $y = 4x + 1$. Write your answer in the form $ax + by + c = 0$.

Solution:

The gradient of a line parallel to $y = 4x + 1$ is 4.
$y - y_1 = m (x - x_1)$
$y - 7 = 4 [x - (-2)]$
$y - 7 = 4 (x + 2)$
$y - 7 = 4x + 8$
$y = 4x + 15$
$0 = 4x + 15 - y$
$4x - y + 15 = 0$
The equation of the line is $4x - y + 15 = 0$.

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane
Exercise E, Question 4

Question:

Find an equation of the line:

(a) parallel to the line $y = -2x - 5$, passing through $\left(-\frac{1}{2}, \frac{3}{2} \right)$.

(b) parallel to the line $x - 2y - 1 = 0$, passing through $(0, 0)$.

(c) perpendicular to the line $y = x - 4$, passing through $(-1, -2)$.

(d) perpendicular to the line $2x + y - 9 = 0$, passing through $(4, -6)$.

Solution:

(a) The gradient of a line parallel to $y = -2x - 5$ is -2.

$y - y_1 = m(x - x_1)$

$y - \frac{3}{2} = -2\left[x - \left(-\frac{1}{2} \right) \right]$

$y - \frac{3}{2} = -2\left(x + \frac{1}{2} \right)$

$y - \frac{3}{2} = -2x - 1$

$y = -2x + \frac{1}{2}$

(b) $x - 2y - 1 = 0$

$x - 1 = 2y$

$2y = x - 1$

$y = \frac{1}{2}x - \frac{1}{2}$

The gradient of $x - 2y - 1 = 0$ is $\frac{1}{2}$.

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{1}{2}\left(x - 0 \right)$

$y = \frac{1}{2}x$

(c) The gradient of $y = x - 4$ is 1.

The gradient of a line perpendicular to $y = x - 4$ is $-\frac{1}{1} = -1$.

$y - y_1 = m(x - x_1)$

$y - (-2) = -1[x - (-1)]$

$y + 2 = -1(x + 1)$

$y + 2 = -x - 1$

$y = -x - 3$

(d) $2x + y - 9 = 0$
$2x + y = 9$
$y = -2x + 9$
The gradient of $2x + y - 9 = 0$ is -2.

The gradient of a line perpendicular to $2x + y - 9 = 0$ is $-\frac{1}{-2} = \frac{1}{2}$.

$y - y_1 = m(x - x_1)$

$y - \left(-6\right) = \frac{1}{2}\left(x - 4\right)$

$y + 6 = \frac{1}{2}\left(x - 4\right)$

$y + 6 = \frac{1}{2}x - 2$

$y = \frac{1}{2}x - 8$

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane
Exercise E, Question 5

Question:

Find an equation of the line:

(a) parallel to the line $y = 3x + 6$, passing through $(-2 , 5)$.

(b) perpendicular to the line $y = 3x + 6$, passing through $(-2 , 5)$.

(c) parallel to the line $4x - 6y + 7 = 0$, passing through $(3 , 4)$.

(d) perpendicular to the line $4x - 6y + 7 = 0$, passing through $(3 , 4)$.

Solution:

(a) The gradient of a line parallel to $y = 3x + 6$ is 3.
$y - y_1 = m (x - x_1)$
$y - 5 = 3 [x - (-2)]$
$y - 5 = 3 (x + 2)$
$y - 5 = 3x + 6$
$y = 3x + 11$

(b) The gradient of a line perpendicular to $y = 3x + 6$ is $-\frac{1}{3}$.

$y - y_1 = m (x - x_1)$

$y - 5 = -\frac{1}{3}\left[x - \left(-2 \right) \right]$

$y - 5 = -\frac{1}{3}\left(x + 2 \right)$

$y - 5 = -\frac{1}{3}x - \frac{2}{3}$

$y = -\frac{1}{3}x + \frac{13}{3}$

(c) $4x - 6y + 7 = 0$
$4x + 7 = 6y$
$6y = 4x + 7$

$y = \frac{4}{6}x + \frac{7}{6}$

$y = \frac{2}{3}x + \frac{7}{6}$

The gradient of a line parallel to $4x - 6y + 7 = 0$ is $\frac{2}{3}$.

$y - y_1 = m (x - x_1)$

$y - 4 = \frac{2}{3}\left(x - 3 \right)$

$y - 4 = \frac{2}{3}x - 2$

$y = \frac{2}{3}x + 2$

(d) The gradient of the line $4x - 6y + 7 = 0$ is $\frac{2}{3}$ [see part (c)].

The gradient of a line perpendicular to $4x - 6y + 7 = 0$ is $-\frac{1}{\frac{2}{3}} = -\frac{3}{2}$.

$y - y_1 = m (x - x_1)$

$y - 4 = -\frac{3}{2}\left(x - 3 \right)$

$y - 4 = -\frac{3}{2}x + \frac{9}{2}$

$y = -\frac{3}{2}x + \frac{17}{2}$

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Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane
Exercise E, Question 6

Question:

Find an equation of the line that passes through the point $(5 , - 5)$ and is perpendicular to the line $y = \frac{2}{3}x + 5$. Write your answer in the form ax + by + $c = 0$, where a, b and c are integers.

Solution:

The gradient of a line perpendicular to $y = \frac{2}{3}x + 5$ is $- \frac{1}{\frac{2}{3}} = - \frac{3}{2}$.

$y - y_1 = m (x - x_1)$

$y - \left(- 5 \right) = - \frac{3}{2} \left(x - 5 \right)$

$y + 5 = - \frac{3}{2} \left(x - 5 \right)$

Multiply each term by 2:
$2y + 10 = - 3 (x - 5)$
$2y + 10 = - 3x + 15$
$3x + 2y + 10 = 15$
$3x + 2y - 5 = 0$
The equation of the line is $3x + 2y - 5 = 0$.

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane
Exercise E, Question 7

Question:

Find an equation of the line that passes through the point $(-2 , -3)$ and is perpendicular to the line $y = -\frac{4}{7}x + 5$.

Write your answer in the form ax + by + c = 0, where a, b and c are integers.

Solution:

The gradient of a line perpendicular to $y = -\frac{4}{7}x + 5$ is $-\frac{1}{-\frac{4}{7}} = \frac{7}{4}$.

$y - y_1 = m (x - x_1)$

$y - \left(-3 \right) = \frac{7}{4} \left[x - \left(-2 \right) \right]$

$y + 3 = \frac{7}{4} \left(x + 2 \right)$

Multiply each term by 4:

$4y + 12 = 7 (x + 2)$

$4y + 12 = 7x + 14$

$4y = 7x + 2$

$0 = 7x + 2 - 4y$

$7x - 4y + 2 = 0$

The equation of the line is $7x - 4y + 2 = 0$.

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Coordinate geometry in the (x, y) plane
Exercise E, Question 8

Question:

The line r passes through the points $(1, 4)$ and $(6, 8)$ and the line s passes through the points $(5, -3)$ and $(20, 9)$. Show that the lines r and s are parallel.

Solution:

The gradient of r is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 4}{6 - 1} = \frac{4}{5}$$

The gradient of s is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - (-3)}{20 - 5} = \frac{12}{15} = \frac{4}{5}$$

The gradients are equal, so the lines are **parallel**.

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Coordinate geometry in the (x, y) plane
Exercise E, Question 9

Question:

The line l passes through the points $(-3, 0)$ and $(3, -2)$ and the line n passes through the points $(1, 8)$ and $(-1, 2)$. Show that the lines l and n are perpendicular.

Solution:

The gradient of l is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 0}{3 - (-3)} = -\frac{2}{6} = -\frac{1}{3}$$

The gradient of n is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 8}{-1 - 1} = \frac{-6}{-2} = 3$$

The product of the gradients is

$$-\frac{1}{3} \times 3 = -1$$

So the lines are **perpendicular**.

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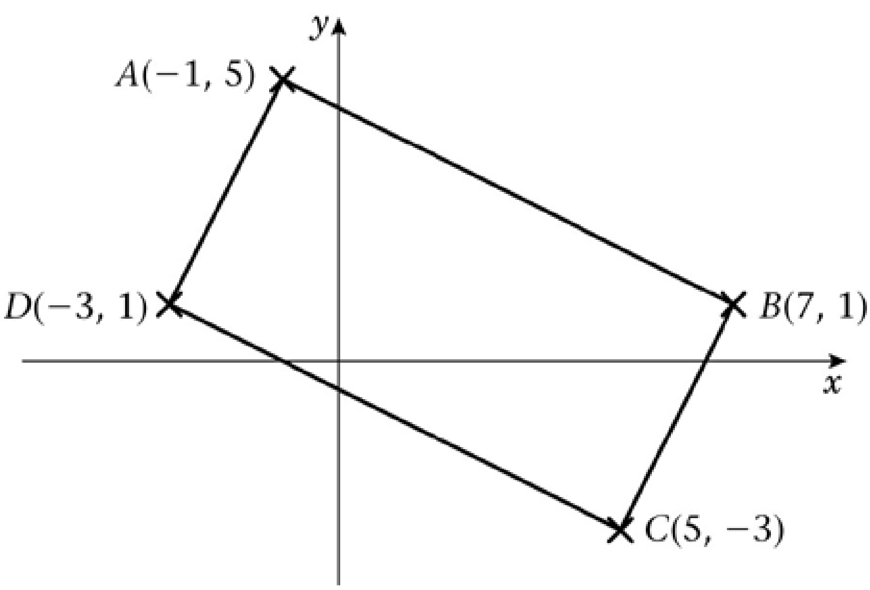
Coordinate geometry in the (x, y) plane
Exercise E, Question 10

Question:

The vertices of a quadrilateral *ABCD* has coordinates $A(-1,5)$, $B(7,1)$, $C(5,-3)$, $D(-3,1)$. Show that the quadrilateral is a rectangle.

Solution:



(1) The gradient of *AB* is

$$\frac{y_2-y_1}{x_2-x_1}=\frac{5-1}{-1-7}=\frac{4}{-8}=-\frac{1}{2}$$

(2) The gradient of *DC* is

$$\frac{y_2-y_1}{x_2-x_1}=\frac{-3-1}{5-(-3)}=-\frac{4}{8}=-\frac{1}{2}$$

The gradient of *AB* is the same as the gradient of *DC*, so the lines are parallel.

(3) The gradient of *AD* is

$$\frac{y_2-y_1}{x_2-x_1}=\frac{5-1}{-1-(-3)}=\frac{4}{-1+3}=\frac{4}{2}=2$$

(4) The gradient of *BC* is

$$\frac{y_2-y_1}{x_2-x_1}=\frac{-3-1}{5-7}=\frac{-4}{-2}=2$$

The gradient of *AD* is the same as the gradient of *BC*, so the lines are parallel.

The line *AD* is perpendicular to the line *AB* as

$$2\times-\frac{1}{2}=-1$$

So *ABCD* is a rectangle.

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Coordinate geometry in the (x, y) plane
Exercise F, Question 1

Question:

The points *A* and *B* have coordinates $(-4, 6)$ and $(2, 8)$ respectively. A line *p* is drawn through *B* perpendicular to *AB* to meet the *y*-axis at the point *C*.

(a) Find an equation of the line *p*.

(b) Determine the coordinates of *C*. **[E]**

Solution:

(a) The gradient of *AB* is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 6}{2 - (-4)} = \frac{2}{6} = \frac{1}{3}$$

The gradient of a line perpendicular to *AB* is

$$-\frac{1}{\frac{1}{3}} = -3$$

The equation of *p* is

$y - y_1 = m(x - x_1)$

$y - 8 = -3(x - 2)$

$y - 8 = -3x + 6$

$y = -3x + 14$

(b) Substitute $x = 0$:

$y = -3(0) + 14 = 14$

The coordinates of *C* are $(0, 14)$.

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Coordinate geometry in the (x, y) plane
Exercise F, Question 2

Question:

The line l has equation $2x - y - 1 = 0$.
The line m passes through the point $A (0 , 4)$ and is perpendicular to the line l.

(a) Find an equation of m and show that the lines l and m intersect at the point $P (2 , 3)$.
The line n passes through the point $B (3 , 0)$ and is parallel to the line m.

(b) Find an equation of n and hence find the coordinates of the point Q where the lines l and n intersect. **[E]**

Solution:

(a) $2x - y - 1 = 0$
$2x - 1 = y$
$y = 2x - 1$
The gradient of $2x - y - 1 = 0$ is 2.

The gradient of a line perpendicular to $2x - y - 1 = 0$ is $-\frac{1}{2}$.

The equation of the line m is
$y - y_1 = m (x - x_1)$

$y - 4 = -\frac{1}{2}\left(x - 0 \right)$

$y - 4 = -\frac{1}{2}x$

$y = -\frac{1}{2}x + 4$

To find P solve $y = -\frac{1}{2}x + 4$ and $2x - y - 1 = 0$ simultaneously.

Substitute:

$2x - \left(-\frac{1}{2}x + 4 \right) - 1 = 0$

$2x + \frac{1}{2}x - 4 - 1 = 0$

$\frac{5}{2}x - 5 = 0$

$\frac{5}{2}x = 5$

$5x = 10$
$x = 2$

Substitute $x = 2$ into $y = -\frac{1}{2}x + 4$:

$y = -\frac{1}{2}\left(2 \right) + 4 = -1 + 4 = 3$

The lines intersect at $P (2 , 3)$, as required.

(b) A line parallel to the line m has gradient $-\frac{1}{2}$.

The equation of the line n is

$y - y_1 = m(x - x_1)$

$y - 0 = -\frac{1}{2}\left(x - 3\right)$

$y = -\frac{1}{2}x + \frac{3}{2}$

To find Q solve $2x - y - 1 = 0$ and $y = -\frac{1}{2}x + \frac{3}{2}$ simultaneously.

Substitute:

$2x - \left(-\frac{1}{2}x + \frac{3}{2}\right) - 1 = 0$

$2x + \frac{1}{2}x - \frac{3}{2} - 1 = 0$

$\frac{5}{2}x - \frac{5}{2} = 0$

$\frac{5}{2}x = \frac{5}{2}$

$x = 1$

Substitute $x = 1$ into $y = -\frac{1}{2}x + \frac{3}{2}$:

$y = -\frac{1}{2}\left(1\right) + \frac{3}{2} = -\frac{1}{2} + \frac{3}{2} = 1$

The lines intersect at $Q(1, 1)$.

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Coordinate geometry in the (x, y) plane
Exercise F, Question 3

Question:

The line L_1 has gradient $\frac{1}{7}$ and passes through the point $A(2, 2)$. The line L_2 has gradient -1 and passes through the point $B(4, 8)$. The lines L_1 and L_2 intersect at the point C.

(a) Find an equation for L_1 and an equation for L_2.

(b) Determine the coordinates of C. **[E]**

Solution:

(a) The equation of L_1 is

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{1}{7}\left(x - 2\right)$

$y - 2 = \frac{1}{7}x - \frac{2}{7}$

$y = \frac{1}{7}x + \frac{12}{7}$

The equation of L_2 is

$y - y_1 = m(x - x_1)$

$y - 8 = -1(x - 4)$

$y - 8 = -x + 4$

$y = -x + 12$

(b) Solve $y = \frac{1}{7}x + \frac{12}{7}$ and $y = -x + 12$ simultaneously.

Substitute:

$-x + 12 = \frac{1}{7}x + \frac{12}{7}$

$12 = \frac{8}{7}x + \frac{12}{7}$

$10\frac{2}{7} = \frac{8}{7}x$

$x = \frac{10\frac{2}{7}}{\frac{8}{7}} = 9$

Substitute $x = 9$ into $y = -x + 12$:

$y = -9 + 12 = 3$

The lines intersect at $C(9, 3)$.

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Coordinate geometry in the (x, y) plane
Exercise F, Question 4

Question:

The straight line passing through the point $P(2, 1)$ and the point $Q(k, 11)$ has gradient $-\frac{5}{12}$.

(a) Find the equation of the line in terms of *x* and *y* only.

(b) Determine the value of *k*. **[E]**

Solution:

(a) $m = -\frac{5}{12}$, $(x_1, y_1) = (2, 1)$

The equation of the line is

$y - y_1 = m(x - x_1)$

$y - 1 = -\frac{5}{12}\left(x - 2\right)$

$y - 1 = -\frac{5}{12}x + \frac{5}{6}$

$y = -\frac{5}{12}x + \frac{11}{6}$

(b) Substitute $(k, 11)$ into $y = -\frac{5}{12}x + \frac{11}{6}$:

$11 = -\frac{5}{12}k + \frac{11}{6}$

$11 - \frac{11}{6} = -\frac{5}{12}k$

$\frac{55}{6} = -\frac{5}{12}k$

Multiply each side by 12:

$110 = -5k$

$k = -22$

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Coordinate geometry in the (x, y) plane
Exercise F, Question 5

Question:

(a) Find an equation of the line l which passes through the points $A(1, 0)$ and $B(5, 6)$.
The line m with equation $2x + 3y = 15$ meets l at the point C.

(b) Determine the coordinates of the point C. **[E]**

Solution:

(a) The equation of l is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 0}{6 - 0} = \frac{x - 1}{5 - 1}$$

$$\frac{y}{6} = \frac{x - 1}{4}$$

Multiply each side by 6:

$$y = 6 \frac{(x - 1)}{4}$$

$$y = \frac{3}{2}\left(x - 1\right)$$

$$y = \frac{3}{2}x - \frac{3}{2}$$

(b) Solve $2x + 3y = 15$ and $y = \frac{3}{2}x - \frac{3}{2}$ simultaneously.

Substitute:

$$2x + 3\left(\frac{3}{2}x - \frac{3}{2}\right) = 15$$

$$2x + \frac{9}{2}x - \frac{9}{2} = 15$$

$$\frac{13}{2}x - \frac{9}{2} = 15$$

$$\frac{13}{2}x = \frac{39}{2}$$

$13x = 39$
$x = 3$

Substitute $x = 3$ into $y = \frac{3}{2}x - \frac{3}{2}$:

$$y = \frac{3}{2}\left(3\right) - \frac{3}{2} = \frac{9}{2} - \frac{3}{2} = \frac{6}{2} = 3$$

The coordinates of C are $(3, 3)$.

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Coordinate geometry in the (x, y) plane
Exercise F, Question 6

Question:

The line L passes through the points $A(1, 3)$ and $B(-19, -19)$.

Find an equation of L in the form $ax + by + c = 0$, where a, b and c are integers. **[E]**

Solution:

$(x_1, y_1) = (1, 3)$, $(x_2, y_2) = (-19, -19)$

The equation of L is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 3}{-19 - 3} = \frac{x - 1}{-19 - 1}$$

$$\frac{y - 3}{-22} = \frac{x - 1}{-20}$$

Multiply each side by -22:

$$y - 3 = \frac{-22}{-20}\left(x - 1\right)$$

$$y - 3 = \frac{11}{10}\left(x - 1\right)$$

Multiply each term by 10:

$10y - 30 = 11(x - 1)$

$10y - 30 = 11x - 11$

$10y = 11x + 19$

$0 = 11x - 10y + 19$

The equation of L is $11x - 10y + 19 = 0$.

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Coordinate geometry in the (x, y) plane
Exercise F, Question 7

Question:

The straight line l_1 passes through the points A and B with coordinates $(2, 2)$ and $(6, 0)$ respectively.

(a) Find an equation of l_1.

The straight line l_2 passes through the point C with coordinates $(-9, 0)$ and has gradient $\frac{1}{4}$.

(b) Find an equation of l_2. **[E]**

Solution:

(a) The equation of l_1 is

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y-2}{0-2} = \frac{x-2}{6-2}$$

$$\frac{y-2}{-2} = \frac{x-2}{4}$$

Multiply each side by -2:

$y-2 = -\frac{1}{2}\left(x-2\right)$ (**Note:** $-\frac{2}{4} = -\frac{1}{2}$)

$$y-2 = -\frac{1}{2}x+1$$

$$y = -\frac{1}{2}x+3$$

(b) The equation of l_2 is

$$y-y_1 = m(x-x_1)$$

$$y-0 = \frac{1}{4}\left[x-\left(-9\right)\right]$$

$$y = \frac{1}{4}\left(x+9\right)$$

$$y = \frac{1}{4}x+\frac{9}{4}$$

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Coordinate geometry in the (x, y) plane
Exercise F, Question 8

Question:

The straight line l_1 passes through the points A and B with coordinates $(0 , - 2)$ and $(6 , 7)$ respectively.

(a) Find the equation of l_1 in the form $y = mx + c$.
The straight line l_2 with equation $x + y = 8$ cuts the y-axis at the point C. The lines l_1 and l_2 intersect at the point D.

(b) Calculate the coordinates of the point D.

(c) Calculate the area of $\triangle ACD$. **[E]**

Solution:

(a) The equation of l_1 is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - (- 2)}{7 - (- 2)} = \frac{x - 0}{6 - 0}$$

$$\frac{y + 2}{9} = \frac{x}{6}$$

Multiply each term by 9:

$$y + 2 = \frac{9}{6}x$$

$$y + 2 = \frac{3}{2}x$$

$$y = \frac{3}{2}x - 2$$

(b) Solve $x + y = 8$ and $y = \frac{3}{2}x - 2$ simultaneously.

Substitute:

$$x + \left(\frac{3}{2}x - 2 \right) = 8$$

$$x + \frac{3}{2}x - 2 = 8$$

$$\frac{5}{2}x - 2 = 8$$

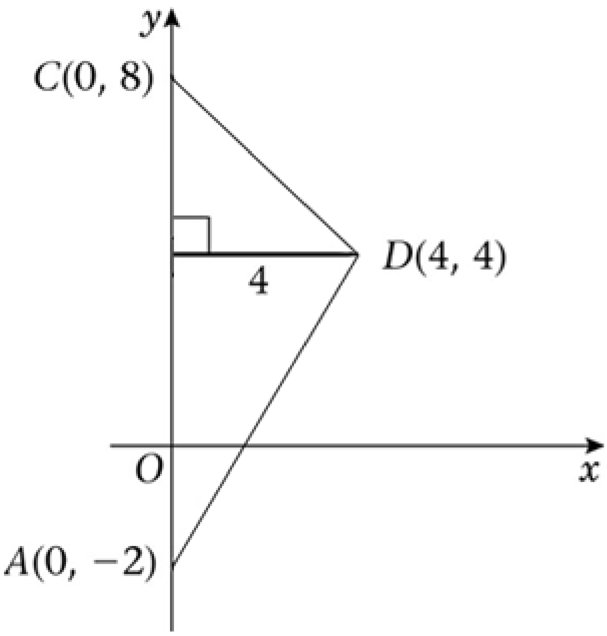
$$\frac{5}{2}x = 10$$

$5x = 20$
$x = 4$
Substitute $x = 4$ into $x + y = 8$:
$(4) + y = 8$
$y = 4$
The coordinates of D are $(4 , 4)$.

(c) $x + y = 8$ cuts the y-axis when $x = 0$.
Substitute $x = 0$:

$0 + y = 8$
$y = 8$
The coordinates of C are $(0, 8)$
$AC = 10$
$h = 4$

Area $= \frac{1}{2} \times 10 \times 4 = 20$



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Coordinate geometry in the (x, y) plane
Exercise F, Question 9

Question:

The points A and B have coordinates $(2, 16)$ and $(12, -4)$ respectively. A straight line l_1 passes through A and B.

(a) Find an equation for l_1 in the form $ax + by = c$.

The line l_2 passes through the point C with coordinates $(-1, 1)$ and has gradient $\frac{1}{3}$.

(b) Find an equation for l_2. **[E]**

Solution:

(a) The equation of l_1 is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 16}{-4 - 16} = \frac{x - 2}{12 - 2}$$

$$\frac{y - 16}{-20} = \frac{x - 2}{10}$$

Multiply each side by -20:

$y - 16 = -2(x - 2)$ (**Note:** $-\frac{20}{10} = -2$)

$y - 16 = -2x + 4$
$y = -2x + 20$
$2x + y = 20$

(b) The equation of l_2 is
$y - y_1 = m(x - x_1)$

$$y - 1 = \frac{1}{3}\left[x - \left(-1\right)\right]$$

$$y - 1 = \frac{1}{3}\left(x + 1\right)$$

$$y - 1 = \frac{1}{3}x + \frac{1}{3}$$

$$y = \frac{1}{3}x + \frac{4}{3}$$

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Coordinate geometry in the (x, y) plane
Exercise F, Question 10

Question:

The points $A(-1, -2)$, $B(7, 2)$ and $C(k, 4)$, where k is a constant, are the vertices of $\triangle ABC$. Angle ABC is a right angle.

(a) Find the gradient of *AB*.

(b) Calculate the value of *k*.

(c) Find an equation of the straight line passing through *B* and *C*. Give your answer in the form $ax + by + c = 0$, where *a*, *b* and *c* are integers. **[E]**

Solution:

(a) The gradient of *AB* is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-2)}{7 - (-1)} = \frac{4}{8} = \frac{1}{2}$$

(b) The gradient of *BC* is

$$\frac{-1}{\frac{1}{2}} = -2$$

So $\frac{y_2 - y_1}{x_2 - x_1} = -2$

$$\Rightarrow \quad \frac{4 - 2}{k - 7} = -2$$

$$\Rightarrow \quad \frac{2}{k - 7} = -2$$

Multiply each side by $(k - 7)$:
$2 = -2(k - 7)$
$2 = -2k + 14$
$-12 = -2k$
$k = 6$

(c) The equation of the line passing through *B* and *C* is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 2}{4 - 2} = \frac{x - 7}{6 - 7}$$

$$\frac{y - 2}{2} = \frac{x - 7}{-1}$$

Multiply each side by 2:

$y - 2 = -2(x - 7)$ (**Note:** $\frac{2}{-1} = -2$)

$y - 2 = -2x + 14$
$y = -2x + 16$
$2x + y = 16$
$2x + y - 16 = 0$

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Coordinate geometry in the (x, y) plane
Exercise F, Question 11

Question:

The straight line l passes through $A\left(1, 3\sqrt{3}\right)$ and $B\left(2+\sqrt{3}, 3+4\sqrt{3}\right)$.

(a) Calculate the gradient of l giving your answer as a surd in its simplest form.

(b) Give the equation of l in the form $y = mx + c$, where constants m and c are surds given in their simplest form.

(c) Show that l meets the x-axis at the point $C\left(-2, 0\right)$. **[E]**

Solution:

(a) The gradient of l is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{\left(3+4\sqrt{3}\right) - 3\sqrt{3}}{\left(2+\sqrt{3}\right) - 1} = \frac{3+\sqrt{3}}{1+\sqrt{3}}$$

Rationalise the denominator:

$$\frac{3+\sqrt{3}}{1+\sqrt{3}} \times \frac{1-\sqrt{3}}{1-\sqrt{3}} = \frac{3-3\sqrt{3}+\sqrt{3}-3}{1-3} = \frac{-2\sqrt{3}}{-2} = \sqrt{3}$$

(b) The equation of l is

$y - y_1 = m\left(x - x_1\right)$

$y - 3\sqrt{3} = \sqrt{3}\left(x - 1\right)$

$y - 3\sqrt{3} = \sqrt{3}x - \sqrt{3}$

$y = \sqrt{3}x + 2\sqrt{3}$

(c) Substitute $y = 0$:

$0 = \sqrt{3}x + 2\sqrt{3}$

$\sqrt{3}x = -2\sqrt{3}$

$x = \frac{-2\sqrt{3}}{\sqrt{3}} = -2$

The coordinates of C are $\left(-2, 0\right)$.

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Coordinate geometry in the (x, y) plane
Exercise F, Question 12

Question:

(a) Find an equation of the straight line passing through the points with coordinates $(-1, 5)$ and $(4, -2)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.
The line crosses the x-axis at the point A and the y-axis at the point B, and O is the origin.

(b) Find the area of $\triangle OAB$. **[E]**

Solution:

(a) The equation of the line is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 5}{-2 - 5} = \frac{x - (-1)}{4 - (-1)}$$

$$\frac{y - 5}{-7} = \frac{x + 1}{5}$$

Multiply each side by -35:

$5(y - 5) = -7(x + 1)$ (**Note:** $\frac{-35}{-7} = 5$ and $\frac{-35}{5} = -7$)

$5y - 25 = -7x - 7$
$7x + 5y - 25 = -7$
$7x + 5y - 18 = 0$

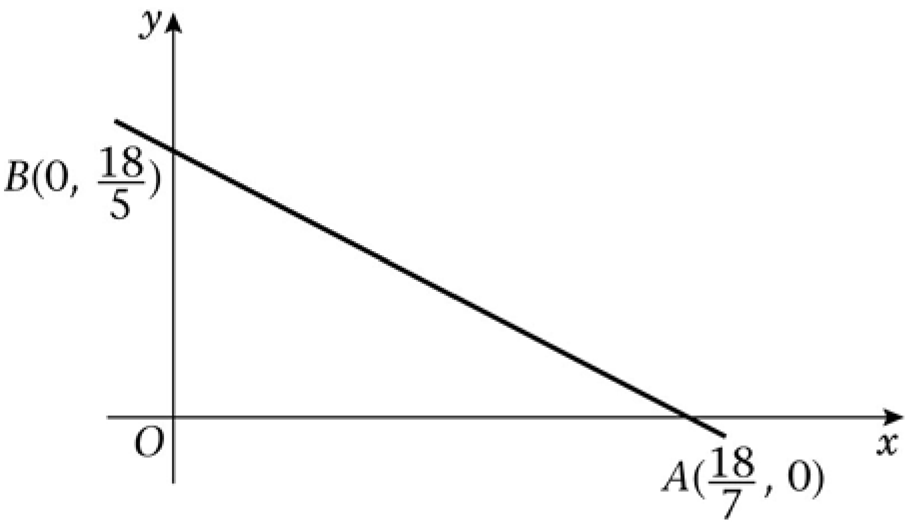
(b) For the coordinates of A substitute $y = 0$:
$7x + 5(0) - 18 = 0$
$7x - 18 = 0$
$7x = 18$
$x = \frac{18}{7}$

The coordinates of A are $\left(\frac{18}{7}, 0\right)$.

For the coordinates of B substitute $x = 0$:
$7(0) + 5y - 18 = 0$
$5y - 18 = 0$
$5y = 18$
$y = \frac{18}{5}$

The coordinates of B are $\left(0, \frac{18}{5}\right)$.

The area of $\triangle OAB$ is
$$\frac{1}{2} \times \frac{18}{7} \times \frac{18}{5} = \frac{162}{35}$$


y
$B(0, \frac{18}{5})$
O
x
$A(\frac{18}{7}, 0)$

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Coordinate geometry in the (x, y) plane
Exercise F, Question 13

Question:

The points A and B have coordinates $(k, 1)$ and $(8, 2k-1)$ respectively, where k is a constant. Given that the gradient of AB is $\frac{1}{3}$,

(a) Show that $k = 2$.

(b) Find an equation for the line through A and B. **[E]**

Solution:

(a) The gradient of AB is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1}{3}$$

$$\frac{(2k-1) - 1}{8 - k} = \frac{1}{3}$$

$$\frac{2k - 1 - 1}{8 - k} = \frac{1}{3}$$

$$\frac{2k - 2}{8 - k} = \frac{1}{3}$$

Multiply each side by $(8 - k)$:

$$2k - 2 = \frac{1}{3}\left(8 - k\right)$$

Multiply each term by 3:
$6k - 6 = 8 - k$
$7k - 6 = 8$
$7k = 14$
$k = 2$

(b) $k = 2$
So A and B have coordinates $(2, 1)$ and $(8, 3)$.
The equation of the line is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 1}{3 - 1} = \frac{x - 2}{8 - 2}$$

$$\frac{y - 1}{2} = \frac{x - 2}{6}$$

Multiply each side by 2:

$$y - 1 = \frac{1}{3}\left(x - 2\right)$$

$$y - 1 = \frac{1}{3}x - \frac{2}{3}$$

$$y = \frac{1}{3}x + \frac{1}{3}$$

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Coordinate geometry in the (x, y) plane

Exercise F, Question 14

Question:

The straight line l_1 has equation $4y + x = 0$.
The straight line l_2 has equation $y = 2x - 3$.

(a) On the same axes, sketch the graphs of l_1 and l_2. Show clearly the coordinates of all points at which the graphs meet the coordinate axes.
The lines l_1 and l_2 intersect at the point A.

(b) Calculate, as exact fractions, the coordinates of A.

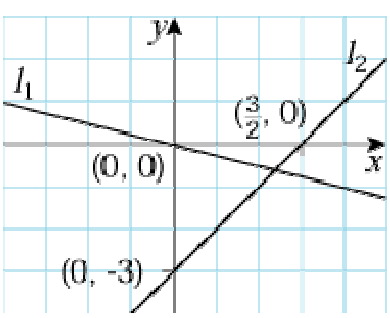
(c) Find an equation of the line through A which is perpendicular to l_1. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. **[E]**

Solution:

(a) (1) Rearrange $4y + x = 0$ into the form $y = mx + c$:
$4y = - x$
$y = - \frac{1}{4}x$

l_1 has gradient $- \frac{1}{4}$ and it meets the coordinate axes at $(0 , 0)$.
(2) l_2 has gradient 2 and it meets the y-axis at $(0 , - 3)$.
l_2 meets the x-axis when $y = 0$.
Substitute $y = 0$:
$0 = 2x - 3$
$2x = 3$
$x = \frac{3}{2}$

l_2 meets the x-axis at $\left(\frac{3}{2} , 0 \right)$.



(b) Solve $4y + x = 0$ and $y = 2x - 3$ simultaneously.
Substitute:
$4 (2x - 3) + x = 0$
$8x - 12 + x = 0$
$9x - 12 = 0$
$9x = 12$

$x = \frac{12}{9}$

$x = \frac{4}{3}$

Substitute $x = \frac{4}{3}$ into $y = 2x - 3$:

$y = 2\left(\frac{4}{3}\right) - 3 = \frac{8}{3} - 3 = -\frac{1}{3}$

The coordinates of A are $\left(\frac{4}{3}, -\frac{1}{3}\right)$.

(c) The gradient of l_1 is $-\frac{1}{4}$.

The gradient of a line perpendicular to l_1 is $-\frac{1}{-\frac{1}{4}} = 4$.

The equation of the line is
$y - y_1 = m(x - x_1)$

$y - \left(-\frac{1}{3}\right) = 4\left(x - \frac{4}{3}\right)$

$y + \frac{1}{3} = 4x - \frac{16}{3}$

$y = 4x - \frac{17}{3}$

Multiply each term by 3:
$3y = 12x - 17$
$0 = 12x - 3y - 17$
The equation of the line is $12x - 3y - 17 = 0$.

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Coordinate geometry in the (x, y) plane
Exercise F, Question 15

Question:

The points A and B have coordinates $(4 , 6)$ and $(12 , 2)$ respectively.

The straight line l_1 passes through A and B.

(a) Find an equation for l_1 in the form $ax + by + c = 0$, where a, b and c are integers.
The straight line l_2 passes through the origin and has gradient -4.

(b) Write down an equation for l_2.
The lines l_1 and l_2 intersect at the point C.

(c) Find the coordinates of C. **[E]**

Solution:

(a) The equation of l_1 is

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y-6}{2-6} = \frac{x-4}{12-4}$$

$$\frac{y-6}{-4} = \frac{x-4}{8}$$

Multiply each side by 8:

$-2 (y - 6) = x - 4$ (**Note:** $\frac{8}{-4} = -2$)

$-2y + 12 = x - 4$
$-2y + 16 = x$
$16 = x + 2y$
$0 = x + 2y - 16$
The equation of the line is $x + 2y - 16 = 0$

(b) The equation of l_2 is
$y - y_1 = m (x - x_1)$
$y - 0 = -4 (x - 0)$
$y = -4x$

(c) Solve $y = -4x$ and $x + 2y = 16$ simultaneously.
Substitute:
$x + 2 (-4x) = 16$
$x - 8x = 16$
$-7x = 16$

$$x = \frac{16}{-7}$$

$$x = -\frac{16}{7}$$

Substitute $x = -\frac{16}{7}$ in $y = -4x$:

$y = -4\left(-\frac{16}{7}\right) = \frac{64}{7}$

The coordinates of C are $\left(-\frac{16}{7}, \frac{64}{7}\right)$.

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Sequences and series
Exercise A, Question 1

Question:

Work out the next three terms of the following sequence. State the rule to find the next term:

4, 9, 14, 19, …

Solution:

$$4 \xrightarrow{+5} 9 \xrightarrow{+5} 14 \xrightarrow{+5} 19 \xrightarrow{+5} 24 \xrightarrow{+5} 29 \xrightarrow{+5} 34$$

“Add 5 to previous term”

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Sequences and series
Exercise A, Question 2

Question:

Work out the next three terms of the following sequence. State the rule to find the next term:

$2, \ -2, 2, \ -2, \quad \ldots$

Solution:

$$2 \xrightarrow{\times -1} -2 \xrightarrow{\times -1} 2 \xrightarrow{\times -1} -2 \xrightarrow{\times -1} 2 \xrightarrow{\times -1} -2 \xrightarrow{\times -1} 2$$

"Multiply previous term by -1"

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Sequences and series
Exercise A, Question 3

Question:

Work out the next three terms of the following sequence. State the rule to find the next term:

30, 27, 24, 21, …

Solution:

$$\begin{tikzcd}
30 \arrow[r, bend right, "-3"'] & 27 \arrow[r, bend right, "-3"'] & 24 \arrow[r, bend right, "-3"'] & 21 \arrow[r, bend right, "-3"'] & 18 \arrow[r, bend right, "-3"'] & 15 \arrow[r, bend right, "-3"'] & 12
\end{tikzcd}$$

"Subtract 3 from previous term"

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Sequences and series
Exercise A, Question 4

Question:

Work out the next three terms of the following sequence. State the rule to find the next term:

2, 6, 18, 54, …

Solution:

$$2 \xrightarrow{\times 3} 6 \xrightarrow{\times 3} 18 \xrightarrow{\times 3} 54 \xrightarrow{\times 3} 162 \xrightarrow{\times 3} 486 \xrightarrow{\times 3} 1458$$

"Multiply previous term by 3"

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Sequences and series
Exercise A, Question 5

Question:

Work out the next three terms of the following sequence. State the rule to find the next term:

$4, \ -2, 1, \ -\frac{1}{2}, \quad \ldots$

Solution:

$$4 \xrightarrow{\times -\frac{1}{2}} -2 \xrightarrow{\times -\frac{1}{2}} 1 \xrightarrow{\times -\frac{1}{2}} -\frac{1}{2} \xrightarrow{\times -\frac{1}{2}} +\frac{1}{4} \xrightarrow{\times -\frac{1}{2}} -\frac{1}{8} \xrightarrow{\times -\frac{1}{2}} +\frac{1}{16}$$

"Multiply previous term by $-\frac{1}{2}$" (or "divide by -2")

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Sequences and series
Exercise A, Question 6

Question:

Work out the next three terms of the following sequence. State the rule to find the next term:

1, 2, 5, 14, …

Solution:

$$1 \xrightarrow{\times 3\ -1} 2 \xrightarrow{\times 3\ -1} 5 \xrightarrow{\times 3\ -1} 14 \xrightarrow{\times 3\ -1} 41 \xrightarrow{\times 3\ -1} 122 \xrightarrow{\times 3\ -1} 365$$

“Multiply previous term by 3 then subtract 1”

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Sequences and series
Exercise A, Question 7

Question:

Work out the next three terms of the following sequence. State the rule to find the next term:

1, 1, 2, 3, 5, …

Solution:

$1 \quad 1 \quad 2 \quad 3 \quad 5 \quad 8 \quad 13 \quad 21$

$1 + 1 = 2$ $\quad$ $3 + 5 = 8$ $\quad$ $8 + 13 = 21$

“Add together the two previous terms”

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Sequences and series

Exercise A, Question 8

Question:

Work out the next three terms of the following sequence. State the rule to find the next term:

$1, \frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \quad \ldots$

Solution:

$1\ ,\ \frac{2}{3}\ ,\ \frac{3}{5}\ ,\ \frac{4}{7}\ ,\ \frac{5}{9}\ ,\ \frac{6}{11}\ ,\ \frac{7}{13}$

“Add 1 to previous numerator, 2 to previous denominator”

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Sequences and series
Exercise A, Question 9

Question:

Work out the next three terms of the following sequence. State the rule to find the next term:

4, 3, 2.5, 2.25, 2.125, …

Solution:

$$4 \xrightarrow{\div 2 + 1} 3 \xrightarrow{\div 2 + 1} 2.5 \xrightarrow{\div 2 + 1} 2.25 \xrightarrow{\div 2 + 1} 2.125 \xrightarrow{\div 2 + 1} 2.0625 \xrightarrow{\div 2 + 1} 2.03125 \xrightarrow{\div 2 + 1} 2.015625$$

“Divide previous term by 2 then add 1”

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Sequences and series
Exercise A, Question 10

Question:

Work out the next three terms of the following sequence. State the rule to find the next term:

0, 3, 8, 15, …

Solution:

$$0 \xrightarrow{+3} 3 \xrightarrow{+5} 8 \xrightarrow{+7} 15 \xrightarrow{+9} 24 \xrightarrow{+11} 35 \xrightarrow{+13} 48$$

"Add consecutive odd numbers to previous term"

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Sequences and series
Exercise B, Question 1

Question:

Find the U_1, U_2, U_3 and U_{10} of the following sequences, where:

(a) $U_n = 3n + 2$

(b) $U_n = 10 - 3n$

(c) $U_n = n^2 + 5$

(d) $U_n = (n - 3)^2$

(e) $U_n = (-2)^n$

(f) $U_n = \frac{n}{n+2}$

(g) $U_n = (-1)^n \frac{n}{n+2}$

(h) $U_n = (n - 2)^3$

Solution:

(a) $U_1 = 3 \times 1 + 2 = 5$, $U_2 = 3 \times 2 + 2 = 8$, $U_3 = 3 \times 3 + 2 = 11$, $U_{10} = 3 \times 10 + 2 = 32$

(b) $U_1 = 10 - 3 \times 1 = 7$, $U_2 = 10 - 3 \times 2 = 4$, $U_3 = 10 - 3 \times 3 = 1$, $U_{10} = 10 - 3 \times 10 = -20$

(c) $U_1 = 1^2 + 5 = 6$, $U_2 = 2^2 + 5 = 9$, $U_3 = 3^2 + 5 = 14$, $U_{10} = 10^2 + 5 = 105$

(d) $U_1 = (1 - 3)^2 = 4$, $U_2 = (2 - 3)^2 = 1$, $U_3 = (3 - 3)^2 = 0$, $U_{10} = (10 - 3)^2 = 49$

(e) $U_1 = (-2)^1 = -2$, $U_2 = (-2)^2 = 4$, $U_3 = (-2)^3 = -8$, $U_{10} = (-2)^{10} = 1024$

(f) $U_1 = \frac{1}{1+2} = \frac{1}{3}$, $U_2 = \frac{2}{2+2} = \frac{2}{4} = \frac{1}{2}$, $U_3 = \frac{3}{3+2} = \frac{3}{5}$, $U_{10} = \frac{10}{10+2} = \frac{10}{12} = \frac{5}{6}$

(g) $U_1 = (-1)^1 \frac{1}{1+2} = -\frac{1}{3}$, $U_2 = (-1)^2 \frac{2}{2+2} = \frac{2}{4} = \frac{1}{2}$, $U_3 = (-1)^3 \frac{3}{3+2} = -\frac{3}{5}$, $U_{10} = (-1)^{10} \frac{10}{10+2} = \frac{10}{12} = \frac{5}{6}$

(h) $U_1 = (1 - 2)^3 = (-1)^3 = -1$, $U_2 = (2 - 2)^3 = 0$, $U_3 = (3 - 2)^3 = 1$, $U_{10} = (10 - 2)^3 = 8^3 = 512$

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Sequences and series
Exercise B, Question 2

Question:

Find the value of n for which U_n has the given value:

(a) $U_n = 2n - 4$, $U_n = 24$

(b) $U_n = (n - 4)^2$, $U_n = 25$

(c) $U_n = n^2 - 9$, $U_n = 112$

(d) $U_n = \frac{2n + 1}{n - 3}$, $U_n = \frac{19}{6}$

(e) $U_n = n^2 + 5n - 6$, $U_n = 60$

(f) $U_n = n^2 - 4n + 11$, $U_n = 56$

(g) $U_n = n^2 + 4n - 5$, $U_n = 91$

(h) $U_n = (- 1)^n \frac{n}{n + 4}$, $U_n = \frac{7}{9}$

(i) $U_n = \frac{n^3 + 3}{5}$, $U_n = 13.4$

(j) $U_n = \frac{n^3}{5} + 3$, $U_n = 28$

Solution:

(a) $24 = 2n - 4$
$28 = 2n \quad (+ 4)$
$14 = n \quad (\div 2)$
$n = 14$

(b) $25 = (n - 4)^2$
$\pm 5 = (n - 4) \quad (\sqrt{\ })$
$9, - 1 = n \quad (+ 4)$
$n = 9$ (it must be positive)

(c) $112 = n^2 - 9$
$121 = n^2 \quad (+ 9)$
$\pm 11 = n \quad (\sqrt{\ })$
$n = 11$

(d) $\frac{19}{6} = \frac{2n + 1}{n - 3}$ (cross multiply)

$19 (n - 3) = 6 (2n + 1)$

$19n - 57 = 12n + 6 \quad (-12n)$
$7n - 57 = 6 \quad (+57)$
$7n = 63$
$n = 9$

(e) $60 = n^2 + 5n - 6 \quad (-60)$
$0 = n^2 + 5n - 66 \quad$ (factorise)
$0 = (n + 11) (n - 6)$
$n = -11, 6$
$n = 6$

(f) $56 = n^2 - 4n + 11 \quad (-56)$
$0 = n^2 - 4n - 45 \quad$ (factorise)
$0 = (n - 9) (n + 5)$
$n = 9, -5$
$n = 9$

(g) $91 = n^2 + 4n - 5 \quad (-91)$
$0 = n^2 + 4n - 96 \quad$ (factorise)
$0 = (n + 12) (n - 8)$
$n = -12, 8$
$n = 8$

(h) $\frac{7}{9} = (-1)^n \frac{n}{n+4}$

n must be even

$\frac{7}{9} = \frac{n}{n+4}$

$7 (n + 4) = 9n$
$7n + 28 = 9n$
$28 = 2n$
$n = 14$

(i) $13.4 = \frac{n^3 + 3}{5} \quad (\times 5)$

$67 = n^3 + 3 \quad (-3)$
$64 = n^3 \quad (\sqrt[3]{\ })$
$n = 4$

(j) $28 = \frac{n^3}{5} + 3 \quad (-3)$

$25 = \frac{n^3}{5} \quad (\times 5)$

$125 = n^3 \quad (\sqrt[3]{\ })$
$n = 5$

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Sequences and series
Exercise B, Question 3

Question:

Prove that the $(2n + 1)$ th term of the sequence $U_n = n^2 - 1$ is a multiple of 4.

Solution:

$(2n + 1)$ th term
$= (2n + 1)^2 - 1$
$= (2n + 1)(2n + 1) - 1$
$= 4n^2 + 4n + 1 - 1$
$= 4n^2 + 4n$
$= 4n(n + 1)$
$= 4 \times n(n + 1)$
= multiple of 4 because it is 4 × whole number.

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Sequences and series
Exercise B, Question 4

Question:

Prove that the terms of the sequence $U_n = n^2 - 10n + 27$ are all positive. For what value of n is U_n smallest?

Solution:

$U_n = n^2 - 10n + 27 = (n - 5)^2 - 25 + 27 = (n - 5)^2 + 2$

$(n - 5)^2$ is always positive (or zero) because it is a square.

$\therefore U_n \geq 0 + 2$

Smallest value of U_n is 2.

(It occurs when $n = 5$.)

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Sequences and series

Exercise B, Question 5

Question:

A sequence is generated according to the formula $U_n = an + b$, where a and b are constants. Given that $U_3 = 14$ and $U_5 = 38$, find the values of a and b.

Solution:

$U_n = an + b$

when $n = 3$, $U_3 = 14 \quad \Rightarrow \quad 14 = 3a + b$ ①

when $n = 5$, $U_5 = 38 \quad \Rightarrow \quad 38 = 5a + b$ ②

② − ①: $24 = 2a \quad \Rightarrow \quad a = 12$

substitute $a = 12$ in ①: $14 = 3 \times 12 + b \quad \Rightarrow \quad 14 = 36 + b \quad \Rightarrow \quad b = -22$

$\therefore U_n = 12n - 22$

(check: when $n = 3$, $U_3 = 12 \times 3 - 22 = 36 - 22 = 14$ ✓)

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Sequences and series
Exercise B, Question 6

Question:

A sequence is generated according to the formula $U_n = an^2 + bn + c$, where a, b and c are constants. If $U_1 = 4$, $U_2 = 10$ and $U_3 = 18$, find the values of a, b and c.

Solution:

$U_n = an^2 + bn + c$
when $n = 1$, $U_n = 4 \quad \Rightarrow \quad 4 = a \times 1^2 + b \times 1 + c \quad \Rightarrow \quad 4 = a + b + c$
when $n = 2$, $U_2 = 10 \quad \Rightarrow \quad 10 = a \times 2^2 + b \times 2 + c \quad \Rightarrow \quad 10 = 4a + 2b + c$
when $n = 3$, $U_3 = 18 \quad \Rightarrow \quad 18 = a \times 3^2 + b \times 3 + c \quad \Rightarrow \quad 18 = 9a + 3b + c$
we need to solve simultaneously
$a + b + c = 4$ ①
$4a + 2b + c = 10$ ②
$9a + 3b + c = 18$ ③
② − ①: $3a + b = 6$ ④
③ − ②: $5a + b = 8$ ⑤
⑤ − ④: $2a = 2 \quad \Rightarrow \quad a = 1$
Substitute $a = 1$ in ④: $3 + b = 6 \quad \Rightarrow \quad b = 3$
Substitute $a = 1$, $b = 3$ in ①: $1 + 3 + c = 4 \quad \Rightarrow \quad c = 0$
$\therefore U_n = 1n^2 + 3n + 0 = n^2 + 3n$

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Sequences and series
Exercise B, Question 7

Question:

A sequence is generated from the formula $U_n = pn^3 + q$, where p and q are constants. Given that $U_1 = 6$ and $U_3 = 19$, find the values of the constants p and q.

Solution:

$U_n = pn^3 + q$
when $n = 1$, $U_1 = 6 \quad \Rightarrow \quad 6 = p \times 1^3 + q \quad \Rightarrow \quad 6 = p + q$
when $n = 3$, $U_3 = 19 \quad \Rightarrow \quad 19 = p \times 3^3 + q \quad \Rightarrow \quad 19 = 27p + q$
Solve simultaneously:
$p + q = 6$ ①
$27p + q = 19$ ②
② − ①: $26p = 13 \quad \Rightarrow \quad p = \frac{1}{2}$

substitute $p = \frac{1}{2}$ in ①: $\frac{1}{2} + q = 6 \quad \Rightarrow \quad q = 5\frac{1}{2}$

$\therefore U_n = \frac{1}{2}n^3 + 5\frac{1}{2}$ or $\frac{1}{2}n^3 + \frac{11}{2}$ or $\frac{n^3 + 11}{2}$

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Sequences and series
Exercise C, Question 1

Question:

Find the first four terms of the following recurrence relationships:

(a) $U_{n+1} = U_n + 3$, $U_1 = 1$

(b) $U_{n+1} = U_n - 5$, $U_1 = 9$

(c) $U_{n+1} = 2U_n$, $U_1 = 3$

(d) $U_{n+1} = 2U_n + 1$, $U_1 = 2$

(e) $U_{n+1} = \frac{U_n}{2}$, $U_1 = 10$

(f) $U_{n+1} = \left(U_n \right)^2 - 1$, $U_1 = 2$

(g) $U_{n+2} = 2U_{n+1} + U_n$, $U_1 = 3$, $U_2 = 5$

Solution:

(a) $U_{n+1} = U_n + 3$, $U_1 = 1$
$n = 1 \quad \Rightarrow \quad U_2 = U_1 + 3 = 1 + 3 = 4$
$n = 2 \quad \Rightarrow \quad U_3 = U_2 + 3 = 4 + 3 = 7$
$n = 3 \quad \Rightarrow \quad U_4 = U_3 + 3 = 7 + 3 = 10$
Terms are 1, 4, 7, 10, ...

(b) $U_{n+1} = U_n - 5$, $U_1 = 9$
$n = 1 \quad \Rightarrow \quad U_2 = U_1 - 5 = 9 - 5 = 4$
$n = 2 \quad \Rightarrow \quad U_3 = U_2 - 5 = 4 - 5 = -1$
$n = 3 \quad \Rightarrow \quad U_4 = U_3 - 5 = -1 - 5 = -6$
Terms are 9, 4, -1, -6, ...

(c) $U_{n+1} = 2U_n$, $U_1 = 3$
$n = 1 \quad \Rightarrow \quad U_2 = 2U_1 = 2 \times 3 = 6$
$n = 2 \quad \Rightarrow \quad U_3 = 2U_2 = 2 \times 6 = 12$
$n = 3 \quad \Rightarrow \quad U_4 = 2U_3 = 2 \times 12 = 24$
Terms are 3, 6, 12, 24, ...

(d) $U_{n+1} = 2U_n + 1$, $U_1 = 2$
$n = 1 \quad \Rightarrow \quad U_2 = 2U_1 + 1 = 2 \times 2 + 1 = 5$
$n = 2 \quad \Rightarrow \quad U_3 = 2U_2 + 1 = 2 \times 5 + 1 = 11$
$n = 3 \quad \Rightarrow \quad U_4 = 2U_3 + 1 = 2 \times 11 + 1 = 23$
Terms are 2, 5, 11, 23, ...

(e) $U_{n+1} = \frac{U_n}{2}$, $U_1 = 10$

$n = 1 \quad \Rightarrow \quad U_2 = \frac{U_1}{2} = \frac{10}{2} = 5$

$n = 2 \quad \Rightarrow \quad U_3 = \frac{U_2}{2} = \frac{5}{2} = 2.5$

$n = 3 \quad \Rightarrow \quad U_4 = \frac{U_3}{2} = \frac{2.5}{2} = 1.25$

Terms are 10, 5, 2.5, 1.25, ...

(f) $U_{n+1} = (U_n)^2 - 1$, $U_1 = 2$
$n = 1 \quad \Rightarrow \quad U_2 = (U_1)^2 - 1 = 2^2 - 1 = 4 - 1 = 3$
$n = 2 \quad \Rightarrow \quad U_3 = (U_2)^2 - 1 = 3^2 - 1 = 9 - 1 = 8$
$n = 3 \quad \Rightarrow \quad U_4 = (U_3)^2 - 1 = 8^2 - 1 = 64 - 1 = 63$
Terms are 2, 3, 8, 63, ...

(g) $U_{n+2} = 2U_{n+1} + U_n$, $U_1 = 3$, $U_2 = 5$
$n = 1 \quad \Rightarrow \quad U_3 = 2U_2 + U_1 = 2 \times 5 + 3 = 13$
$n = 2 \quad \Rightarrow \quad U_4 = 2U_3 + U_2 = 2 \times 13 + 5 = 31$
Terms are 3, 5, 13, 31, ...

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Sequences and series
Exercise C, Question 2

Question:

Suggest possible recurrence relationships for the following sequences (remember to state the first term):

(a) 3, 5, 7, 9, …

(b) 20, 17, 14, 11, …

(c) 1, 2, 4, 8, …

(d) 100, 25, 6.25, 1.5625, …

(e) 1, − 1, 1, − 1, 1, …

(f) 3, 7, 15, 31, …

(g) 0, 1, 2, 5, 26, …

(h) 26, 14, 8, 5, 3.5, …

(i) 1, 1, 2, 3, 5, 8, 13, …

(j) 4, 10, 18, 38, 74, …

Solution:

(a) $$3 \xrightarrow{+2} 5 \xrightarrow{+2} 7 \xrightarrow{+2} 9 \ldots$$

$U_{n+1} = U_n + 2$, $U_1 = 3$

(b) $$20 \xrightarrow{-3} 17 \xrightarrow{-3} 14 \xrightarrow{-3} 11 \ldots$$

$U_{n+1} = U_n - 3$, $U_1 = 20$

(c) $$1 \xrightarrow{\times 2} 2 \xrightarrow{\times 2} 4 \xrightarrow{\times 2} 8 \ldots$$

$U_{n+1} = 2 \times U_n$, $U_1 = 1$

(d) $100 \xrightarrow{\div 4} 25 \xrightarrow{\div 4} 6.25 \xrightarrow{\div 4} 1.5625 \ldots$

$U_{n+1} = \frac{U_n}{4}, U_1 = 100$

(e) $1 \xrightarrow{\times -1} -1 \xrightarrow{\times -1} 1 \xrightarrow{\times -1} -1 \ldots$

$U_{n+1} = (-1) \times U_n, U_1 = 1$

(f) $3 \xrightarrow{\times 2 + 1} 7 \xrightarrow{\times 2 + 1} 15 \xrightarrow{\times 2 + 1} 31 \ldots$

$U_{n+1} = 2U_n + 1, U_1 = 3$

(g) $0 \xrightarrow{0^2 + 1} 1 \xrightarrow{1^2 + 1} 2 \xrightarrow{2^2 + 1} 5 \xrightarrow{5^2 + 1} 26 \ldots$

$U_{n+1} = (U_n)^2 + 1, U_1 = 0$

(h) $26 \xrightarrow{+2 \div 2} 14 \xrightarrow{+2 \div 2} 8 \xrightarrow{+2 \div 2} 5 \xrightarrow{+2 \div 2} 3.5 \ldots$

$U_{n+1} = \frac{U_n + 2}{2}, U_1 = 26$

(i) $1, \quad 1, \quad \underset{\uparrow \atop 1+1}{2}, \quad \underset{\uparrow \atop 2+1}{3}, \quad \underset{\uparrow \atop 3+2}{5}, \quad \underset{\uparrow \atop 5+3}{8}, \quad \underset{\uparrow \atop 8+5}{13}, \ldots$

$U_{n+2} = U_{n+1} + U_n, U_1 = 1, U_2 = 1$

(j) $4, \quad 10, \quad \underset{\nearrow \atop 2 \times 4 + 10}{18}, \quad \underset{\nearrow \atop 2 \times 10 + 18}{38}, \quad \underset{\nearrow \atop 2 \times 18 + 38}{74}, \ldots$

$U_{n+2} = U_{n+1} + 2U_n, U_1 = 4, U_2 = 10$

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Sequences and series
Exercise C, Question 3

Question:

By writing down the first four terms or otherwise, find the recurrence formula that defines the following sequences:

(a) $U_n = 2n - 1$

(b) $U_n = 3n + 2$

(c) $U_n = n + 2$

(d) $U_n = \frac{n+1}{2}$

(e) $U_n = n^2$

(f) $U_n = (-1)^n n$

Solution:

(a) $U_n = 2n - 1$. Substituting $n = 1, 2, 3$ and 4 gives

$$U_1 = 1 \xrightarrow{+2} U_2 = 3 \xrightarrow{+2} U_3 = 5 \xrightarrow{+2} U_4 = 7$$

Recurrence formula is $U_{n+1} = U_n + 2$, $U_1 = 1$.

(b) $U_n = 3n + 2$. Substituting $n = 1, 2, 3$ and 4 gives

$$U_1 = 5 \xrightarrow{+3} U_2 = 8 \xrightarrow{+3} U_3 = 11 \xrightarrow{+3} U_4 = 14$$

Recurrence formula is $U_{n+1} = U_n + 3$, $U_1 = 5$.

(c) $U_n = n + 2$. Substituting $n = 1, 2, 3$ and 4 gives

$$U_1 = 3 \xrightarrow{+1} U_2 = 4 \xrightarrow{+1} U_3 = 5 \xrightarrow{+1} U_4 = 6$$

Recurrence formula is $U_{n+1} = U_n + 1$, $U_1 = 3$.

(d) $U_n = \frac{n+1}{2}$. Substituting $n = 1, 2, 3$ and 4 gives

$$U_1 = 1 \xrightarrow{+\frac{1}{2}} U_2 = 1\tfrac{1}{2} \xrightarrow{+\frac{1}{2}} U_3 = 2 \xrightarrow{+\frac{1}{2}} U_4 = 2\tfrac{1}{2}$$

Recurrence formula is $U_{n+1} = U_n + \frac{1}{2}$, $U_1 = 1$.

(e) $U_n = n^2$. Substituting $n = 1, 2, 3$ and 4 gives

$$U_1 = 1 \xrightarrow[= 2 \times 1 + 1]{+3} U_2 = 4 \xrightarrow[= 2 \times 2 + 1]{+5} U_3 = 9 \xrightarrow[= 2 \times 3 + 1]{+7} U_4 = 16$$

$U_{n+1} = U_n + 2n + 1$, $U_1 = 1$.

(f) $U_n = (-1)^n n$. Substituting $n = 1, 2, 3$ and 4 gives

$$U_1 = -1 \xrightarrow[= 2 \times 1 + 1]{+3} U_2 = 2 \xrightarrow[= -(2 \times 2 + 1)]{-5} U_3 = -3 \xrightarrow[= 2 \times 3 + 1]{+7} U_4 = 4$$

$U_{n+1} = U_n - (-1)^n (2n + 1)$, $U_1 = 1$.

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Sequences and series
Exercise C, Question 4

Question:

A sequence of terms $\{ U_n \}$ is defined $n \geq 1$ by the recurrence relation $U_{n+1} = kU_n + 2$, where k is a constant. Given that $U_1 = 3$:

(a) Find an expression in terms of k for U_2.

(b) Hence find an expression for U_3.

Given that $U_3 = 42$:

(c) Find possible values of k.

Solution:

$U_{n+1} = kU_n + 2$

(a) Substitute $n = 1 \Rightarrow U_2 = kU_1 + 2$

As $U_1 = 3 \Rightarrow U_2 = 3k + 2$

(b) Substitute $n = 2 \Rightarrow U_3 = kU_2 + 2$

As $U_2 = 3k + 2 \Rightarrow U_3 = k (3k + 2) + 2$

$\Rightarrow U_3 = 3k^2 + 2k + 2$

(c) We are given $U_3 = 42$

$\Rightarrow 3k^2 + 2k + 2 = 42 \quad (- 42)$

$\Rightarrow 3k^2 + 2k - 40 = 0$

$\Rightarrow (3k - 10) (k + 4) = 0$

$\Rightarrow k = \frac{10}{3} , - 4$

Possible values of k are $\frac{10}{3} , - 4$.

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Sequences and series
Exercise C, Question 5

Question:

A sequence of terms $\{ U_k \{$ is defined $k \geq 1$ by the recurrence relation $U_{k+2} = U_{k+1} - pU_k$, where p is a constant. Given that $U_1 = 2$ and $U_2 = 4$:

(a) Find an expression in terms of p for U_3.

(b) Hence find an expression in terms of p for U_4.

Given also that U_4 is twice the value of U_3:

(c) Find the value of p.

Solution:

(a) $U_{k+2} = U_{k+1} - pU_k$
Let $k = 1$, then $U_3 = U_2 - pU_1$
Substitute $U_1 = 2$, $U_2 = 4$: $U_3 = 4 - p \times 2 \Rightarrow U_3 = 4 - 2p$

(b) $U_{k+2} = U_{k+1} - pU_k$
Let $k = 2$, then $U_4 = U_3 - pU_2$
Substitute $U_2 = 4$, $U_3 = 4 - 2p$: $U_4 = (4 - 2p) - p \times 4 = 4 - 2p - 4p = 4 - 6p$

(c) We are told U_4 is twice U_3, so
$U_4 = 2 \times U_3$
$4 - 6p = 2 (4 - 2p)$
$4 - 6p = 8 - 4p$
$-4 = 2p$
$-2 = p$
Hence $p = -2$.

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Sequences and series
Exercise D, Question 1

Question:

Which of the following sequences are arithmetic?

(a) 3, 5, 7, 9, 11, …

(b) 10, 7, 4, 1, …

(c) y, $2y$, $3y$, $4y$, …

(d) 1, 4, 9, 16, 25, …

(e) 16, 8, 4, 2, 1, …

(f) 1, − 1, 1, − 1, 1, …

(g) y, y^2, y^3, y^4, …

(h) $U_{n+1} = U_n + 2$, $U_1 = 3$

(i) $U_{n+1} = 3U_n - 2$, $U_1 = 4$

(j) $U_{n+1} = \left(U_n \right)^2$, $U_1 = 2$

(k) $U_n = n \left(n + 1 \right)$

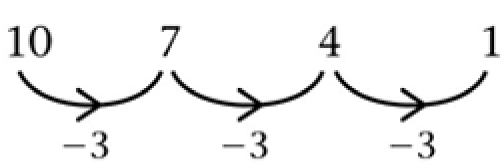
(l) $U_n = 2n + 3$

Solution:

(a)

Arithmetic (+ 2)

(b)



Arithmetic (− 3)

(c)

$$y \xrightarrow{+y} 2y \xrightarrow{+y} 3y \xrightarrow{+y} 4y$$

Arithmetic ($+y$)

(d)

$$1 \xrightarrow{+3} 4 \xrightarrow{+5} 9 \xrightarrow{+7} 16$$

Not arithmetic

(e)

$$16 \xrightarrow{\div 2} 8 \xrightarrow{\div 2} 4 \xrightarrow{\div 2} 2 \xrightarrow{\div 2} 1$$

Not arithmetic

(f)

$$1 \xrightarrow{\times -1} -1 \xrightarrow{\times -1} 1 \xrightarrow{\times -1} -1 \xrightarrow{\times -1} 1$$

Not arithmetic

(g)

$$y \xrightarrow{\times y} y^2 \xrightarrow{\times y} y^3 \xrightarrow{\times y} y^4$$

Not arithmetic

(h) $U_{n+1} = U_n + 2$

$$3 \xrightarrow{+2} 5 \xrightarrow{+2} 7 \xrightarrow{+2} 9 \xrightarrow{+2} 11$$

Arithmetic ($+2$)

(i) $U_{n+1} = 3U_n - 2$

$$4 \xrightarrow{\times 3 - 2} 10 \xrightarrow{\times 3 - 2} 28$$

Not arithmetic

(j) $U_{n+1} = (U_n)^2, U_1 = 2$
2, 4, 16, 256
Not arithmetic

(k) $U_n = n(n+1)$
2, 6, 12, 20
Not arithmetic

(l) $U_n = 2n + 3$

$$5 \xrightarrow{+2} 7 \xrightarrow{+2} 9 \xrightarrow{+2} 11$$

Arithmetic (+ 2)

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Sequences and series
Exercise D, Question 2

Question:

Find the 10th and *n*th terms in the following arithmetic progressions:

(a) 5, 7, 9, 11, …

(b) 5, 8, 11, 14, …

(c) 24, 21, 18, 15, …

(d) − 1, 3, 7, 11, …

(e) x, $2x$, $3x$, $4x$, …

(f) a, $a + d$, $a + 2d$, $a + 3d$, …

Solution:

(a)

$$5 \xrightarrow{+2} 7 \xrightarrow{+2} 9 \xrightarrow{+2} 11$$

10th term = $5 + 9 \times 2 = 5 + 18 = 23$
nth term = $5 + (n - 1) \times 2 = 5 + 2n - 2 = 2n + 3$

(b)

$$5 \xrightarrow{+3} 8 \xrightarrow{+3} 11 \xrightarrow{+3} 14$$

10th term = $5 + 9 \times 3 = 5 + 27 = 32$
nth term = $5 + (n - 1) \times 3 = 5 + 3n - 3 = 3n + 2$

(c)

$$24 \xrightarrow{-3} 21 \xrightarrow{-3} 18 \xrightarrow{-3} 15$$

10th term = $24 + 9 \times - 3 = 24 - 27 = - 3$
nth term = $24 + (n - 1) \times - 3 = 24 - 3n + 3 = 27 - 3n$

(d)

$$-1 \xrightarrow{+4} 3 \xrightarrow{+4} 7 \xrightarrow{+4} 11$$

10th term = $-1 + 9 \times 4 = -1 + 36 = 35$
nth term = $-1 + (n - 1) \times 4 = -1 + 4n - 4 = 4n - 5$

(e)

$$x \xrightarrow{+x} 2x \xrightarrow{+x} 3x \xrightarrow{+x} 4x$$

10th term = $x + 9 \times x = 10x$
nth term = $x + (n - 1)x = nx$

(f)

$$a \xrightarrow{+d} a + d \xrightarrow{+d} a + 2d \xrightarrow{+d} a + 3d$$

10th term = $a + 9d$
nth term = $a + (n - 1)d$

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Sequences and series
Exercise D, Question 3

Question:

An investor puts £4000 in an account. Every month thereafter she deposits another £200. How much money in total will she have invested at the start of **a** the 10th month and **b** the *m*th month? (Note that at the start of the 6th month she will have made only 5 deposits of £200.)

Solution:

(a) Initial amount $=$ £ 4000 (start of month 1)
Start of month 2 $=$ £ $(4000 + 200)$
Start of month 3 $=$ £ $(4000 + 200 + 200)$ $=$ £ $(4000 + 2 \times 200)$
$\vdots$
Start of month 10 $=$ £ $(4000 + 9 \times 200)$ $=$ £ $(4000 + 1800)$ $=$ £ 5800

(b) Start of mth month
$=$ £ $[4000 + (m - 1) \times 200]$
$=$ £ $(4000 + 200m - 200)$
$=$ £ $(3800 + 200m)$

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Sequences and series
Exercise D, Question 4

Question:

Calculate the number of terms in the following arithmetic sequences:

(a) 3, 7, 11, … , 83, 87

(b) 5, 8, 11, … , 119, 122

(c) 90, 88, 86, … , 16, 14

(d) 4, 9, 14, … , 224, 229

(e) x, $3x$, $5x$, … , $35x$

(f) a, $a + d$, $a + 2d$, … , $a + (n - 1) d$

Solution:

(a) $3 \xrightarrow{+4} 7 \xrightarrow{+4} 11 \ \ldots\ 83 \xrightarrow{+4} 87$

number of jumps $= \frac{87-3}{4} = 21$

therefore number of terms $= 21 + 1 = 22$.

(b) $5 \xrightarrow{+3} 8 \xrightarrow{+3} 11 \ \ldots\ 119 \xrightarrow{+3} 122$

number of jumps $= \frac{122-5}{3} = 39$

therefore number of terms $= 40$

(c) $90 \xrightarrow{-2} 88 \xrightarrow{-2} 86 \ \ldots\ 16 \xrightarrow{-2} 14$

number of jumps $= \frac{90-14}{2} = 38$

therefore number of terms $= 39$

(d) $4 \xrightarrow{+5} 9 \xrightarrow{+5} 14 \ \ldots\ 224 \xrightarrow{+5} 229$

number of jumps $= \frac{229-4}{5} = 45$

therefore number of terms $= 46$

(e) $x \xrightarrow{+2x} 3x \xrightarrow{+2x} 5x \quad \ldots \quad 35x$

number of jumps $= \frac{35x - x}{2x} = 17$

number of terms $= 18$

(f) $a \xrightarrow{+d} a + d \xrightarrow{+d} a + 2d \quad \ldots \quad a + (n - 1)d$

number of jumps $= \frac{a + (n - 1)d - a}{d} = \frac{(n - 1)d}{d} = n - 1$

number of terms $= n$

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Sequences and series
Exercise E, Question 1

Question:

Find **i** the 20th and **ii** the nth terms of the following arithmetic series:

(a) $2 + 6 + 10 + 14 + 18 \quad \ldots$

(b) $4 + 6 + 8 + 10 + 12 + \quad \ldots$

(c) $80 + 77 + 74 + 71 + \quad \ldots$

(d) $1 + 3 + 5 + 7 + 9 + \quad \ldots$

(e) $30 + 27 + 24 + 21 + \quad \ldots$

(f) $2 + 5 + 8 + 11 + \quad \ldots$

(g) $p + 3p + 5p + 7p + \quad \ldots$

(h) $5x + x + (- 3x) + (- 7x) + \quad \ldots$

Solution:

(a) $2 + 6 + 10 + 14 + 18$
$a = 2, d = 4$
(i) 20th term $= a + 19d = 2 + 19 \times 4 = 78$
(ii) nth term $= a + (n - 1) d = 2 + (n - 1) \times 4 = 4n - 2$

(b) $4 + 6 + 8 + 10 + 12$
$a = 4, d = 2$
(i) 20th term $= a + 19d = 4 + 19 \times 2 = 42$
(ii) nth term $= a + (n - 1) d = 4 + (n - 1) \times 2 = 2n + 2$

(c) $80 + 77 + 74 + 71 +$
$a = 80, d = - 3$
(i) 20th term $= a + 19d = 80 + 19 \times - 3 = 23$
(ii) nth term $= a + (n - 1) d = 80 + (n - 1) \times - 3 = 83 - 3n$

(d) $1 + 3 + 5 + 7 + 9$
$a = 1, d = 2$
(i) 20th term $= a + 19d = 1 + 19 \times 2 = 39$
(ii) nth term $= a + (n - 1) d = 1 + (n - 1) \times 2 = 2n - 1$

(e) $30 + 27 + 24 + 21$
$a = 30, d = - 3$
(i) 20th term $= a + 19d = 30 + 19 \times - 3 = - 27$
(ii) nth term $= a + (n - 1) d = 30 + (n - 1) \times - 3 = 33 - 3n$

(f) $2 + 5 + 8 + 11$
$a = 2, d = 3$
(i) 20th term $= a + 19d = 2 + 19 \times 3 = 59$
(ii) nth term $= a + (n - 1) d = 2 + (n - 1) \times 3 = 3n - 1$

(g) $p + 3p + 5p + 7p$
$a = p, d = 2p$

(i) 20th term $= a + 19d = p + 19 \times 2p = 39p$
(ii) nth term $= a + (n - 1)d = p + (n - 1) \times 2p = 2pn - p = (2n - 1)p$

(h) $5x + x + (-3x) + (-7x)$
$a = 5x, d = -4x$
(i) 20th term $= a + 19d = 5x + 19 \times -4x = -71x$
(ii) nth term $= a + (n - 1)d = 5x + (n - 1) \times -4x = 9x - 4nx = (9 - 4n)x$

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Sequences and series
Exercise E, Question 2

Question:

Find the number of terms in the following arithmetic series:

(a) $5 + 9 + 13 + 17 + \ldots + 121$

(b) $1 + 1.25 + 1.5 + 1.75 \ldots + 8$

(c) $-4 + -1 + 2 + 5 \ldots + 89$

(d) $70 + 61 + 52 + 43 \ldots + (-200)$

(e) $100 + 95 + 90 + \ldots + (-1000)$

(f) $x + 3x + 5x \ldots + 153x$

Solution:

(a) $5 + 9 + 13 + 17 + \ldots + 121$
nth term $= a + (n-1)d$
$121 = 5 + (n-1) \times 4$
$116 = (n-1) \times 4$
$29 = (n-1)$
$30 = n$
$n = 30$ (30 terms)

(b) $1 + 1.25 + 1.5 + 1.75 + \ldots + 8$
nth term $= a + (n-1)d$
$8 = 1 + (n-1) \times 0.25$
$7 = (n-1) \times 0.25$
$28 = (n-1)$
$29 = n$
$n = 29$ (29 terms)

(c) $-4 + -1 + 2 + 5 + \ldots + 89$
nth term $= a + (n-1)d$
$89 = -4 + (n-1) \times 3$
$93 = (n-1) \times 3$
$31 = (n-1)$
$32 = n$
$n = 32$ (32 terms)

(d) $70 + 61 + 52 + 43 + \ldots + (-200)$
nth term $= a + (n-1)d$
$-200 = 70 + (n-1) \times -9$
$-270 = (n-1) \times -9$
$+30 = (n-1)$
$31 = n$
$n = 31$ (31 terms)

(e) $100 + 95 + 90 + \ldots + (-1000)$
nth term $= a + (n-1)d$
$-1000 = 100 + (n-1) \times -5$
$-1100 = (n-1) \times -5$
$+220 = (n-1)$

$221 = n$
$n = 221$ (221 terms)

(f) $x + 3x + 5x + \quad \ldots \quad + 153x$
nth term $= a + (n - 1)d$
$153x = x + (n - 1) \times 2x$
$152x = (n - 1) \times 2x$
$76 = (n - 1)$
$77 = n$
$n = 77$ (77 terms)

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Sequences and series
Exercise E, Question 3

Question:

The first term of an arithmetic series is 14. If the fourth term is 32, find the common difference.

Solution:

Let the common difference be d.
4th term $= a + 3d = 14 + 3d$ (first term = 14)
we are told the 4th term is 32

$\Rightarrow \quad 14 + 3d = 32$

$\Rightarrow \quad 3d = 18$

$\Rightarrow \quad d = 6$

Common difference is 6.

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Sequences and series
Exercise E, Question 4

Question:

Given that the 3rd term of an arithmetic series is 30 and the 10th term is 9 find a and d.
Hence find which term is the first one to become negative.

Solution:

Let $a =$ first term and $d =$ common difference in the arithmetic series.

If 3rd term $= 30 \quad \Rightarrow \quad a + 2d = 30$ ①

If 10th term $= 9 \quad \Rightarrow \quad a + 9d = 9$ ②

② − ①: $7d = -21 \quad \Rightarrow \quad d = -3$

Substitute $d = -3$ into equation ①:

$a + 2 \times -3 = 30 \quad \Rightarrow \quad a = 36$

nth term in series $= 36 + (n - 1) \times -3 = 36 - 3n + 3 = 39 - 3n$

when $n = 13$, nth term $= 39 - 39 = 0$

when $n = 14$, nth term $= 39 - 42 = -3$

The 14th term is the first to be negative.

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Sequences and series
Exercise E, Question 5

Question:

In an arithmetic series the 20th term is 14 and the 40th term is -6. Find the 10th term.

Solution:

Let $a =$ first term in the series and $d =$ common difference in the series.

20th term in series is 14 $\Rightarrow$ $a + 19d = 14$ ①

40th term in series is -6 $\Rightarrow$ $a + 39d = -6$ ②

Equation ② − ①: $20d = -20$ $\Rightarrow$ $d = -1$

Substitute $d = -1$ into equation ①:

$a + 19 \times -1 = 14$ $\Rightarrow$ $a = 33$

10th term $= a + 9d = 33 + 9 \times -1 = 33 - 9 = 24$

The 10th term in the series is 24.

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Sequences and series
Exercise E, Question 6

Question:

The first three terms of an arithmetic series are $5x$, 20 and $3x$. Find the value of x and hence the values of the three terms.

Solution:

$5x, 20, 3x, \quad \ldots$
Term2 − Term1 = Term3 − Term2
$20 - 5x = 3x - 20$
$40 = 8x$
$5 = x$
Substituting $x = 5$ into the expressions gives
$5 \times 5, 20, 3 \times 5$
25, 20, 15
1st, 2nd, 3rd term

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Sequences and series
Exercise E, Question 7

Question:

For which values of x would the expression -8, x^2 and $17x$ form the first three terms of an arithmetic series?

Solution:

$-8, x^2, 17x$
Term2 − Term1 = Term3 − Term2
$x^2 - (-8) = 17x - x^2$
$x^2 + 8 = 17x - x^2$
$2x^2 - 17x + 8 = 0$
$(2x - 1)(x - 8) = 0$
$x = +\frac{1}{2}, +8$

Values of x are $+\frac{1}{2}$ or $+8$

Check:
$x = \frac{1}{2}$ gives terms

$$-8 \xrightarrow{+8\frac{1}{4}} \frac{1}{4} \xrightarrow{+8\frac{1}{4}} 8\frac{1}{2}$$

$x = 8$ gives terms

$$-8 \xrightarrow{+72} 64 \xrightarrow{+72} 136$$

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Sequences and series

Exercise F, Question 1

Question:

Find the sums of the following series:

(a) $3 + 7 + 11 + 14 + \ldots$ (20 terms)

(b) $2 + 6 + 10 + 14 + \ldots$ (15 terms)

(c) $30 + 27 + 24 + 21 + \ldots$ (40 terms)

(d) $5 + 1 + -3 + -7 + \ldots$ (14 terms)

(e) $5 + 7 + 9 + \ldots + 75$

(f) $4 + 7 + 10 + \ldots + 91$

(g) $34 + 29 + 24 + 19 + \ldots + -111$

(h) $(x + 1) + (2x + 1) + (3x + 1) + \ldots + (21x + 1)$

Solution:

(a) $3 + 7 + 11 + 14 + \ldots$ (for 20 terms)
Substitute $a = 3$, $d = 4$ and $n = 20$ into

$$S_n = \frac{n}{2}\left[2a + \left(n - 1\right)d\right] = \frac{20}{2}\left(6 + 19 \times 4\right) = 10 \times 82 = 820$$

(b) $2 + 6 + 10 + 14 + \ldots$ (for 15 terms)
Substitute $a = 2$, $d = 4$ and $n = 15$ into

$$S_n = \frac{n}{2}\left[2a + \left(n - 1\right)d\right] = \frac{15}{2}\left(4 + 14 \times 4\right) = \frac{15}{2} \times 60 = 450$$

(c) $30 + 27 + 24 + 21 + \ldots$ (for 40 terms)
Substitute $a = 30$, $d = -3$ and $n = 40$ into

$$S_n = \frac{n}{2}\left[2a + \left(n - 1\right)d\right] = \frac{40}{2}\left(60 + 39 \times -3\right) = 20 \times -57 = -1140$$

(d) $5 + 1 + -3 + -7 + \ldots$ (for 14 terms)
Substitute $a = 5$, $d = -4$ and $n = 14$ into

$$S_n = \frac{n}{2}\left[2a + \left(n - 1\right)d\right] = \frac{14}{2}\left(10 + 13 \times -4\right) = 7 \times -42 = -294$$

(e) $5 + 7 + 9 + \ldots + 75$
Here $a = 5$, $d = 2$ and $L = 75$.
Use $L = a + (n - 1)d$ to find the number of terms n.
$75 = 5 + (n - 1) \times 2$
$70 = (n - 1) \times 2$
$35 = n - 1$
$n = 36$ (36 terms)

Substitute $a = 5$, $d = 2$, $n = 36$ and $L = 75$ into

$$S_n = \frac{n}{2}\left(a + L\right) = \frac{36}{2}\left(5 + 75\right) = 18 \times 80 = 1440$$

(f) $4 + 7 + 10 + \ldots + 91$
Here $a = 4$, $d = 3$ and $L = 91$.
Use $L = a + (n - 1)d$ to find the number of terms n.
$91 = 4 + (n - 1) \times 3$
$87 = (n - 1) \times 3$
$29 = (n - 1)$
$n = 30$ (30 terms)
Substitute $a = 4$, $d = 3$, $L = 91$ and $n = 30$ into

$$S_n = \frac{n}{2}\left(a + L\right) = \frac{30}{2}\left(4 + 91\right) = 15 \times 95 = 1425$$

(g) $34 + 29 + 24 + 19 + \ldots + - 111$
Here $a = 34$, $d = -5$ and $L = -111$.
Use $L = a + (n - 1)d$ to find the number of terms n.
$-111 = 34 + (n - 1) \times -5$
$-145 = (n - 1) \times -5$
$29 = (n - 1)$
$30 = n$ (30 terms)
Substitute $a = 34$, $d = -5$, $L = -111$ and $n = 30$ into

$$S_n = \frac{n}{2}\left(a + L\right) = \frac{30}{2}\left(34 + -111\right) = 15 \times -77 = -1155$$

(h) $(x + 1) + (2x + 1) + (3x + 1) + \ldots + (21x + 1)$
Here $a = x + 1$, $d = x$ and $L = 21x + 1$.
Use $L = a + (n - 1)d$ to find the number of terms n.
$21x + 1 = x + 1 + (n - 1) \times x$
$20x = (n - 1) \times x$
$20 = (n - 1)$
$21 = n$ (21 terms)
Substitute $a = x + 1$, $d = x$, $L = 21x + 1$ and $n = 21$ into

$$S_n = \frac{n}{2}\left(a + L\right) = \frac{21}{2}\left(x + 1 + 21x + 1\right) = \frac{21}{2} \times \left(22x + 2\right) = 21\left(11x + 1\right)$$

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Sequences and series
Exercise F, Question 2

Question:

Find how many terms of the following series are needed to make the given sum:

(a) $5 + 8 + 11 + 14 + \ldots = 670$

(b) $3 + 8 + 13 + 18 + \ldots = 1575$

(c) $64 + 62 + 60 + \ldots = 0$

(d) $34 + 30 + 26 + 22 + \ldots = 112$

Solution:

(a) $5 + 8 + 11 + 14 + \ldots = 670$
Substitute $a = 5$, $d = 3$, $S_n = 670$ into

$$S_n = \frac{n}{2}\left[2a + \left(n - 1 \right) d \right]$$

$$670 = \frac{n}{2}\left[10 + \left(n - 1 \right) \times 3 \right]$$

$$670 = \frac{n}{2}\left(3n + 7 \right)$$

$1340 = n (3n + 7)$
$0 = 3n^2 + 7n - 1340$
$0 = (n - 20) (3n + 67)$

$n = 20$ or $-\frac{67}{3}$

Number of terms is 20

(b) $3 + 8 + 13 + 18 + \ldots = 1575$
Substitute $a = 3$, $d = 5$, $S_n = 1575$ into

$$S_n = \frac{n}{2}\left[2a + \left(n - 1 \right) d \right]$$

$$1575 = \frac{n}{2}\left[6 + \left(n - 1 \right) \times 5 \right]$$

$$1575 = \frac{n}{2}\left(5n + 1 \right)$$

$3150 = n (5n + 1)$
$0 = 5n^2 + n - 3150$
$0 = (5n + 126) (n - 25)$

$n = -\frac{126}{5}, 25$

Number of terms is 25

(c) $64 + 62 + 60 + \ldots = 0$

Substitute $a = 64$, $d = -2$ and $S_n = 0$ into

$$S_n = \frac{n}{2}\left[2a + \left(n-1\right)d\right]$$

$$0 = \frac{n}{2}\left[128 + \left(n-1\right) \times -2\right]$$

$$0 = \frac{n}{2}\left(130 - 2n\right)$$

$0 = n(65 - n)$
$n = 0$ or 65
Number of terms is 65

(d) $34 + 30 + 26 + 22 + \ldots = 112$
Substitute $a = 34$, $d = -4$ and $S_n = 112$ into

$$S_n = \frac{n}{2}\left[2a + \left(n-1\right)d\right]$$

$$112 = \frac{n}{2}\left[68 + \left(n-1\right) \times -4\right]$$

$$112 = \frac{n}{2}\left(72 - 4n\right)$$

$112 = n(36 - 2n)$
$2n^2 - 36n + 112 = 0$
$n^2 - 18n + 56 = 0$
$(n - 4)(n - 14) = 0$
$n = 4$ or 14
Number of terms is 4 or 14

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Sequences and series
Exercise F, Question 3

Question:

Find the sum of the first 50 even numbers.

Solution:

$$S = \underbrace{2 + 4 + 6 + 8 + \cdots}_{50\text{ terms}}$$

This is an arithmetic series with $a = 2$, $d = 2$ and $n = 50$.

Use $S_n = \frac{n}{2}\left[2a + (n-1)d \right]$

So $S = \frac{50}{2}(4 + 49 \times 2) = 25 \times 102 = 2550$

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Sequences and series
Exercise F, Question 4

Question:

Carol starts a new job on a salary of £20000. She is given an annual wage rise of £500 at the end of every year until she reaches her maximum salary of £25000. Find the total amount she earns (assuming no other rises),

(a) in the first 10 years and

(b) over 15 years.

Solution:

Total salary

$$= \underset{\text{Year 1}}{\underset{\uparrow}{20000}} + \underset{\text{Year 2}}{\underset{\uparrow}{20500}} + \underset{\text{Year 3}}{\underset{\uparrow}{21000}} + \underset{\text{Year 4}}{\underset{\uparrow}{21500}} + \ldots$$

1st increment 2nd increment 3rd increment

Carol will reach her maximum salary after

$\frac{25000 - 20000}{500} = 10$ increments

This will be after 11 years.

(a) Total amount after 10 years

$$= \underbrace{20000 + 20500 + 21000 + \ldots}$$

This is an arithmetic series with $a = 20000$, $d = 500$ and $n = 10$. Use $S_n = \frac{n}{2}\left[2a + \left(n - 1\right)d\right]$.

$= \frac{10}{2}\left(40000 + 9 \times 500\right)$

$= 5 \times 44500$

$=$ £ 222 500

(b) From year 11 to year 15 she will continue to earn £ 25 000.
Total in this time $= 5 \times 25000 =$ £ 125000.
Total amount in the first 15 years is
£ 222 500 + £ 125000 = £ 347 500

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Sequences and series
Exercise F, Question 5

Question:

Find the sum of the multiples of 3 less than 100. Hence or otherwise find the sum of the numbers less than 100 which are not multiples of 3.

Solution:

Sum of multiples of 3 less than 100

$= \underbrace{3 + 6 + 9 + 12 \ldots + 96 + 99}$

This is an arithmetic series with $a = 3$, $d = 3$ and $n = \frac{99-3}{3} + 1 = 33$ terms.

Use $S_n = \frac{n}{2} [2a + (n - 1) d]$

$= \frac{33}{2} \left[2 \times 3 + \left(33 - 1 \right) \times 3 \right]$

$= \frac{33}{2} (6 + 96)$

$= 33 \times 51$
$= 1683$

Sum of numbers less than 100 that are not multiples of 3

$= 1 + 2 + 4 + 5 + 7 + 8 + 10 + 11 + \ldots + 97 + 98$

$= (1 + 2 + 3 + \ldots + 97 + 98 + 99) - (3 + 6 + \ldots 96 + 99)$

$= \frac{99}{2} \left[2 + \left(99 - 1 \right) \times 1 \right] - 1683$

$= \frac{99}{2} \times 100 - 1683$

$= 4950 - 1683$
$= 3267$

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Sequences and series
Exercise F, Question 6

Question:

James decides to save some money during the six-week holiday. He saves 1p on the first day, 2p on the second, 3p on the third and so on. How much will he have at the end of the holiday (42 days)? If he carried on, how long would it be before he has saved £100?

Solution:

Amount saved by James

$= \underbrace{1 + 2 + 3 + \ldots 42}$

This is an arithmetic series with $a = 1$, $d = 1$, $n = 42$ and $L = 42$.

Use $S_n = \frac{n}{2}\left(a + L\right)$

$= \frac{42}{2}\left(1 + 42\right)$

$= 21 \times 43$
$= 903\text{p}$
$= £\ 9.03$

To save £100 we need

$\underbrace{1 + 2 + 3 + \ldots}_{\text{Sum to } n \text{ terms}} = 10000$

$\frac{n}{2}\left[2 \times 1 + \left(n - 1\right) \times 1\right] = 10000$

$\frac{n}{2}\left(n + 1\right) = 10000$

$n\left(n + 1\right) = 20000$

$n^2 + n - 20000 = 0$

$n = \frac{-1 \pm \sqrt{\left(1\right)^2 - 4 \times 1 \times \left(-20000\right)}}{2}$

$n = 140.9$ or -141.9

It takes James 141 days to save £100.

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Sequences and series

Exercise F, Question 7

Question:

The first term of an arithmetic series is 4. The sum to 20 terms is -15. Find, in any order, the common difference and the 20th term.

Solution:

Let common difference = d.
Substitute $a = 4$, $n = 20$, and $S_{20} = -15$ into

$$S_n = \frac{n}{2}\left[2a + \left(n - 1\right)d\right]$$

$$-15 = \frac{20}{2}\left[8 + \left(20 - 1\right)d\right]$$

$-15 = 10(8 + 19d)$
$-1.5 = 8 + 19d$
$19d = -9.5$
$d = -0.5$
The common difference is -0.5.
Use nth term $= a + (n - 1)d$ to find
20th term $= a + 19d = 4 + 19 \times -0.5 = 4 - 9.5 = -5.5$
20th term is -5.5.

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Sequences and series
Exercise F, Question 8

Question:

The sum of the first three numbers of an arithmetic series is 12. If the 20th term is -32, find the first term and the common difference.

Solution:

Let the first term be a and the common difference d.
Sum of first three terms is 12, so
$a + (a + d) + (a + 2d) = 12$
$3a + 3d = 12$
$a + d = 4$ ①
20th term is -32, so
$a + 19d = -32$ ②
Equation ② − equation ①:
$18d = -36$
$d = -2$
Substitute $d = -2$ into equation ①:
$a + -2 = 4$
$a = 6$
Therefore, first term is 6 and common difference is -2.

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Sequences and series
Exercise F, Question 9

Question:

Show that the sum of the first $2n$ natural numbers is $n(2n+1)$.

Solution:

Sum required

$$= \underbrace{1 + 2 + 3 + \ldots 2n}$$

Arithmetic series with $a = 1$, $d = 1$ and $n = 2n$.

Use $S_n = \frac{n}{2}\left[2a + \left(n-1\right)d\right]$

$$= \frac{2n}{2}\left[2 \times 1 + \left(2n-1\right) \times 1\right]$$

$$= \frac{\cancel{2}n}{\cancel{2}}\left(2n+1\right)$$

$$= n\left(2n+1\right)$$

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Sequences and series
Exercise F, Question 10

Question:

Prove that the sum of the first n odd numbers is n^2.

Solution:

Required sum

$$= \underbrace{1 + 3 + 5 + 7 + \ldots}_{n \text{ terms}}$$

This is an arithmetic series with $a = 1$, $d = 2$ and $n = n$.

Use $S_n = \frac{n}{2}\left[2a + \left(n - 1\right)d\right]$

$$= \frac{n}{2}\left[2 \times 1 + \left(n - 1\right) \times 2\right]$$

$$= \frac{n}{2}\left(2 + 2n - 2\right)$$

$$= \frac{n \times \cancel{2}n}{\cancel{2}}$$

$$= n \times n$$

$$= n^2$$

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Sequences and series
Exercise G, Question 1

Question:

Rewrite the following sums using Σ notation:

(a) $4 + 7 + 10 + \ldots + 31$

(b) $2 + 5 + 8 + 11 + \ldots + 89$

(c) $40 + 36 + 32 + \ldots + 0$

(d) The multiples of 6 less than 100

Solution:

(a) $4 + 7 + 10 + \ldots + 31$
Here $a = 4$ and $d = 3$,
nth term $= 4 + (n - 1) \times 3 = 3n + 1$
4 is the 1st term $(3 \times 1 + 1)$
31 is the 10th term $(3 \times 10 + 1)$

Hence series is $\sum_{r=1}^{10} (3r + 1)$.

(b) $2 + 5 + 8 + 11 + \ldots + 89$
Here $a = 2$ and $d = 3$,
nth term $= 2 + (n - 1) \times 3 = 3n - 1$
2 is the 1st term $(3 \times 1 - 1)$
89 is the 30th term $(3 \times 30 - 1)$

Hence series is $\sum_{r=1}^{30} (3r - 1)$.

(c) $40 + 36 + 32 + \ldots + 0$
Here $a = 40$ and $d = -4$,
nth term $= 40 + (n - 1) \times -4 = 44 - 4n$
40 is the 1st term $(44 - 4 \times 1)$
0 is the 11th term $(44 - 4 \times 11)$

Hence series is $\sum_{r=1}^{11} (44 - 4r)$.

(d) Multiples of 6 less than 100 $= 6 + 12 + 18 + \ldots + 96$
6 is the 1st multiple
96 is the 16th multiple

Hence series is $\sum_{r=1}^{16} 6r$.

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Sequences and series
Exercise G, Question 2

Question:

Calculate the following:

(a) <semantics> $\sum_{r=1}^{5} 3r$</semantics>

(b) <semantics> $\sum_{r=1}^{10} (4r-1)$ </semantics>

(c) <semantics> $\sum_{r=1}^{20} (5r-2)$ </semantics>

(d) <semantics> $\sum_{r=0}^{5} r(r+1)$ </semantics>

Solution:

(a) <semantics> $\sum_{r=1}^{5} 3r = 3 + 6 + \ldots + 15$</semantics>

Arithmetic series with $a = 3$, $d = 3$, $n = 5$, $L = 15$

Use $S_n = \frac{n}{2}\left(a + L\right)$

$= \frac{5}{2}\left(3 + 15\right)$

$= 45$

(b) <semantics> $\sum_{r=1}^{10} (4r-1) = 3 + 7 + 11 + \ldots + 39$ </semantics>

Arithmetic series with $a = 3$, $d = 4$, $n = 10$, $L = 39$

Use $S_n = \frac{n}{2}\left(a + L\right)$

$= \frac{10}{2}\left(3 + 39\right)$

$= 5 \times 42$

$= 210$

(c) <semantics> $\sum_{r=1}^{20} (5r - 2) = (5 \times 1 - 2) + (5 \times 2 - 2) + (5 \times 3 - 2) + \ldots + (5 \times 20 - 2)$
</semantics>
$= 3 + 8 + 13 + \ldots + 98$
Arithmetic series with $a = 3$, $d = 5$, $n = 20$, $L = 98$

Use $S_n = \frac{n}{2}\left(a + L\right)$

$= \frac{20}{2}\left(3 + 98\right)$

$= 10 \times 101$
$= 1010$

(d) <semantics> $\sum_{r=0}^{5} r(r + 1)$ </semantics> is not an arithmetic series, so simply add the terms

<semantics> $\sum_{r=0}^{5} r(r + 1) = 0 + 2 + 6 + 12 + 20 + 30$ </semantics>

$= 70$

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Sequences and series
Exercise G, Question 3

Question:

For what value of n does $\sum_{r=1}^{n} (5r + 3)$ first exceed 1000?

Solution:

$$\sum_{r=1}^{n} (5r + 3)$$

$= (5 \times 1 + 3) + (5 \times 2 + 3) + (5 \times 3 + 3) + \ldots + (5 \times n + 3)$

$= \underbrace{8 + 13 + 18 + \ldots + 5n + 3}$

Arithmetic series with $a = 8$, $d = 5$ and $n = n$.

Use $S_n = \frac{n}{2}\left[2a + (n - 1)d\right]$

$= \frac{n}{2}\left[16 + (n - 1) \times 5\right]$

$= \frac{n}{2}(5n + 11)$

If sum exceeds 1000 then

$\frac{n}{2}\left(5n + 11\right) > 1000$

$n(5n + 11) > 2000$

$5n^2 + 11n - 2000 > 0$

Solve equality $5n^2 + 11n - 2000 = 0$

$n = \frac{-11 \pm \sqrt{(11)^2 - 4 \times 5 \times -2000}}{2 \times 5} = \frac{-11 \pm 200.30\ldots}{10} = 18.93$ or -21.13

The sum has to be bigger than 1000

$\Rightarrow$ $n = 19$

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Sequences and series
Exercise G, Question 4

Question:

For what value of n would $\sum_{r=1}^{n} (100 - 4r) = 0$?

Solution:

$\sum_{r=1}^{n} (100 - 4r)$

$= (100 - 4 \times 1) + (100 - 4 \times 2) + (100 - 4 \times 3) + \ldots + (100 - 4n)$

$= \underbrace{96 + 92 + 88 + \ldots + (100 - 4n)}$

Arithmetic series with $a = 96$, $d = -4$ and $n = n$.

Use the sum formula $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$= \frac{n}{2}\left[192 + (n-1) \times -4\right]$

$= \frac{n}{2}(196 - 4n)$

$= n(98 - 2n)$

we require the sum to be zero, so

$n(98 - 2n) = 0 \Rightarrow n = 0 \text{ or } \frac{98}{2}$

Hence the value of n is 49.

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Sequences and series
Exercise H, Question 1

Question:

The rth term in a sequence is $2 + 3r$. Find the first three terms of the sequence.

Solution:

Substitute $r = 1$ in $2 + 3r = 2 + 3 \times 1 = 5$
1st term $= 5$
Substitute $r = 2$ in $2 + 3r = 2 + 3 \times 2 = 2 + 6 = 8$
2nd term $= 8$
Substitute $r = 3$ in $2 + 3r = 2 + 3 \times 3 = 2 + 9 = 11$
3rd term $= 11$

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Sequences and series
Exercise H, Question 2

Question:

The rth term in a sequence is $(r+3)(r-4)$. Find the value of r for the term that has the value 78.

Solution:

rth term $=(r+3)(r-4)$
when rth term $=78$
$78=(r+3)(r-4)$
$78=r^2-1r-12$
$0=r^2-1r-90$
$0=(r-10)(r+9)$
$r=10,\ -9$
r must be 10.
[Check: Substitute $r=10$ in $(r+3)(r-4)$
$\Rightarrow\quad (10+3)(10-4)=13\times 6=78$ ✓]

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Sequences and series
Exercise H, Question 3

Question:

A sequence is formed from an inductive relationship:

$U_{n+1} = 2U_n + 5$

Given that $U_1 = 2$, find the first four terms of the sequence.

Solution:

$U_{n+1} = 2U_n + 5$
Substitute $n = 1 \Rightarrow U_2 = 2U_1 + 5$
$U_1 = 2 \Rightarrow U_2 = 2 \times 2 + 5 = 9$
Substitute $n = 2 \Rightarrow U_3 = 2U_2 + 5$
$U_2 = 9 \Rightarrow U_3 = 2 \times 9 + 5 = 23$
Substitute $n = 3 \Rightarrow U_4 = 2U_3 + 5$
$U_3 = 23 \Rightarrow U_4 = 2 \times 23 + 5 = 51$

The first four terms of the sequence are 2, 9, 23 and 51.

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Sequences and series
Exercise H, Question 4

Question:

Find a rule that describes the following sequences:

(a) 5, 11, 17, 23, …

(b) 3, 6, 9, 12, …

(c) 1, 3, 9, 27, …

(d) 10, 5, 0, − 5, …

(e) 1, 4, 9, 16, …

(f) 1, 1.2, 1.44, 1.728

Which of the above are arithmetic sequences?

For the ones that are, state the values of *a* and *d*.

Solution:

(a) $5 \xrightarrow{+6} 11 \xrightarrow{+6} 17 \xrightarrow{+6} 23 \ldots$

"Add 6 to the previous term."

(b) $3 \xrightarrow{+3} 6 \xrightarrow{+3} 9 \xrightarrow{+3} 12 \ldots$

"Add 3 to the previous term."

(c) $1 \xrightarrow{\times 3} 3 \xrightarrow{\times 3} 9 \xrightarrow{\times 3} 27 \ldots$

"Multiply the previous term by 3."

(d) $10 \xrightarrow{-5} 5 \xrightarrow{-5} 0 \xrightarrow{-5} -5 \ldots$

"Subtract 5 from the previous term."

(e) $1 \xrightarrow{+3} 4 \xrightarrow{+5} 9 \xrightarrow{+7} 16 \ldots$

“Add consecutive odd numbers to each term.” or “They are the square numbers.”

(f) $1 \xrightarrow{\times 1.2} 1.2 \xrightarrow{\times 1.2} 1.44 \xrightarrow{\times 1.2} 1.728 \ldots$

“Multiply the previous term by 1.2.”

The arithmetic sequences are (a) where $a = 5$, $d = 6$, (b) where $a = 3$, $d = 3$,
(d) where $a = 10$, $d = -5$.
Alternatively you could give the nth terms of the series as (a) $6n - 1$ (b) $3n$ (c) 3^{n-1} (d) $15 - 5n$ (e) n^2 (f) 1.2^{n-1}

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Sequences and series
Exercise H, Question 5

Question:

For the arithmetic series 5 + 9 + 13 + 17 + …
Find **a** the 20th term, and **b** the sum of the first 20 terms.

Solution:

$$5 \xrightarrow{+4} 9 \xrightarrow{+4} 13 \xrightarrow{+4} 17 \ldots$$

The above sequence is arithmetic with $a = 5$ and $d = 4$.

(a) As nth term $= a + (n - 1)d$
20th term $= a + (20 - 1)d = a + 19d$
Substitute $a = 5, d = 4 \Rightarrow$ 20th term $= 5 + 19 \times 4 = 5 + 76 = 81$

(b) As sum to n terms $S_n = \frac{n}{2}\left[2a + \left(n - 1\right)d\right]$

$S_{20} = \frac{20}{2}\left[2a + \left(20 - 1\right)d\right] = 10\left(2a + 19d\right)$

Substitute $a = 5, d = 4 \Rightarrow S_{20} = 10(2 \times 5 + 19 \times 4) = 10 \times (10 + 76) = 10 \times 86 = 860$

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Sequences and series
Exercise H, Question 6

Question:

(a) Prove that the sum of the first n terms in an arithmetic series is

$$S = \frac{n}{2}\left[2a + \left(n-1\right)d\right]$$

where a = first term and d = common difference.

(b) Use this to find the sum of the first 100 natural numbers.

Solution:

(a) $S = a + (a+d) + (a+2d) + \ldots [a + (n-2)d] + [a + (n-1)d]$
Turning series around:
$S = [a + (n-1)d] + [a + (n-2)d] + \ldots (a+d) + a$
Adding the two sums:
$2S = [2a + (n-1)d] + [2a + (n-1)d] + \ldots [2a + (n-1)d] + [2a + (n-1)d]$
There are n lots of $[2a + (n-1)d]$:
$2S = n \times [2a + (n-1)d]$

$$(\div 2)\ \ S = \frac{n}{2}\left[2a + \left(n-1\right)d\right]$$

(b) The first 100 natural numbers are 1,2,3, ... 100.
We need to find $S = 1 + 2 + 3 + \ldots 99 + 100$.
This series is arithmetic with $a = 1$, $d = 1$, $n = 100$.

Using $S = \frac{n}{2}\left[2a + \left(n-1\right)d\right]$ with $a = 1$, $d = 1$ and $n = 100$ gives

$$S = \frac{100}{2}\left[2 \times 1 + \left(100 - 1\right) \times 1\right] = \frac{100}{2}\left(2 + 99 \times 1\right) = 50 \times 101 = 5050$$

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Sequences and series
Exercise H, Question 7

Question:

Find the least value of n for which $\sum_{r=1}^{n} (4r-3) > 2000$.

Solution:

$$\sum_{r=1}^{n} (4r-3) = (4\times 1-3)+(4\times 2-3)+(4\times 3-3)....(4\times n-3)$$

$$= \underbrace{1 + 5 + 9 + \ldots + 4n-3}$$

Arithmetic series with $a=1$, $d=4$.

Using $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ with $a=1$, $d=4$ gives

$$S_n = \frac{n}{2}\left[2\times 1+(n-1)\times 4\right] = \frac{n}{2}(2+4n-4) = \frac{n}{2}(4n-2) = n(2n-1)$$

Solve $S_n = 2000$:

$n(2n-1) = 2000$

$2n^2 - n = 2000$

$2n^2 - n - 2000 = 0$

$n = \frac{1 \pm \sqrt{1 - 4\times 2\times -2000}}{2\times 2} = 31.87$ or -31.37

n must be positive, so $n = 31.87$.

If the sum has to be greater than 2000 then $n = 32$.

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Sequences and series
Exercise H, Question 8

Question:

A salesman is paid commission of £10 per week for each life insurance policy that he has sold. Each week he sells one new policy so that he is paid £10 commission in the first week, £20 commission in the second week, £30 commission in the third week and so on.

(a) Find his total commission in the first year of 52 weeks.

(b) In the second year the commission increases to £11 per week on new policies sold, although it remains at £10 per week for policies sold in the first year. He continues to sell one policy per week. Show that he is paid £542 in the second week of his second year.

(c) Find the total commission paid to him in the second year. **[E]**

Solution:

(a) Total commission

$= \underbrace{10 + 20 + 30 + \ldots + 520}$

Arithmetic series with $a = 10$, $d = 10$, $n = 52$.

$= \frac{52}{2} \left[2 \times 10 + (52 - 1) \times 10 \right]$ using $S_n = \frac{n}{2} \left[2a + (n - 1)d \right]$

$= 26 \left(20 + 51 \times 10 \right)$
$= 26 \left(20 + 510 \right)$
$= 26 \times 530$
$= £\ 13780$

(b) Commission = policies for year 1 + policies for 2nd week of year 2 = 520 + 22 = £ 542

(c) Total commission for year 2
= Commission for year 1 policies + Commission for year 2 policies
$= 520 \times 52 + \left(11 + 22 + 33 + \ldots \quad 52 \times 11 \right)$

Use $S_n = \frac{n}{2} = \left[2a + (n - 1)d \right]$ with $n = 52$, $a = 11$, $d = 11$

$= 27040 + \frac{52}{2} \left[2 \times 11 + (52 - 1) \times 11 \right]$

$= £\ 27040 + 26 \times \left(22 + 51 \times 11 \right)$
$= £\ 27\ 040 + £\ 15\ 158$
$= £\ 42\ 198$

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Sequences and series
Exercise H, Question 9

Question:

The sum of the first two terms of an arithmetic series is 47.
The thirtieth term of this series is -62. Find:

(a) The first term of the series and the common difference.

(b) The sum of the first 60 terms of the series. **[E]**

Solution:

Let $a =$ first term and $d =$ common difference.
Sum of the first two terms $= 47$

$\Rightarrow \quad a + a + d = 47$

$\Rightarrow \quad 2a + d = 47$

30th term $= -62$
Using nth term $= a + (n-1)d$

$\Rightarrow \quad a + 29d = -62$ (Note: $a + 12d$ is a common error here)

Our two simultaneous equations are

$2a + d = 47$ ①

$a + 29d = -62$ ②

$2a + 58d = -124$ ③ (② $\times 2$)

$57d = -171$ (③ − ①)

$d = -3$ ($\div 57$)

Substitute $d = -3$ into ①: $2a - 3 = 47 \quad \Rightarrow \quad 2a = 50 \quad \Rightarrow \quad a = 25$

Therefore, (a) first term $= 25$ and common difference $= -3$

(b) using $S_n = \frac{n}{2}\left[2a + \left(n-1\right)d\right]$

$$S_{60} = \frac{60}{2}\left[2a + \left(60-1\right)d\right] = 30\left(2a + 59d\right)$$

Substituting $a = 25$, $d = -3$ gives

$S_{60} = 30(2 \times 25 + 59 \times -3) = 30(50 - 177) = 30 \times -127 = -3810$

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Sequences and series
Exercise H, Question 10

Question:

(a) Find the sum of the integers which are divisible by 3 and lie between 1 and 400.

(b) Hence, or otherwise, find the sum of the integers, from 1 to 400 inclusive, which are **not** divisible by 3. **[E]**

Solution:

(a) Sum of integers divisible by 3 which lie between 1 and 400
$= 3 + 6 + 9 + 12 + \ldots + 399$
This is an arithmetic series with $a = 3$, $d = 3$ and $L = 399$.
Using $L = a + (n - 1) d$
$399 = 3 + (n - 1) \times 3$
$399 = 3 + 3n - 3$
$399 = 3n$
$n = 133$
Therefore, there are 133 of these integers up to 400.

$$S_n = \frac{n}{2}\left(a + L \right) = \frac{133}{2}\left(3 + 399 \right) = \frac{133}{2} \times 402 = 26\ 733$$

(b) Sum of integers not divisible by 3
$= 1 + 2 + 4 + 5 + 7 + 8 + 10 + 11 ..... 400$

$$= \underbrace{(1 + 2 + 3 + 4 \ldots + 399 + 400)}_{\text{Arithmetic series with } a = 1,\ d = 1,\ L = 400,\ n = 400} - \underbrace{(3 + 6 + 9 + \ldots + 399)}_{\text{From part (a). This equals 26733}}$$

$$Sn = \frac{400}{2}(1 + 400)$$
$$= 200 \times 401$$
$$= 80200$$

$= 80200 - 26733$
$= 53467$

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Sequences and series
Exercise H, Question 11

Question:

A polygon has 10 sides. The lengths of the sides, starting with the smallest, form an arithmetic series. The perimeter of the polygon is 675 cm and the length of the longest side is twice that of the shortest side. Find, for this series:

(a) The common difference.

(b) The first term. **[E]**

Solution:

If we let the smallest side be a, the other sides would be $a + d$, a + 2d, ... The longest side would be $a + 9d$.
If perimeter = 675, then
$a + (a + d) + (a + 2d) + \ldots + (a + 9d) = 675$
$\frac{10}{2}\left[2a + \left(10 - 1\right)d\right] = 675$ (Sum to 10 terms of an arithmetic series)
$5(2a + 9d) = 675 \quad (\div 5)$
$2a + 9d = 135$
The longest side is double the shortest side
$\Rightarrow \quad a + 9d = 2 \times a \qquad (-a)$
$\Rightarrow \quad 9d = a$
The simultaneous equations we need to solve are
$2a + 9d = 135$ ①
$9d = a$ ②
Substitute $9d = a$ into ①:
$2a + a = 135$
$3a = 135$
$a = 45$
Substitute back into ②:
$9d = 45$
$d = 5$
Therefore (a) the common difference = 5 and (b) the first term = 45.

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Sequences and series
Exercise H, Question 12

Question:

A sequence of terms $\{ U_n \}$ is defined for $n \geq 1$, by the recurrence relation $U_{n+2} = 2kU_{n+1} + 15U_n$, where k is a constant. Given that $U_1 = 1$ and $U_2 = -2$:
(a) Find an expression, in terms of k, for U_3.
(b) Hence find an expression, in terms of k, for U_4.
(c) Given also that $U_4 = -38$, find the possible values of k. **[E]**

Solution:

$U_{n+2} = 2kU_{n+1} + 15U_n$

(a) Replacing n by 1 gives
$U_3 = 2kU_2 + 15U_1$
We know $U_1 = 1$ and $U_2 = -2$, therefore
$U_3 = 2k \times -2 + 15 \times 1$
$U_3 = -4k + 15$

(b) Replacing n by 2 gives
$U_4 = 2kU_3 + 15U_2$
We know $U_2 = -2$ and $U_3 = -4k + 15$, therefore
$U_4 = 2k(-4k + 15) + 15 \times -2$
$U_4 = -8k^2 + 30k - 30$

(c) We are told that $U_4 = -38$, therefore
$-8k^2 + 30k - 30 = -38 \quad (+38)$
$-8k^2 + 30k + 8 = 0 \quad (\div -2)$
$4k^2 - 15k - 4 = 0$ (factorise)
$(4k + 1)(k - 4) = 0$
$k = -\frac{1}{4}, 4$

Possible values of k are $-\frac{1}{4}$, 4.

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Sequences and series
Exercise H, Question 13

Question:

Prospectors are drilling for oil. The cost of drilling to a depth of 50 m is £500. To drill a further 50 m costs £640 and, hence, the total cost of drilling to a depth of 100 m is £1140. Each subsequent extra depth of 50 m costs £140 more to drill than the previous 50 m.
(a) Show that the cost of drilling to a depth of 500 m is £11300.
(b) The total sum of money available for drilling is £76000. Find, to the earnest 50 m, the greatest depth that can be drilled. **[E]**

Solution:

(a) Cost of drilling to 500 m

$$= \underset{\substack{\uparrow \\ \text{1st} \\ 50\text{ m}}}{500} + \underset{\substack{\uparrow \\ \text{2nd} \\ 50\text{ m}}}{640} + \underset{\substack{\uparrow \\ \text{3rd} \\ 50\text{ m}}}{780} + \ldots$$

There would be 10 terms because there are 10 lots of 50 m in 500 m.
Arithmetic series with $a = 500$, $d = 140$ and $n = 10$.

Using $S_n = \frac{n}{2} \left[2a + \left(n - 1 \right) d \right]$

$= \frac{10}{2} \left[2 \times 500 + \left(10 - 1 \right) \times 140 \right]$

$= 5 \left(1000 + 9 \times 140 \right)$
$= 5 \times 2260$
$= £\ 11300$

(b) This time we are given $S = 76\ 000$. The first term will still be 500 and d remains 140.

Use $S = \frac{n}{2} \left[2a + \left(n - 1 \right) d \right]$ with $S = 76000$, $a = 500$, $d = 140$ and solve for n.

$76000 = \frac{n}{2} \left[2 \times 500 + \left(n - 1 \right) \times 140 \right]$

$76000 = \frac{n}{2} \left[1000 + 140 \left(n - 1 \right) \right]$

$76000 = n \left[500 + 70 \left(n - 1 \right) \right]$
$76000 = n \left(500 + 70n - 70 \right)$
$76000 = n \left(70n + 430 \right)$ (multiply out)
$76000 = 70n^2 + 430n \quad \left(\div 10 \right)$
$7600 = 7n^2 + 43n$
$0 = 7n^2 + 43n - 7600$

$n = \frac{-43 \pm \sqrt{\left(43 \right)^2 - 4 \times 7 \times \left(-7600 \right)}}{2 \times 7}$ (using $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$)

$n = 30.02, \ \left(-36.16 \right)$
only accept the positive answer.
There are 30 terms (to the nearest term).
So the greatest depth that can be drilled is $30 \times 50 = 1500$ m (to the nearest 50 m)

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Sequences and series
Exercise H, Question 14

Question:

Prove that the sum of the first $2n$ multiples of 4 is $4n(2n+1)$. **[E]**

Solution:

$$\text{Sum} = \underset{\substack{\uparrow \\ \text{1st}}}{4} + \underset{\substack{\uparrow \\ \text{2nd}}}{8} + \underset{\substack{\uparrow \\ \text{3rd}}}{12} + \ldots + \underset{\substack{\uparrow \\ 2n\text{th}}}{8n}$$

This is an arithmetic series with $a = 4$, $d = 4$ and $n = 2n$.

Using $S_n = \frac{n}{2}\left[2a + \left(n-1\right)d\right]$

$$S_{2n} = \frac{\cancel{2}n}{\cancel{2}}[2\times 4 + (2n-1)\times 4]$$
$= n(8 + 8n - 4)$
$= n(8n + 4)$
$= n \times 4(2n + 1)$
$= 4n(2n + 1)$

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Sequences and series
Exercise H, Question 15

Question:

A sequence of numbers $\{ U_n \{$ is defined, for $n \geq 1$, by the recurrence relation $U_{n+1} = kU_n - 4$, where k is a constant. Given that $U_1 = 2$:

(a) Find expressions, in terms of k, for U_2 and U_3.

(b) Given also that $U_3 = 26$, use algebra to find the possible values of k. **[E]**

Solution:

(a) Replacing n with 1 $\Rightarrow$ $U_2 = kU_1 - 4$
$U_1 = 2 \Rightarrow U_2 = 2k - 4$
Replacing n with 2 $\Rightarrow$ $U_3 = kU_2 - 4$
$U_2 = 2k - 4 \Rightarrow U_3 = k (2k - 4) - 4 \Rightarrow U_3 = 2k^2 - 4k - 4$

(b) Substitute $U_3 = 26$
$\Rightarrow 2k^2 - 4k - 4 = 26$
$\Rightarrow 2k^2 - 4k - 30 = 0$ ($\div 2$)
$\Rightarrow k^2 - 2k - 15 = 0$ (factorise)
$\Rightarrow (k - 5) (k + 3) = 0$
$\Rightarrow k = 5, - 3$

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Sequences and series
Exercise H, Question 16

Question:

Each year, for 40 years, Anne will pay money into a savings scheme. In the first year she pays in £500. Her payments then increase by £50 each year, so that she pays in £550 in the second year, £600 in the third year, and so on.

(a) Find the amount that Anne will pay in the 40th year.

(b) Find the total amount that Anne will pay in over the 40 years.

(c) Over the same 40 years, Brian will also pay money into the savings scheme. In the first year he pays in £890 and his payments then increase by £d each year. Given that Brian and Anne will pay in exactly the same amount over the 40 years, find the value of d. **[E]**

Solution:

(a) 1^{st} year = £ 500
2^{nd} year = £ 550 = £ (500 + 1 × 50)
3^{rd} year = £ 600 = £ (500 + 2 × 50)
⋮
40^{th} year = £ 500 + 39 × 50 = £ 2450

(b) Total amount paid in

$$= \underbrace{£500 + £550 + £600 + \ldots + £2450}$$

This is an arithmetic series with $a = 500$, $d = 50$, $L = 2450$ and $n = 40$.

$$= \frac{n}{2}\left(a + L\right)$$

$$= \frac{40}{2}\left(500 + 2450\right)$$

= 20 × 2950
= £ 59000

(c) Brian's amount

$$= \underbrace{890 + (890 + d) + (890 + 2d) + \ldots}_{40 \text{ years}}$$

Use $S_n = \frac{n}{2}\left[2a + \left(n - 1\right)d\right]$ with $n = 40$, $a = 890$ and d.

$$= \frac{40}{2}\left[2 \times 890 + \left(40 - 1\right)d\right]$$

= 20 (1780 + 39d)
Use the fact that
Brian's savings = Anne's savings
20 (1780 + 39d) = 59000 (÷ 20)
1780 + 39d = 2950 (− 1780)
39d = 1170 (÷ 39)
d = 30

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Sequences and series
Exercise H, Question 17

Question:

The fifth term of an arithmetic series is 14 and the sum of the first three terms of the series is -3.

(a) Use algebra to show that the first term of the series is -6 and calculate the common difference of the series.

(b) Given that the nth term of the series is greater than 282, find the least possible value of n. **[E]**

Solution:

(a) Use nth term $= a + (n - 1) d$:
5th term is 14 $\Rightarrow$ $a + 4d = 14$
Use 1st term $= a$, 2nd term $= a + d$, 3rd term $= a + 2d$:
sum of 1st three terms $= -3$
$\Rightarrow$ $a + a + d + a + 2d = -3$
$\Rightarrow$ $3a + 3d = -3$ $(\div 3)$
$\Rightarrow$ $a + d = -1$
Our simultaneous equations are
$a + 4d = 14$①
$a + d = -1$②
① − ②: $3d = 15$ $(\div 3)$
$d = 5$
Common difference $= 5$
Substitute $d = 5$ back in ②:
$a + 5 = -1$
$a = -6$
First term $= -6$

(b) nth term must be greater than 282
$\Rightarrow$ $a + (n - 1) d > 282$
$\Rightarrow$ $-6 + 5 (n - 1) > 282$ $(+ 6)$
$\Rightarrow$ $5 (n - 1) > 288$ $(\div 5)$
$\Rightarrow$ $(n - 1) > 57.6$ $(+ 1)$
$n > 58.6$
$\therefore$ least value of $n = 59$

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Sequences and series
Exercise H, Question 18

Question:

The fourth term of an arithmetic series is $3k$, where k is a constant, and the sum of the first six terms of the series is $7k + 9$.

(a) Show that the first term of the series is $9 - 8k$.

(b) Find an expression for the common difference of the series in terms of k.
Given that the seventh term of the series is 12, calculate:

(c) The value of k.

(d) The sum of the first 20 terms of the series. **[E]**

Solution:

(a) We know nth term $= a + (n - 1) d$
4th term is $3k$ $\Rightarrow$ $a + (4 - 1) d = 3k$ $\Rightarrow$ $a + 3d = 3k$

We know $S_n = \frac{n}{2} \left[2a + \left(n - 1 \right) d \right]$

Sum to 6 terms is $7k + 9$, therefore

$\frac{6}{2} \left[2a + \left(6 - 1 \right) d \right] = 7k + 9$

$3 (2a + 5d) = 7k + 9$
$6a + 15d = 7k + 9$
The simultaneous equations are
$a + 3d = 3k$ ①
$6a + 15d = 7k + 9$ ②
① × 5: $5a + 15d = 15k$ ③
② − ③: $1a = - 8k + 9$ $\Rightarrow$ $a = 9 - 8k$
First term is $9 - 8k$

(b) Substituting this is ① gives
$9 - 8k + 3d = 3k$
$3d = 11k - 9$
$d = \frac{11k - 9}{3}$

Common difference is $\frac{11k - 9}{3}$.

(c) If the 7th term is 12, then
$a + 6d = 12$
Substitute values of a and d:

$- 8k + 9 + 6 \times \left(\frac{11k - 9}{3} \right) = 12$

$- 8k + 9 + 2 (11k - 9) = 12$
$- 8k + 9 + 22k - 18 = 12$
$14k - 9 = 12$

$14k = 21$

$k = \frac{21}{14} = 1.5$

(d) Calculate values of a and d first:
$a = 9 - 8k = 9 - 8 \times 1.5 = 9 - 12 = -3$

$d = \frac{11k - 9}{3} = \frac{11 \times 1.5 - 9}{3} = \frac{16.5 - 9}{3} = \frac{7.5}{3} = 2.5$

$S_{20} = \frac{20}{2} \left[2a + \left(20 - 1 \right) d \right]$
$= 10 (2a + 19d)$
$= 10 (2 \times -3 + 19 \times 2.5)$
$= 10 (-6 + 47.5)$
$= 10 \times 41.5$
$= 415$
Sum to 20 terms is 415.

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Differentiation
Exercise A, Question 1

Question:

F is the point with co-ordinates $(3, 9)$ on the curve with equation $y = x^2$.

(a) Find the gradients of the chords joining the point F to the points with coordinates:

(i) $(4, 16)$

(ii) $(3.5, 12.25)$

(iii) $(3.1, 9.61)$

(iv) $(3.01, 9.0601)$

(v) $(3 + h, (3 + h)^2)$

(b) What do you deduce about the gradient of the tangent at the point $(3, 9)$?

Solution:

a (i) Gradient $= \frac{16 - 9}{4 - 3} = \frac{7}{1} = 7$

(ii) Gradient $= \frac{12.25 - 9}{3.5 - 3} = \frac{3.25}{0.5} = 6.5$

(iii) Gradient $= \frac{9.61 - 9}{3.1 - 3} = \frac{0.61}{0.1} = 6.1$

(iv) Gradient $= \frac{9.0601 - 9}{3.01 - 3} = \frac{0.0601}{0.01} = 6.01$

(v) Gradient $= \frac{(3 + h)^2 - 9}{(3 + h) - 3} = \frac{9 + 6h + h^2 - 9}{h} = \frac{6h + h^2}{h} = \frac{h(6 + h)}{h} = 6 + h$

(b) The gradient at the point $(3, 9)$ is the value of $6 + h$ as h becomes very small, i.e. the gradient is 6.

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Differentiation
Exercise A, Question 2

Question:

G is the point with coordinates $(4, 16)$ on the curve with equation $y = x^2$.

(a) Find the gradients of the chords joining the point G to the points with coordinates:

(i) $(5, 25)$

(ii) $(4.5, 20.25)$

(iii) $(4.1, 16.81)$

(iv) $(4.01, 16.0801)$

(v) $(4 + h, (4 + h)^2)$

(b) What do you deduce about the gradient of the tangent at the point $(4, 16)$?

Solution:

(a) (i) Gradient $= \frac{25 - 16}{5 - 4} = \frac{9}{1} = 9$

(ii) Gradient $= \frac{20.25 - 16}{4.5 - 4} = \frac{4.25}{0.5} = 8.5$

(iii) Gradient $= \frac{16.81 - 16}{4.1 - 4} = \frac{0.81}{0.1} = 8.1$

(iv) Gradient $= \frac{16.0801 - 16}{4.01 - 4} = \frac{0.0801}{0.01} = 8.01$

(v) Gradient $= \frac{(4 + h)^2 - 16}{4 + h - 4} = \frac{16 + 8h + h^2 - 16}{h} = \frac{8h + h^2}{h} = \frac{h(8 + h)}{h} = 8 + h$

(b) When h is small the gradient of the chord is close to the gradient of the tangent, and $8 + h$ is close to the value 8.
So the gradient of the tangent at $(4, 16)$ is 8.

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Differentiation
Exercise B, Question 1

Question:

Find the derived function, given that f(x) equals:

x^7

Solution:

$f(x) = x^7$
$f'(x) = 7x^6$

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Differentiation
Exercise B, Question 2

Question:

Find the derived function, given that f(x) equals:

x^8

Solution:

$f(x) = x^8$
$f'(x) = 8x^7$

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Differentiation
Exercise B, Question 3

Question:

Find the derived function, given that f(x) equals:

x^4

Solution:

$f(x) = x^4$
$f'(x) = 4x^3$

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Differentiation
Exercise B, Question 4

Question:

Find the derived function, given that f(x) equals:

$x^{\frac{1}{3}}$

Solution:

$f(x) = x^{\frac{1}{3}}$

$f'(x) = \frac{1}{3}x^{\frac{1}{3} - 1} = \frac{1}{3}x^{-\frac{2}{3}}$

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Differentiation
Exercise B, Question 5

Question:

Find the derived function, given that f(x) equals:

$x^{\frac{1}{4}}$

Solution:

$f(x) = x^{\frac{1}{4}}$

$f'(x) = \frac{1}{4}x^{\frac{1}{4} - 1} = \frac{1}{4}x^{-\frac{3}{4}}$

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Differentiation
Exercise B, Question 6

Question:

Find the derived function, given that f(x) equals:

$\sqrt[3]{x}$

Solution:

$f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$

$f'(x) = \frac{1}{3}x^{\frac{1}{3} - 1} = \frac{1}{3}x^{-\frac{2}{3}}$

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Differentiation
Exercise B, Question 7

Question:

Find the derived function, given that f(x) equals:

x^{-3}

Solution:

$f(x) = x^{-3}$
$f'(x) = -3x^{-3-1} = -3x^{-4}$

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Differentiation
Exercise B, Question 8

Question:

Find the derived function, given that f(x) equals:

x^{-4}

Solution:

$f(x) = x^{-4}$
$f'(x) = -4x^{-4-1} = -4x^{-5}$

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Differentiation
Exercise B, Question 9

Question:

Find the derived function, given that f(x) equals:

$$\frac{1}{x^2}$$

Solution:

$f(x) = \frac{1}{x^2} = x^{-2}$

$f'(x) = -2x^{-2-1} = -2x^{-3}$

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Differentiation

Exercise B, Question 10

Question:

Find the derived function, given that f(x) equals:

$\frac{1}{x^5}$

Solution:

$\mathrm{f}(x) = \frac{1}{x^5} = x^{-5}$

$\mathrm{f}'(x) = -5x^{-5-1} = -5x^{-6}$

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Differentiation
Exercise B, Question 11

Question:

Find the derived function, given that f(x) equals:

$$\frac{1}{\sqrt[3]{x}}$$

Solution:

$$\text{f}(x) = \frac{1}{\sqrt[3]{x}} = x^{-\frac{1}{3}}$$

$$\text{f}'(x) = -\frac{1}{3}x^{-\frac{1}{3}-1} = -\frac{1}{3}x^{-\frac{4}{3}}$$

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Differentiation
Exercise B, Question 12

Question:

Find the derived function, given that f(x) equals:

$\frac{1}{\sqrt{x}}$

Solution:

$f(x) = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$

$f'(x) = -\frac{1}{2}x^{-\frac{1}{2}-1} = -\frac{1}{2}x^{-\frac{3}{2}}$

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Differentiation

Exercise B, Question 13

Question:

Find the derived function, given that f(x) equals:

$$\frac{x^2}{x^4}$$

Solution:

$$f(x) = \frac{x^2}{x^4} = x^{2-4} = x^{-2}$$

$$f'(x) = -2x^{-2-1} = -2x^{-3}$$

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Differentiation
Exercise B, Question 14

Question:

Find the derived function, given that f(x) equals:

$$\frac{x^3}{x^2}$$

Solution:

$$f(x) = \frac{x^3}{x^2} = x^{3-2} = x^1$$

$$f'(x) = 1x^{1-1} = 1x^0 = 1$$

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Differentiation
Exercise B, Question 15

Question:

Find the derived function, given that f(x) equals:

$\frac{x^6}{x^3}$

Solution:

$f(x) = \frac{x^6}{x^3} = x^{6-3} = x^3$

$f'(x) = 3x^2$

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Differentiation
Exercise B, Question 16

Question:

Find the derived function, given that f(x) equals:

$x^3 \times x^6$

Solution:

$f(x) = x^3 \times x^6 = x^{3 + 6} = x^9$
$f'(x) = 9x^8$

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Differentiation
Exercise B, Question 17

Question:

Find the derived function, given that f(x) equals:

$x^2 \times x^3$

Solution:

$\mathrm{f}(x) = x^2 \times x^3 = x^{2+3} = x^5$
$\mathrm{f}'(x) = 5x^4$

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Differentiation
Exercise B, Question 18

Question:

Find the derived function, given that f(x) equals:

$x \times x^2$

Solution:

$f(x) = x \times x^2 = x^{1+2} = x^3$
$f'(x) = 3x^2$

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Differentiation
Exercise C, Question 1

Question:

Find $\frac{dy}{dx}$ when y equals:

(a) $2x^2 - 6x + 3$

(b) $\frac{1}{2}x^2 + 12x$

(c) $4x^2 - 6$

(d) $8x^2 + 7x + 12$

(e) $5 + 4x - 5x^2$

Solution:

(a) $y = 2x^2 - 6x + 3$

$\frac{dy}{dx} = 2(2x) - 6(1) + 0 = 4x - 6$

(b) $y = \frac{1}{2}x^2 + 12x$

$\frac{dy}{dx} = \frac{1}{2}(2x) + 12(1) = x + 12$

(c) $y = 4x^2 - 6$

$\frac{dy}{dx} = 4(2x) - 0 = 8x$

(d) $y = 8x^2 + 7x + 12$

$\frac{dy}{dx} = 8(2x) + 7 + 0 = 16x + 7$

(e) $y = 5 + 4x - 5x^2$

$\frac{dy}{dx} = 0 + 4(1) - 5(2x) = 4 - 10x$

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Differentiation
Exercise C, Question 2

Question:

Find the gradient of the curve whose equation is

(a) $y = 3x^2$ at the point $(2 , 12)$

(b) $y = x^2 + 4x$ at the point $(1 , 5)$

(c) $y = 2x^2 - x - 1$ at the point $(2 , 5)$

(d) $y = \frac{1}{2}x^2 + \frac{3}{2}x$ at the point $(1 , 2)$

(e) $y = 3 - x^2$ at the point $(1 , 2)$

(f) $y = 4 - 2x^2$ at the point $(- 1 , 2)$

Solution:

(a) $y = 3x^2$

$\frac{dy}{dx} = 6x$

At the point $(2 , 12)$, $x = 2$.

Substitute $x = 2$ into the gradient expression $\frac{dy}{dx} = 6x$ to give

gradient $= 6 \times 2 = 12$.

(b) $y = x^2 + 4x$

$\frac{dy}{dx} = 2x + 4$

At the point $(1 , 5)$, $x = 1$.

Substitute $x = 1$ into $\frac{dy}{dx} = 2x + 4$ to give

gradient $= 2 \times 1 + 4 = 6$

(c) $y = 2x^2 - x - 1$

$\frac{dy}{dx} = 4x - 1$

At the point $(2 , 5)$, $x = 2$.

Substitute $x = 2$ into $\frac{dy}{dx} = 4x - 1$ to give

gradient $= 4 \times 2 - 1 = 7$

(d) $y = \frac{1}{2}x^2 + \frac{3}{2}x$

$\frac{dy}{dx} = x + \frac{3}{2}$

At the point $(1 , 2)$, $x = 1$.

Substitute $x = 1$ into $\frac{dy}{dx} = x + \frac{3}{2}$ to give

gradient $= 1 + \frac{3}{2} = 2\frac{1}{2}$

(e) $y = 3 - x^2$

$\frac{dy}{dx} = -2x$

At $(1, 2)$, $x = 1$.

Substitute $x = 1$ into $\frac{dy}{dx} = -2x$ to give

gradient $= -2 \times 1 = -2$

(f) $y = 4 - 2x^2$

$\frac{dy}{dx} = -4x$

At $(-1, 2)$, $x = -1$.

Substitute $x = -1$ into $\frac{dy}{dx} = -4x$ to give

gradient $= -4 \times -1 = +4$

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Differentiation
Exercise C, Question 3

Question:

Find the y-coordinate and the value of the gradient at the point P with x-coordinate 1 on the curve with equation $y = 3 + 2x - x^2$.

Solution:

$y = 3 + 2x - x^2$
When $x = 1$, $y = 3 + 2 - 1$
$\Rightarrow \quad y = 4$ when $x = 1$
Differentiate to give
$\frac{dy}{dx} = 0 + 2 - 2x$

When $x = 1$, $\frac{dy}{dx} = 2 - 2$

$\Rightarrow \quad \frac{dy}{dx} = 0$ when $x = 1$

Therefore, the y-coordinate is 4 and the gradient is 0 when the x-coordinate is 1 on the given curve.

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Differentiation
Exercise C, Question 4

Question:

Find the coordinates of the point on the curve with equation $y = x^2 + 5x - 4$ where the gradient is 3.

Solution:

$y = x^2 + 5x - 4$

$\frac{dy}{dx} = 2x + 5$

Put $\frac{dy}{dx} = 3$

Then $2x + 5 = 3$

$\Rightarrow \quad 2x = -2$

$\Rightarrow \quad x = -1$

Substitute $x = -1$ into $y = x^2 + 5x - 4$:

$y = (-1)^2 + 5(-1) - 4 = 1 - 5 - 4 = -8$

Therefore, $(-1, -8)$ is the point where the gradient is 3.

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Differentiation
Exercise C, Question 5

Question:

Find the gradients of the curve $y = x^2 - 5x + 10$ at the points A and B where the curve meets the line $y = 4$.

Solution:

The curve $y = x^2 - 5x + 10$ meets the line $y = 4$ when
$x^2 - 5x + 10 = 4$
$x^2 - 5x + 6 = 0$
$(x - 3) (x - 2) = 0$
$x = 3$ or $x = 2$
The gradient function for the curve is given by
$\frac{dy}{dx} = 2x - 5$

when $x = 3$, $\frac{dy}{dx} = 2 \times 3 - 5 = 1$

when $x = 2$, $\frac{dy}{dx} = 2 \times 2 - 5 = - 1$

So the gradients are $- 1$ and 1 at $(2 , 4)$ and $(3 , 4)$ respectively.

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Differentiation
Exercise C, Question 6

Question:

Find the gradients of the curve $y = 2x^2$ at the points C and D where the curve meets the line $y = x + 3$.

Solution:

The curve $y = 2x^2$ meets the line $y = x + 3$ when
$2x^2 = x + 3$
$2x^2 - x - 3 = 0$
$(2x - 3)(x + 1) = 0$
$x = 1.5$ or -1

The gradient of the curve is given by the equation $\frac{dy}{dx} = 4x$.

The gradient at the point where $x = -1$ is $4 \times -1 = -4$.
The gradient at the point where $x = 1.5$ is $4 \times 1.5 = 6$.
So the gradient is -4 at $(-1, 2)$ and is 6 at $(1.5, 4.5)$.

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Differentiation
Exercise D, Question 1

Question:

Use standard results to differentiate:

(a) $x^4 + x^{-1}$

(b) $\frac{1}{2}x^{-2}$

(c) $2x^{-\frac{1}{2}}$

Solution:

(a) $f(x) = x^4 + x^{-1}$
$f'(x) = 4x^3 + (-1)x^{-2}$

(b) $f(x) = \frac{1}{2}x^{-2}$

$f'(x) = \frac{1}{2}(-2)x^{-3} = -x^{-3}$

(c) $f(x) = 2x^{-\frac{1}{2}}$

$f'(x) = 2\left(-\frac{1}{2}\right)x^{-1\frac{1}{2}} = -x^{-\frac{3}{2}}$

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Differentiation
Exercise D, Question 2

Question:

Find the gradient of the curve with equation $y = f(x)$ at the point A where:

(a) $f(x) = x^3 - 3x + 2$ and A is at $(-1, 4)$

(b) $f(x) = 3x^2 + 2x^{-1}$ and A is at $(2, 13)$

Solution:

(a) $f(x) = x^3 - 3x + 2$
$f'(x) = 3x^2 - 3$
At $(-1, 4)$, $x = -1$.
Substitute $x = -1$ to find $f'(-1) = 3(-1)^2 - 3 = 0$
Therefore, gradient $= 0$.

(b) $f(x) = 3x^2 + 2x^{-1}$
$f'(x) = 6x + 2(-1)x^{-2} = 6x - 2x^{-2}$
At $(2, 13)$, $x = 2$.
$f'(2) = 6(2) - 2(2)^{-2} = 12 - \frac{2}{4} = 11\frac{1}{2}$

Therefore, gradient $= 11\frac{1}{2}$.

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Differentiation
Exercise D, Question 3

Question:

Find the point or points on the curve with equation $y = f(x)$, where the gradient is zero:

(a) $f(x) = x^2 - 5x$

(b) $f(x) = x^3 - 9x^2 + 24x - 20$

(c) $f(x) = x^{\frac{3}{2}} - 6x + 1$

(d) $f(x) = x^{-1} + 4x$

Solution:

(a) $f(x) = x^2 - 5x$
$f'(x) = 2x - 5$
When gradient is zero, $f'(x) = 0$.
$\Rightarrow \quad 2x - 5 = 0$
$\Rightarrow \quad x = 2.5$
As $y = f(x)$, $y = f(2.5)$ when $x = 2.5$.
$\Rightarrow \quad y = (2.5)^2 - 5(2.5) = -6.25$
Therefore, $(2.5, -6.25)$ is the point on the curve where the gradient is zero.

(b) $f(x) = x^3 - 9x^2 + 24x - 20$
$f'(x) = 3x^2 - 18x + 24$
When gradient is zero, $f'(x) = 0$.
$\Rightarrow \quad 3x^2 - 18x + 24 = 0$
$\Rightarrow \quad 3(x^2 - 6x + 8) = 0$
$\Rightarrow \quad 3(x-4)(x-2) = 0$
$\Rightarrow \quad x = 4$ or $x = 2$
As $y = f(x)$, $y = f(4)$ when $x = 4$.
$\Rightarrow \quad y = 4^3 - 9 \times 4^2 + 24 \times 4 - 20 = -4$
Also $y = f(2)$ when $x = 2$.
$\Rightarrow \quad y = 2^3 - 9 \times 2^2 + 24 \times 2 - 20 = 0$.
Therefore, at $(4, -4)$ and at $(2, 0)$ the gradient is zero.

(c) $f(x) = x^{\frac{3}{2}} - 6x + 1$

$f'(x) = \frac{3}{2}x^{\frac{1}{2}} - 6$

When gradient is zero, $f'(x) = 0$.

$\Rightarrow \quad \frac{3}{2}x^{\frac{1}{2}} - 6 = 0$

$\Rightarrow \quad x^{\frac{1}{2}} = 4$

$\Rightarrow \quad x = 16$
As $y = f(x)$, $y = f(16)$ when $x = 16$.

$\Rightarrow \quad y = 16^{\frac{3}{2}} - 6 \times 16 + 1 = -31$

Therefore, at $(16, -31)$ the gradient is zero.

(d) $f(x) = x^{-1} + 4x$

$f'(x) = -1x^{-2} + 4$

For zero gradient, $f'(x) = 0$.

$\Rightarrow \quad -x^{-2} + 4 = 0$

$\Rightarrow \quad \frac{1}{x^2} = 4$

$\Rightarrow \quad x = \pm \frac{1}{2}$

When $x = \frac{1}{2}, y = f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^{-1} + 4\left(\frac{1}{2}\right) = 2 + 2 = 4$

When $x = -\frac{1}{2}, y = f\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^{-1} + 4\left(-\frac{1}{2}\right) = -2 - 2 = -4$

Therefore, $\left(\frac{1}{2}, 4\right)$ and $\left(-\frac{1}{2}, -4\right)$ are points on the curve where the gradient is zero.

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Differentiation
Exercise E, Question 1

Question:

Use standard results to differentiate:

(a) $2\sqrt{x}$

(b) $\frac{3}{x^2}$

(c) $\frac{1}{3x^3}$

(d) $\frac{1}{3}x^3(x-2)$

(e) $\frac{2}{x^3}+\sqrt{x}$

(f) $\sqrt[3]{x}+\frac{1}{2x}$

(g) $\frac{2x+3}{x}$

(h) $\frac{3x^2-6}{x}$

(i) $\frac{2x^3+3x}{\sqrt{x}}$

(j) $x(x^2-x+2)$

(k) $3x^2(x^2+2x)$

(l) $(3x-2)\left(4x+\frac{1}{x}\right)$

Solution:

(a) $y=2\sqrt{x}=2x^{\frac{1}{2}}$

$$\frac{dy}{dx}=2\left(\frac{1}{2}\right)x^{-\frac{1}{2}}=x^{-\frac{1}{2}}$$

(b) $y = \frac{3}{x^2} = 3x^{-2}$

$\frac{dy}{dx} = 3(-2)x^{-3} = -6x^{-3}$

(c) $y = \frac{1}{3x^3} = \frac{1}{3}x^{-3}$

$\frac{dy}{dx} = \frac{1}{3}(-3)x^{-4} = -x^{-4}$

(d) $y = \frac{1}{3}x^3(x-2) = \frac{1}{3}x^4 - \frac{2}{3}x^3$

$\frac{dy}{dx} = \frac{4}{3}x^3 - \frac{2}{3} \times 3x^2 = \frac{4}{3}x^3 - 2x^2$

(e) $y = \frac{2}{x^3} + \sqrt{x} = 2x^{-3} + x^{\frac{1}{2}}$

$\frac{dy}{dx} = -6x^{-4} + \frac{1}{2}x^{-\frac{1}{2}}$

(f) $y = \sqrt[3]{x} + \frac{1}{2x} = x^{\frac{1}{3}} + \frac{1}{2}x^{-1}$

$\frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}} - \frac{1}{2}x^{-2}$

(g) $y = \frac{2x+3}{x} = \frac{2x}{x} + \frac{3}{x} = 2 + 3x^{-1}$

$\frac{dy}{dx} = 0 - 3x^{-2}$

(h) $y = \frac{3x^2-6}{x} = \frac{3x^2}{x} - \frac{6}{x} = 3x - 6x^{-1}$

$\frac{dy}{dx} = 3 + 6x^{-2}$

(i) $y = \frac{2x^3+3x}{\sqrt{x}} = \frac{2x^3}{x^{\frac{1}{2}}} + \frac{3x}{x^{\frac{1}{2}}} = 2x^{2\frac{1}{2}} + 3x^{\frac{1}{2}}$

$\frac{dy}{dx} = 5x^{1\frac{1}{2}} + 1.5x^{-\frac{1}{2}}$

(j) $y = x(x^2 - x + 2) = x^3 - x^2 + 2x$

$\frac{dy}{dx} = 3x^2 - 2x + 2$

(k) $y = 3x^2(x^2 + 2x) = 3x^4 + 6x^3$

$\frac{dy}{dx} = 12x^3 + 18x^2$

(l) $y = (3x - 2)\left(4x + \frac{1}{x}\right) = 12x^2 - 8x + 3 - \frac{2}{x} = 12x^2 - 8x + 3 - 2x^{-1}$

$\frac{dy}{dx} = 24x - 8 + 2x^{-2}$

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Differentiation
Exercise E, Question 2

Question:

Find the gradient of the curve with equation $y = f(x)$ at the point A where:

(a) $f(x) = x(x + 1)$ and A is at $(0, 0)$

(b) $f(x) = \frac{2x - 6}{x^2}$ and A is at $(3, 0)$

(c) $f(x) = \frac{1}{\sqrt{x}}$ and A is at $\left(\frac{1}{4}, 2\right)$

(d) $f(x) = 3x - \frac{4}{x^2}$ and A is at $(2, 5)$

Solution:

(a) $f(x) = x(x + 1) = x^2 + x$
$f'(x) = 2x + 1$
At $(0, 0)$, $x = 0$.
Therefore, gradient $= f'(0) = 1$

(b) $f(x) = \frac{2x - 6}{x^2} = \frac{2x}{x^2} - \frac{6}{x^2} = \frac{2}{x} - 6x^{-2} = 2x^{-1} - 6x^{-2}$
$f'(x) = -2x^{-2} + 12x^{-3}$
At $(3, 0)$, $x = 3$.
Therefore, gradient $= f'(3) = -\frac{2}{3^2} + \frac{12}{3^3} = -\frac{2}{9} + \frac{12}{27} = \frac{2}{9}$

(c) $f(x) = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$

$f'(x) = -\frac{1}{2}x^{-\frac{3}{2}}$

At $\left(\frac{1}{4}, 2\right)$, $x = \frac{1}{4}$.

Therefore, gradient $= f'\left(\frac{1}{4}\right) = -\frac{1}{2}\left(\frac{1}{4}\right)^{-\frac{3}{2}} = -\frac{1}{2} \times 2^3 = -4$

(d) $f(x) = 3x - \frac{4}{x^2} = 3x - 4x^{-2}$
$f'(x) = 3 + 8x^{-3}$
At $(2, 5)$, $x = 2$.
Therefore, gradient $= f'(2) = 3 + 8(2)^{-3} = 3 + \frac{8}{8} = 4$.

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Differentiation
Exercise F, Question 1

Question:

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when y equals:

$12x^2 + 3x + 8$

Solution:

$y = 12x^2 + 3x + 8$

$\frac{dy}{dx} = 24x + 3$

$\frac{d^2y}{dx^2} = 24$

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Differentiation
Exercise F, Question 2

Question:

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when y equals:

$15x + 6 + \frac{3}{x}$

Solution:

$y = 15x + 6 + \frac{3}{x} = 15x + 6 + 3x^{-1}$

$\frac{dy}{dx} = 15 - 3x^{-2}$

$\frac{d^2y}{dx^2} = 0 + 6x^{-3}$

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Differentiation
Exercise F, Question 3

Question:

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when y equals:

$9\sqrt{x} - \frac{3}{x^2}$

Solution:

$y = 9\sqrt{x} - \frac{3}{x^2} = 9x^{\frac{1}{2}} - 3x^{-2}$

$\frac{dy}{dx} = 4\frac{1}{2}x^{-\frac{1}{2}} + 6x^{-3}$

$\frac{d^2y}{dx^2} = -2\frac{1}{4}x^{-\frac{3}{2}} - 18x^{-4}$

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Differentiation
Exercise F, Question 4

Question:

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when y equals:

$(5x + 4)(3x - 2)$

Solution:

$y = (5x + 4)(3x - 2) = 15x^2 + 2x - 8$

$\frac{dy}{dx} = 30x + 2$

$\frac{d^2y}{dx^2} = 30$

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Differentiation
Exercise F, Question 5

Question:

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when y equals:

$\frac{3x+8}{x^2}$

Solution:

$y = \frac{3x+8}{x^2} = \frac{3x}{x^2} + \frac{8}{x^2} = \frac{3}{x} + 8x^{-2} = 3x^{-1} + 8x^{-2}$

$\frac{dy}{dx} = -3x^{-2} - 16x^{-3}$

$\frac{d^2y}{dx^2} = 6x^{-3} + 48x^{-4}$

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Differentiation
Exercise G, Question 1

Question:

Find $\frac{d\theta}{dt}$ where $\theta = t^2 - 3t$

Solution:

$\theta = t^2 - 3t$

$\frac{d\theta}{dt} = 2t - 3$

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Differentiation
Exercise G, Question 2

Question:

Find $\frac{\mathrm{d}A}{\mathrm{d}r}$ where $A = 2\pi r$

Solution:

$A = 2\pi r$

$\frac{\mathrm{d}A}{\mathrm{d}r} = 2\pi$

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Differentiation
Exercise G, Question 3

Question:

Find $\frac{\mathrm{d}r}{\mathrm{d}t}$ where $r = \frac{12}{t}$

Solution:

$r = \frac{12}{t} = 12t^{-1}$

$\frac{\mathrm{d}r}{\mathrm{d}t} = -12t^{-2}$

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Differentiation
Exercise G, Question 4

Question:

Find $\frac{\mathrm{d}v}{\mathrm{d}t}$ where $v = 9.8t + 6$

Solution:

$v = 9.8t + 6$

$\frac{\mathrm{d}v}{\mathrm{d}t} = 9.8$

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Differentiation
Exercise G, Question 5

Question:

Find $\frac{dR}{dr}$ where $R = r + \frac{5}{r}$

Solution:

$R = r + \frac{5}{r} = r + 5r^{-1}$

$\frac{dR}{dr} = 1 - 5r^{-2}$

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Differentiation
Exercise G, Question 6

Question:

Find $\frac{dx}{dt}$ where $x = 3 - 12t + 4t^2$

Solution:

$x = 3 - 12t + 4t^2$

$\frac{dx}{dt} = 0 - 12 + 8t$

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Differentiation
Exercise G, Question 7

Question:

Find $\frac{\mathrm{d}A}{\mathrm{d}x}$ where $A = x\left(10 - x\right)$

Solution:

$A = x(10 - x) = 10x - x^2$

$\frac{\mathrm{d}A}{\mathrm{d}x} = 10 - 2x$

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Differentiation
Exercise H, Question 1

Question:

Find the equation of the tangent to the curve:

(a) $y = x^2 - 7x + 10$ at the point $(2 , 0)$

(b) $y = x + \frac{1}{x}$ at the point $\left(2 , 2 \frac{1}{2} \right)$

(c) $y = 4 \sqrt{x}$ at the point $(9 , 12)$

(d) $y = \frac{2x - 1}{x}$ at the point $(1 , 1)$

(e) $y = 2x^3 + 6x + 10$ at the point $(- 1 , 2)$

(f) $y = x^2 + \frac{- 7}{x^2}$ at the point $(1 , - 6)$

Solution:

(a) $y = x^2 - 7x + 10$

$\frac{dy}{dx} = 2x - 7$

At $(2 , 0)$, $x = 2$, gradient $= 2 \times 2 - 7 = - 3$.
Therefore, equation of tangent is
$y - 0 = - 3 (x - 2)$
$y = - 3x + 6$
$y + 3x - 6 = 0$

(b) $y = x + \frac{1}{x} = x + x^{- 1}$

$\frac{dy}{dx} = 1 - x^{- 2}$

At $\left(2 , 2 \frac{1}{2} \right)$, $x = 2$, gradient $= 1 - 2^{- 2} = \frac{3}{4}$.

Therefore, equation of tangent is

$y - 2 \frac{1}{2} = \frac{3}{4}(x - 2)$

$y = \frac{3}{4}x - 1 \frac{1}{2} + 2 \frac{1}{2}$

$y = \frac{3}{4}x + 1$

$4y - 3x - 4 = 0$

(c) $y = 4 \sqrt{x} = 4x^{\frac{1}{2}}$

$\frac{dy}{dx} = 2x^{-\frac{1}{2}}$

At $(9, 12)$, $x = 9$, gradient $= 2 \times 9^{-\frac{1}{2}} = \frac{2}{3}$.

Therefore, equation of tangent is

$y - 12 = \frac{2}{3}(x - 9)$

$y = \frac{2}{3}x - 6 + 12$

$y = \frac{2}{3}x + 6$

$3y - 2x - 18 = 0$

(d) $y = \frac{2x - 1}{x} = \frac{2x}{x} - \frac{1}{x} = 2 - x^{-1}$

$\frac{dy}{dx} = 0 + x^{-2}$

At $(1, 1)$, $x = 1$, gradient $= 1^{-2} = 1$.
Therefore, equation of tangent is
$y - 1 = 1 \times (x - 1)$
$y = x$

(e) $y = 2x^3 + 6x + 10$
$\frac{dy}{dx} = 6x^2 + 6$

At $(-1, 2)$, $x = -1$, gradient $= 6(-1)^2 + 6 = 12$.
Therefore, equation of tangent is
$y - 2 = 12[x - (-1)]$
$y - 2 = 12x + 12$
$y = 12x + 14$

(f) $y = x^2 - \frac{7}{x^2} = x^2 - 7x^{-2}$

$\frac{dy}{dx} = 2x + 14x^{-3}$

At $(1, -6)$, $x = 1$, gradient $= 2 + 14 = 16$.
Therefore, equation of tangent is
$y - (-6) = 16(x - 1)$
$y + 6 = 16x - 16$
$y = 16x - 22$

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Differentiation
Exercise H, Question 2

Question:

Find the equation of the normal to the curves:

(a) $y = x^2 - 5x$ at the point $(6 , 6)$

(b) $y = x^2 - \frac{8}{\sqrt{x}}$ at the point $(4 , 12)$

Solution:

(a) $y = x^2 - 5x$

$\frac{dy}{dx} = 2x - 5$

At $(6 , 6)$, $x = 6$, gradient of curve is $2 \times 6 - 5 = 7$.

Therefore, gradient of normal is $-\frac{1}{7}$.

The equation of the normal is

$y - 6 = -\frac{1}{7}(x - 6)$

$7y - 42 = -x + 6$
$7y + x - 48 = 0$

(b) $y = x^2 - \frac{8}{\sqrt{x}} = x^2 - 8x^{-\frac{1}{2}}$

$\frac{dy}{dx} = 2x + 4x^{-\frac{3}{2}}$

At $(4 , 12)$, $x = 4$, gradient of curve is $2 \times 4 + 4 (4)^{-\frac{3}{2}} = 8 + \frac{4}{8} = \frac{17}{2}$

Therefore, gradient of normal is $-\frac{2}{17}$.

The equation of the normal is

$y - 12 = -\frac{2}{17}(x - 4)$

$17y - 204 = -2x + 8$
$17y + 2x - 212 = 0$

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Differentiation
Exercise H, Question 3

Question:

Find the coordinates of the point where the tangent to the curve $y = x^2 + 1$ at the point $(2 , 5)$ meets the normal to the same curve at the point $(1 , 2)$.

Solution:

$y = x^2 + 1$

$\frac{dy}{dx} = 2x$

At $(2 , 5)$, $x = 2$, $\frac{dy}{dx} = 4$.

The tangent at $(2 , 5)$ has gradient 4.
Its equation is
$y - 5 = 4 (x - 2)$
$y = 4x - 3$ ①
The curve has gradient 2 at the point $(1 , 2)$.

The normal is perpendicular to the curve. Its gradient is $-\frac{1}{2}$.

The equation of the normal is

$y - 2 = -\frac{1}{2}(x - 1)$

$y = -\frac{1}{2}x + 2\frac{1}{2}$ ②

Solve Equations ① and ② to find where the tangent and the normal meet.
Equation ① − Equation ②:

$0 = 4\frac{1}{2}x - 5\frac{1}{2}$

$x = \frac{11}{9}$

Substitute into Equation ① to give $y = \frac{44}{9} - 3 = \frac{17}{9}$.

Therefore, the tangent at $(2 , 5)$ meets the normal at $(1 , 2)$ at $\left(\frac{11}{9} , \frac{17}{9} \right)$.

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Differentiation
Exercise H, Question 4

Question:

Find the equations of the normals to the curve $y = x + x^3$ at the points $(0 , 0)$ and $(1 , 2)$, and find the coordinates of the point where these normals meet.

Solution:

$y = x + x^3$

$\frac{dy}{dx} = 1 + 3x^2$

At $(0 , 0)$ the curve has gradient $1 + 3 \times 0^2 = 1$.

The gradient of the normal at $(0 , 0)$ is $- \frac{1}{1} = - 1$.

The equation of the normal at $(0 , 0)$ is

$y - 0 = - 1 (x - 0)$

$y = - x$ ①

At $(1 , 2)$ the curve has gradient $1 + 3 \times 1^2 = 4$.

The gradient of the normal at $(1 , 2)$ is $- \frac{1}{4}$.

The equation of the normal at $(1 , 2)$ is

$y - 2 = - \frac{1}{4}(x - 1)$

$4y - 8 = - x + 1$

$4y + x - 9 = 0$ ②

Solve Equations ① and ② to find where the normals meet.

Substitute $y = - x$ into Equation ②:

$- 4x + x = 9 \quad \Rightarrow \quad x = - 3$ and $y = + 3$.

Therefore, the normals meet at $(- 3 , 3)$.

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Differentiation

Exercise H, Question 5

Question:

For $f(x) = 12 - 4x + 2x^2$, find an equation of the tangent and normal at the point where $x = -1$ on the curve with equation $y = f(x)$. **[E]**

Solution:

$y = 12 - 4x + 2x^2$

$\frac{dy}{dx} = 0 - 4 + 4x$

when $x = -1$, $\frac{dy}{dx} = -4 - 4 = -8$.

The gradient of the curve is -8 when $x = -1$.
As $y = f(x)$, when $x = -1$
$y = f(-1) = 12 + 4 + 2 = 18$
The tangent at $(-1, 18)$ has gradient -8. So its equation is
$y - 18 = -8(x + 1)$
$y - 18 = -8x - 8$
$y = 10 - 8x$

The normal at $(-1, 18)$ has gradient $\frac{-1}{-8} = \frac{1}{8}$. So its equation is

$y - 18 = \frac{1}{8}\left(x + 1\right)$

$8y - 144 = x + 1$
$8y - x - 145 = 0$

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Differentiation
Exercise I, Question 1

Question:

A curve is given by the equation $y = 3x^2 + 3 + \frac{1}{x^2}$, where $x > 0$.

At the points A, B and C on the curve, $x = 1$, 2 and 3 respectively.
Find the gradients at A, B and C. **[E]**

Solution:

$y = 3x^2 + 3 + \frac{1}{x^2} = 3x^2 + 3 + x^{-2}$

$\frac{dy}{dx} = 6x - 2x^{-3} = 6x - \frac{2}{x^3}$

When $x = 1$, $\frac{dy}{dx} = 6 \times 1 - \frac{2}{1^3} = 4$

When $x = 2$, $\frac{dy}{dx} = 6 \times 2 - \frac{2}{2^3} = 12 - \frac{2}{8} = 11\frac{3}{4}$

When $x = 3$, $\frac{dy}{dx} = 6 \times 3 - \frac{2}{3^3} = 18 - \frac{2}{27} = 17\frac{25}{27}$

The gradients at points A, B and C are 4, $11\frac{3}{4}$ and $17\frac{25}{27}$, respectively.

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Differentiation
Exercise I, Question 2

Question:

Taking $f(x) = \frac{1}{4}x^4 - 4x^2 + 25$, find the values of x for which $f'(x) = 0$. **[E]**

Solution:

$f(x) = \frac{1}{4}x^4 - 4x^2 + 25$

$f'(x) = x^3 - 8x$
When $f'(x) = 0$,
$x^3 - 8x = 0$
$x(x^2 - 8) = 0$
$x = 0$ or $x^2 = 8$
$x = 0$ or $\pm\sqrt{8}$
$x = 0$ or $\pm 2\sqrt{2}$

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Differentiation
Exercise I, Question 3

Question:

A curve is drawn with equation $y = 3 + 5x + x^2 - x^3$. Find the coordinates of the two points on the curve where the gradient of the curve is zero. **[E]**

Solution:

$y = 3 + 5x + x^2 - x^3$

$\frac{dy}{dx} = 5 + 2x - 3x^2$

Put $\frac{dy}{dx} = 0$. Then

$5 + 2x - 3x^2 = 0$

$(5 - 3x) (1 + x) = 0$

$x = -1$ or $x = \frac{5}{3}$

Substitute to obtain

$y = 3 - 5 + 1 - (-1)^3$ when $x = -1$, i.e.

$y = 0$ when $x = -1$

and

$y = 3 + 5 \left(\frac{5}{3} \right) + \left(\frac{5}{3} \right)^2 - \left(\frac{5}{3} \right)^3$ when $x = \frac{5}{3}$, i.e.

$y = 3 + \frac{25}{3} + \frac{25}{9} - \frac{125}{27} = 9 \frac{13}{27}$ when $x = \frac{5}{3}$

So the points have coordinates $(-1 , 0)$ and $\left(1 \frac{2}{3} , 9 \frac{13}{27} \right)$.

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Differentiation

Exercise I, Question 4

Question:

Calculate the x-coordinates of the points on the curve with equation $y = 7x^2 - x^3$ at which the gradient is equal to 16. **[E]**

Solution:

$y = 7x^2 - x^3$

$\frac{dy}{dx} = 14x - 3x^2$

Put $\frac{dy}{dx} = 16$, i.e.

$14x - 3x^2 = 16$

$3x^2 - 14x + 16 = 0$

$(3x - 8) (x - 2) = 0$

$x = \frac{8}{3}$ or $x = 2$

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Differentiation

Exercise I, Question 5

Question:

Find the x-coordinates of the two points on the curve with equation $y = x^3 - 11x + 1$ where the gradient is 1. Find the corresponding y-coordinates. **[E]**

Solution:

$y = x^3 - 11x + 1$

$\frac{dy}{dx} = 3x^2 - 11$

As gradient is 1, put $\frac{dy}{dx} = 1$, then

$3x^2 - 11 = 1$
$3x^2 = 12$
$x^2 = 4$
$x = \pm 2$

Substitute these values into $y = x^3 - 11x + 1$:
$y = 2^3 - 11 \times 2 + 1 = -13$ when $x = 2$ and
$y = (-2)^3 - 11(-2) + 1 = 15$ when $x = -2$
The gradient is 1 at the points $(2, -13)$ and $(-2, 15)$.

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Differentiation
Exercise I, Question 6

Question:

The function f is defined by $f(x) = x + \frac{9}{x}, x \in \mathbb{R}, x \neq 0$.

(a) Find $f'(x)$.

(b) Solve $f'(x) = 0$. **[E]**

Solution:

(a) $f(x) = x + \frac{9}{x} = x + 9x^{-1}$

$f'(x) = 1 - 9x^{-2} = 1 - \frac{9}{x^2}$

(b) When $f'(x) = 0$,

$1 - \frac{9}{x^2} = 0$

$\frac{9}{x^2} = 1$

$x^2 = 9$

$x = \pm 3$

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Differentiation
Exercise I, Question 7

Question:

Given that

$y = x^{\frac{3}{2}} + \frac{48}{x}$, $x > 0$,

find the value of x and the value of y when $\frac{dy}{dx} = 0$. **[E]**

Solution:

$y = x^{\frac{3}{2}} + \frac{48}{x} = x^{\frac{3}{2}} + 48x^{-1}$

$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} - 48x^{-2}$

Put $\frac{dy}{dx} = 0$, then

$\frac{3}{2}x^{\frac{1}{2}} - \frac{48}{x^2} = 0$

$\frac{3}{2}x^{\frac{1}{2}} = \frac{48}{x^2}$

Multiply both sides by x^2:

$\frac{3}{2}x^{2\frac{1}{2}} = 48$

$x^{2\frac{1}{2}} = 32$

$x = (32)^{\frac{2}{5}}$

$x = 4$

Substitute to give $y = 4^{\frac{3}{2}} + \frac{48}{4} = 8 + 12 = 20$

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Differentiation
Exercise I, Question 8

Question:

Given that

$y = 3x^{\frac{1}{2}} - 4x^{-\frac{1}{2}}, x > 0,$

find $\frac{\mathrm{d}y}{\mathrm{d}x}$. **[E]**

Solution:

$y = 3x^{\frac{1}{2}} - 4x^{-\frac{1}{2}}$

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{2}x^{-\frac{1}{2}} + \frac{4}{2}x^{-\frac{3}{2}} = \frac{3}{2}x^{-\frac{1}{2}} + 2x^{-\frac{3}{2}}$

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Differentiation
Exercise I, Question 9

Question:

A curve has equation $y = 12x^{\frac{1}{2}} - x^{\frac{3}{2}}$.

(a) Show that $\frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}}(4 - x)$

(b) Find the coordinates of the point on the curve where the gradient is zero. **[E]**

Solution:

(a) $y = 12x^{\frac{1}{2}} - x^{\frac{3}{2}}$

$$\frac{dy}{dx} = 12\left(\frac{1}{2}\right)x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} = 6x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2}x^{-\frac{1}{2}}(4 - x)$$

(b) The gradient is zero when $\frac{dy}{dx} = 0$:

$\frac{3}{2}x^{-\frac{1}{2}}(4 - x) = 0$

$x = 4$

Substitute into $y = 12x^{\frac{1}{2}} - x^{\frac{3}{2}}$ to obtain

$y = 12 \times 2 - 2^3 = 16$

The gradient is zero at the point with coordinates $(4, 16)$.

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Differentiation
Exercise I, Question 10

Question:

(a) Expand $\left(x^{\frac{3}{2}} - 1 \right) \left(x^{-\frac{1}{2}} + 1 \right)$.

(b) A curve has equation $y = \left(x^{\frac{3}{2}} - 1 \right) \left(x^{-\frac{1}{2}} + 1 \right)$, $x > 0$. Find $\frac{dy}{dx}$.

(c) Use your answer to **b** to calculate the gradient of the curve at the point where $x = 4$. **[E]**

Solution:

(a) $\left(x^{\frac{3}{2}} - 1 \right) \left(x^{-\frac{1}{2}} + 1 \right) = x + x^{\frac{3}{2}} - x^{-\frac{1}{2}} - 1$

(b) $y = x + x^{\frac{3}{2}} - x^{-\frac{1}{2}} - 1$

$\frac{dy}{dx} = 1 + \frac{3}{2}x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}}$

(c) When $x = 4$, $\frac{dy}{dx} = 1 + \frac{3}{2} \times 2 + \frac{1}{2} \times \frac{1}{4^{\frac{3}{2}}} = 1 + 3 + \frac{1}{16} = 4\frac{1}{16}$

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Differentiation
Exercise I, Question 11

Question:

Differentiate with respect to x:

$2x^3 + \sqrt{x} + \frac{x^2 + 2x}{x^2}$ **[E]**

Solution:

Let $y = 2x^3 + \sqrt{x} + \frac{x^2 + 2x}{x^2}$

$\Rightarrow \quad y = 2x^3 + x^{\frac{1}{2}} + \frac{x^2}{x^2} + \frac{2x}{x^2}$

$\Rightarrow \quad y = 2x^3 + x^{\frac{1}{2}} + 1 + 2x^{-1}$

$\frac{dy}{dx} = 6x^2 + \frac{1}{2}x^{-\frac{1}{2}} - 2x^{-2} = 6x^2 + \frac{1}{2\sqrt{x}} - \frac{2}{x^2}$

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Differentiation
Exercise I, Question 12

Question:

The volume, V cm^3, of a tin of radius r cm is given by the formula $V = \pi (40r - r^2 - r^3)$. Find the positive value of r for which $\frac{dV}{dr} = 0$, and find the value of V which corresponds to this value of r. **[E]**

Solution:

$V = \pi (40r - r^2 - r^3)$

$\frac{dV}{dr} = 40 \pi - 2 \pi r - 3 \pi r^2$

Put $\frac{dV}{dr} = 0$, then

$\pi (40 - 2r - 3r^2) = 0$

$(4 + r) (10 - 3r) = 0$

$r = \frac{10}{3}$ or -4

As r is positive, $r = \frac{10}{3}$.

Substitute into the given expression for V:

$V = \pi \left(40 \times \frac{10}{3} - \frac{100}{9} - \frac{1000}{27} \right) = \frac{2300}{27} \pi$

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Differentiation
Exercise I, Question 13

Question:

The total surface area of a cylinder Acm^2 with a fixed volume of 1000 cubic cm is given by the formula $A = 2\pi x^2 + \frac{2000}{x}$, where x cm is the radius. Show that when the rate of change of the area with respect to the radius is zero, $x^3 = \frac{500}{\pi}$. **[E]**

Solution:

$A = 2\pi x^2 + \frac{2000}{x} = 2\pi x^2 + 2000x^{-1}$

$\frac{\mathrm{d}A}{\mathrm{d}x} = 4\pi x - 2000x^{-2} = 4\pi x - \frac{2000}{x^2}$

When $\frac{\mathrm{d}A}{\mathrm{d}x} = 0$,

$4\pi x = \frac{2000}{x^2}$

$x^3 = \frac{2000}{4\pi} = \frac{500}{\pi}$

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Differentiation
Exercise I, Question 14

Question:

The curve with equation $y = ax^2 + bx + c$ passes through the point $(1 , 2)$. The gradient of the curve is zero at the point $(2 , 1)$. Find the values of a, b and c. **[E]**

Solution:

The point $(1 , 2)$ lies on the curve with equation $y = ax^2 + bx + c$.
Therefore, substitute $x = 1$, $y = 2$ into the equation to give

$2 = a + b + c$ ①

The point $(2 , 1)$ also lies on the curve.
Therefore, substitute $x = 2$, $y = 1$ to give

$1 = 4a + 2b + c$ ②

Eliminate c by subtracting Equation ② − Equation ①:

$-1 = 3a + b$ ③

The gradient of the curve is zero at $(2 , 1)$ so substitute $x = 2$ into the expression for $\frac{dy}{dx} = 0$.

As $y = ax^2 + bx + c$

$\frac{dy}{dx} = 2ax + b$

At $(2 , 1)$

$0 = 4a + b$ ④

Solve Equations ③ and ④ by subtracting ④ − ③:
$1 = a$
Substitute $a = 1$ into Equation ③ to give $b = -4$.
Then substitute a and b into Equation ① to give $c = 5$.
Therefore, $a = 1$, $b = -4$, $c = 5$.

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Edexcel Modular Mathematics for AS and A-Level

Differentiation

Exercise I, Question 15

Question:

A curve C has equation $y = x^3 - 5x^2 + 5x + 2$.

(a) Find $\frac{dy}{dx}$ in terms of x.

(b) The points P and Q lie on C. The gradient of C at both P and Q is 2. The x-coordinate of P is 3.

(i) Find the x-coordinate of Q.

(ii) Find an equation for the tangent to C at P, giving your answer in the form $y = mx + c$, where m and c are constants.

(iii) If this tangent intersects the coordinate axes at the points R and S, find the length of RS, giving your answer as a surd. **[E]**

Solution:

$y = x^3 - 5x^2 + 5x + 2$

(a) $\frac{dy}{dx} = 3x^2 - 10x + 5$

(b) Given that the gradient is 2, $\frac{dy}{dx} = 2$

$3x^2 - 10x + 5 = 2$
$3x^2 - 10x + 3 = 0$
$(3x - 1)(x - 3) = 0$
$x = \frac{1}{3}$ or 3

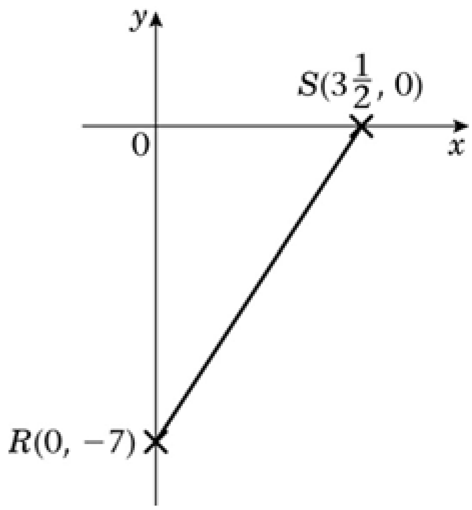
(i) At P, $x = 3$. Therefore, at Q, $x = \frac{1}{3}$.

(ii) At the point P, $x = 3$, $y = 3^3 - 5 \times 3^2 + 5 \times 3 + 2 = 27 - 45 + 15 + 2 = -1$
The gradient of the curve is 2.
The equation of the tangent at P is
$y - (-1) = 2(x - 3)$
$y + 1 = 2x - 6$
$y = 2x - 7$

(iii) This tangent meets the axes when $x = 0$ and when $y = 0$.

When $x = 0$, $y = -7$. When $y = 0$, $x = 3\frac{1}{2}$.

The tangent meets the axes at $(0, -7)$ and $\left(3\frac{1}{2}, 0\right)$.



The distance $RS = \sqrt{\left(3\frac{1}{2} - 0\right)^2 + [0 - (-7)]^2} = \sqrt{\frac{49}{4} + 49} = \frac{7}{2}\sqrt{1+4} = \frac{7}{2}\sqrt{5}$.

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Differentiation
Exercise I, Question 16

Question:

Find an equation of the tangent and the normal at the point where $x = 2$ on the curve with equation $y = \frac{8}{x} - x + 3x^2$, $x > 0$. **[E]**

Solution:

$y = \frac{8}{x} - x + 3x^2 = 8x^{-1} - x + 3x^2$

$\frac{\mathrm{d}y}{\mathrm{d}x} = -8x^{-2} - 1 + 6x = -\frac{8}{x^2} - 1 + 6x$

when $x = 2$, $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{8}{4} - 1 + 12 = 9$

At $x = 2$, $y = \frac{8}{2} - 2 + 3 \times 2^2 = 14$

So the equation of the tangent through the point $(2, 14)$ with gradient 9 is
$y - 14 = 9(x - 2)$
$y = 9x - 18 + 14$
$y = 9x - 4$

The gradient of the normal is $-\frac{1}{9}$, as the normal is at right angles to the tangent.

So the equation of the normal is

$y - 14 = -\frac{1}{9}(x - 2)$

$9y + x = 128$

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Differentiation
Exercise I, Question 17

Question:

The normals to the curve $2y = 3x^3 - 7x^2 + 4x$, at the points $O(0, 0)$ and $A(1, 0)$, meet at the point N.

(a) Find the coordinates of N.

(b) Calculate the area of triangle OAN. **[E]**

Solution:

(a) $2y = 3x^3 - 7x^2 + 4x$

$y = \frac{3}{2}x^3 - \frac{7}{2}x^2 + 2x$

$\frac{dy}{dx} = \frac{9}{2}x^2 - 7x + 2$

At $(0, 0)$, $x = 0$, gradient of curve is $0 - 0 + 2 = 2$.

The gradient of the normal at $(0, 0)$ is $-\frac{1}{2}$.

The equation of the normal at $(0, 0)$ is $y = -\frac{1}{2}x$.

At $(1, 0)$, $x = 1$, gradient of curve is $\frac{9}{2} - 7 + 2 = -\frac{1}{2}$.

The gradient of the normal at $(1, 0)$ is 2.
The equation of the normal at $(1, 0)$ is $y = 2(x - 1)$.

The normals meet when $y = 2x - 2$ and $y = -\frac{1}{2}x$:

$2x - 2 = -\frac{1}{2}x$

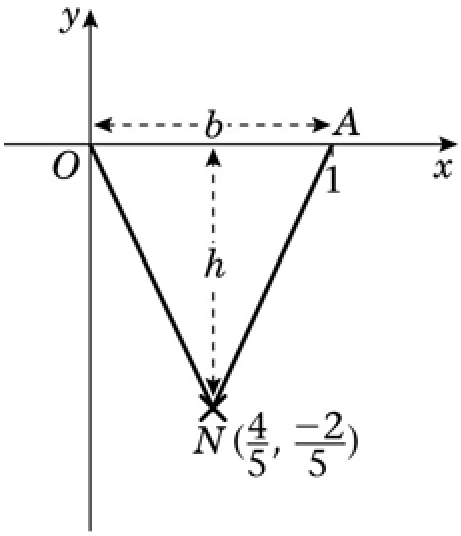
$2\frac{1}{2}x = 2$

$x = 2 \div 2\frac{1}{2} = \frac{4}{5}$

Substitute into $y = 2x - 2$ to obtain $y = -\frac{2}{5}$ and check in $y = -\frac{1}{2}x$.

N has coordinates $\left(\frac{4}{5}, -\frac{2}{5}\right)$.

(b)



The area of $\triangle OAN = \frac{1}{2}$base × height

base (b) = 1

height(h) = $\frac{2}{5}$

Area = $\frac{1}{2} \times 1 \times \frac{2}{5} = \frac{1}{5}$

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Integration
Exercise A, Question 1

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

x^5

Solution:

$\frac{dy}{dx} = x^5$

$y = \frac{x^6}{6} + c$

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Integration
Exercise A, Question 2

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$10x^4$

Solution:

$\frac{dy}{dx} = 10x^4$

$y = 10 \frac{x^5}{5} + c$

$y = 2x^5 + c$

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Integration
Exercise A, Question 3

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$3x^2$

Solution:

$\frac{dy}{dx} = 3x^2$

$y = 3 \frac{x^3}{3} + c$

$y = x^3 + c$

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Integration
Exercise A, Question 4

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$-x^{-2}$

Solution:

$\frac{dy}{dx} = -x^{-2}$

$y = -\frac{x^{-1}}{-1} + c$

$y = x^{-1} + c$ or

$y = \frac{1}{x} + c$

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Integration
Exercise A, Question 5

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$-4x^{-3}$

Solution:

$\frac{dy}{dx} = -4x^{-3}$

$y = -4\frac{x^{-2}}{-2} + c$

$y = 2x^{-2} + c$ or

$y = \frac{2}{x^2} + c$

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Integration
Exercise A, Question 6

Question:

Find an expression for y when $\frac{\mathrm{d}y}{\mathrm{d}x}$ is:

$x^{\frac{2}{3}}$

Solution:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x^{\frac{2}{3}}$$

$$y = \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + c$$

$$y = \frac{3}{5}x^{\frac{5}{3}} + c$$

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Integration
Exercise A, Question 7

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$4x^{\frac{1}{2}}$

Solution:

$$\frac{dy}{dx} = 4x^{\frac{1}{2}}$$

$$y = 4 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$y = \frac{8}{3}x^{\frac{3}{2}} + c$$

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Integration

Exercise A, Question 8

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$-2x^6$

Solution:

$\frac{dy}{dx} = -2x^6$

$y = -2\frac{x^7}{7} + c$

$y = -\frac{2}{7}x^7 + c$

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Integration

Exercise A, Question 9

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$3x^5$

Solution:

$\frac{dy}{dx} = 3x^5$

$y = 3\frac{x^6}{6} + c$

$y = \frac{1}{2}x^6 + c$

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Integration
Exercise A, Question 10

Question:

Find an expression for y when $\frac{\mathrm{d}y}{\mathrm{d}x}$ is:

$3x^{-4}$

Solution:

$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^{-4}$

$y = 3\frac{x^{-3}}{-3} + c$

$y = -x^{-3} + c$ or

$y = -\frac{1}{x^3} + c$

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Integration
Exercise A, Question 11

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$x^{-\frac{1}{2}}$

Solution:

$\frac{dy}{dx} = x^{-\frac{1}{2}}$

$y = \frac{x^{+\frac{1}{2}}}{\frac{1}{2}} + c$

$y = 2x^{\frac{1}{2}} + c$ or
$y = 2\sqrt{x} + c$

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Integration
Exercise A, Question 12

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$5x^{-\frac{3}{2}}$

Solution:

$\frac{dy}{dx} = 5x^{-\frac{3}{2}}$

$y = 5\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + c$

$y = -10x^{-\frac{1}{2}} + c$ or

$y = \frac{-10}{\sqrt{x}} + c$

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Integration
Exercise A, Question 13

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$-2x^{-\frac{3}{2}}$

Solution:

$$\frac{dy}{dx} = -2x^{-\frac{3}{2}}$$

$$y = -2\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + c$$

$y = 4x^{-\frac{1}{2}} + c$ or

$$y = \frac{4}{\sqrt{x}} + c$$

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Integration
Exercise A, Question 14

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$6x^{\frac{1}{3}}$

Solution:

$\frac{dy}{dx} = 6x^{\frac{1}{3}}$

$y = 6 \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + c$

$y = \frac{18}{4}x^{\frac{4}{3}} + c$

$y = \frac{9}{2}x^{\frac{4}{3}} + c$

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Integration
Exercise A, Question 15

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$36x^{11}$

Solution:

$\frac{dy}{dx} = 36x^{11}$

$y = 36 \frac{x^{12}}{12} + c$

$y = 3x^{12} + c$

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Integration
Exercise A, Question 16

Question:

Find an expression for y when $\frac{\mathrm{d}y}{\mathrm{d}x}$ is:

$-14x^{-8}$

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Integration
Exercise A, Question 17

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$-3x^{-\frac{2}{3}}$

Solution:

$$\frac{dy}{dx} = -3x^{-\frac{2}{3}}$$

$$y = -3\frac{x^{\frac{1}{3}}}{\frac{1}{3}} + c$$

$$y = -9x^{\frac{1}{3}} + c$$

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Integration
Exercise A, Question 18

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

-5

Solution:

$\frac{dy}{dx} = -5 = -5x^0$

$y = -5\frac{x^1}{1} + c$

$y = -5x + c$

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Integration
Exercise A, Question 19

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$6x$

Solution:

$\frac{dy}{dx} = 6x$

$y = 6 \frac{x^2}{2} + c$

$y = 3x^2 + c$

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Integration
Exercise A, Question 20

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$2x^{-0.4}$

Solution:

$\frac{dy}{dx} = 2x^{-0.4}$

$y = 2\frac{x^{0.6}}{0.6} + c$

$y = \frac{20}{6}x^{0.6} + c$

$y = \frac{10}{3}x^{0.6} + c$

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Integration
Exercise B, Question 1

Question:

Find y when $\frac{dy}{dx}$ is given by the following expressions. In each case simplify your answer:

(a) $4x - x^{-2} + 6x^{\frac{1}{2}}$

(b) $15x^2 + 6x^{-3} - 3x^{-\frac{5}{2}}$

(c) $x^3 - \frac{3}{2}x^{-\frac{1}{2}} - 6x^{-2}$

(d) $4x^3 + x^{-\frac{2}{3}} - x^{-2}$

(e) $4 - 12x^{-4} + 2x^{-\frac{1}{2}}$

(f) $5x^{\frac{2}{3}} - 10x^4 + x^{-3}$

(g) $-\frac{4}{3}x^{-\frac{4}{3}} - 3 + 8x$

(h) $5x^4 - x^{-\frac{3}{2}} - 12x^{-5}$

Solution:

(a) $\frac{dy}{dx} = 4x - x^{-2} + 6x^{\frac{1}{2}}$

$y = 4\frac{x^2}{2} - \frac{x^{-1}}{-1} + 6\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$

$y = 2x^2 + x^{-1} + 4x^{\frac{3}{2}} + c$

(b) $\frac{dy}{dx} = 15x^2 + 6x^{-3} - 3x^{-\frac{5}{2}}$

$y = 15\frac{x^3}{3} + 6\frac{x^{-2}}{-2} - 3\frac{x^{-\frac{3}{2}}}{-\frac{3}{2}} + c$

$y = 5x^3 - 3x^{-2} + 2x^{-\frac{3}{2}} + c$

(c) $\frac{dy}{dx} = x^3 - \frac{3}{2}x^{-\frac{1}{2}} - 6x^{-2}$

$y = \frac{x^4}{4} - \frac{3}{2}\frac{x^{+\frac{1}{2}}}{\frac{1}{2}} - 6\frac{x^{-1}}{-1} + c$

$y = \frac{1}{4}x^4 - 3x^{\frac{1}{2}} + 6x^{-1} + c$

(d) $\frac{dy}{dx} = 4x^3 + x^{-\frac{2}{3}} - x^{-2}$

$y = 4\frac{x^4}{4} + \frac{x^{\frac{1}{3}}}{\frac{1}{3}} - \frac{x^{-1}}{-1} + c$

$y = x^4 + 3x^{\frac{1}{3}} + x^{-1} + c$

(e) $\frac{dy}{dx} = 4 - 12x^{-4} + 2x^{-\frac{1}{2}}$

$y = 4x - 12\frac{x^{-3}}{-3} + 2\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$y = 4x + 4x^{-3} + 4x^{\frac{1}{2}} + c$

(f) $\frac{dy}{dx} = 5x^{\frac{2}{3}} - 10x^4 + x^{-3}$

$y = 5\frac{x^{\frac{5}{3}}}{\frac{5}{3}} - 10\frac{x^5}{5} + \frac{x^{-2}}{-2} + c$

$y = 3x^{\frac{5}{3}} - 2x^5 - \frac{1}{2}x^{-2} + c$

(g) $\frac{dy}{dx} = -\frac{4}{3}x^{-\frac{4}{3}} - 3 + 8x$

$y = -\frac{4}{3}\frac{x^{-\frac{1}{3}}}{-\frac{1}{3}} - 3x + 8\frac{x^2}{2} + c$

$y = 4x^{-\frac{1}{3}} - 3x + 4x^2 + c$

(h) $\frac{dy}{dx} = 5x^4 - x^{-\frac{3}{2}} - 12x^{-5}$

$$y = 5\,\frac{x^5}{5} - \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} - 12\,\frac{x^{-4}}{-4} + c$$

$$y = x^5 + 2x^{-\frac{1}{2}} + 3x^{-4} + c$$

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Integration
Exercise B, Question 2

Question:

Find f(x) when f′(x) is given by the following expressions. In each case simplify your answer:

(a) $12x + \frac{3}{2}x^{-\frac{3}{2}} + 5$

(b) $6x^5 + 6x^{-7} - \frac{1}{6}x^{-\frac{7}{6}}$

(c) $\frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}}$

(d) $10x + 8x^{-3}$

(e) $2x^{-\frac{1}{3}} + 4x^{-\frac{5}{3}}$

(f) $9x^2 + 4x^{-3} + \frac{1}{4}x^{-\frac{1}{2}}$

(g) $x^2 + x^{-2} + x^{\frac{1}{2}}$

(h) $-2x^{-3} - 2x + 2x^{\frac{1}{2}}$

Solution:

(a) $f'(x) = 12x + \frac{3}{2}x^{-\frac{3}{2}} + 5$

$f(x) = 12\frac{x^2}{2} + \frac{3}{2}\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + 5x + c$

$f(x) = 6x^2 - 3x^{-\frac{1}{2}} + 5x + c$

(b) $f'(x) = 6x^5 + 6x^{-7} - \frac{1}{6}x^{-\frac{7}{6}}$

$f(x) = 6\frac{x^6}{6} + 6\frac{x^{-6}}{-6} - \frac{1}{6}\frac{x^{-\frac{1}{6}}}{-\frac{1}{6}} + c$

$f(x) = x^6 - x^{-6} + x^{-\frac{1}{6}} + c$

(c) $f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}}$

$f(x) = \frac{1}{2}\frac{x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{1}{2}\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + c$

$f(x) = x^{\frac{1}{2}} + x^{-\frac{1}{2}} + c$

(d) $f'(x) = 10x + 8x^{-3}$

$f(x) = 10\frac{x^2}{2} + 8\frac{x^{-2}}{-2} + c$

$f(x) = 5x^2 - 4x^{-2} + c$

(e) $f'(x) = 2x^{-\frac{1}{3}} + 4x^{-\frac{5}{3}}$

$f(x) = 2\frac{x^{\frac{2}{3}}}{\frac{2}{3}} + 4\frac{x^{-\frac{2}{3}}}{-\frac{2}{3}} + c$

$f(x) = 3x^{\frac{2}{3}} - 6x^{-\frac{2}{3}} + c$

(f) $f'(x) = 9x^2 + 4x^{-3} + \frac{1}{4}x^{-\frac{1}{2}}$

$f(x) = 9\frac{x^3}{3} + 4\frac{x^{-2}}{-2} + \frac{1}{4}\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$f(x) = 3x^3 - 2x^{-2} + \frac{1}{2}x^{\frac{1}{2}} + c$

(g) $f'(x) = x^2 + x^{-2} + x^{\frac{1}{2}}$

$f(x) = \frac{x^3}{3} + \frac{x^{-1}}{-1} + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$

$f(x) = \frac{1}{3}x^3 - x^{-1} + \frac{2}{3}x^{\frac{3}{2}} + c$

(h) $f'(x) = -2x^{-3} - 2x + 2x^{\frac{1}{2}}$

$f(x) = -2\frac{x^{-2}}{-2} - 2\frac{x^2}{2} + 2\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$

$f(x) = x^{-2} - x^2 + \frac{4}{3}x^{\frac{3}{2}} + c$

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Integration
Exercise C, Question 1

Question:

Find the following integral:

$\int (x^3 + 2x) \, dx$

Solution:

$\int (x^3 + 2x) \, dx$

$= \frac{x^4}{4} + 2 \frac{x^2}{2} + c$

$= \frac{1}{4}x^4 + x^2 + c$

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Integration
Exercise C, Question 2

Question:

Find the following integral:

$\int (2x^{-2} + 3) \, dx$

Solution:

$\int (2x^{-2} + 3) \, dx$

$= 2 \frac{x^{-1}}{-1} + 3x + c$

$= -2x^{-1} + 3x + c$

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Integration
Exercise C, Question 3

Question:

Find the following integral:

$$\int \left(5x^{\frac{3}{2}} - 3x^2 \right) dx$$

Solution:

$$\int \left(5x^{\frac{3}{2}} - 3x^2 \right) dx$$

$$= 5 \frac{x^{\frac{5}{2}}}{\frac{5}{2}} - 3 \frac{x^3}{3} + c$$

$$= 2x^{\frac{5}{2}} - x^3 + c$$

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Integration
Exercise C, Question 4

Question:

Find the following integral:

$$\int \left(2x^{\frac{1}{2}} - 2x^{-\frac{1}{2}} + 4 \right) \mathrm{d}x$$

Solution:

$$\int \left(2x^{\frac{1}{2}} - 2x^{-\frac{1}{2}} + 4 \right) \mathrm{d}x$$

$$= 2\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - 2\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + 4x + c$$

$$= \frac{4}{3}x^{\frac{3}{2}} - 4x^{\frac{1}{2}} + 4x + c$$

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Integration
Exercise C, Question 5

Question:

Find the following integral:
$\int (4x^3 - 3x^{-4} + r) \, dx$

Solution:

$\int (4x^3 - 3x^{-4} + r) \, dx$

$= 4 \frac{x^4}{4} - 3 \frac{x^{-3}}{-3} + rx + c$

$= x^4 + x^{-3} + rx + c$

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Integration
Exercise C, Question 6

Question:

Find the following integral:

$\int (3t^2 - t^{-2}) \, dt$

Solution:

$\int (3t^2 - t^{-2}) \, dt$

$= 3 \frac{t^3}{3} - \frac{t^{-1}}{-1} + c$

$= t^3 + t^{-1} + c$

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Integration
Exercise C, Question 7

Question:

Find the following integral:

$$\int \left(2t^2 - 3t^{-\frac{3}{2}} + 1 \right) \mathrm{d}t$$

Solution:

$$\int \left(2t^2 - 3t^{-\frac{3}{2}} + 1 \right) \mathrm{d}t$$

$$= 2 \frac{t^3}{3} - 3 \frac{t^{-\frac{1}{2}}}{-\frac{1}{2}} + t + c$$

$$= \frac{2}{3}t^3 + 6t^{-\frac{1}{2}} + t + c$$

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Integration
Exercise C, Question 8

Question:

Find the following integral:

$$\int \left(x + x^{-\frac{1}{2}} + x^{-\frac{3}{2}} \right) \mathrm{d}x$$

Solution:

$$\int \left(x + x^{-\frac{1}{2}} + x^{-\frac{3}{2}} \right) \mathrm{d}x$$

$$= \frac{x^2}{2} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + c$$

$$= \frac{1}{2}x^2 + 2x^{\frac{1}{2}} - 2x^{-\frac{1}{2}} + c$$

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Integration
Exercise C, Question 9

Question:

Find the following integral:

$\int \left(px^4 + 2t + 3x^{-2} \right) dx$

Solution:

$\int \left(px^4 + 2t + 3x^{-2} \right) dx$

$= p \frac{x^5}{5} + 2tx + 3 \frac{x^{-1}}{-1} + c$

$= \frac{p}{5} x^5 + 2tx - 3x^{-1} + c$

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Integration
Exercise C, Question 10

Question:

Find the following integral:

$\int (pt^3 + q^2 + px^3) \, dt$

Solution:

$\int (pt^3 + q^2 + px^3) \, dt$

$= p \frac{t^4}{4} + q^2 t + px^3 t + c$

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Edexcel Modular Mathematics for AS and A-Level

Integration
Exercise D, Question 1

Question:

Find the following integrals:

(a) $\int (2x+3)x^2 dx$

(b) $\int \frac{(2x^2+3)}{x^2} dx$

(c) $\int (2x+3)^2 dx$

(d) $\int (2x+3)(x-1) dx$

(e) $\int (2x+3)\sqrt{x} dx$

Solution:

(a) $\int (2x+3)x^2 dx$

$= \int (2x^3+3x^2) dx$

$= 2\frac{x^4}{4} + 3\frac{x^3}{3} + c$

$= \frac{1}{2}x^4 + x^3 + c$

(b) $\int \frac{(2x^2+3)}{x^2} dx$

$= \int \left(\frac{2x^2}{x^2} + \frac{3}{x^2} \right) dx$

$= \int (2+3x^{-2}) dx$

$= 2x + 3\frac{x^{-1}}{-1} + c$

$= 2x - 3x^{-1} + c$

or $= 2x - \frac{3}{x} + c$

(c) $\int (2x+3)^2 dx$

$= \int (4x^2+12x+9) dx$

$= 4\frac{x^3}{3} + 12\frac{x^2}{2} + 9x + c$

$= \frac{4}{3}x^3 + 6x^2 + 9x + c$

(d) $\int (2x+3)(x-1) dx$

$= \int (2x^2+x-3) dx$

$= 2\frac{x^3}{3} + \frac{x^2}{2} - 3x + c$

$= \frac{2}{3}x^3 + \frac{1}{2}x^2 - 3x + c$

(e) $\int (2x + 3)\sqrt{x}\ dx$

$= \int \left(2x + 3\right) x^{\frac{1}{2}} dx$

$= \int \left(2x^{\frac{3}{2}} + 3x^{\frac{1}{2}}\right) dx$

$= 2\frac{x^{\frac{5}{2}}}{\frac{5}{2}} + 3\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$

$= \frac{4}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + c$

or $= \frac{4}{5}\sqrt{x^5} + 2\sqrt{x^3} + c$

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Integration
Exercise D, Question 2

Question:

Find $\int f(x)dx$ when $f(x)$ is given by the following:

(a) $(x+2)^2$

(b) $\left(x+\frac{1}{x}\right)^2$

(c) $(\sqrt{x}+2)^2$

(d) $\sqrt{x}(x+2)$

(e) $\left(\frac{x+2}{\sqrt{x}}\right)$

(f) $\left(\frac{1}{\sqrt{x}}+2\sqrt{x}\right)$

Solution:

(a) $\int (x+2)^2 dx$
$= \int (x^2+4x+4)\,dx$
$= \frac{1}{3}x^3+\frac{4}{2}x^2+4x+c$
$= \frac{1}{3}x^3+2x^2+4x+c$

(b) $\int \left(x+\frac{1}{x}\right)^2 dx$
$= \int \left(x^2+2+\frac{1}{x^2}\right) dx$
$= \int (x^2+2+x^{-2})\,dx$
$= \frac{1}{3}x^3+2x+\frac{x^{-1}}{-1}+c$
$= \frac{1}{3}x^3+2x-x^{-1}+c$
or $= \frac{1}{3}x^3+2x-\frac{1}{x}+c$

(c) $\int (\sqrt{x}+2)^2 dx$
$= \int (x+4\sqrt{x}+4)\,dx$

$= \int \left(x + 4x^{\frac{1}{2}} + 4 \right) dx$

$= \frac{1}{2}x^2 + 4\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 4x + c$

$= \frac{1}{2}x^2 + \frac{8}{3}x^{\frac{3}{2}} + 4x + c$

(d) $\int \sqrt{x}\,(x + 2)\, dx$

$= \int \left(x^{\frac{3}{2}} + 2x^{\frac{1}{2}} \right) dx$

$= \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + 2\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$

$= \frac{2}{5}x^{\frac{5}{2}} + \frac{4}{3}x^{\frac{3}{2}} + c$

or $= \frac{2}{5}\sqrt{x^5} + \frac{4}{3}\sqrt{x^3} + c$

(e) $\int \left(\frac{x+2}{\sqrt{x}} \right) dx$

$= \int \left(\frac{x}{x^{\frac{1}{2}}} + \frac{2}{x^{\frac{1}{2}}} \right) dx$

$= \int \left(x^{\frac{1}{2}} + 2x^{-\frac{1}{2}} \right) dx$

$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 2\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= \frac{2}{3}x^{\frac{3}{2}} + 4x^{\frac{1}{2}} + c$

or $= \frac{2}{3}\sqrt{x^3} + 4\sqrt{x} + c$

(f) $\int \left(\frac{1}{\sqrt{x}} + 2\sqrt{x} \right) dx$

$= \int \left(x^{-\frac{1}{2}} + 2x^{\frac{1}{2}} \right) dx$

$= \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + 2\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$

$= 2x^{\frac{1}{2}} + \frac{4}{3}x^{\frac{3}{2}} + c$

or $= 2\sqrt{x} + \frac{4}{3}\sqrt{x^3} + c$

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Integration
Exercise D, Question 3

Question:

Find the following integrals:

(a) $\int \left(3\sqrt{x} + \frac{1}{x^2} \right) dx$

(b) $\int \left(\frac{2}{\sqrt{x}} + 3x^2 \right) dx$

(c) $\int \left(x^{\frac{2}{3}} + \frac{4}{x^3} \right) dx$

(d) $\int \left(\frac{2+x}{x^3} + 3 \right) dx$

(e) $\int (x^2 + 3)(x - 1)\, dx$

(f) $\int \left(\frac{2}{\sqrt{x}} + 3x\sqrt{x} \right) dx$

(g) $\int (x - 3)^2 dx$

(h) $\int \frac{(2x+1)^2}{\sqrt{x}} dx$

(i) $\int \left(3 + \frac{\sqrt{x} + 6x^3}{x} \right) dx$

(j) $\int \sqrt{x} (\sqrt{x} + 3)^2 dx$

Solution:

(a) $\int \left(3\sqrt{x} + \frac{1}{x^2} \right) dx$

$= \int \left(3x^{\frac{1}{2}} + x^{-2} \right) dx$

$= 3 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{-1}}{-1} + c$

$= 2x^{\frac{3}{2}} - x^{-1} + c$

or $= 2\sqrt{x^3} - \frac{1}{x} + c$

(b) $\int \left(\frac{2}{\sqrt{x}} + 3x^2 \right) dx$

$= \int \left(2x^{-\frac{1}{2}} + 3x^2 \right) dx$

$= 2 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{3}{3}x^3 + c$

$= 4x^{\frac{1}{2}} + x^3 + c$

or $= 4\sqrt{x} + x^3 + c$

(c) $\int \left(x^{\frac{2}{3}} + \frac{4}{x^3} \right) dx$

$= \int \left(x^{\frac{2}{3}} + 4x^{-3} \right) dx$

$= \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + 4 \frac{x^{-2}}{-2} + c$

$= \frac{3}{5}x^{\frac{5}{3}} - 2x^{-2} + c$

or $= \frac{3}{5}x^{\frac{5}{3}} - \frac{2}{x^2} + c$

(d) $\int \left(\frac{2+x}{x^3} + 3 \right) dx$

$= \int \left(2x^{-3} + x^{-2} + 3 \right) dx$

$= 2 \frac{x^{-2}}{-2} + \frac{x^{-1}}{-1} + 3x + c$

$= -x^{-2} - x^{-1} + 3x + c$

or $= -\frac{1}{x^2} - \frac{1}{x} + 3x + c$

(e) $\int \left(x^2 + 3 \right) \left(x - 1 \right) dx$

$= \int \left(x^3 - x^2 + 3x - 3 \right) dx$

$= \frac{1}{4}x^4 - \frac{1}{3}x^3 + \frac{3}{2}x^2 - 3x + c$

(f) $\int \left(\frac{2}{\sqrt{x}} + 3x\sqrt{x} \right) dx$

$= \int \left(2x^{-\frac{1}{2}} + 3x^{\frac{3}{2}} \right) dx$

$= 2\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + 3\frac{x^{\frac{5}{2}}}{\frac{5}{2}} + c$

$= 4x^{\frac{1}{2}} + \frac{6}{5}x^{\frac{5}{2}} + c$

or $= 4\sqrt{x} + \frac{6}{5}x^2\sqrt{x} + c$

(g) $\int (x-3)^2 dx$
$= \int (x^2 - 6x + 9)\, dx$
$= \frac{1}{3}x^3 - \frac{6}{2}x^2 + 9x + c$

$= \frac{1}{3}x^3 - 3x^2 + 9x + c$

(h) $\int \frac{(2x+1)^2}{\sqrt{x}} dx$

$= \int x^{-\frac{1}{2}} \left(4x^2 + 4x + 1 \right) dx$

$= \int \left(4x^{\frac{3}{2}} + 4x^{\frac{1}{2}} + x^{-\frac{1}{2}} \right) dx$

$= 4\frac{x^{\frac{5}{2}}}{\frac{5}{2}} + 4\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= \frac{8}{5}x^{\frac{5}{2}} + \frac{8}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$

or $= \frac{8}{5}\sqrt{x^5} + \frac{8}{3}\sqrt{x^3} + 2\sqrt{x} + c$

(i) $\int \left(3 + \frac{\sqrt{x} + 6x^3}{x} \right) dx$

$= \int \left(3 + x^{-\frac{1}{2}} + 6x^2 \right) dx$

$= 3x + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{6}{3}x^3 + c$

$= 3x + 2x^{\frac{1}{2}} + 2x^3 + c$
or $= 3x + 2\sqrt{x} + 2x^3 + c$

(j) $\int \sqrt{x}\,(\sqrt{x} + 3)^2 dx$

$$= \int x^{\frac{1}{2}}\left(x + 6x^{\frac{1}{2}} + 9\right) dx$$

$$= \int \left(x^{\frac{3}{2}} + 6x + 9x^{\frac{1}{2}}\right) dx$$

$$= \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{6}{2}x^2 + 9\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= \frac{2}{5}x^{\frac{5}{2}} + 3x^2 + 6x^{\frac{3}{2}} + c$$

or $= \frac{2}{5}\sqrt{x^5} + 3x^2 + 6\sqrt{x^3} + c$

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Integration
Exercise E, Question 1

Question:

Find the equation of the curve with the given $\frac{dy}{dx}$ that passes through the given point:

(a) $\frac{dy}{dx} = 3x^2 + 2x$; point $(2 , 10)$

(b) $\frac{dy}{dx} = 4x^3 + \frac{2}{x^3} + 3$; point $(1 , 4)$

(c) $\frac{dy}{dx} = \sqrt{x} + \frac{1}{4}x^2$; point $(4 , 11)$

(d) $\frac{dy}{dx} = \frac{3}{\sqrt{x}} - x$; point $(4 , 0)$

(e) $\frac{dy}{dx} = (x + 2)^2$; point $(1 , 7)$

(f) $\frac{dy}{dx} = \frac{x^2 + 3}{\sqrt{x}}$; point $(0 , 1)$

Solution:

(a) $\frac{dy}{dx} = 3x^2 + 2x$

$\Rightarrow \quad y = \frac{3}{3}x^3 + \frac{2}{2}x^2 + c$

So $y = x^3 + x^2 + c$

$x = 2, y = 10 \quad \Rightarrow \quad 10 = 8 + 4 + c$

So $c = - 2$

So equation is $y = x^3 + x^2 - 2$

(b) $\frac{dy}{dx} = 4x^3 + \frac{2}{x^3} + 3$

$\Rightarrow \quad y = \frac{4}{4}x^4 - \frac{2}{2}x^{-2} + 3x + c$

So $y = x^4 - x^{-2} + 3x + c$

$x = 1, y = 4 \quad \Rightarrow \quad 4 = 1 - 1 + 3 + c$

So $c = 1$

So equation is $y = x^4 - x^{-2} + 3x + 1$

(c) $\frac{dy}{dx} = \sqrt{x} + \frac{1}{4}x^2$

$$\Rightarrow \quad y = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{1}{4}\frac{x^3}{3} + c$$

So $y = \frac{2}{3}x^{\frac{3}{2}} + \frac{1}{12}x^3 + c$

$x = 4, y = 11 \quad \Rightarrow \quad 11 = \frac{2}{3} \times 2^3 + \frac{1}{12} \times 4^3 + c$

So $c = \frac{33}{3} - \frac{32}{3} = \frac{1}{3}$

So equation is $y = \frac{2}{3}x^{\frac{3}{2}} + \frac{1}{12}x^3 + \frac{1}{3}$

(d) $\frac{dy}{dx} = \frac{3}{\sqrt{x}} - x$

$$\Rightarrow \quad y = 3\frac{x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{1}{2}x^2 + c$$

So $y = 6\sqrt{x} - \frac{1}{2}x^2 + c$

$x = 4, y = 0 \quad \Rightarrow \quad 0 = 6 \times 2 - \frac{1}{2} \times 16 + c$

So $c = -4$

So equation is $y = 6\sqrt{x} - \frac{1}{2}x^2 - 4$

(e) $\frac{dy}{dx} = (x + 2)^2 = x^2 + 4x + 4$

$$\Rightarrow \quad y = \frac{1}{3}x^3 + 2x^2 + 4x + c$$

$x = 1, y = 7 \quad \Rightarrow \quad 7 = \frac{1}{3} + 2 + 4 + c$

So $c = \frac{2}{3}$

So equation is $y = \frac{1}{3}x^3 + 2x^2 + 4x + \frac{2}{3}$

(f) $\frac{dy}{dx} = \frac{x^2 + 3}{\sqrt{x}} = x^{\frac{3}{2}} + 3x^{-\frac{1}{2}}$

$$\Rightarrow \quad y = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + 3\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

So $y = \frac{2}{5}x^{\frac{5}{2}} + 6x^{\frac{1}{2}} + c$

$x = 0, y = 1 \quad \Rightarrow \quad 1 = \frac{2}{5} \times 0 + 6 \times 0 + c$

So $c = 1$

So equation of curve is $y = \frac{2}{5}x^{\frac{5}{2}} + 6x^{\frac{1}{2}} + 1$

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Integration
Exercise E, Question 2

Question:

The curve C, with equation $y = \mathrm{f}(x)$, passes through the point $(1, 2)$ and $\mathrm{f}'(x) = 2x^3 - \frac{1}{x^2}$. Find the equation of C in the form $y = \mathrm{f}(x)$.

Solution:

$\mathrm{f}'(x) = 2x^3 - \frac{1}{x^2} = 2x^3 - x^{-2}$

So $\mathrm{f}(x) = \frac{2}{4}x^4 - \frac{x^{-1}}{-1} + c = \frac{1}{2}x^4 + \frac{1}{x} + c$

But $\mathrm{f}(1) = 2$

So $2 = \frac{1}{2} + 1 + c$

$\Rightarrow \quad c = \frac{1}{2}$

So $\mathrm{f}(x) = \frac{1}{2}x^4 + \frac{1}{x} + \frac{1}{2}$

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Integration
Exercise E, Question 3

Question:

The gradient of a particular curve is given by $\frac{dy}{dx} = \frac{\sqrt{x} + 3}{x^2}$. Given that the curve passes through the point $(9, 0)$, find an equation of the curve.

Solution:

$$\frac{dy}{dx} = \frac{\sqrt{x} + 3}{x^2} = x^{-\frac{3}{2}} + 3x^{-2}$$

$$\Rightarrow \quad y = \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + 3\frac{x^{-1}}{-1} + c$$

So $y = -2x^{-\frac{1}{2}} - 3x^{-1} + c = -\frac{2}{\sqrt{x}} - \frac{3}{x} + c$

$x = 9, y = 0 \quad \Rightarrow \quad 0 = -\frac{2}{3} - \frac{3}{9} + c$

So $c = \frac{2}{3} + \frac{1}{3} = 1$

So equation is $y = 1 - \frac{2}{\sqrt{x}} - \frac{3}{x}$

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Integration
Exercise E, Question 4

Question:

A set of curves, that each pass through the origin, have equations $y = f_1(x)$, $y = f_2(x)$, $y = f_3(x)$... where $f_n'(x) = f_{n-1}(x)$ and $f_1(x) = x^2$.

(a) Find $f_2(x)$, $f_3(x)$.

(b) Suggest an expression for $f_n(x)$.

Solution:

(a) $f_2'(x) = f_1(x) = x^2$

So $f_2(x) = \frac{1}{3}x^3 + c$

The curve passes through (0 , 0) so $f_2(0) = 0 \Rightarrow c = 0$.

So $f_2(x) = \frac{1}{3}x^3$

$f_3'(x) = \frac{1}{3}x^3$

$f_3(x) = \frac{1}{12}x^4 + c$, but $c = 0$ since $f_3(0) = 0$.

So $f_3(x) = \frac{1}{12}x^4$

(b) $f_2(x) = \frac{x^3}{3}$, $f_3(x) = \frac{x^4}{3 \times 4}$

So power of x is $n + 1$ for $f_n(x)$, denominator is $3 \times 4 \times$ ... up to $n + 1$:

$f_n(x) = \frac{x^{n+1}}{3 \times 4 \times 5 \times \ldots \times (n+1)}$

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Integration
Exercise E, Question 5

Question:

A set of curves, with equations $y = f_1(x)$, $y = f_2(x)$, $y = f_3(x)$ ... all pass through the point $(0, 1)$ and they are related by the property $f_n'(x) = f_{n-1}(x)$ and $f_1(x) = 1$.

Find $f_2(x)$, $f_3(x)$, $f_4(x)$.

Solution:

$f_2'(x) = f_1(x) = 1$

$\Rightarrow \quad f_2(x) = x + c$

But $f_2(0) = 1 \quad \Rightarrow \quad 1 = 0 + c \quad \Rightarrow \quad c = 1$

So $f_2(x) = x + 1$

$f_3'(x) = f_2(x) = x + 1$

$\Rightarrow \quad f_3(x) = \frac{1}{2}x^2 + x + c$

But $f_3(0) = 1 \quad \Rightarrow \quad 1 = 0 + c \quad \Rightarrow \quad c = 1$

So $f_3(x) = \frac{1}{2}x^2 + x + 1$

$f_4'(x) = f_3(x) = \frac{1}{2}x^2 + x + 1$

$\Rightarrow \quad f_4(x) = \frac{1}{6}x^3 + \frac{1}{2}x^2 + x + c$

But $f_4(0) = 1 \quad \Rightarrow \quad 1 = 0 + c \quad \Rightarrow \quad c = 1$

So $f_4(x) = \frac{1}{6}x^3 + \frac{1}{2}x^2 + x + 1$

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Integration
Exercise F, Question 1

Question:

Find:

(a) $\int (x + 1) (2x - 5) \, dx$

(b) $\int \left(x^{\frac{1}{3}} + x^{-\frac{1}{3}} \right) dx$.

Solution:

(a) $\int (x + 1) (2x - 5) \, dx$

$= \int (2x^2 - 3x - 5) \, dx$

$= 2 \frac{x^3}{3} - 3 \frac{x^2}{2} - 5x + c$

$= \frac{2}{3}x^3 - \frac{3}{2}x^2 - 5x + c$

(b) $\int \left(x^{\frac{1}{3}} + x^{-\frac{1}{3}} \right) dx$

$= \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + \frac{x^{\frac{2}{3}}}{\frac{2}{3}} + c$

$= \frac{3}{4}x^{\frac{4}{3}} + \frac{3}{2}x^{\frac{2}{3}} + c$

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Integration
Exercise F, Question 2

Question:

The gradient of a curve is given by $f'(x) = x^2 - 3x - \frac{2}{x^2}$. Given that the curve passes through the point $(1, 1)$, find the equation of the curve in the form $y = f(x)$.

Solution:

$f'(x) = x^2 - 3x - \frac{2}{x^2} = x^2 - 3x - 2x^{-2}$

So $f(x) = \frac{x^3}{3} - 3\frac{x^2}{2} - 2\frac{x^{-1}}{-1} + c$

So $f(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 + \frac{2}{x} + c$

But $f\left(1\right) = 1 \quad \Rightarrow \quad \frac{1}{3} - \frac{3}{2} + 2 + c = 1$

So $c = \frac{1}{6}$

So the equation is $y = \frac{1}{3}x^3 - \frac{3}{2}x^2 + \frac{2}{x} + \frac{1}{6}$

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Integration
Exercise F, Question 3

Question:

Find

(a) $\int (8x^3 - 6x^2 + 5) \, dx$

(b) $\int \left(5x + 2 \right) x^{\frac{1}{2}} dx$.

Solution:

(a) $\int (8x^3 - 6x^2 + 5) \, dx$

$= 8 \frac{x^4}{4} - 6 \frac{x^3}{3} + 5x + c$

$= 2x^4 - 2x^3 + 5x + c$

(b) $\int \left(5x + 2 \right) x^{\frac{1}{2}} dx$

$= \int \left(5x^{\frac{3}{2}} + 2x^{\frac{1}{2}} \right) dx$

$= 5 \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + 2 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$

$= 2x^{\frac{5}{2}} + \frac{4}{3} x^{\frac{3}{2}} + c$

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Integration
Exercise F, Question 4

Question:

Given $y = \frac{(x+1)(2x-3)}{\sqrt{x}}$, find $\int y\,dx$.

Solution:

$$y = \frac{(x+1)(2x-3)}{\sqrt{x}}$$

$$y = \left(2x^2 - x - 3\right)x^{-\frac{1}{2}}$$

$$y = 2x^{\frac{3}{2}} - x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$$

$$\int y\,dx = \int \left(2x^{\frac{3}{2}} - x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}\right) dx$$

$$= 2\frac{x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - 3\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= \frac{4}{5}x^{\frac{5}{2}} - \frac{2}{3}x^{\frac{3}{2}} - 6x^{\frac{1}{2}} + c$$

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Integration
Exercise F, Question 5

Question:

Given that $\frac{dx}{dt} = 3t^2 - 2t + 1$ and that $x = 2$ when $t = 1$, find the value of x when $t = 2$.

Solution:

$\frac{dx}{dt} = 3t^2 - 2t + 1$

$\Rightarrow \quad x = 3\frac{t^3}{3} - 2\frac{t^2}{2} + t + c$

So $x = t^3 - t^2 + t + c$
But when $t = 1$, $x = 2$.
So $2 = 1 - 1 + 1 + c$
$\Rightarrow \quad c = 1$
So $x = t^3 - t^2 + t + 1$
When $t = 2$, $x = 8 - 4 + 2 + 1$
So $x = 7$

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Integration
Exercise F, Question 6

Question:

Given $y = 3x^{\frac{1}{2}} + 2x^{-\frac{1}{2}}$, $x > 0$, find $\int y\,\mathrm{d}x$.

Solution:

$$\int y\,\mathrm{d}x = \int \left(3x^{\frac{1}{2}} + 2x^{-\frac{1}{2}} \right) \mathrm{d}x$$

$$= 3\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 2\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= 2x^{\frac{3}{2}} + 4x^{\frac{1}{2}} + c$$

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Integration
Exercise F, Question 7

Question:

Given that $\frac{dx}{dt} = (t+1)^2$ and that $x = 0$ when $t = 2$, find the value of x when $t = 3$.

Solution:

$\frac{dx}{dt} = (t+1)^2 = t^2 + 2t + 1$

$\Rightarrow \quad x = \frac{t^3}{3} + 2\frac{t^2}{2} + t + c$

But $x = 0$ when $t = 2$.

So $0 = \frac{8}{3} + 4 + 2 + c$

$\Rightarrow \quad c = -\frac{26}{3}$

So $x = \frac{1}{3}t^3 + t^2 + t - \frac{26}{3}$

When $t = 3$, $x = \frac{27}{3} + 9 + 3 - \frac{26}{3}$

So $x = 12\frac{1}{3}$ or $\frac{37}{3}$

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Integration
Exercise F, Question 8

Question:

Given that $y^{\frac{1}{2}} = x^{\frac{1}{3}} + 3$:

(a) Show that $y = x^{\frac{2}{3}} + Ax^{\frac{1}{3}} + B$, where A and B are constants to be found.

(b) Hence find $\int y \, dx$. **[E]**

Solution:

(a) $y^{\frac{1}{2}} = x^{\frac{1}{3}} + 3$

So $y = \left(x^{\frac{1}{3}} + 3 \right)^2$

So $y = \left(x^{\frac{1}{3}} \right)^2 + 6x^{\frac{1}{3}} + 9$

So $y = x^{\frac{2}{3}} + 6x^{\frac{1}{3}} + 9$
$(A = 6 , B = 9)$

(b) $\int y dx = \int \left(x^{\frac{2}{3}} + 6x^{\frac{1}{3}} + 9 \right) dx$

$$= \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + 6 \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + 9x + c$$

$$= \frac{3}{5}x^{\frac{5}{3}} + \frac{9}{2}x^{\frac{4}{3}} + 9x + c$$

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Integration
Exercise F, Question 9

Question:

Given that $y = 3x^{\frac{1}{2}} - 4x^{-\frac{1}{2}} \left(x > 0 \right)$:

(a) Find $\frac{dy}{dx}$.

(b) Find $\int y dx$. **[E]**

Solution:

$y = 3x^{\frac{1}{2}} - 4x^{-\frac{1}{2}}$

(a) $\frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}} - 4 \times \left(- \frac{1}{2} \right) x^{-\frac{3}{2}}$

So $\frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}} + 2x^{-\frac{3}{2}}$

(b) $\int y dx = \int \left(3x^{\frac{1}{2}} - 4x^{-\frac{1}{2}} \right) dx$

$= 3 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - 4 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= 2x^{\frac{3}{2}} - 8x^{\frac{1}{2}} + c$

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Integration
Exercise F, Question 10

Question:

Find $\int \left(x^{\frac{1}{2}} - 4 \right) \left(x^{-\frac{1}{2}} - 1 \right) \mathrm{d}x$. **[E]**

Solution:

$\int \left(x^{\frac{1}{2}} - 4 \right) \left(x^{-\frac{1}{2}} - 1 \right) \mathrm{d}x$

$= \int \left(1 - 4x^{-\frac{1}{2}} - x^{\frac{1}{2}} + 4 \right) \mathrm{d}x$

$= \int \left(5 - 4x^{-\frac{1}{2}} - x^{\frac{1}{2}} \right) \mathrm{d}x$

$= 5x - 4 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$

$= 5x - 8x^{\frac{1}{2}} - \frac{2}{3}x^{\frac{3}{2}} + c$

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Practice paper C1
Exercise 1, Question 1

Question:

(a) Write down the value of $16^{\frac{1}{2}}$. (1)

(b) Hence find the value of $16^{\frac{3}{2}}$. (2)

Solution:

(a) $16^{\frac{1}{2}} = \sqrt{16} = 4$

(b) $16^{\frac{3}{2}} = \left(16^{\frac{1}{2}}\right)^3 = 4^3 = 64$

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Practice paper C1
Exercise 1, Question 2

Question:

Find $\int \left(6x^2 + \sqrt{x} \right) dx$. (4)

Solution:

$$\int \left(6x^2 + x^{\frac{1}{2}} \right) dx$$

$$= 6 \frac{x^3}{3} + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= 2x^3 + \frac{2}{3} x^{\frac{3}{2}} + c$$

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Practice paper C1
Exercise 1, Question 3

Question:

A sequence $a_1, a_2, a_3, \ldots\ a_n$ is defined by
$a_1 = 2, a_{n+1} = 2a_n - 1$.

(a) Write down the value of a_2 and the value of a_3. (2)

(b) Calculate $\sum_{r=1}^{5} a_r$. (2)

Solution:

(a) $a_2 = 2a_1 - 1 = 4 - 1 = 3$
$a_3 = 2a_2 - 1 = 6 - 1 = 5$

(b) $a_4 = 2a_3 - 1 = 10 - 1 = 9$
$a_5 = 2a_4 - 1 = 18 - 1 = 17$

$$\sum_{r=1}^{5} a_r = a_1 + a_2 + a_3 + a_4 + a_5 = 2 + 3 + 5 + 9 + 17 = 36$$

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Practice paper C1
Exercise 1, Question 4

Question:

(a) Express $(5+\sqrt{2})^2$ in the form $a+b\sqrt{2}$, where a and b are integers. (3)

(b) Hence, or otherwise, simplify $(5+\sqrt{2})^2-(5-\sqrt{2})^2$. (2)

Solution:

(a) $(5+\sqrt{2})^2=(5+\sqrt{2})(5+\sqrt{2})=25+10\sqrt{2}+2=27+10\sqrt{2}$

(b) $(5-\sqrt{2})^2=(5-\sqrt{2})(5-\sqrt{2})=25-10\sqrt{2}+2=27-10\sqrt{2}$

$(5+\sqrt{2})^2-(5-\sqrt{2})^2$
$=(27+10\sqrt{2})-(27-10\sqrt{2})$
$=27+10\sqrt{2}-27+10\sqrt{2}$
$=20\sqrt{2}$

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Practice paper C1
Exercise 1, Question 5

Question:

Solve the simultaneous equations:
$x - 3y = 6$
$3xy + x = 24$ (7)

Solution:

$x - 3y = 6$
$x = 6 + 3y$
Substitute into $3xy + x = 24$:
$3y (6 + 3y) + (6 + 3y) = 24$
$18y + 9y^2 + 6 + 3y = 24$
$9y^2 + 21y - 18 = 0$
Divide by 3:
$3y^2 + 7y - 6 = 0$
$(3y - 2) (y + 3) = 0$
$y = \frac{2}{3}, y = -3$
Substitute into $x = 6 + 3y$:
$y = \frac{2}{3} \Rightarrow x = 6 + 2 = 8$
$y = -3 \Rightarrow x = 6 - 9 = -3$
$x = -3, y = -3$ or $x = 8, y = \frac{2}{3}$

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Practice paper C1
Exercise 1, Question 6

Question:

The points A and B have coordinates $(-3, 8)$ and $(5, 4)$ respectively.
The straight line l_1 passes through A and B.

(a) Find an equation for l_1, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. (4)

(b) Another straight line l_2 is perpendicular to l_1 and passes through the origin. Find an equation for l_2. (2)

(c) The lines l_1 and l_2 intersect at the point P. Use algebra to find the coordinates of P. (3)

Solution:

(a) Gradient of $l_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 8}{5 - (-3)} = - \frac{4}{8} = - \frac{1}{2}$

Equation for l_1:
$y - y_1 = m (x - x_1)$

$y - 4 = - \frac{1}{2} \left(x - 5 \right)$

$y - 4 = - \frac{1}{2} x + \frac{5}{2}$

$\frac{1}{2} x + y - \frac{13}{2} = 0$

$x + 2y - 13 = 0$

(b) For perpendicular lines, $m_1 m_2 = -1$

$m_1 = - \frac{1}{2}$, so $m_2 = 2$

Equation for l_2 is $y = 2x$

(c) Substitute $y = 2x$ into $x + 2y - 13 = 0$:
$x + 4x - 13 = 0$
$5x = 13$

$x = 2 \frac{3}{5}$

$y = 2x = 5 \frac{1}{5}$

Coordinates of P are $\left(2 \frac{3}{5} , 5 \frac{1}{5} \right)$

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Practice paper C1
Exercise 1, Question 7

Question:

On separate diagrams, sketch the curves with equations:

(a) $y = \frac{2}{x}$, $-2 \le x \le 2, x \neq 0$ (2)

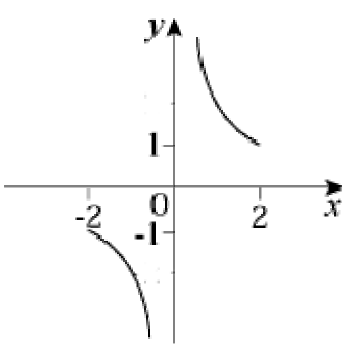
(b) $y = \frac{2}{x} - 4$, $-2 \le x \le 2, x \neq 0$ (3)

(c) $y = \frac{2}{x+1}$, $-2 \le x \le 2, x \neq -1$ (3)

In each part, show clearly the coordinates of any point at which the curve meets the *x*-axis or the *y*-axis.

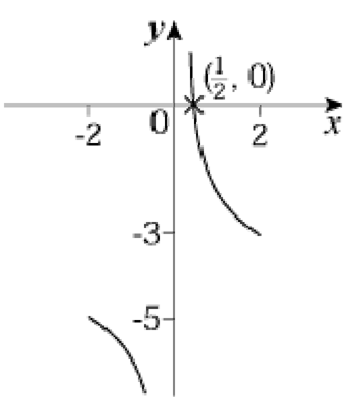
Solution:

(a)



$y = \frac{2}{x}$

(b) Translation of -4 units parallel to the *y*-axis.



$y = \frac{2}{x} - 4$

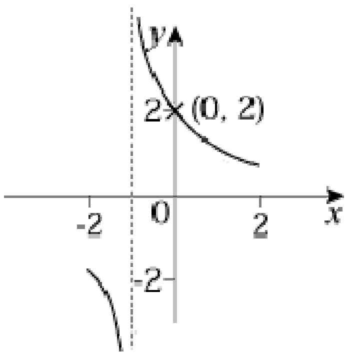
Curve crosses the x-axis where $y = 0$:

$$\frac{2}{x} - 4 = 0$$

$$\frac{2}{x} = 4$$

$$x = \frac{1}{2}$$

(c) Translation of -1 unit parallel to the x-axis.



$$y = \frac{2}{x+1}$$

The line $x = -1$ is an asymptote.
Curve crosses the y-axis where $x = 0$:

$$y = \frac{2}{0+1} = 2$$

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Practice paper C1
Exercise 1, Question 8

Question:

In the year 2007, a car dealer sold 400 new cars. A model for future sales assumes that sales will increase by x cars per year for the next 10 years, so that $(400 + x)$ cars are sold in 2008, $(400 + 2x)$ cars are sold in 2009, and so on. Using this model with $x = 30$, calculate:

(a) The number of cars sold in the year 2016. (2)

(b) The total number of cars sold over the 10 years from 2007 to 2016. (3)
The dealer wants to sell at least 6000 cars over the 10-year period.
Using the same model:

(c) Find the least value of x required to achieve this target. (4)

Solution:

(a) $a = 400$, $d = x = 30$
$T_{10} = a + 9d = 400 + 270 = 670$
670 cars sold in 2010

(b) $S_n = \frac{1}{2}n\left[2a + \left(n - 1\right)d\right]$

So $S_{10} = 5 \left[\left(2 \times 400 \right) + \left(9 \times 30 \right) \right] = 5 \times 1070 = 5350$
5350 cars sold from 2001 to 2010

(c) S_{10} required to be at least 6000:

$\frac{1}{2}n\left[2a + \left(n - 1\right)d\right] \geq 6000$

$5 \left(800 + 9x \right) \geq 6000$
$4000 + 45x \geq 6000$
$45x \geq 2000$
$x \geq 44\frac{4}{9}$

To achieve the target, $x = 45$.

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Practice paper C1
Exercise 1, Question 9

Question:

(a) Given that
$x^2 + 4x + c = (x + a)^2 + b$
where a, b and c are constants:

(i) Find the value of a. (1)

(ii) Find b in terms of c. (2)
Given also that the equation $x^2 + 4x + c = 0$ has unequal real roots:

(iii) Find the range of possible values of c. (2)

(b) Find the set of values of x for which:

(i) $3x < 20 - x$, (2)

(ii) $x^2 + 4x - 21 > 0$, (4)

(iii) both $3x < 20 - x$ and $x^2 + 4x - 21 > 0$. (2)

Solution:

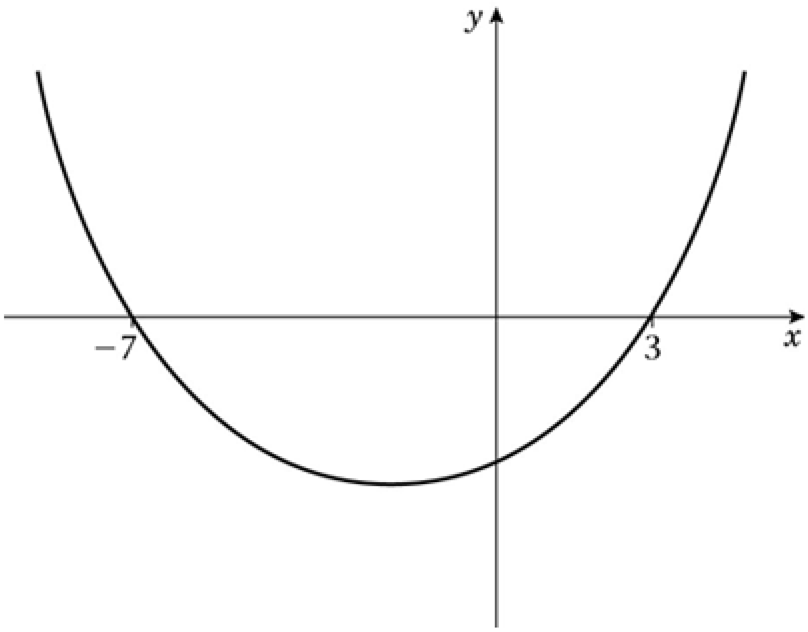
(a) (i) $x^2 + 4x + c = (x + 2)^2 - 4 + c = (x + 2)^2 + (c - 4)$
So $a = 2$

(ii) $b = c - 4$

(iii) For unequal real roots:
$(x + 2)^2 - 4 + c = 0$
$(x + 2)^2 = 4 - c$
$4 - c > 0$
$c < 4$

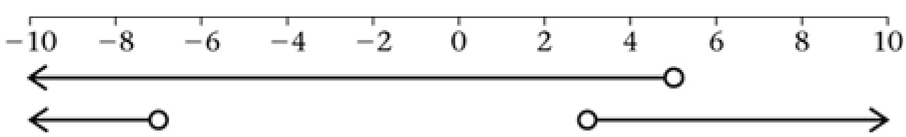
(b) (i) $3x < 20 - x$
$3x + x < 20$
$4x < 20$
$x < 5$

(ii) Solve $x^2 + 4x - 21 = 0$:
$(x + 7) (x - 3) = 0$
$x = - 7, x = 3$
Sketch of $y = x^2 + 4x - 21$:



$x^2 + 4x - 21 > 0$ when $x < -7$ or $x > 3$

(c)



Both inequalties are true when
$x < -7$ or $3 < x < 5$

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Practice paper C1
Exercise 1, Question 10

Question:

(a) Show that $\frac{(3x-4)^2}{x^2}$ may be written as $P+\frac{Q}{x}+\frac{R}{x^2}$, where P, Q and R are constants to be found. (3)

(b) The curve C has equation $y=\frac{(3x-4)^2}{x^2}$, $x \neq 0$. Find the gradient of the tangent to C at the point on C where $x=-2$. (5)

(c) Find the equation of the normal to C at the point on C where $x=-2$, giving your answer in the form $ax+by+c=0$, where a, b and c are integers. (5)

Solution:

(a) $(3x-4)^2=(3x-4)(3x-4)=9x^2-24x+16$

$$\frac{(3x-4)^2}{x^2}=\frac{9x^2-24x+16}{x^2}=9-\frac{24}{x}+\frac{16}{x^2}$$

$P=9$, $Q=-24$, $R=16$

(b) $y=9-24x^{-1}+16x^{-2}$

$$\frac{dy}{dx}=24x^{-2}-32x^{-3}$$

Where $x=-2$, $\frac{dy}{dx}=\frac{24}{(-2)^2}-\frac{32}{(-2)^3}=\frac{24}{4}+\frac{32}{8}=10$

Gradient of the tangent is 10.

(c) Where $x=-2$, $y=9-\frac{24}{(-2)}+\frac{16}{(-2)^2}=9+12+4=25$

Gradient of the normal $=\frac{-1}{\text{Gradient of tangent}}=-\frac{1}{10}$

The equation of the normal at $(-2, 25)$ is

$$y-25=-\frac{1}{10}\left[x-\left(-2\right)\right]$$

Multiply by 10:
$10y-250=-x-2$
$x+10y-248=0$

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Algebraic fractions
Exercise A, Question 1

Question:

Factorise completely

(a) $2x^3 - 13x^2 - 7x$

(b) $9x^2 - 16$

(c) $x^4 + 7x^2 - 8$

Solution:

(a)

$2x^3 - 13x^2 - 7x$ — x is a common factor

$= x (2x^2 - 13x - 7)$ — So take x outside the bracket.

$= x (2x^2 + x - 14x - 7)$ — For the quadratic, $ac = - 14$ and $1 - 14 = - 13 = b$

$= x [x (2x + 1) - 7 (2x + 1)]$ — Factorise

$= x (2x + 1) (x - 7)$

(b)

$9x^2 - 16$ — This is a difference of two squares, $(3x)^2$ and 4^2

$= (3x)^2 - 4^2$

$= (3x + 4) (3x - 4)$ — Use $x^2 - y^2 = (x + y) (x - y)$

(c)

$x^4 + 7x^2 - 8$ — Let $y = x^2$

$= y^2 + 7y - 8$

$= y^2 - y + 8y - 8$ — $ac = - 8$ and $- 1 + 8 = + 7 = b$

$= y (y - 1) + 8 (y - 1)$ — Factorise

$= (y - 1) (y + 8)$

$= (x^2 - 1) (x^2 + 8)$ — Replace y by x^2

$= (x + 1) (x - 1) (x^2 + 8)$ — $x^2 - 1$ is a difference of two squares, so use $x^2 - y^2 = (x + y) (x - y)$

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Algebraic fractions
Exercise A, Question 2

Question:

Find the value of

(a) $81^{\frac{1}{2}}$

(b) $81^{\frac{3}{4}}$

(c) $81^{\frac{3}{4}}$.

Solution:

(a)

$81^{1/2}$

$= \sqrt{81}$

$= 9$

Use $a^{\frac{1}{m}} = \sqrt[m]{a}$, so $a^{\frac{1}{2}} = \sqrt{a}$

(b)

$81^{\frac{3}{4}}$

$= (\sqrt[4]{81})^3$ then cube this

$= 3^3$

$= 27$

$a^{\frac{n}{m}} = \sqrt[m]{(a^n)}$ or $(\sqrt[m]{a})^n$

It is easier to find the fourth root,

$\sqrt[4]{81} = 3$ because $3 \times 3 \times 3 \times 3 = 81$

(c)

$81^{-\frac{3}{4}} = \frac{1}{81^{3/4}}$

$= \frac{1}{27}$

Use $a^{-m} = \frac{1}{a^m}$

Use the answer from part (b)

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Algebraic fractions

Exercise A, Question 3

Question:

(a) Write down the value of $8^{\frac{1}{3}}$.

(b) Find the value of $8^{-\frac{2}{3}}$.

Solution:

(a)

$8^{\frac{1}{3}}$

$= \sqrt[3]{8}$

$= 2$

Use $a^{\frac{1}{m}} = \sqrt[m]{a}$, so $a^{\frac{1}{3}} = \sqrt[3]{a}$

$\sqrt[3]{8} = 2$ because $2 \times 2 \times 2 = 8$

(b)

$8^{-\frac{2}{3}}$

$8^{\frac{2}{3}} = (\sqrt[3]{8})^2$

$= 2^2 = 4$

First find $8^{\frac{2}{3}}$ $a^{\frac{n}{m}} = \sqrt[m]{(a^n)}$ or $(\sqrt[m]{a})^n$

$8^{-\frac{2}{3}} = \frac{1}{8^{\frac{2}{3}}}$

$= \frac{1}{4}$

Use $a^{-m} = \frac{1}{a^m}$

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Algebraic fractions
Exercise A, Question 4

Question:

(a) Find the value of $125^{\frac{4}{3}}$.

(b) Simplify $24x^2 \div 18x^{\frac{4}{3}}$.

Solution:

(a)

$125^{\frac{4}{3}}$

$= \left(\sqrt[3]{125} \right)^4$

$= 5^4$

$= 625$

$a^{\frac{n}{m}} = \sqrt[m]{(a^n)}$ or $\left(\sqrt[m]{a} \right)^n$

It is easier to find the cube root, then the fourth power

$\sqrt[3]{125} = 5$ because $5 \times 5 \times 5 = 125$

(b)

$24x^2 \div 18x^{\frac{4}{3}}$

$= \frac{24x^2}{18x^{\frac{4}{3}}} = \frac{4x^2}{3x^{\frac{4}{3}}}$

Divide by 6

$= \frac{4x^{\frac{2}{3}}}{3}$

Use $a^m \div a^n = a^{m-n}$

(or $\frac{4}{3}x^{\frac{2}{3}}$)

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Algebraic fractions
Exercise A, Question 5

Question:

(a) Express $\sqrt{80}$ in the form $a\sqrt{5}$, where a is an integer.

(b) Express $(4-\sqrt{5})^2$ in the form $b+c\sqrt{5}$, where b and c are integers.

Solution:

(a)
$$\sqrt{80} = \sqrt{16}\times\sqrt{5}$$
$$= 4\sqrt{5} \quad (a = 4)$$

Use $\sqrt{(ab)} = \sqrt{a}\sqrt{b}$

(b)
$$(4-\sqrt{5})^2 = (4-\sqrt{5})(4-\sqrt{5})$$
$$= 4(4-\sqrt{5}) - \sqrt{5}(4-\sqrt{5})$$
$$= 16 - 4\sqrt{5} - 4\sqrt{5} + 5$$
$$= 21 - 8\sqrt{5}$$
$(b = 21$ and $c = -8)$

Multiply the brackets.
$\sqrt{5}\times\sqrt{5} = 5$

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Algebraic fractions
Exercise A, Question 6

Question:

(a) Expand and simplify $(4+\sqrt{3})(4-\sqrt{3})$.

(b) Express $\frac{26}{4+\sqrt{3}}$ in the form $a+b\sqrt{3}$, where a and b are integers.

Solution:

(a)

$(4+\sqrt{3})(4-\sqrt{3})$

$= 4(4-\sqrt{3}) + \sqrt{3}(4-\sqrt{3})$ Multiply the brackets.

$= 16 - 4\sqrt{3} + 4\sqrt{3} - 3$ $\sqrt{3} \times \sqrt{3} = 3$

$= 13$

(b)

$\frac{26}{4+\sqrt{3}} \times \frac{4-\sqrt{3}}{4-\sqrt{3}}$ To rationalise the denominator, multiply top and bottom by $4-\sqrt{3}$

$= \frac{26(4-\sqrt{3})}{(4+\sqrt{3})(4-\sqrt{3})}$

$= \frac{26(4-\sqrt{3})}{13}$ Use the answer from part (a)

$= 2(4-\sqrt{3})$ Divide by 13

$= 8 - 2\sqrt{3}$

$(a = 8 \text{ and } b = -2)$

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Algebraic fractions
Exercise A, Question 7

Question:

(a) Express $\sqrt{108}$ in the form $a\sqrt{3}$, where a is an integer.

(b) Express $(2-\sqrt{3})^2$ in the form $b+c\sqrt{3}$, where b and c are integers to be found.

Solution:

(a)

$$\sqrt{108} = \sqrt{36}\times\sqrt{3} = 6\sqrt{3} \quad (a=6)$$

Use $\sqrt{(ab)}=\sqrt{a}\sqrt{b}$

(b)

$$(2-\sqrt{3})^2 = (2-\sqrt{3})(2-\sqrt{3})$$
$$= 2(2-\sqrt{3}) - \sqrt{3}(2-\sqrt{3})$$
$$= 4 - 2\sqrt{3} - 2\sqrt{3} + 3$$
$$= 7 - 4\sqrt{3}$$

$(b=7$ and $c=-4)$

Multiply the brackets

$\sqrt{3}\times\sqrt{3}=3$

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Algebraic fractions
Exercise A, Question 8

Question:

(a) Express $(2\sqrt{7})^3$ in the form $a\sqrt{7}$, where a is an integer.

(b) Express $(8+\sqrt{7})(3-2\sqrt{7})$ in the form $b+c\sqrt{7}$, where b and c are integers.

(c) Express $\frac{6+2\sqrt{7}}{3-\sqrt{7}}$ in the form $d+e\sqrt{7}$, where d and e are integers.

Solution:

(a)

$$(2\sqrt{7})^3 = 2\sqrt{7}\times 2\sqrt{7}\times 2\sqrt{7}$$ Multiply the 2s.

$$= 8(\sqrt{7}\times\sqrt{7}\times\sqrt{7})$$

$$= 8(7\sqrt{7})$$ $\sqrt{7}\times\sqrt{7}=7$

$$= 56\sqrt{7} \quad (a=56)$$

(b)

$$(8+\sqrt{7})(3-2\sqrt{7})$$

$$= 8(3-2\sqrt{7})+\sqrt{7}(3-2\sqrt{7})$$ $\sqrt{7}\times 2\sqrt{7}=2\times 7$

$$= 24-16\sqrt{7}+3\sqrt{7}-14$$

$$= 10-13\sqrt{7}$$

$(b=10 \text{ and } c=-13)$

(c)

$$\frac{6+2\sqrt{7}}{3-\sqrt{7}}\times\frac{3+\sqrt{7}}{3+\sqrt{7}}$$

To rationalise the denominator, multiply top and bottom by $3+\sqrt{7}$

$$= \frac{(6+2\sqrt{7})(3+\sqrt{7})}{(3-\sqrt{7})(3+\sqrt{7})}$$

$$= \frac{6(3+\sqrt{7})+2\sqrt{7}(3+\sqrt{7})}{3(3+\sqrt{7})-\sqrt{7}(3+\sqrt{7})}$$

$$= \frac{18+6\sqrt{7}+6\sqrt{7}+14}{9+3\sqrt{7}-3\sqrt{7}-7}$$

$$= \frac{32+12\sqrt{7}}{2} = 16+6\sqrt{7}$$ Divide by 2

$(d=16 \text{ and } e=6)$

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Edexcel Modular Mathematics for AS and A-Level

Algebraic fractions
Exercise A, Question 9

Question:

Solve the equations

(a) $x^2 - x - 72 = 0$

(b) $2x^2 + 7x = 0$

(c) $10x^2 + 9x - 9 = 0$

Solution:

(a)

$x^2 - x - 72 = 0$

$(x + 8)(x - 9) = 0$ Factorise

$x + 8 = 0 , x - 9 = 0$ Although the equation could be solved using the quadratic formula or 'completing the square', factorisation is quicker.

$x = -8 , x = 9$

(b)

$2x^2 + 7x = 0$ Use the factor x.

$x(2x + 7) = 0$

$x = 0 , 2x + 7 = 0$ Don't forget the $x = 0$ solution.

$x = 0 , x = -\frac{7}{2}$

(b)

$2x^2 + 7x = 0$ Use the factor x.

$x(2x + 7) = 0$

$x = 0 , 2x + 7 = 0$ Don't forget the $x = 0$ solution.

$x = 0 , x = -\frac{7}{2}$

(c)

$10x^2 + 9x - 9 = 0$

$(2x + 3)(5x - 3) = 0$ Factorise

$2x + 3 = 0 , 5x - 3 = 0$

$x = -\frac{3}{2} , x = \frac{3}{5}$

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Algebraic fractions
Exercise A, Question 10

Question:

Solve the equations, giving your answers to 3 significant figures

(a) $x^2 + 10x + 17 = 0$

(b) $2x^2 - 5x - 1 = 0$

(c) $(2x - 3)^2 = 7$

Solution:

(a)

$x^2 + 10x + 17 = 0$ Since the question requires answers to

$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$ 3 significant figures, you know that the

quadratic will not factorise.

$a = 1$, $b = 10$, $c = 17$

$x = \frac{-10 \pm \sqrt{(100 - 68)}}{2}$ Use the quadratic formula, quoting

the formula first.

$= \frac{-10 \pm \sqrt{32}}{2}$

$= \frac{-10 \pm 5.656\ldots}{2}$ Intermediate working should be to

at least 4 sig. figs.

$= \frac{-10 + 5.656\ldots}{2}$, $\frac{-10 - 5.656\ldots}{2}$

$x = -2.17$, $x = -7.83$ Divide by 2, and round to 3 sig. figs.

Alternative method:

$x^2 + 10x + 17 = 0$

$x^2 + 10x = -17$ Subtract 17 to get LHS in the required form.

$(x + 5)^2 - 25 = -17$ Complete the square for $x^2 + 10x$

$(x + 5)^2 = -17 + 25$ Add 25 to both sides

$(x + 5)^2 = 8$

$x + 5 = \pm \sqrt{8}$ Square root both sides.

$x = -5 \pm \sqrt{8}$ Subtract 5 from both sides.

$x = -5 + \sqrt{8}$, $x = -5 - \sqrt{8}$

$x = -2.17$, $x = -7.83$

(b)

$2x^2 - 5x - 1 = 0$

$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

$a = 2, b = -5, c = -1$

$x = \frac{5 \pm \sqrt{(-5)^2 - (4 \times 2 \times -1)}}{4}$ Use the quadratic formula, quoting the formula first.

$= \frac{5 \pm \sqrt{(25 + 8)}}{4} = \frac{5 \pm \sqrt{33}}{4}$

$= \frac{5 + 5.744\ldots}{4}, \frac{5 - 5.744\ldots}{4}$

$x = 2.69, x = -0.186$ Divide by 4, and round to 3 sig. figs.

(c)

$(2x - 3)^2 = 7$

$2x - 3 = \pm\sqrt{7}$ The quickest method is to take the square root of both sides.

$2x = 3 \pm \sqrt{7}$ Add 3 to both sides.

$x = \frac{3 + \sqrt{7}}{2}, x = \frac{3 - \sqrt{7}}{2}$ Divide both sides by 2

$x = 2.82, x = 0.177$

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Algebraic fractions
Exercise A, Question 11

Question:

$$x^2 - 8x - 29 \equiv (x + a)^2 + b,$$
where a and b are constants.

(a) Find the value of a and the value of b.

(b) Hence, or otherwise, show that the roots of
$$x^2 - 8x - 29 = 0$$
are $c \pm d\sqrt{5}$, where c and d are integers to be found.

Solution:

(a)

$x^2 - 8x = (x - 4)^2 - 16$ Complete the square for $x^2 - 8x$

$x^2 - 8x - 29 = (x - 4)^2 - 16 - 29$

$= (x - 4)^2 - 45$

$(a = -4 \text{ and } b = -45)$

(b)

$x^2 - 8x - 29 = 0$

$(x - 4)^2 - 45 = 0$ Use the result from part (a)

$(x - 4)^2 = 45$

$x - 4 = \pm\sqrt{45}$ Take the square root of both sides.

$x = 4 \pm \sqrt{45}$

$\sqrt{45} = \sqrt{9} \times \sqrt{5} = 3\sqrt{5}$ Use $\sqrt{(ab)} = \sqrt{a}\sqrt{b}$

Roots are $4 \pm 3\sqrt{5}$

$(c = 4 \text{ and } d = 3)$

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Algebraic fractions

Exercise A, Question 12

Question:

Given that

$$f(x) = x^2 - 6x + 18, \quad x \geq 0,$$

(a) express f(x) in the form $(x - a)^2 + b$, where a and b are integers.

The curve C with equation $y = f(x)$, $x \geq 0$, meets the y-axis at P and has a minimum point at Q.

(b) Sketch the graph of C, showing the coordinates of P and Q.

The line $y = 41$ meets C at the point R.

(c) Find the x-coordinate of R, giving your answer in the form $p + q\sqrt{2}$, where p and q are integers.

Solution:

(a)

$f(x) = x^2 - 6x + 18$

$x^2 - 6x = (x - 3)^2 - 9$ for $x^2 - 6x$ Complete the square

$x^2 - 6x + 18 = (x - 3)^2 - 9 + 18$

$= (x - 3)^2 + 9$

$(a = 3$ and $b = 9)$

(b)

$y = x^2 - 6x + 18$

$y = (x - 3)^2 + 9$

$(x - 3)^2 \geq 0$ Squaring a number cannot give a negative result

The minimum value of $(x - 3)^2$ is zero, when $x = 3$.

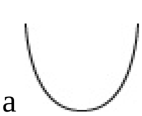
So the minimum value of y is $0 + 9 = 9$, when $x = 3$.

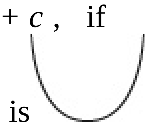
Q is the point (3, 9)

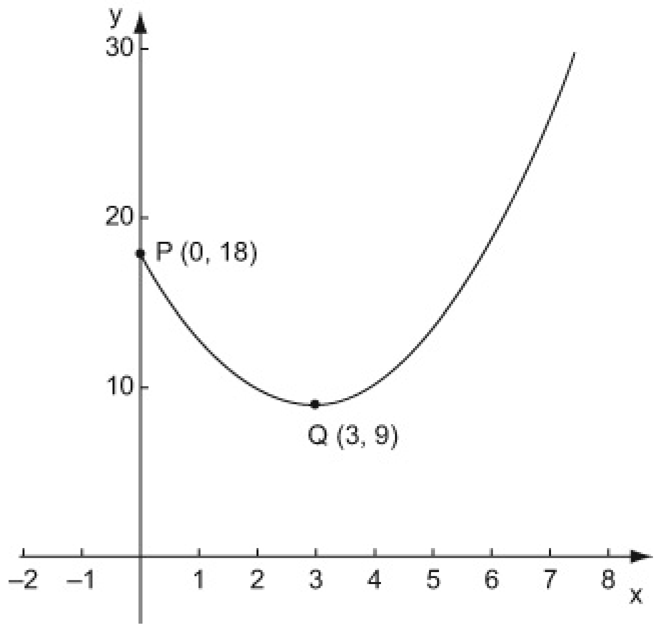
The curve crosses the y-axis where $x = 0$.

For $x = 0$, $y = 18$

P is the point (0, 18)

The graph of $y = x^2 - 6x + 18$ is a  shape.

For $y = ax^2 + bx + c$, if $a > 0$, the shape is 



Use the information about P and Q to sketch the curve $x \geq 0$, so the part where $x < 0$ is not needed.

(c)

$y = (x - 3)^2 + 9$

$41 = (x - 3)^2 + 9$ — Put $y = 41$ into the equation of C.

$32 = (x - 3)^2$ — Subtract 9 from both sides.

$(x - 3)^2 = 32$

$x - 3 = \pm\sqrt{32}$ — Take the square root of both sides.

$x = 3 \pm \sqrt{32}$

$\sqrt{32} = \sqrt{16} \times \sqrt{2} = 4\sqrt{2}$ — Use $\sqrt{(ab)} = \sqrt{a}\sqrt{b}$

$x = 3 \pm 4\sqrt{2}$

x-coordinate of R is $3 + 4\sqrt{2}$ — The other value $3 - 4\sqrt{2}$ is less than 0, so not needed

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Algebraic fractions
Exercise A, Question 13

Question:

Given that the equation $kx^2 + 12x + k = 0$, where k is a positive constant, has equal roots, find the value of k.

Solution:

$Kx^2 + 12x + K = 0$

$a = K$, $b = 12$, $c = K$ — Write down the values of a, b and c for the quadratic equation.

For equal roots, $b^2 = 4ac$
(or $b^2 - 4ac = 0$)

$12^2 = 4 \times K \times K$

$4K^2 = 144$

$K^2 = 36$

$K = \pm 6$

So $K = 6$ — The question says that K is a positive constant.

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Algebraic fractions
Exercise A, Question 14

Question:

Given that

$$x^2 + 10x + 36 \equiv (x + a)^2 + b,$$

where a and b are constants,

(a) find the value of a and the value of b.

(b) Hence show that the equation $x^2 + 10x + 36 = 0$ has no real roots.

The equation $x^2 + 10x + k = 0$ has equal roots.

(c) Find the value of k.

(d) For this value of k, sketch the graph of $y = x^2 + 10x + k$, showing the coordinates of any points at which the graph meets the coordinate axes.

Solution:

(a)

$x^2 + 10x + 36$

$x^2 + 10x = (x + 5)^2 - 25$ Complete the square for $x^2 + 10x$

$x^2 + 10x + 36 = (x + 5)^2 - 25 + 36$

$= (x + 5)^2 + 11$

$a = 5$ and $b = 11$

(b)

$x^2 + 10x + 36 = 0$

$(x + 5)^2 + 11 = 0$ 'Hence' implies that part (a) must be used

$(x + 5)^2 = -11$

A real number squared cannot

be negative, ∴ no real roots

(c)

$x^2 + 10x + K = 0$

$a = 1, b = 10, c = K$

For equal roots, $b^2 = 4ac$

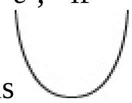
$10^2 = 4 \times 1 \times K$

$4K = 100$

$K = 25$

(d)

The graph of $y = x^2 + 10x + 25$ is a  shape.

For $y = ax^2 + bx + c$, if $a > 0$, the shape is 

$x = 0$: $y = 0 + 0 + 25 = 25$

Meets y-axis at $(0, 25)$

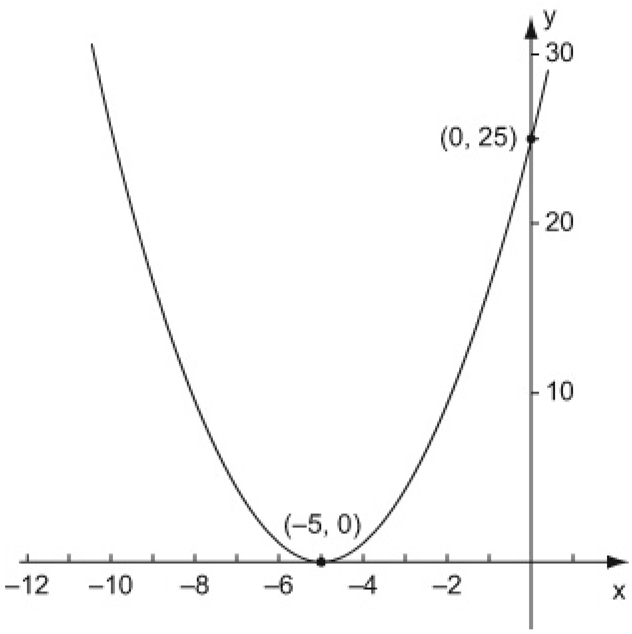
$y = 0$: $x^2 + 10x + 25 = 0$

$(x + 5)(x + 5) = 0$

$x = -5$

Meets x-axis at $(-5, 0)$

Put $x = 0$ to find intersections with the y-axis, and $y = 0$ to find intersections with the x-axis.



The graph meets the x-axis at just one point, so it ‘touches’ the x-axis

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Algebraic fractions
Exercise A, Question 15

Question:

$$x^2 + 2x + 3 \equiv (x + a)^2 + b .$$

(a) Find the values of the constants a and b.

(b) Sketch the graph of $y = x^2 + 2x + 3$, indicating clearly the coordinates of any intersections with the coordinate axes.

(c) Find the value of the discriminant of $x^2 + 2x + 3$.
Explain how the sign of the discriminant relates to your sketch in part (b).

The equation $x^2 + kx + 3 = 0$, where k is a constant, has no real roots.

(d) Find the set of possible values of k, giving your answer in surd form.

Solution:

(a)

$x^2 + 2x + 3$

Complete the square

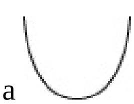
$x^2 + 2x = (x + 1)^2 - 1$ for $x^2 + 2x$

$x^2 + 2x + 3 = (x + 1)^2 - 1 + 3$

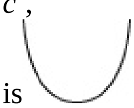
$= (x + 1)^2 + 2$

$a = 1$ and $b = 2$

(b)

The graph of $y = x^2 + 2x + 3$ is a  shape

For $y = ax^2 + bx + c$,

if $a > 0$, the shape is 

$x = 0$: $y = 0 + 0 + 3$

Meets y-axis at $(0, 3)$

Put $x = 0$ to find intersections with the y-axis,

$y = 0$: $x^2 + 2x + 3 = 0$

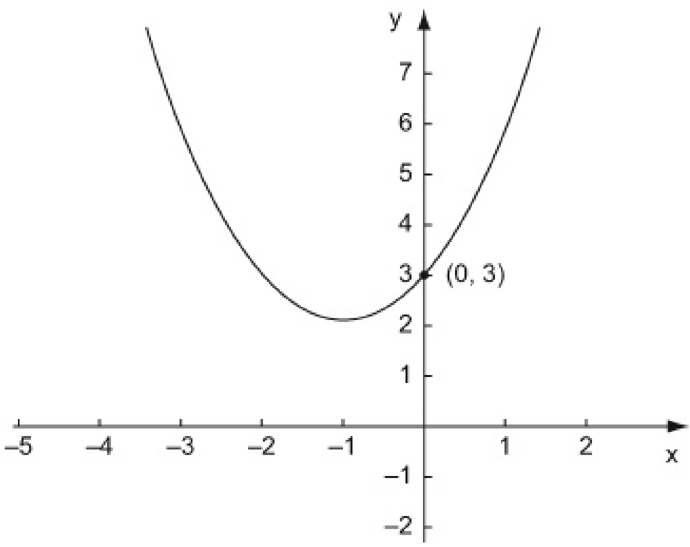
and $y = 0$ to find intersections with the x-axis.

$(x + 1)^2 + 2 = 0$

$(x + 1)^2 = -2$

A real number squared cannot be negative, $\therefore$

no real roots, so no intersection with x-axis.



The minimum value of $(x+1)^2$ is zero, when $x = -1$, so the minimum point on the graph is at $x = -1$

(c)

$x^2 + 2x + 3$

$a = 1$, $b = 2$, $c = 3$

$b^2 - 4ac = 2^2 - 4 \times 1 \times 3$

$= -8$

The discriminant is $b^2 - 4ac$

Since the discriminant is negative $(b^2 - 4ac < 0)$, $x^2 + 2x + 3 = 0$ has no real roots, so the graph does not cross the x-axis.

No real roots: $b^2 < 4ac$

(d)

$x^2 + kx + 3 = 0$

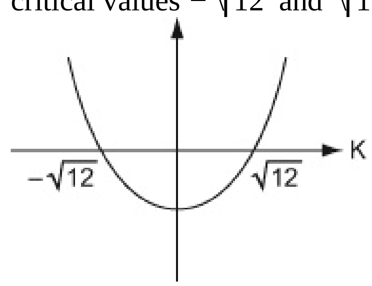
$a = 1$, $b = k$, $c = 3$

For no real roots, $b^2 < 4ac$

$K^2 < 12$

$K^2 - 12 < 0$

This is a quadratic inequality with critical values $-\sqrt{12}$ and $\sqrt{12}$



$(K + \sqrt{12})(K - \sqrt{12}) < 0$

Critical values:

$K = -\sqrt{12}$, $K = \sqrt{12}$

$-\sqrt{12} < K < \sqrt{12}$

$\left(\begin{array}{l} \sqrt{12} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3} \\ -2\sqrt{3} < K < 2\sqrt{3} \end{array} \right)$

The surds can be simplified using

$\sqrt{(ab)} = \sqrt{a}\sqrt{b}$

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Algebraic fractions

Exercise A, Question 16

Question:

Solve the simultaneous equations

$$x + y = 2$$
$$x^2 + 2y = 12$$

Solution:

$y = 2 - x$ Rearrange the linear equation to get $y = \ldots$

$x^2 + 2 (2 - x) = 12$ Substitute into the quadratic equation

$x^2 + 4 - 2x = 12$

$x^2 - 2x + 4 - 12 = 0$

$x^2 - 2x - 8 = 0$

$(x + 2) (x - 4) = 0$ Solve for x using factorisation

$x = -2$ or $x = 4$

$x = -2 :\quad y = 2 - (-2) = 4$ Substitute the x values back into $y = 2 - x$

$x = 4 :\quad y = 2 - 4 = -2$

Solution: $x = -2 , y = 4$ and $x = 4 , y = -2$

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Algebraic fractions
Exercise A, Question 17

Question:

(a) By eliminating y from the equations

$$y = x - 4\,,$$
$$2x^2 - xy = 8\,,$$

show that

$$x^2 + 4x - 8 = 0.$$

(b) Hence, or otherwise, solve the simultaneous equations

$$y = x - 4\,,$$
$$2x^2 - xy = 8\,,$$

giving your answers in the form $a \pm b\sqrt{3}$, where a and b are integers.

Solution:

(a)

$$2x^2 - x(x-4) = 8$$

Substitute $y = x - 4$ into the quadratic equation.

$$2x^2 - x^2 + 4x = 8$$
$$x^2 + 4x - 8 = 0$$

(b)

Solve the equation found in part (a). The $\sqrt{3}$ in the given answer suggests that factorisation will not be possible, so use the quadratic formula, or complete the square.

$x^2 + 4x - 8 = 0$

$x^2 + 4x = (x + 2)^2 - 4$

Complete the square for $x^2 + 4x$

$(x + 2)^2 - 4 - 8 = 0$

$(x + 2)^2 = 12$

$x + 2 = \pm \sqrt{12}$

$x = -2 \pm \sqrt{12}$

$\sqrt{12} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$

Use $\sqrt{(ab)} = \sqrt{a}\sqrt{b}$

$x = -2 \pm 2\sqrt{3}$

$(a = -2 \text{ and } b = 2)$

Using $y = x - 4$,

$y = (-2 \pm 2\sqrt{3}) - 4$

$= -6 \pm 2\sqrt{3}$

Solution: $x = -2 \pm 2\sqrt{3}$

$y = -6 \pm 2\sqrt{3}$

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Algebraic fractions
Exercise A, Question 18

Question:

Solve the simultaneous equations

$$2x - y - 5 = 0$$
$$x^2 + xy - 2 = 0$$

Solution:

$y = 2x - 5$ Rearrange the linear equation to get $y = \ldots$

$x^2 + x(2x - 5) - 2 = 0$ Substitute into the quadratic equation.

$x^2 + 2x^2 - 5x - 2 = 0$

$3x^2 - 5x - 2 = 0$

$(3x + 1)(x - 2) = 0$ Solve for x using factorisation

$x = -\frac{1}{3}$ or $x = 2$

$x = -\frac{1}{3}: \ y = -\frac{2}{3} - 5 = -\frac{17}{3}$ Substitute the x values back into $y = 2x - 5$

$x = 2: \ y = 4 - 5 = -1$

Solution $x = -\frac{1}{3}, \ y = -\frac{17}{3}$

and $x = 2, \ y = -1$

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Algebraic fractions
Exercise A, Question 19

Question:

Find the set of values of x for which

(a) $3(2x+1) > 5 - 2x$,

(b) $2x^2 - 7x + 3 > 0$,

(c) both $3(2x+1) > 5 - 2x$ and $2x^2 - 7x + 3 > 0$.

Solution:

(a)

$3(2x+1) > 5 - 2x$

$6x + 3 > 5 - 2x$ Multiply out

$6x + 2x + 3 > 5$ Add $2x$ to both sides.

$8x > 2$ Subtract 3 from both sides

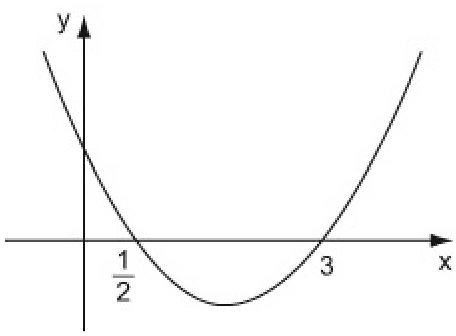
$x > \frac{1}{4}$ Divide both sides by 8

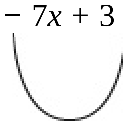
(b)

$2x^2 - 7x + 3 = 0$

$(2x - 1)(x - 3) = 0$ Factorise to solve the quadratic equation.

$x = \frac{1}{2}$, $x = 3$

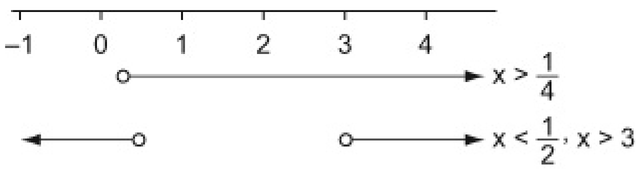


Sketch the graph of $y = 2x^2 - 7x + 3$. The shape is  The sketch does not need to be accurate.

$2x^2 - 7x + 3 > 0$ where $2x^2 - 7x + 3 > 0$ $(y > 0)$ for the part of the graph above the x-axis

$x < \frac{1}{2}$ or $x > 3$

(c)



$\frac{1}{4} < x < \frac{1}{2}$, $x > 3$

Use a number line. The two sets of values (from part (a) and part (b)) overlap for $\frac{1}{4} < x < \frac{1}{2}$ and $x > 3$

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Algebraic fractions
Exercise A, Question 20

Question:

Find the set of values of x for which

(a) $x(x-5) < 7x - x^2$

(b) $x(3x+7) > 20$

Solution:

(a)
$x(x-5) < 7x - x^2$

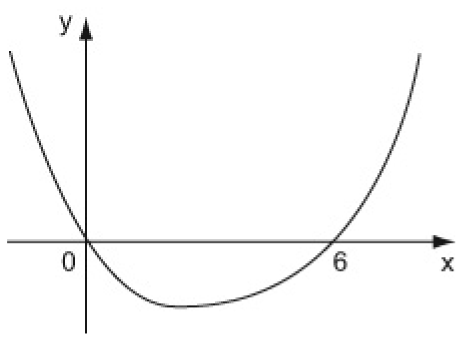
$x^2 - 5x < 7x - x^2$ Multiply out

$2x^2 - 12x < 0$

$2x(x-6) < 0$ Factorise using the common factor $2x$

$2x(x-6) = 0$ Solve the quadratic equation

$x = 0,\ x = 6$ to find the critical values



Sketch the graph of $y = 2x^2 - 12x$

$2x^2 - 12x < 0$ where $0 < x < 6$

$2x^2 - 12x < 0$ $(y < 0)$ for the part of the graph below the x-axis

(b)
$x(3x+7) > 20$

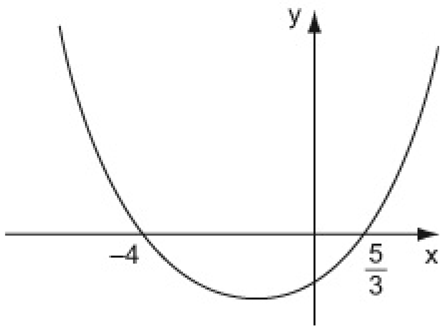
$3x^2 + 7x > 20$ Multiply out

$3x^2 + 7x - 20 > 0$

$(3x-5)(x+4) > 0$ Factorise

$(3x-5)(x+4) = 0$ Solve the quadratic equation to

$x = \frac{5}{3},\ x = -4$ find the critical values



Sketch the graph of
$y = 3x^2 + 7x - 20$

$3x^2 + 7x - 20 > 0$ where

$x < -4$ or $x > \frac{5}{3}$

$3x^2 + 7x - 20 > 0$ ($y > 0$)

for the part of the graph

above the x-axis.

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Algebraic fractions
Exercise A, Question 21

Question:

(a) Solve the simultaneous equations

$$y + 2x = 5 ,$$
$$2x^2 - 3x - y = 16.$$

(b) Hence, or otherwise, find the set of values of x for which

$$2x^2 - 3x - 16 > 5 - 2x .$$

Solution:

(a)

$y = 5 - 2x$ Rearrange the linear equation to get $y = \ldots$

$2x^2 - 3x - (5 - 2x) = 16$ Substitute into the quadratic equation.

$2x^2 - 3x - 5 + 2x = 16$

$2x^2 - x - 21 = 0$

$(2x - 7) (x + 3) = 0$ Solve for x using factorisation.

$x = 3\frac{1}{2}$ or $x = -3$

$x = 3\frac{1}{2}$: $y = 5 - 7 = -2$ Substitute the x-values back into $y = 5 - 2x$

$x = -3$: $y = 5 + 6 = 11$

Solution $x = 3\frac{1}{2}$, $y = -2$

and $x = -3$, $y = 11$

(b)

The equations in (a) could be written as

$y = 5 - 2x$ and $y = 2x^2 - 3x - 16$.

The solutions to $2x^2 - 3x - 16 = 5 - 2x$ are the x solutions from (a). These are the critical values for $2x^2 - 3x - 16 > 5 - 2x$.

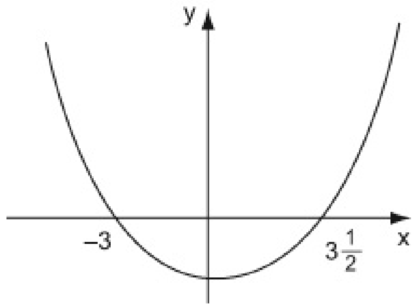
Critical values

$x = 3\frac{1}{2}$ and $x = -3$.

$2x^2 - 3x - 16 > 5 - 2x$

$(2x^2 - 3x - 16 - 5 + 2x > 0)$

$2x^2 - x - 21 > 0$



Sketch the graph of
$y = 2x^2 - x - 21$

$x < -3$ or $x > 3\frac{1}{2}$

$2x^2 - x - 21 > 0$ ($y > 0$) for the part of the graph above the x-axis.

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Algebraic fractions

Exercise A, Question 22

Question:

The equation $x^2 + kx + (k + 3) = 0$, where k is a constant, has different real roots.

(a) Show that $k^2 - 4k - 12 > 0$.

(b) Find the set of possible values of k.

Solution:

(a)

$x^2 + kx + (k + 3) = 0$

$a = 1 , b = k , c = k + 3$ Write down a , b and c for the equation

$b^2 > 4ac$

$k^2 > 4 (k + 3)$ For different real roots, $b^2 > 4ac$

$k^2 > 4k + 12$

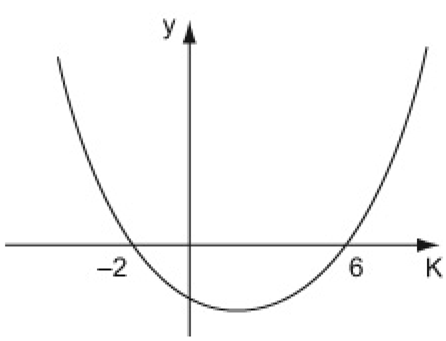
$k^2 - 4k - 12 > 0$

(b)

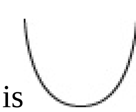
$k^2 - 4k - 12 = 0$ Factorise to solve the quadratic equation.

$(k + 2) (k - 6) = 0$

$k = -2 , k = 6$



Sketch the graph of $y = k^2 - 4k - 12$.

The shape is  The sketch does not need to be accurate

$k^2 - 4k - 12 > 0$ where $k < -2$ or $k > 6$

$k^2 - 4k - 12 > 0$ $(y > 0)$ for the part of the graph above the k-axis.

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Algebraic fractions
Exercise A, Question 23

Question:

Given that the equation $kx^2 + 3kx + 2 = 0$, where k is a constant, has no real roots, find the set of possible values of k.

Solution:

$kx^2 + 3kx + 2 = 0$

$a = k$, $b = 3k$, $c = 2$ Write down a, b and c for the equation.

$b^2 < 4ac$

$(3k)^2 < 4 \times k \times 2$ For no real roots, $b^2 < 4ac$.

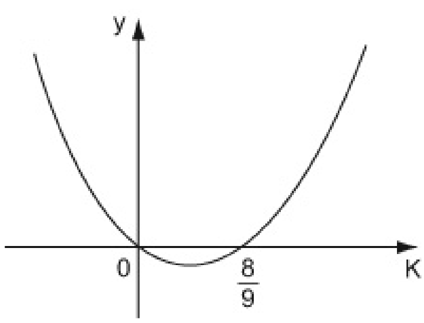
$9k^2 < 8k$

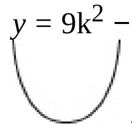
$9k^2 - 8k < 0$

$9k^2 - 8k = 0$

$k(9k - 8) = 0$ Factorise to solve the quadratic equation

$k = 0$, $k = \frac{8}{9}$



Sketch the graph of $y = 9k^2 - 8k$. The shape is . The sketch does not need to be accurate.

$9k^2 - 8k < 0$ where $0 < k < \frac{8}{9}$

$9k^2 - 8k < 0$ $(y < 0)$ for the part of the graph below the k-axis.

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Algebraic fractions
Exercise A, Question 24

Question:

The equation $(2p+5)x^2+px+1=0$, where p is a constant, has different real roots.

(a) Show that $p^2-8p-20>0$

(b) Find the set of possible values of p.

Given that $p=-3$,

(c) find the exact roots of $(2p+5)x^2+px+1=0$.

Solution:

(a)

$(2p+5)x^2+px+1=0$

$a=2p+5$, $b=p$, $c=1$ Write down a, b and c for the equation.

$b^2>4ac$

$p^2>4(2p+5)$ For different real roots, $b^2>4ac$

$p^2>8p+20$

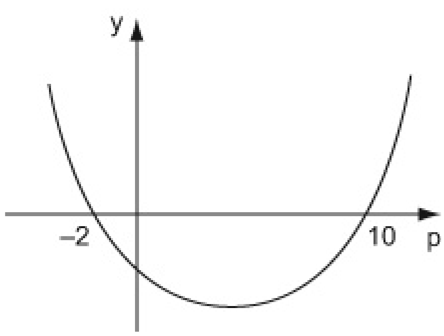
$p^2-8p-20>0$

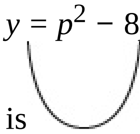
(b)

$p^2-8p-20=0$

$(p+2)(p-10)=0$ Factorise to solve the quadratic equation.

$p=-2$, $p=10$



Sketch the graph of $y=p^2-8p-20$ The shape is . The sketch does not need to be accurate

$p^2-8p-20>0$ where $p<-2$ or $p>10$

$p^2-8p-20>0$ $(y>0)$ for the part of the graph above the p-axis

(c)

For $p = -3$

$(-6+5)x^2 - 3x + 1 = 0$ Substitute $p = -3$ into the equation.

$-x^2 - 3x + 1 = 0$ Multiply by -1

$x^2 + 3x - 1 = 0$ The equation does not factorise,

$a = 1$, $b = 3$, $c = -1$ so use the quadratics formula.

$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$ Quote the formula.

$x = \frac{-3 \pm \sqrt{9+4}}{2}$

$x = \frac{1}{2}(-3 \pm \sqrt{13})$ Exact roots are required.

$\sqrt{13}$ cannot be simplified.

$x = \frac{1}{2}(-3 + \sqrt{13})$ or $x = \frac{1}{2}(-3 - \sqrt{13})$

Solutionbank C1

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Algebraic fractions

Exercise A, Question 25

Question:

(a) Factorise completely $x^3 - 4x$

(b) Sketch the curve with equation $y = x^3 - 4x$, showing the coordinates of the points where the curve crosses the x-axis.

(c) On a separate diagram, sketch the curve with equation

$$y = (x - 1)^3 - 4(x - 1)$$

showing the coordinates of the points where the curve crosses the x-axis.

Solution:

(a)

$x^3 - 4x$ — x is a common factor

$= x(x^2 - 4)$ — $(x^2 - 4)$ is a difference of squares

$= x(x + 2)(x - 2)$

(b)

Curve crosses x-axis where $y = 0$

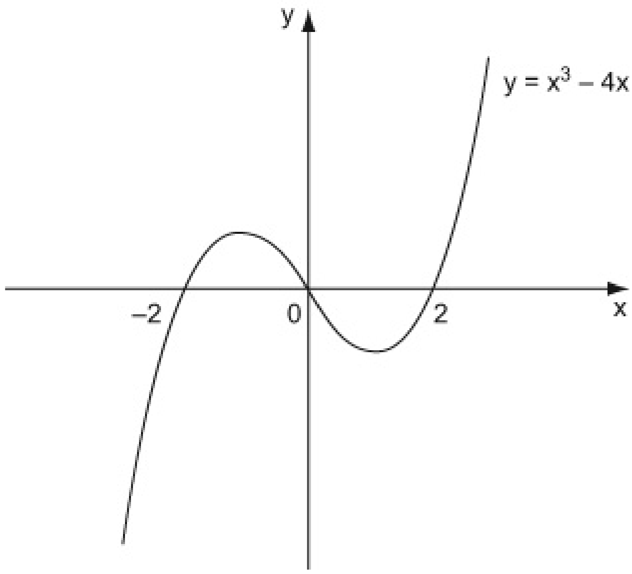
$x(x + 2)(x - 2) = 0$ — Put $y = 0$ and solve for x

$x = 0$, $x = -2$, $x = 2$

When $x = 0$, $y = 0$ — Put $x = 0$ to find where the curve crosses the y-axis.

When $x \to \infty$, $y \to \infty$ — Check what happens to y for large positive and negative values of x

When $x \to -\infty$, $y \to -\infty$



Crosses at (0, 0)

Crosses x-axis at (−2, 0), (2, 0).

(c)

$y = x^3 - 4x$

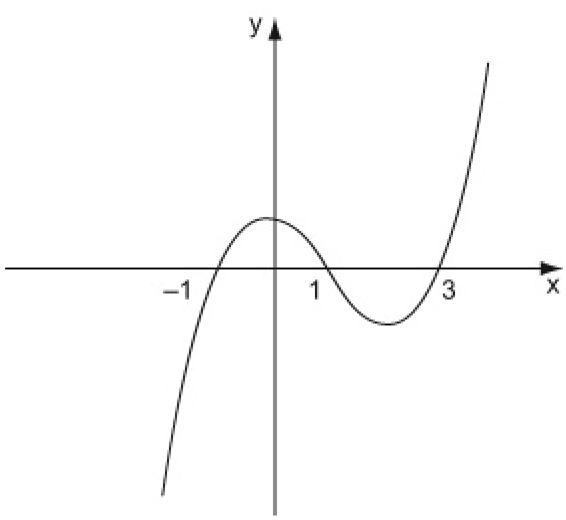
$y = (x - 1)^3 - 4(x - 1)$

This is a translation of $+1$ in the x-direction.

Compare with the equation from part (b).

x has been replaced by $x - 1$.

$f(x + a)$ is a translation of $-a$ in the x-direction.



The shape is the same as in part (b).

Crosses x-axis at $(-1, 0)$, $(1, 0)$, $(3, 0)$

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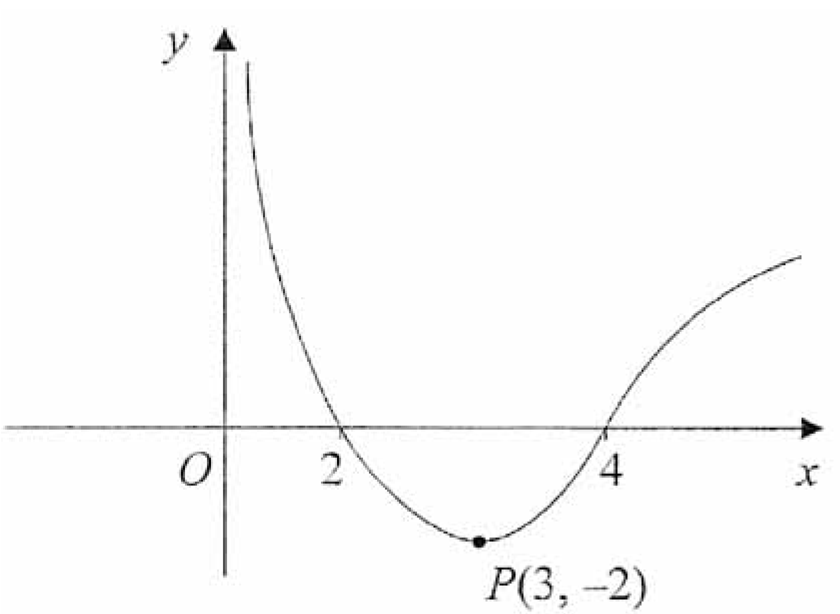
Algebraic fractions
Exercise A, Question 26

Question:



The figure shows a sketch of the curve with equation $y = f(x)$. The curve crosses the x-axis at the points (2, 0) and (4, 0). The minimum point on the curve is $P(3, -2)$.

In separate diagrams, sketch the curve with equation

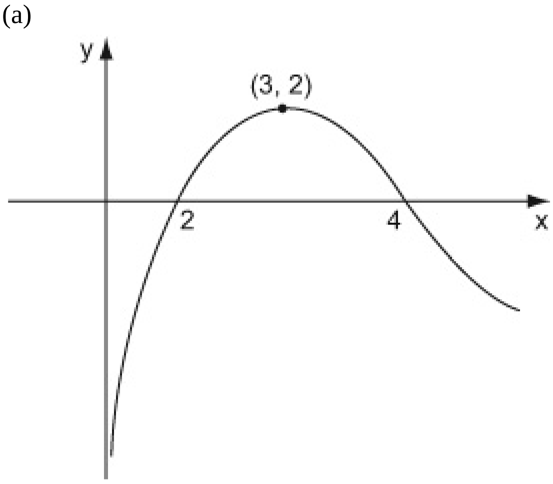
(a) $y = -f(x)$

(b) $y = f(2x)$

On each diagram, give the coordinates of the points at which the curve crosses the x-axis, and the coordinates of the image of P under the given transformation.

Solution:

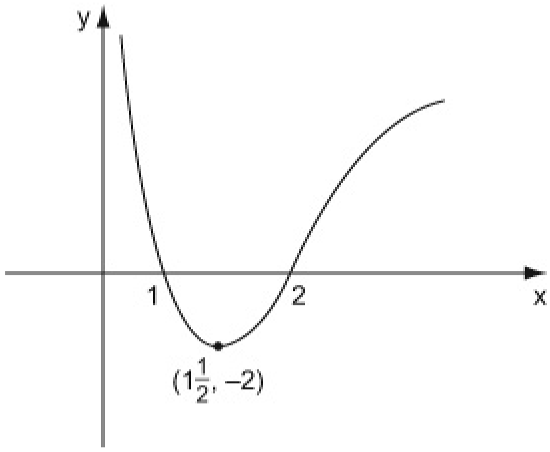
(a)



The transformation $-f(x)$ multiplies the y-coordinates by -1. This turns the graph upside-down.

Crosses the x-axis at $(2, 0)$, $(4, 0)$
Image of P is $(3, 2)$

(b)



$f(2x)$ is a stretch of $\frac{1}{2}$ in the x-direction. (Multiply x-coordinates by $\frac{1}{2}$.)

Crosses the x-axis at $(1, 0)$, $(2, 0)$

Image of P is $\left(1\frac{1}{2}, -2\right)$

y-coordinates are unchanged.

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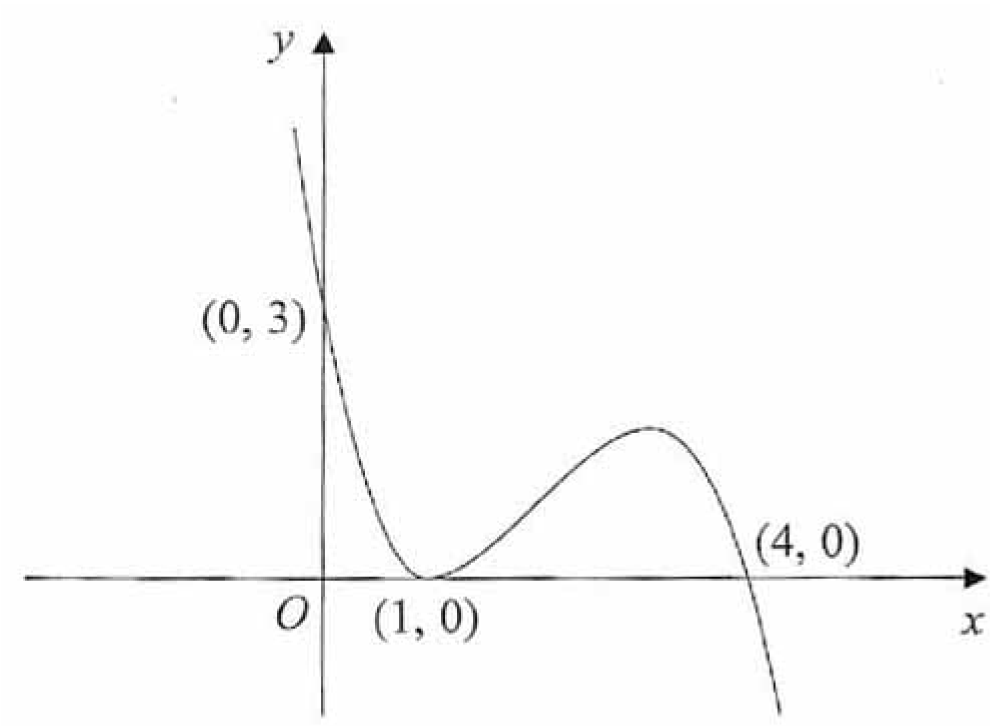
Algebraic fractions
Exercise A, Question 27

Question:



The figure shows a sketch of the curve with equation $y = f(x)$. The curve passes through the points (0, 3) and (4, 0) and touches the x-axis at the point (1, 0).

On separate diagrams, sketch the curve with equation

(a) $y = f(x + 1)$

(b) $y = 2f(x)$

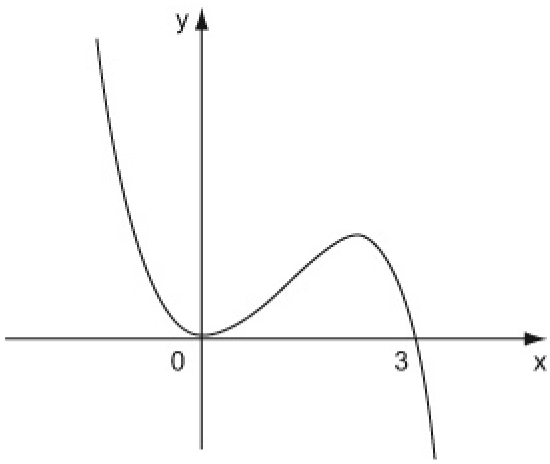
(c) $y = f\left(\frac{1}{2}x\right)$

On each diagram, show clearly the coordinates of all the points where the curve meets the axes.

Solution:

(a)

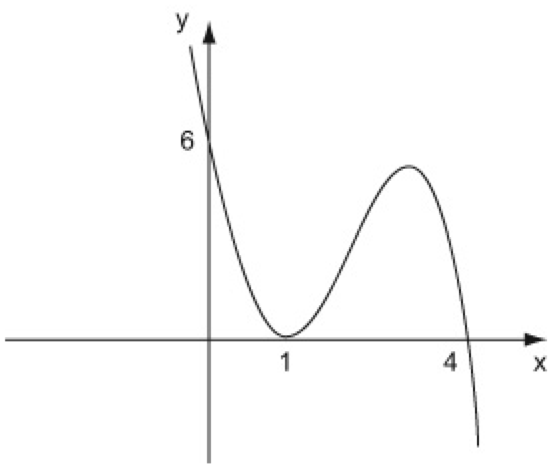
$f(x + 1)$ is a translation of



–1 in the x-direction.

Meets the x-axis at $(0, 0)$, $(3, 0)$
Meets the y-axis at $(0, 0)$

(b)

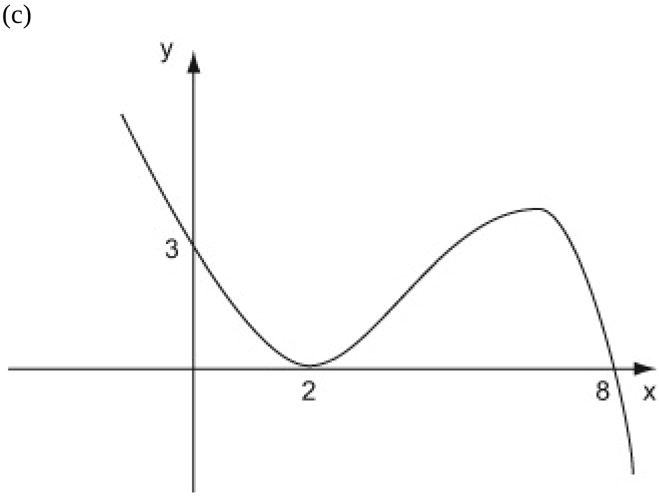


$2f(x)$ is a stretch of scale factor 2 in the y-direction (Multiply y-coordinates by 2)

Meets the x-axis at $(1, 0)$, $(4, 0)$
Meets the y-axis at $(0, 6)$

x-coordinates are unchanged.

(c)



$f\left(\frac{1}{2}x\right)$ is a stretch of scale factor $\frac{1}{\left(\frac{1}{2}\right)} = 2$ in the x-direction. (Multiply x-coordinates by 2)

Meets the x-axis at $(2, 0)$, $(8, 0)$

Meets the y-axis at $(0, 3)$

y-coordinates are unchanged.

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Algebraic fractions
Exercise A, Question 28

Question:

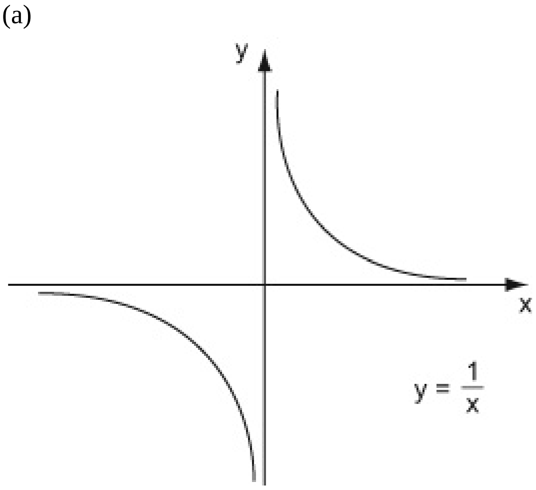
Given that $f\left(x\right)=\frac{1}{x}$, $x \neq 0$,

(a) sketch the graph of $y = f(x) + 3$ and state the equations of the asymptotes.

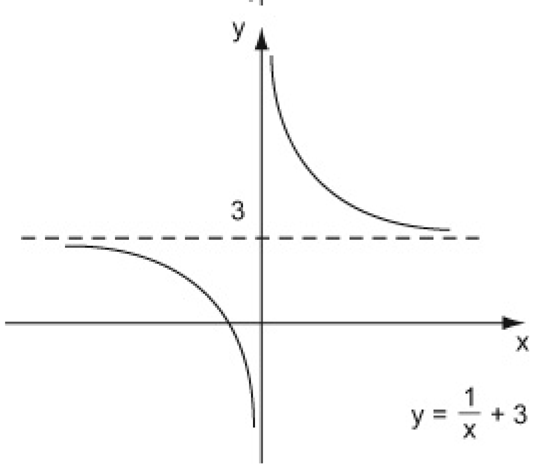
(b) Find the coordinates of the point where $y = f(x) + 3$ crosses a coordinate axis.

Solution:

(a)



You should know the shape of this curve.



$f(x) + 3$ is a translation of $+3$ in the y-direction.

$y = 3$ is an asymptote

$x = 0$ is an asymptote

The equation of the y-axis is $x = 0$

(b)

The graph does not cross get

the y-axis (see sketch in (a)) .

undefined ,

Crosses the x-axis where $y = 0$:

$\frac{1}{x} + 3 = 0$

$\frac{1}{x} = -3$

$x = -\frac{1}{3}$ $\left(-\frac{1}{3}, 0 \right)$

If you used $x = 0$ you would

$y = \frac{1}{0} + 3$ but $\frac{1}{0}$ is

or infinite.

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Algebraic fractions
Exercise A, Question 29

Question:

Given that $f(x) = (x^2 - 6x)(x - 2) + 3x$,

(a) express f(x) in the form $x(ax^2 + bx + c)$, where a, b and c are constants

(b) hence factorise f(x) completely

(c) sketch the graph of $y = f(x)$, showing the coordinates of each point at which the graph meets the axes

Solution:

(a)

$f(x) = (x^2 - 6x)(x - 2) + 3x$ Multiply out the bracket

$= x^2(x - 2) - 6x(x - 2) + 3x$

$= x^3 - 2x^2 - 6x^2 + 12x + 3x$

$= x^3 - 8x^2 + 15x$ x is a common factor

$= x(x^2 - 8x + 15)$

$(a = 1, b = -8, c = 15)$

(b)

$x(x^2 - 8x + 15)$ Factorise the quadratic

$f(x) = x(x - 3)(x - 5)$

(c)

Curve meets x-axis where $y = 0$.

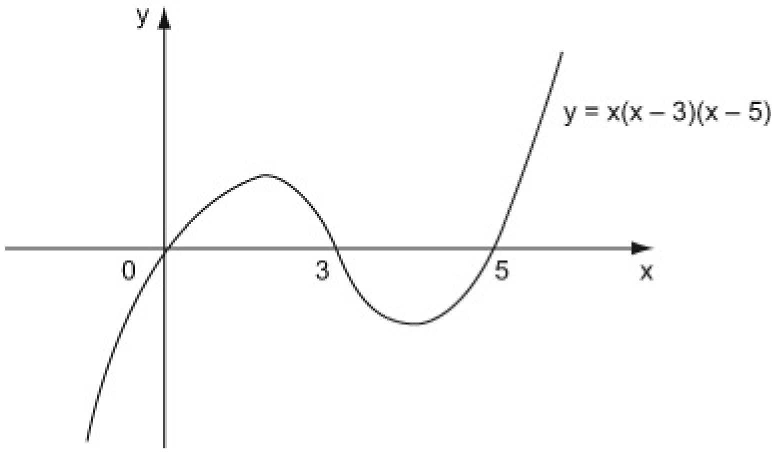
$x(x - 3)(x - 5) = 0$ Put $y = 0$ and solve for x

$x = 0, x = 3, x = 5$

When $x = 0, y = 0$ Put $x = 0$ to find where the curve crosses the y-axis

When $x \to \infty, y \to \infty$ Check what happens to y for large positive and negative values of x.

When $x \to -\infty, y \to -\infty$



Meets x-axis at (0 , 0) , (3 , 0) , (5 , 0)
Meets y-axis at (0 , 0)

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Algebraic fractions
Exercise A, Question 30

Question:

(a) Sketch on the same diagram the graph of $y = x (x + 2) (x - 4)$ and the graph of $y = 3x - x^2$, showing the coordinates of the points at which each graph meets the x-axis.

(b) Find the exact coordinates of each of the intersection points of $y = x (x + 2) (x - 4)$ and $y = 3x - x^2$.

Solution:

(a)

$y = x (x + 2) (x - 4)$

Curve meets x-axis where $y = 0$.

$x (x + 2) (x - 4) = 0$ — Put $y = 0$ and solve for x.

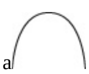
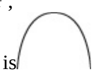
$x = 0 , x = - 2 , x = 4$

When $x = 0 , y = 0$ — Put $x = 0$ to find where the curve crosses the y-axis

When $x \to \infty , y \to \infty$

When $x \to - \infty , y \to - \infty$ — Check what happens to y for large positive and negative values of x.

$y = 3x - x^2$

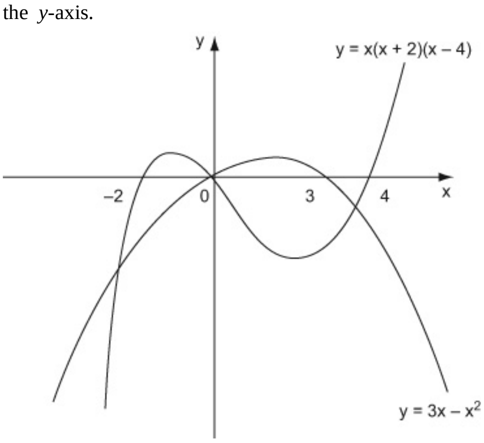
The graph of $y = 3x - x^2$ is a  shape — For $y = ax^2 + bx + c$, if $a < 0$, the shape is 

$3x - x^2 = 0$ — Put $y = 0$ and solve for x.

$x (3 - x) = 0$

$x = 0 , x = 3$

When $x = 0 , y = 0$ — Put $x = 0$ to find where the curve crosses the y-axis.



$y = x (x + 2) (x - 4)$ meets the x-axis at $(- 2 , 0) , (0 , 0) , (4 , 0)$

$y = 3x - x^2$ meets the x-axis at $(0 , 0) , (3 , 0)$

(b)

$x (x + 2) (x - 4) = 3x - x^2$

$x (x + 2) (x - 4) = x (3 - x)$ — To find where the graphs intersect, equate the two expressions for y to give an equation in x.

$(x + 2) (x - 4) = 3 - x$ — If you divide by x, remember that $x = 0$ is a solution.

One solution is $x = 0$

$x^2 - 2x - 8 = 3 - x$

$x^2 - 2x + x - 8 - 3 = 0$

$x^2 - x - 11 = 0$ — The equation does not factorise, so use the quadratic formula.

$a = 1 , b = - 1 , c = - 11$

$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$ Quote the formula

$x = \frac{1 \pm \sqrt{(-1)^2 - (4 \times 1 \times -11)}}{2}$

$= \frac{1 \pm \sqrt{45}}{2}$ Exact values are required, not rounded

$\sqrt{45} = \sqrt{9}\sqrt{5} = 3\sqrt{5}$ decimals, so leave the answers in surd form.

$x = \frac{1}{2}(1 \pm 3\sqrt{5})$

$x = \frac{1}{2}(1 + 3\sqrt{5})$ or $x = \frac{1}{2}(1 - 3\sqrt{5})$

$x = 0 : y = 0$ The y-coordinates for the intersection

$x = \frac{1}{2}(1 + 3\sqrt{5})$ points are also needed.

$y = \frac{3(1 + 3\sqrt{5})}{2} - \frac{(1 + 3\sqrt{5})^2}{4}$ Use $y = 3x - x^2$, the simpler equation

$(1 + 3\sqrt{5})^2 = (1 + 3\sqrt{5})(1 + 3\sqrt{5})$

$= 1(1 + 3\sqrt{5}) + 3\sqrt{5}(1 + 3\sqrt{5})$

$= 1 + 3\sqrt{5} + 3\sqrt{5} + 45$ $\sqrt{5} \times \sqrt{5} = 5$

$= 46 + 6\sqrt{5}$

$y = \frac{6(1 + 3\sqrt{5})}{4} - \frac{46 + 6\sqrt{5}}{4}$ Use a common denominator 4.

$= \frac{6 + 18\sqrt{5} - 46 - 6\sqrt{5}}{4}$

$= \frac{-40 + 12\sqrt{5}}{4} = -10 + 3\sqrt{5}$

$x = \frac{1}{2}(1 - 3\sqrt{5})$

$y = \frac{3(1 - 3\sqrt{5})}{2} - \frac{(1 - 3\sqrt{5})^2}{4}$

$y = \frac{6(1 - 3\sqrt{5})}{4} - \frac{46 - 6\sqrt{5}}{4}$ The working will be similar to that for $1 + 3\sqrt{5}$, so need not be fully repeated.

$= \frac{6 - 18\sqrt{5} - 46 + 6\sqrt{5}}{4}$

$= \frac{-40 - 12\sqrt{5}}{4} = -10 - 3\sqrt{5}$

Intersection points are : Finally, write down the coordinates of all the

$(0, 0)$, $\left(\frac{1}{2}(1 + 3\sqrt{5}), -10 + 3\sqrt{5}\right)$ points you have found. You can compare these with your sketch, as a rough check.

and $\left(\frac{1}{2}(1 - 3\sqrt{5}), -10 - 3\sqrt{5}\right)$

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Algebraic fractions
Exercise A, Question 1

Question:

The line L has equation $y = 5 - 2x$.

(a) Show that the point $P(3, -1)$ lies on L.

(b) Find an equation of the line, perpendicular to L, which passes through P. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

Solution:

(a)

For $x = 3$,
$y = 5 - (2 \times 3) = 5 - 6 = -1$

Substitute $x = 3$ into the equation of L .

So $(3 , -1)$ lies on L .

Give a conclusion.

(b)

$y = -2x + 5$
Gradient of L is -2 .

Compare with $y = mx + c$ to find the gradient m

Perpendicular to L ,
gradient is $\frac{1}{2}$ ($\frac{1}{2} \times -2 = -1$)

For a perpendicular line, the gradient is $-\frac{1}{m}$

$y - (-1) = \frac{1}{2} (x - 3)$

Use $y - y_1 = m (x - x_1)$

$y + 1 = \frac{1}{2}x - \frac{3}{2}$

Multiply by 2

$2y + 2 = x - 3$
$0 = x - 2y - 5$
$x - 2y - 5 = 0$
$(a = 1 , b = -2 , c = -5)$

This is the required form $ax + by + c = 0$, where a , b and c are integers.

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Algebraic fractions
Exercise A, Question 2

Question:

The points *A* and *B* have coordinates (−2, 1) and (5, 2) respectively.

(a) Find, in its simplest surd form, the length *AB*.

(b) Find an equation of the line through *A* and *B*, giving your answer in the form $ax + by + c = 0$, where *a*, *b* and *c* are integers.

The line through *A* and *B* meets the *y*-axis at the point *C*.

(c) Find the coordinates of *C*.

Solution:

(a)

A : (−2 , 1) , B (5 , 2)

$AB = \sqrt{(5 - (-2))^2 + (2 - 1)^2}$ — The distance between two points is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(7^2 + 1^2)} = \sqrt{50}$ (Pythagoras's Theorem)

$\sqrt{50} = \sqrt{(25 \times 2)} = 5\sqrt{2}$ — Use $\sqrt{(ab)} = \sqrt{a}\sqrt{b}$

$AB = 5\sqrt{2}$

(b)

$m = \frac{2 - 1}{5 - (-2)} = \frac{1}{7}$ — Find the gradient of the line, using $m = \frac{y_2 - y_1}{x_2 - x_1}$

$y - 1 = \frac{1}{7}(x - (-2))$ — Use $y - y_1 = m(x - x_1)$

$y - 1 = \frac{1}{7}x + \frac{2}{7}$ — Multiply by 7

$7y - 7 = x + 2$

$0 = x - 7y + 9$

$x - 7y + 9 = 0$ — This is the required form $ax + by + c = 0$, where a, b and c are integers.

$(a = 1 , b = -7 , c = 9)$

(c)

$x = 0$: — Use $x = 0$ to find where the line meets the y-axis.

$0 - 7y + 9 = 0$

$9 = 7y$

$y = \frac{9}{7}$ or $y = 1\frac{2}{7}$

C is the point $\left(0 , 1\frac{2}{7}\right)$

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Algebraic fractions
Exercise A, Question 3

Question:

The line l_1 passes through the point (9, −4) and has gradient $\frac{1}{3}$.

(a) Find an equation for l_1 in the form $ax + by + c = 0$, where *a*, *b* and *c* are integers.

The line l_2 passes through the origin *O* and has gradient −2. The lines l_1 and l_2 intersect at the point *P*.

(b) Calculate the coordinates of *P*.

Given that l_1 crosses the *y*-axis at the point *C*,

(c) calculate the exact area of Δ*OCP*.

Solution:

(a)

$y - (- 4) = \frac{1}{3} (x - 9)$ Use $y - y_1 = m (x - x_1)$

$y + 4 = \frac{1}{3} (x - 9)$

$y + 4 = \frac{1}{3}x - 3$ Multiply by 3

$3y + 12 = x - 9$
$0 = x - 3y - 21$

$x - 3y - 21 = 0$ This is the required form $ax + by + c = 0$, where a, b and c are integers.

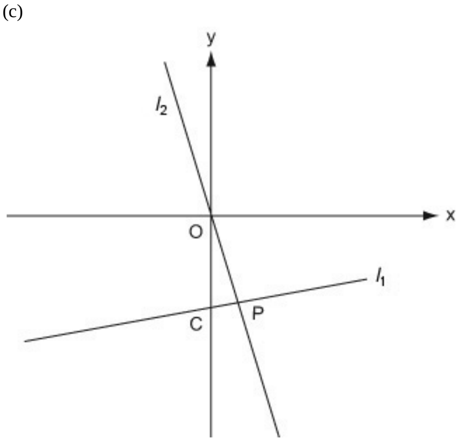
$(a = 1 , b = - 3 , c = - 21)$

(b)
Equation of $l_2 : y = - 2x$ The equation of a straight line through the origin is $y = mx$.

$l_1 : \quad x - 3y - 21 = 0$
$x - 3 (- 2x) - 21 = 0$ Substitute $y = - 2x$ into the equation of l_1
$x + 6x - 21 = 0$
$7x = 21$
$x = 3$
$y = - 2 \times 3 = - 6$ Substitute back into $y = - 2x$

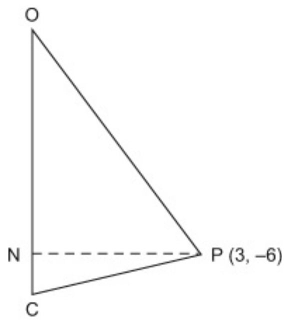
Coordinates of P : $(3 , - 6)$

(c)



Use a rough sketch to show the given information

Be careful not to make any wrong assumptions. Here, for example, ∠ OPC is *not* 90 °



Use OC as the base and PN as the perpendicular height

Where l_1 meets the y-axis, $x = 0$.

$0 - 3y - 21$	$= 0$	Put $x = 0$ in the equation of l_1
$3y$	$= -21$	
y	$= -7$	

So OC $= 7$ and PN $= 3$

The distance of P from the y-axis is the same as its x-coordinate

Area of Δ OCP $= \frac{1}{2}$ (base × height)

$= \frac{1}{2} (7 \times 3)$

$= 10 \frac{1}{2}$

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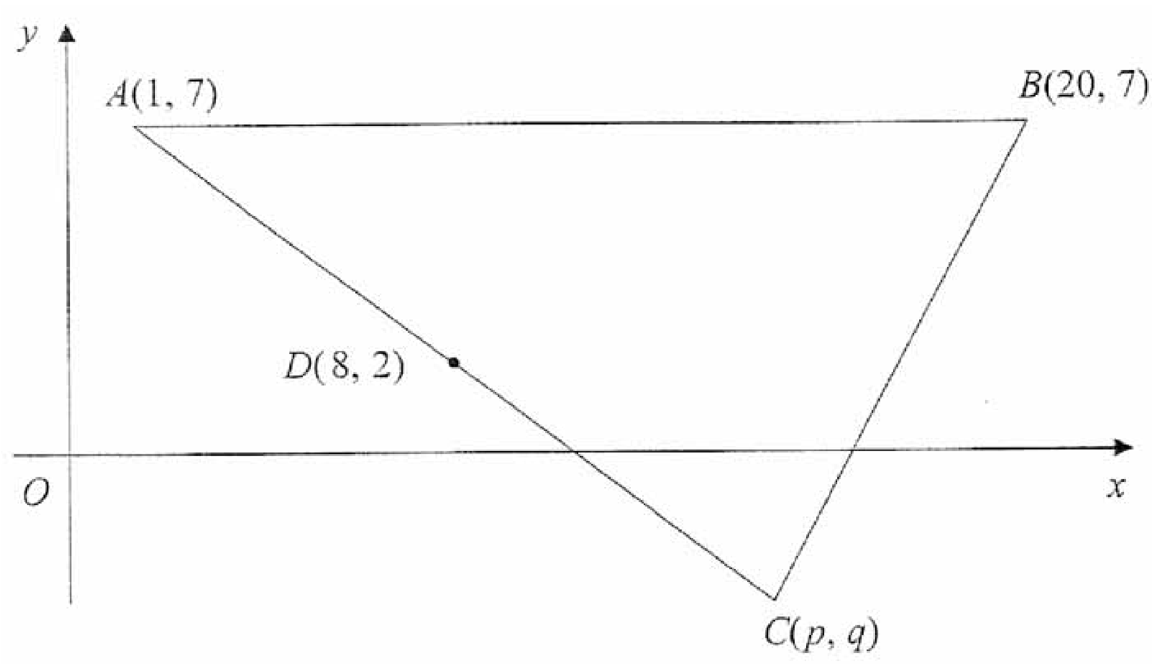
Algebraic fractions
Exercise A, Question 4

Question:



The points $A(1, 7)$, $B(20, 7)$ and $C(p, q)$ form the vertices of a triangle ABC, as shown in the figure. The point $D(8, 2)$ is the mid-point of AC.

(a) Find the value of p and the value of q.

The line l, which passes through D and is perpendicular to AC, intersects AB at E.

(b) Find an equation for l, in the form $ax + by + c = 0$, where a, b and c are integers.

(c) Find the exact x-coordinate of E.

Solution:

(a)

$\left(\frac{1+p}{2} , \frac{7+q}{2} \right) = (8, 2)$

$\left(\frac{x_1+x_2}{2} , \frac{y_1+y_2}{2} \right)$ is the mid-point of the line from (x_1, y_1) to (x_2, y_2)

$\frac{1+p}{2} = 8$ Equate the x-coordinates

$1 + p = 16$

$p = 15$

$\frac{7+q}{2} = 2$ Equate the y-coordinates

$7 + q = 4$

$q = -3$

(b)

Gradient of AC :

$m = \frac{2-7}{8-1} = \frac{-5}{7}$

Use the points A and D, with $m = \frac{y_2 - y_1}{x_2 - x_1}$, to find the gradient of AC (or AD) .

Gradient of l is

$-\frac{1}{\left(\frac{-5}{7}\right)} = \frac{7}{5}$

For a perpendicular line, the gradient is $-\frac{1}{m}$

$y - 2 = \frac{7}{5}(x - 8)$

The line l passes through $D(8, 2)$. So use this point in $y - y_1 = m(x - x_1)$

$y - 2 = \frac{7x}{5} - \frac{56}{5}$

Multiply by 5

$5y - 10 = 7x - 56$

$0 = 7x - 5y - 46$

$7x - 5y - 46 = 0$

$(a = 7, b = -5, c = -46)$

This is in the required form $ax + by + c = 0$, where a, b and c are integers.

(c)

The equation of AB is $y = 7$

At E :

Substitute $y = 7$ into the equation of l to find the point E.

$7x - (5 \times 7) - 46 = 0$

$7x - 35 - 46 = 0$

$7x = 81$

$x = 11\frac{4}{7}$

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Algebraic fractions
Exercise A, Question 5

Question:

The straight line l_1 has equation $y = 3x - 6$.

The straight line l_2 is perpendicular to l_1 and passes through the point (6, 2).

(a) Find an equation for l_2 in the form $y = mx + c$, where m and c are constants.

The lines l_1 and l_2 intersect at the point C.

(b) Use algebra to find the coordinates of C.

The lines l_1 and l_2 cross the x-axis at the point A and B respectively.

(c) Calculate the exact area of triangle ABC.

Solution:

(a)

The gradient of l_1 is 3. Compare with $y = mx + c$.

So the gradient of l_2 is $-\frac{1}{3}$ For a perpendicular line, the gradient is $-\frac{1}{m}$

Eqn. of l_2:

$y - 2 = -\frac{1}{3}(x - 6)$ Use $y - y_1 = m(x - x_1)$

$y - 2 = -\frac{1}{3}x + 2$

$y = -\frac{1}{3}x + 4$ This is the required form $y = mx + c$.

(b)

$y = 3x - 6$ Solve these equations simultaneously

$y = -\frac{1}{3}x + 4$

$3x - 6 = -\frac{1}{3}x + 4$

$3x + \frac{1}{3}x = 4 + 6$

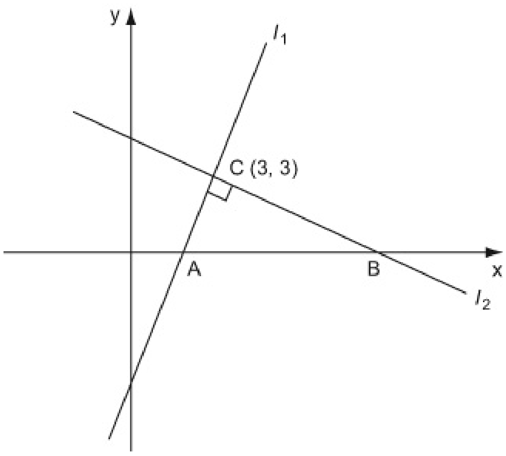
$\frac{10}{3}x = 10$ Multiply by 3 and divide by 10

$x = 3$

$y = (3 \times 3) - 6 = 3$ Substitute back into $y = 3x - 6$

The point C is $(3, 3)$

(c)



Use a rough sketch to show the given information.

Where l_1 meets the x-axis , $y = 0$:

Put $y = 0$ to find where the lines meet the x-axis

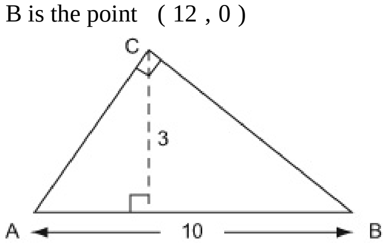
$$0 = 3x - 6$$
$$3x = 6$$
$$x = 2$$

A is the point (2 , 0)

Where l_2 meets the x-axis , $y = 0$:

$$0 = -\frac{1}{3}x + 4$$
$$\frac{1}{3}x = 4$$
$$x = 12$$

B is the point (12 , 0)



Although $\angle$C is a right-angle, it is easier to use AB as the base.

AB = 10 (12 − 2)

The perpendicular height, using AB as the base, is 3

The distance of C from the x-axis is the same as its y-coordinate.

$$\text{Area of } \Delta \text{ ABC} = \frac{1}{2} (\text{ base} \times \text{height })$$
$$= \frac{1}{2} (10 \times 3)$$
$$= 15$$

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Algebraic fractions
Exercise A, Question 6

Question:

The line l_1 has equation $6x - 4y - 5 = 0$.

The line l_2 has equation $x + 2y - 3 = 0$.

(a) Find the coordinates of P, the point of intersection of l_1 and l_2.

The line l_1 crosses the y-axis at the point M and the line l_2 crosses the y-axis at the point N.

(b) Find the area of ΔMNP.

Solution:

(a)

$6x - 4y - 5$	$= 0 \quad (\text{i})$	Solve the equations
$x + 2y - 3$	$= 0 \quad (\text{ii})$	simultaneously
x	$= 3 - 2y$	Find x in terms of y from equation (ii)
$6 (3 - 2y) - 4y - 5$	$= 0$	Substitute into equation (i)
$18 - 12y - 4y - 5$	$= 0$	
$18 - 5$	$= 12y + 4y$	
$16y$	$= 13$	
y	$= \frac{13}{16}$	
$x = 3 - 2 (\frac{13}{16})$	$= 3 - \frac{26}{16}$	Substitute back into $x = 3 - 2y$
x	$= 1 \frac{3}{8}$	

P is the point $(1 \frac{3}{8} , \frac{13}{16})$

(b)

Where l_1 meets the y-axis, $x = 0$

Put $x = 0$ to find where the lines meet the y-axis.

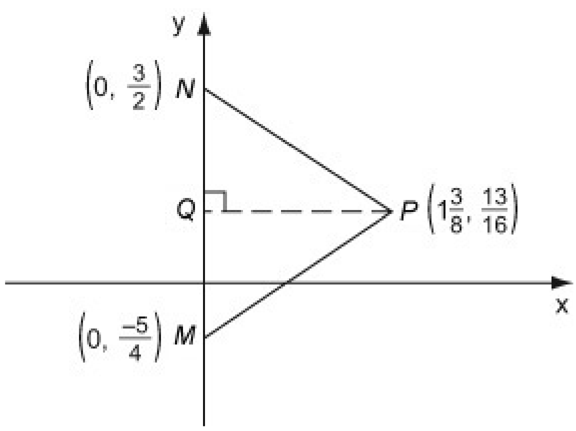
$$0 - 4y - 5 = 0$$
$$4y = -5$$
$$y = -\frac{5}{4}$$

M is the point $\left(0, \frac{-5}{4}\right)$

Where l_2 meets the y-axis, $x = 0$:

$$0 + 2y - 3 = 0$$
$$2y = 3$$
$$y = \frac{3}{2}$$

N is the point $\left(0, \frac{3}{2}\right)$



Use a rough sketch to show the information

Use MN as the base and PQ as the perpendicular height.

$$MN = \frac{3}{2} + \frac{5}{4} = \frac{11}{4}$$

The distance of P from the y-axis is the same as its x-coordinate

$$PQ = 1\frac{3}{8} = \frac{11}{8}$$

Area of ΔMNP $= \frac{1}{2}$ (base × height)

$$= \frac{1}{2}\left(\frac{11}{4} \times \frac{11}{8}\right)$$
$$= \frac{121}{64}$$
$$= 1\frac{57}{64}$$

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Algebraic fractions
Exercise A, Question 7

Question:

The 5th term of an arithmetic series is 4 and the 15th term of the series is 39.

(a) Find the common difference of the series.

(b) Find the first term of the series.

(c) Find the sum of the first 15 terms of the series.

Solution:

(a)

n^{th} term $= a + (n-1)d$

$n = 5: \quad a + 4d = 4 \quad (\text{i})$

$n = 15: \quad a + 14d = 39 \quad (\text{ii})$

Substitute the given values into the n^{th} term formula.

Subtract (ii)-(i)

$10d = 35$

$d = 3\frac{1}{2}$

Solve simultaneously.

Common difference is $3\frac{1}{2}$

(b)

$a + (4 \times 3\frac{1}{2}) = 4$

Substitute back into equation (i).

$a + 14 = 4$

$a = -10$

First term is −10

(c)

$S_n = \frac{1}{2}n\left(2a + (n-1)d\right)$

$n = 15$, $a = -10$, $d = 3\frac{1}{2}$

Substitute the values into the sum formula.

$S_{15} = \frac{1}{2} \times 15\left(-20 + \left(14 \times 3\frac{1}{2}\right)\right)$

$= \frac{15}{2}\left(-20 + 49\right)$

$= \frac{15}{2} \times 29$

$= 217\frac{1}{2}$

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Algebraic fractions

Exercise A, Question 8

Question:

An athlete prepares for a race by completing a practice run on each of 11 consecutive days. On each day after the first day, he runs farther than he ran on the previous day. The lengths of his 11 practice runs form an arithmetic sequence with first term *a* km and common difference *d* km.

He runs 9 km on the 11th day, and he runs a total of 77 km over the 11 day period.

Find the value of *a* and the value of *d*.

Solution:

n^{th} term $= a + (n - 1) d$ — The distance run on the 11th day is the 11th term of the arithmetic sequence.

$n = 11 : \quad a + 10d = 9$

$S_n = \frac{1}{2} n (2a + (n - 1) d)$ — The total distance run is the sum of the arithmetic series.

$S_n = 77 , n = 11 :$

$\frac{1}{2} \times 11 (2a + 10d) = 77$

$\frac{1}{2} (2a + 10d) = 7$ — It is simpler to divide each side of the equation by 11.

$a + 5d = 7$

$a + 10d = 9 \quad (i)$ — Solve simultaneously

$a + 5d = 7 \quad (ii)$

Subtract (i)-(ii):

$5d = 2$

$d = \frac{2}{5}$

$a + (10 \times \frac{2}{5}) = 9$ — Substitute back into equation (i).

$a + 4 = 9$

$a = 5$

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Algebraic fractions
Exercise A, Question 9

Question:

The rth term of an arithmetic series is $(2r - 5)$.

(a) Write down the first three terms of this series.

(b) State the value of the common difference.

(c) Show that $\sum_{r=1}^{n} \left(2r - 5 \right) = n \left(n - 4 \right)$.

Solution:

(a)

$r = 1 : \quad 2r - 5 = -3$

$r = 2 : \quad 2r - 5 = -1$

$r = 3 : \quad 2r - 5 = 1$

First three terms are $-3, -1, 1$

(b)

Common difference $d = 2$

The terms increase by 2 each time $(U_{k+1} = U_{k+2})$

(c)

$\sum_{r=1}^{n} (2r - 5) = S_n$

$\sum_{r=1}^{n} (2r - 5)$ is just the sum of the series to n terms

$S_n = \frac{1}{2} n (2a + (n - 1) d)$

$a = -3, d = 2$

$S_n = \frac{1}{2} n (-6 + 2 (n - 1))$

$= \frac{1}{2} n (-6 + 2n - 2)$

$= \frac{1}{2} n (2n - 8)$

$= \frac{1}{2} n 2 (n - 4)$

$= n (n - 4)$

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Algebraic fractions

Exercise A, Question 10

Question:

Ahmed plans to save £250 in the year 2001, £300 in 2002, £350 in 2003, and so on until the year 2020. His planned savings form an arithmetic sequence with common difference £50.

(a) Find the amount he plans to save in the year 2011.

(b) Calculate his total planned savings over the 20 year period from 2001 to 2020.

Ben also plans to save money over the same 20 year period. He saves £*A* in the year 2001 and his planned yearly savings form an arithmetic sequence with common difference £60.

Given that Ben's total planned savings over the 20 year period are equal to Ahmed's total planned savings over the same period,

(c) calculate the value of *A*.

Solution:

(a)

$a = 250$ (Year 2001)

$d = 50$

Write down the values of a and d for the arithmetic series

Taking 2001 as Year 1 ($n = 1$) ,

2011 is Year 11 ($n = 11$).

Year 11 savings:

$a + (n - 1) d = 250 + (11 - 1) 50$

$= 250 + (10 \times 50)$

$= 750$

Use the term formula $a + (n - 1) d$

Year 11 savings : £ 750

(b)

$S_n = \frac{1}{2}n(2a + (n-1)d)$

Using $n = 20$,

$S_{20} = \frac{1}{2} \times 20(500 + (19 \times 50))$

$= 10(500 + 950)$

$= 10 \times 1450$

$= 14500$

The total savings will be the sum of the arithmetic series.

Total savings : £ 14 500

(c)

$a = A$ (Year2001)

$d = 60$

Write down the values of a and d for Ben's series.

$S_{20} = \frac{1}{2} \times 20(2A + (19 \times 60))$

Use the sum formula.

$S_{20} = 10(2A + 1140)$

$= 20A + 11400$

$20A + 11400 = 14500$

$20A = 14500 - 11400$

$20A = 3100$

$A = 155$

Equate Ahmed's and Ben's total savings.

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Algebraic fractions
Exercise A, Question 11

Question:

A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 3,$$
$$a_{n+1} = 3a_n - 5, \quad n \geq 1.$$

(a) Find the value of a_2 and the value of a_3.

(b) Calculate the value of $\sum_{r=1}^{5} a_r$.

Solution:

(a)

$$a_{n+1} = 3a_n - 5$$
$$n = 1: \quad a_2 = 3a_1 - 5$$
$$a_1 = 3, \text{ so } a_2 = 9 - 5$$
$$a_2 = 4$$
$$n = 2: a_3 = 3a_2 - 5$$
$$a_2 = 4, \text{ so } a_3 = 12 - 5$$
$$a_3 = 7$$

Use the given formula, with $n = 1$ and $n = 2$

(b)

$\sum_{a=1}^{5} a_r = a_1 + a_2 + a_3 + a_4 + a_5$

$n = 3 : a_4$	$= 3a_3 - 5$	This is not an arithmetic series.
$a_3 = 7$, so a_4	$= 21 - 5$	The first three terms are 3, 4, 7.
a_4	$= 16$	The differences between
$n = 4 : a_5$	$= 3a_4 - 5$	the terms are not the same.
$a_4 = 16$, so a_5	$= 48 - 5$	You cannot use a standard formula,
a_5	$= 43$	so work out each separate term and
$\sum_{a=1}^{5} a_r$	$= 3 + 4 + 7 + 16 + 43$	then add them together to find
	$= 73$	the required sum.

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Algebraic fractions
Exercise A, Question 12

Question:

A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = k ,$$
$$a_{n+1} = 3a_n + 5 , \qquad n \geq 1 ,$$

where k is a positive integer.

(a) Write down an expression for a_2 in terms of k.

(b) Show that $a_3 = 9k + 20$.

(c) (i) Find $\sum_{r=1}^{4} a_r$ in terms of k.

(ii) Show that $\sum_{r=1}^{4} a_r$ is divisible by 10.

Solution:

(a)

$$a_{n+1} = 3a_n + 5$$
$$n = 1 : a_2 = 3a_1 + 5$$
$$a_2 = 3k + 5$$

Use the given formula with $n = 1$

(b)

$$n = 2 : a_3 = 3a_2 + 5$$
$$= 3 (3k + 5) + 5$$
$$= 9k + 15 + 5$$
$$a_3 = 9k + 20$$

(c)(i)

$\sum_{r=1}^{4} a_r = a_1 + a_2 + a_3 + a_4$

This is *not* an arithmetic series.

$n = 3 : a_4 = 3a_3 + 5$

$= 3 (9k + 20) + 5$

$= 27k + 65$

You cannot use a standard formula, so work out each separate term and then add them together

$\sum_{r=1}^{4} a_r = k + (3k + 5) + (9k + 20) + (27k + 65)$

$= 40k + 90$

to find the required sum.

(ii)

$\sum_{r=1}^{4} a_r = 10 (4k + 9)$

There is a factor 10 , so the sum is divisible by 10.

Give a conclusion.

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Algebraic fractions
Exercise A, Question 13

Question:

A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = k$$
$$a_{n+1} = 2a_n - 3, \qquad n \geq 1$$

(a) Show that $a_5 = 16k - 45$

Given that $a_5 = 19$, find the value of

(b) k

(c) $\sum_{r=1}^{6} a_r$

Solution:

(a)

$$\begin{aligned}
a_{n+1} &= 2a_n - 3 \\
n = 1: \quad a_2 &= 2a_1 - 3 \\
&= 2k - 3 \\
n = 2: \quad a_3 &= 2a_2 - 3 \\
&= 2(2k - 3) - 3 \\
&= 4k - 6 - 3 \\
&= 4k - 9 \\
n = 3: \quad a_4 &= 2a_3 - 3 \\
&= 2(4k - 9) - 3 \\
&= 8k - 18 - 3 \\
&= 8k - 21 \\
n = 4: \quad a_5 &= 2a_4 - 3 \\
&= 2(8k - 21) - 3 \\
&= 16k - 42 - 3 \\
a_5 &= 16k - 45
\end{aligned}$$

Use the given formula with $n = 1, 2, 3$ and 4.

(b)

$a_5 = 19$,

so $16k - 45 = 19$

$16k = 19 + 45$

$16k = 64$

$k = 4$

(c)

$$\sum_{r=1}^{6} a_r = a_1 + a_2 + a_3 + a_4 + a_5 + a_6$$

This is *not* an arithmetic series.

You cannot use a standard formula, so work out each separate term and then add them together to find the required sum.

$a_1 = k = 4$

$a_2 = 2k - 3 = 5$

$a_3 = 4k - 9 = 7$

$a_4 = 8k - 21 = 11$

$a_5 = 16k - 45 = 19$

From the original formula,

$a_6 = 2a_5 - 3 = (2 \times 19) - 3$

$= 35$

$$\sum_{r=1}^{6} a_r = 4 + 5 + 7 + 11 + 19 + 35$$

$= 81$

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Algebraic fractions
Exercise A, Question 14

Question:

An arithmetic sequence has first term a and common difference d.

(a) Prove that the sum of the first n terms of the series is

$$\frac{1}{2}n\left[2a+\left(n-1\right)d\right]$$

Sean repays a loan over a period of n months. His monthly repayments form an arithmetic sequence.

He repays £149 in the first month, £147 in the second month, £145 in the third month, and so on. He makes his final repayment in the nth month, where $n > 21$.

(b) Find the amount Sean repays in the 21st month.

Over the n months, he repays a total of £5000.

(c) Form an equation in n, and show that your equation may be written as

$$n^2 - 150n + 5000 = 0$$

(d) Solve the equation in part (c).

(e) State, with a reason, which of the solutions to the equation in part (c) is not a sensible solution to the repayment problem.

Solution:

(a)

S_n	$= a + (a+d) + (a+2d) + \ldots .. + (a + (n-1)d)$	You need to know this proof . Make
Reversing the sum :	sure that you understand it, and do	
S_n	$= (a+(n-1)d) + \ldots .. + (a+2d) + (a+d) + a$	not miss out any of the steps.
Adding these two :	When you add, each pair of terms	
$2S_n$	$= (2a+(n-1)d) + \ldots \ldots + (2a + (n-1)d)$	
$2S_n$	$= n(2a+(n-1)d)$	adds up to $2a + (n-1)d$, and there are n pairs of terms.
S_n	$= \frac{1}{2}n(2a+(n-1)d)$	

(b)

$a = 149$ (First month)

$d = -2$

Write down the values of a and d for the arithmetic series.

21st month:

$a + (n-1)d = 149 + (20 \times -2)$
$= 149 - 40$
$= 109$

Use the term formula $a + (n-1)d$

He repays £ 109 in the 21st month

(c)

$S_n = \frac{1}{2}n(2a + (n-1)d)$

The total he repays will be the sum of the arithmetic series.

$= \frac{1}{2}n(298 - 2(n-1))$

$= \frac{1}{2}n(298 - 2n + 2)$

$= \frac{1}{2}n(300 - 2n)$

$= \frac{1}{2}n2(150 - n)$

$= n(150 - n)$

$n(150 - n) = 5000$

Equate S_n to 5000

$150n - n^2 = 5000$

$n^2 - 150n + 5000 = 0$

(d)

$(n-50)(n-100) = 0$

$n = 50$ or $n = 100$

Always try to factorise the quadratic. The quadratic formula would be awkward here with such large numbers.

(e)

$n = 100$ is not sensible.
For example, his repayment in month 100 $(n = 100)$ would be $a + (n - 1)d$

$= 149 + (99 \times -2)$
$= 149 - 198$
$= -49$

Check back in the context of the problem to see if the solution is sensible.

A negative repayment is not sensible.

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Algebraic fractions

Exercise A, Question 15

Question:

A sequence is given by

$$a_1 = 2$$
$$a_{n+1} = a_n^2 - ka_n, \qquad n \geq 1,$$

where k is a constant.

(a) Show that $a_3 = 6k^2 - 20k + 16$

Given that $a_3 = 2$,

(b) find the possible values of k.

For the larger of the possible values of k, find the value of

(c) a_2

(d) a_5

(e) a_{100}

Solution:

(a)

$$a_{n+1} = a_n^2 - ka_n$$

$n = 1: \quad a_2 = a_1^2 - ka_1$

$= 4 - 2k$

Use the given formula with $n = 1$ and 2.

$n = 2: \quad a_3 = a_2^2 - ka_2$

$= (4 - 2k)^2 - k(4 - 2k)$

$= 16 - 16k + 4k^2 - 4k + 2k^2$

$a_3 = 6k^2 - 20k + 16$

(b)

$a_3 = 2:$

$6k^2 - 20k + 16 = 2$

$6k^2 - 20k + 14 = 0$

Divide by 2 to make solution easier

$3k^2 - 10k + 7 = 0$

$(3k - 7)(k - 1) = 0$

$k = \frac{7}{3}$ or $k = 1$

Try to factorise the quadratic rather than using the quadratic formula.

(c)

The larger k value is $\frac{7}{3}$

$a_2 = 4 - 2k = 4 - \left(2 \times \frac{7}{3}\right)$

$= 4 - \frac{14}{3} = -\frac{2}{3}$

(d)

$a_{n+1} = a_n^2 - \frac{7}{3}a_n$

$n = 3 : a_4 = a_3^2 - \frac{7}{3}a_3$

But $a_3 = 2$ is given, so

$a_4 = 2^2 - \left(\frac{7}{3} \times 2\right)$

$= 4 - \frac{14}{3} = \frac{-2}{3}$

$n = 4 : \quad a_5 = a_4^2 - \frac{7}{3}a_4$

$= \left(\frac{-2}{3}\right)^2 - \left(\frac{7}{3} \times \frac{-2}{3}\right)$

$= \frac{4}{9} + \frac{14}{9} = \frac{18}{9}$

$a_5 = 2$

Use the formula with $k = \frac{7}{3}$, for $n = 3$ and 4.

(e)

$a_2 = \frac{-2}{3}, a_3 = 2$

$a_4 = \frac{-2}{3}, a_5 = 2$

For even values of n, $a_n = \frac{-2}{3}$.

So $a_{100} = \frac{-2}{3}$

Notice that the sequence is "oscillating" between the values $\frac{-2}{3}$ and 2.

If n is even, $a_n = \frac{-2}{3}$.

If n is odd, $a_n = 2$.

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Algebraic fractions
Exercise A, Question 16

Question:

Given that

$$y = 4x^3 - 1 + 2x^{\frac{1}{2}} , \quad x > 0 ,$$

find $\frac{dy}{dx}$.

Solution:

$y = 4x^3 - 1 + 2x^{\frac{1}{2}}$

For $y = x^n$, $\frac{dy}{dx} = nx^{n-1}$

$\frac{dy}{dx} = (4 \times 3x^2) + (2 \times \frac{1}{2}x^{-\frac{1}{2}})$

Differentiating the constant -1 gives zero.

$\frac{dy}{dx} = 12x^2 + x^{-\frac{1}{2}}$

It is better to write down an unsimplified version of the answer first (in case you make a mistake when simplifying).

(Or: $\frac{dy}{dx} = 12x^2 + \frac{1}{x^{\frac{1}{2}}}$

Or: $\frac{dy}{dx} = 12x^2 + \frac{1}{\sqrt{x}}$)

It is not necessary to change your answer into one of these forms.

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Algebraic fractions
Exercise A, Question 17

Question:

Given that $y = 2x^2 - \frac{6}{x^3}$, $x \neq 0$,

(a) find $\frac{dy}{dx}$,

(b) find $\int y \, dx$.

Solution:

(a)

$y = 2x^2 - \frac{6}{x^3}$

Use $\frac{1}{x^n} = x^{-n}$

$= 2x^2 - 6x^{-3}$

$\frac{dy}{dx} = (2 \times 2x^1) - (6 \times -3x^{-4})$

For $y = x^n$, $\frac{dy}{dx} = nx^{n-1}$

$\frac{dy}{dx} = 4x + 18x^{-4}$

Write down an unsimplified version of the answer first.

(Or: $\frac{dy}{dx} = 4x + \frac{18}{x^4}$)

It is not necessary to change your answer into this form.

(b)

$\int (2x^2 - 6x^{-3}) \, dx$

Use $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

$= \frac{2x^3}{3} - \frac{6x^{-2}}{-2} + C$

Do not forget to include the constant of integration, C.

$= \frac{2x^3}{3} + 3x^{-2} + C$

Write down an unsimplified version of the answer first

(Or: $\frac{2x^3}{3} + \frac{3}{x^2} + C$)

It is not necessary to change your answer into this form.

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Algebraic fractions
Exercise A, Question 18

Question:

Given that $y = 3x^2 + 4\sqrt{x}$, $x > 0$, find

(a) $\frac{dy}{dx}$,

(b) $\frac{d^2y}{dx^2}$,

(c) $\int y \ dx$.

Solution:

(a)

$y = 3x^2 + 4\sqrt{x}$ Use $\sqrt{x} = x^{\frac{1}{2}}$

$= 3x^2 + 4x^{\frac{1}{2}}$

$\frac{dy}{dx} = (3 \times 2x^1) + (4 \times \frac{1}{2}x^{-\frac{1}{2}})$ For $y = x^n$, $\frac{dy}{dx} = nx^{n-1}$

$\frac{dy}{dx} = 6x + 2x^{-\frac{1}{2}}$ Write down an unsimplified version of the answer first.

(Or: $\frac{dy}{dx} = 6x + \frac{2}{x^{\frac{1}{2}}}$

Or: $\frac{dy}{dx} = 6x + \frac{2}{\sqrt{x}}$) It is not necessary to change your answer into one of these forms

(b)

$\frac{dy}{dx} = 6x + 2x^{-\frac{1}{2}}$

Differentiate again

$\frac{d^2y}{dx^2} = 6 + \left(2 \times \frac{-1}{2} x^{-\frac{3}{2}} \right)$

$= 6 - x^{-\frac{3}{2}}$

(Or: $\frac{d^2y}{dx^2} = 6 - \frac{1}{x^{\frac{3}{2}}}$)

Or: $\frac{d^2y}{dx^2} = 6 - \frac{1}{x\sqrt{x}}$

It is not necessary to change your answer into one of these forms.

$x^{\frac{3}{2}} = x^1 \times x^{\frac{1}{2}} = x\sqrt{x}$

(c)

$\int \left(3x^2 + 4x^{\frac{1}{2}} \right) dx$

Use $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ Do not forget to include the constant of integration, C

$= \frac{3x^3}{3} + \frac{4x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + C$

Write down an unsimplified version of the answer first.

$= x^3 + 4\left(\frac{2}{3}\right) x^{\frac{3}{2}} + C$

$= x^3 + \frac{8}{3} x^{\frac{3}{2}} + C$

(Or: $x^3 + \frac{8}{3} x\sqrt{x} + C$)

It is not necessary to change your answer into this form.

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Algebraic fractions
Exercise A, Question 19

Question:

(i) Given that $y = 5x^3 + 7x + 3$, find

(a) $\frac{dy}{dx}$,

(b) $\frac{d^2y}{dx^2}$.

(ii) Find $\int \left(1 + 3\sqrt{x} - \frac{1}{x^2} \right) dx$.

Solution:

(i)

$y = 5x^3 + 7x + 3$

(a)

$$\frac{dy}{dx} = (5 \times 3x^2) + (7 \times 1x^0)$$

For $y = x^n$, $\frac{dy}{dx} = nx^{n-1}$.

Differentiating the constant 3 gives zero.

$$\frac{dy}{dx} = 15x^2 + 7$$

Use $x^0 = 1$

Differentiating Kx gives K .

(b)

$$\frac{dy}{dx} = 15x^2 + 7$$

Differentiate again

$$\frac{d^2y}{dx^2} = (15 \times 2x^1)$$

$$= 30x$$

(ii)

$\int \left(1 + 3\sqrt{x} - \frac{1}{x^2} \right) dx$

Use $\sqrt{x} = x^{\frac{1}{2}}$ and $\frac{1}{x^n} = x^{-n}$

$= \int \left(1 + 3x^{\frac{1}{2}} - x^{-2} \right) dx$

Use $\int x^n dx = \frac{x^{n+1}}{n+1} + C$.

Do not forget to include the constant of integration C.

$= x + \frac{3x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} - \frac{x^{-1}}{(-1)} + C$

$= x + \left(3 \times \frac{2}{3} x^{\frac{3}{2}} \right) + x^{-1} + C$

$= x + 2x^{\frac{3}{2}} + x^{-1} + C$

(Or: $x + 2x\sqrt{x} + \frac{1}{x} + C$)

It is not necessary to change your answer into this form.

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Algebraic fractions
Exercise A, Question 20

Question:

The curve C has equation $y = 4x + 3x^{\frac{3}{2}} - 2x^2$, $x > 0$.

(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(b) Show that the point $P(4, 8)$ lies on C.

(c) Show that an equation of the normal to C at the point P is

$$3y = x + 20 .$$

The normal to C at P cuts the x-axis at the point Q.

(d) Find the length PQ, giving your answer in a simplified surd form.

Solution:

(a)

$y = 4x + 3x^{\frac{3}{2}} - 2x^2$

$\frac{dy}{dx} = (4 \times 1x^0) + (3 \times \frac{3}{2}x^{\frac{1}{2}}) - (2 \times 2x^1)$

For $y = x^n$, $\frac{dy}{dx} = nx^{n-1}$

$\frac{dy}{dx} = 4 + \frac{9}{2}x^{\frac{1}{2}} - 4x$

(b)

For $x = 4$,

$y = (4 \times 4) + (3 \times 4^{\frac{3}{2}}) - (2 \times 4^2)$

$= 16 + (3 \times 4 \times 2) - 32$

$= 16 + 24 - 32 = 8$

$x^{\frac{3}{2}} = x^1 \times x^{\frac{1}{2}} = x\sqrt{x}$

So P (4 , 8) lies on C

(c)

For $x = 4$, The value of $\frac{dy}{dx}$ is the gradient of the tangent.

$\frac{dy}{dx} = 4 + \left(\frac{9}{2} \times 4^{\frac{1}{2}} \right) - (4 \times 4)$

$= 4 + \left(\frac{9}{2} \times 2 \right) - 16$

$= 4 + 9 - 16 = -3$

The gradient of the normal at P is $\frac{1}{3}$

The normal is perpendicular to the tangent, so the gradient is $-\frac{1}{m}$

Equation of the normal :

$y - 8 = \frac{1}{3}(x - 4)$ Use $y - y_1 = m(x - x_1)$

$y - 8 = \frac{x}{3} - \frac{4}{3}$ Multiply by 3

$3y - 24 = x - 4$

$3y = x + 20$

(d)

$y = 0$: $0 = x + 20$

$x = -20$

Use $y = 0$ to find where the normal cuts the x-axis.

Q is the point $(-20, 0)$

$PQ = \sqrt{(4 - -20)^2 + (8 - 0)^2}$

$= \sqrt{24^2 + 8^2}$

$= \sqrt{576 + 64}$

$= \sqrt{640}$

$= \sqrt{64} \times \sqrt{10}$

$= 8\sqrt{10}$

The distance between two points is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

To simplify the surd, find a factor which is an exact square (here $64 = 8^2$)

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Algebraic fractions
Exercise A, Question 21

Question:

The curve C has equation $y = 4x^2 + \frac{5 - x}{x}$, $x \neq 0$. The point P on C has x-coordinate 1.

(a) Show that the value of $\frac{dy}{dx}$ at P is 3.

(b) Find an equation of the tangent to C at P.

This tangent meets the x-axis at the point $(k, 0)$.

(c) Find the value of k.

Solution:

(a)

$y = 4x^2 + \frac{5 - x}{x}$ Divide $5 - x$ by x

$= 4x^2 + 5x^{-1} - 1$ For $y = x^n$, $\frac{dy}{dx} = nx^{n-1}$

$\frac{dy}{dx} = (4 \times 2x^1) + (5x - 1x^{-2})$ Differentiating the constant -1 gives zero

$\frac{dy}{dx} = 8x - 5x^{-2}$

At P, $x = 1$, so

$\frac{dy}{dx} = (8 \times 1) - (5 \times 1^{-2})$ $1^{-2} = \frac{1}{1^2} = \frac{1}{1} = 1$

$= 8 - 5 = 3$

(b)

At $x = 1$, $\frac{dy}{dx} = 3$ The value of $\frac{dy}{dx}$ is the gradient of the tangent

At $x = 1$, $y = (4 \times 1^2) + \frac{5 - 1}{1}$

$y = 4 + 4 = 8$

Equation of the tangent:

$y - 8 = 3(x - 1)$ Use $y - y_1 = m(x - x_1)$

$y = 3x + 5$

(c)

$y = 0$: $0 = 3x + 5$ Use $y = 0$ to find where the tangent meets the x-axis

$3x = -5$

$x = -\frac{5}{3}$

So K $= -\frac{5}{3}$

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Algebraic fractions
Exercise A, Question 22

Question:

The curve C has equation $y = \frac{1}{3}x^3 - 4x^2 + 8x + 3$.

The point P has coordinates (3, 0).

(a) Show that P lies on C.

(b) Find the equation of the tangent to C at P, giving your answer in the form $y = mx + c$, where m and c are constants.

Another point Q also lies on C. The tangent to C at Q is parallel to the tangent to C at P.

(c) Find the coordinates of Q.

Solution:

(a)

$$y = \frac{1}{3}x^3 - 4x^2 + 8x + 3$$

At $x = 3$,

$$y = \left(\frac{1}{3} \times 3^3 \right) - \left(4 \times 3^2 \right) + \left(8 \times 3 \right) + 3$$
$$= 9 - 36 + 24 + 3$$
$$= 0$$

So P (3 , 0) lies on C

(b)

$\frac{dy}{dx} = \left(\frac{1}{3} \times 3x^2 \right) - \left(4 \times 2x^1 \right) + \left(8 \times 1x^0 \right)$

For $y = x^n$, $\frac{dy}{dx} = nx^{n-1}$

Differentiating the constant 3 gives zero.

$= x^2 - 8x + 8$

At $x = 3$,

$\frac{dy}{dx} = 3^2 - \left(8 \times 3 \right) + 8$

$= 9 - 24 + 8 = -7$

The value of $\frac{dy}{dx}$ is the gradient of the tangent.

Equation of the tangent :

$y - 0 = -7 \left(x - 3 \right)$

Use $y - y_1 = m \left(x - x_1 \right)$

$y = -7x + 21$

This is in the required form $y = mx + c$

(c)

At Q, $\frac{dy}{dx} = -7$

If the tangents are parallel, they have the same gradient.

$x^2 - 8x + 8 = -7$

$x^2 - 8x + 15 = 0$

$\left(x - 3 \right) \left(x - 5 \right) = 0$

$x = 3$ or $x = 5$ $x = 3$ at the point P

For Q, $x = 5$

$y = \left(\frac{1}{3} \times 5^3 \right) - \left(4 \times 5^2 \right) + \left(8 \times 5 \right) + 3$

Substitute $x = 5$ back into the equation of C

$= \frac{125}{3} - 100 + 40 + 3$

$= -15 \frac{1}{3}$

Q is the point $\left(5, -15 \frac{1}{3} \right)$

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Algebraic fractions
Exercise A, Question 23

Question:

$$f\left(x\right)=\frac{(2x+1)(x+4)}{\sqrt{x}}, \quad x>0$$

(a) Show that f(x) can be written in the form $Px^{\frac{3}{2}}+Qx^{\frac{1}{2}}+Rx^{-\frac{1}{2}}$, stating the values of the constants P, Q and R.

(b) Find $f'(x)$.

(c) Show that the tangent to the curve with equation $y=f(x)$ at the point where $x=1$ is parallel to the line with equation $2y=11x+3$.

Solution:

(a)

$$f(x)=\frac{(2x+1)(x+4)}{\sqrt{x}}$$

$$=\frac{2x^2+9x+4}{\sqrt{x}}$$

Divide each term by $x^{\frac{1}{2}}$, remembering that $x^m \div x^n = x^{m-n}$

$$=2x^{\frac{3}{2}}+9x^{\frac{1}{2}}+4x^{\frac{-1}{2}}.$$

$P=2, \quad Q=9, \quad R=4$

(b)

$$f'(x)=\left(2\times\frac{3}{2}x^{\frac{1}{2}}\right)+\left(9\times\frac{1}{2}x^{\frac{-1}{2}}\right)+\left(4\times\frac{-1}{2}x^{\frac{-3}{2}}\right)$$

$f'(x)$ is the derivative of $f(x)$, so differentiate

$$f'(x)=3x^{\frac{1}{2}}+\frac{9}{2}x^{\frac{-1}{2}}-2x^{\frac{-3}{2}}$$

(c)

At $x = 1$,

$f'(1) = \left(3 \times 1^{\frac{1}{2}} \right) + \left(\frac{9}{2} \times 1^{\frac{-1}{2}} \right) - \left(2 \times 1^{\frac{-3}{2}} \right)$

$f'(1)$ is the gradient of the tangent at $x = 1$

$= 3 + \frac{9}{2} - 2 = \frac{11}{2}$

$1^n = 1$ for any n .

The line $2y = 11x + 3$ is

$y = \frac{11}{2}x + \frac{3}{2}$

Compare with $y = mx + c$

The gradient is $\frac{11}{2}$

So the tangent to the curve where $x = 1$ is parallel to this line , since the gradients are equal .

Give a conclusion , with a reason .

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Algebraic fractions
Exercise A, Question 24

Question:

The curve C with equation $y = \mathrm{f}(x)$ passes through the point (3, 5).

Given that $\mathrm{f}'(x) = x^2 + 4x - 3$, find f($x$).

Solution:

$f'(x) = x^2 + 4x - 3$

To find $f(x)$ from $f'(x)$, integrate. Use $\int x^n dx = \frac{x^{n+1}}{n+1} + C$.

$f(x) = \frac{x^3}{3} + \frac{4x^2}{2} - 3x + C$

$= \frac{x^3}{3} + 2x^2 - 3x + C$

Do not forget to include the constant of integration C.

When $x = 3$, $f(x) = 5$, so

The curve passes through $(3, 5)$, so $f(3) = 5$.

$\frac{3^3}{3} + (2 \times 3^2) - (3 \times 3) + C = 5$

$9 + 18 - 9 + C = 5$

$C = -13$

$f(x) = \frac{x^3}{3} + 2x^2 - 3x - 13$

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Algebraic fractions
Exercise A, Question 25

Question:

The curve with equation $y = f(x)$ passes through the point (1, 6). Given that

$$f'(x) = 3 + \frac{5x^2 + 2}{x^{\frac{1}{2}}}, \ x > 0,$$

find f(x) and simplify your answer.

Solution:

$f'(x) = 3 + \dfrac{5x^2 + 2}{x^{\frac{1}{2}}}$

Divide $5x^2 + 2$ by $x^{\frac{1}{2}}$, remembering that $x^m \div x^n = x^{m-n}$

$= 3 + 5x^{\frac{3}{2}} + 2x^{-\frac{1}{2}}$

To find $f(x)$ from $f'(x)$, integrate.

$f(x) = 3x + \dfrac{5x^{\frac{5}{2}}}{\left(\frac{5}{2}\right)} + \dfrac{2x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} + C$

Use $\int x^n dx = \dfrac{x^{n+1}}{n+1} + C$.

$= 3x + \left(5 \times \frac{2}{5}x^{\frac{5}{2}}\right) + \left(2 \times \frac{2}{1}x^{\frac{1}{2}}\right) + C$

$= 3x + 2x^{\frac{5}{2}} + 4x^{\frac{1}{2}} + C$

Do not forget to include the constant of integration C.

When $x = 1$, $f(x) = 6$, so

The curve passes through (1, 6), so $f(1) = 6$

$(3 \times 1) + \left(2 \times 1^{\frac{5}{2}}\right) + \left(4 \times 1^{\frac{1}{2}}\right) + C = 6$

$3 + 2 + 4 + C = 6$

$1^n = 1$ for any n.

$C = -3$

$f(x) = 3x + 2x^{\frac{5}{2}} + 4x^{\frac{1}{2}} - 3$

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Algebraic fractions
Exercise A, Question 26

Question:

For the curve C with equation $y = f(x)$,

$$\frac{dy}{dx} = x^3 + 2x - 7$$

(a) Find $\frac{d^2y}{dx^2}$

(b) Show that $\frac{d^2y}{dx^2} \geq 2$ for all values of x.

Given that the point $P(2, 4)$ lies on C,

(c) find y in terms of x,

(d) find an equation for the normal to C at P in the form $ax + by + c = 0$, where a, b and c are integers.

Solution:

(a)

$\frac{dy}{dx} = x^3 + 2x - 7$ Differentiate to find

$\frac{d^2y}{dx^2} = 3x^2 + 2$ the second derivative

(b)

$x^2 \geq 0$ for any (real) x. The square of a

So $3x^2 \geq 0$ real number

So $3x^2 + 2 \geq 2$ cannot be negative .

So $\frac{d^2y}{dx^2} \geq 2$ for all values of x. Give a conclusion .

(c)

$\frac{dy}{dx} = x^3 + 2x - 7$ Integrate $\frac{dy}{dx}$ to find y in terms of x.

$y = \frac{x^4}{4} + \frac{2x^2}{2} - 7x + C$ Do not forget to include the constant of integration C.

$= \frac{x^4}{4} + x^2 - 7x + C$

When $x = 2$, $y = 4$, so Use the fact that $P(2, 4)$ lies on the curve.

$4 = \frac{2^4}{4} + 2^2 - (7 \times 2) + C$

$4 = 4 + 4 - 14 + C$

$C = +10$

$y = \frac{x^4}{4} + x^2 + 7x + 10$

(d)

For $x = 2$,

$\frac{dy}{dx} = 2^3 + (2 \times 2) - 7$

$= 8 + 4 - 7 = 5$

The value of $\frac{dy}{dx}$ is the gradient of the tangent .

The gradient of the normal at P is $\frac{-1}{5}$

The normal is perpendicular to the tangent, so the gradient is $-\frac{1}{m}$

Equation of the normal :

$y - 4 = \frac{-1}{5}(x - 2)$

Use $y - y_1 = m(x - x_1)$

$y - 4 = \frac{-x}{5} + \frac{2}{5}$

Multiply by 5

$5y - 20 = -x + 2$

$x + 5y - 22 = 0$

This is in the required form $ax + by + c = 0$, where a, b and c are integers .

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Algebraic fractions
Exercise A, Question 27

Question:

For the curve C with equation $y = f(x)$,

$$\frac{dy}{dx} = \frac{1 - x^2}{x^4}$$

Given that C passes through the point $\left(\frac{1}{2}, \frac{2}{3} \right)$,

(a) find y in terms of x.

(b) find the coordinates of the point on C at which $\frac{dy}{dx} = 0$.

Solution:

(a)

$\frac{dy}{dx} = \frac{1 - x^2}{x^4}$ Divide $1 - x^2$ by x^4

$= x^{-4} - x^{-2}$

$y = \frac{x^{-3}}{-3} - \frac{x^{-1}}{-1} + C$ Integrate $\frac{dy}{dx}$ to find y in terms of x. Do not forget to include the constant of integration C.

$= \frac{-x^{-3}}{3} + x^{-1} + C$

$y = \frac{-1}{3x^3} + \frac{1}{x} + C$ Use $x^{-n} = \frac{1}{x^n}$. This will make it easier to calculate values at the next stage.

When $x = \frac{1}{2}$, $y = \frac{2}{3}$, so

$\frac{2}{3} = -\frac{8}{3} + 2 + C$ Use the fact that $\left(\frac{1}{2}, \frac{2}{3} \right)$ lies on the curve.

$C = \frac{2}{3} + \frac{8}{3} - 2 = \frac{4}{3}$

$y = \frac{-1}{3x^3} + \frac{1}{x} + \frac{4}{3}$

(b)

$\frac{1 - x^2}{x^4} = 0$

If a fraction is equal to zero, its numerator must be zero.

$1 - x^2 = 0$

$x^2 = 1$

$x = 1$ or $x = -1$

$x = 1: \quad y = \frac{-1}{3} + 1 + \frac{4}{3}$

$y = 2$

$x = -1: \quad y = \frac{1}{3} - 1 + \frac{4}{3}$

$y = \frac{2}{3}$

The points are $(1, 2)$ and $\left(-1, \frac{2}{3}\right)$

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Algebraic fractions
Exercise A, Question 28

Question:

The curve C with equation $y = f(x)$ passes through the point (5, 65).

Given that $f'(x) = 6x^2 - 10x - 12$,

(a) use integration to find f (x).

(b) Hence show that $f(x) = x(2x + 3)(x - 4)$.

(c) Sketch C, showing the coordinates of the points where C crosses the x-axis.

Solution:

(a)

$f'(x) = 6x^2 - 10x - 12$

To find $f(x)$ from $f'(x)$, integrate

$f(x) = \frac{6x^3}{3} - \frac{10x^2}{2} - 12x + C$

Do not forget to include the constant of integration C.

When $x = 5$, $y = 65$, so

$65 = \frac{6 \times 125}{3} - \frac{10 \times 25}{2} - 60 + C$

Use the fact that the curve passes through $(5, 65)$

$65 = 250 - 125 - 60 + C$

$C = 65 + 125 + 60 - 250$

$C = 0$

$f(x) = 2x^3 - 5x^2 - 12x$

(b)

$f(x) = x(2x^2 - 5x - 12)$

$f(x) = x(2x + 3)(x - 4)$

(c)

Curve meets x-axis where $y = 0$

$x(2x + 3)(x - 4) = 0$

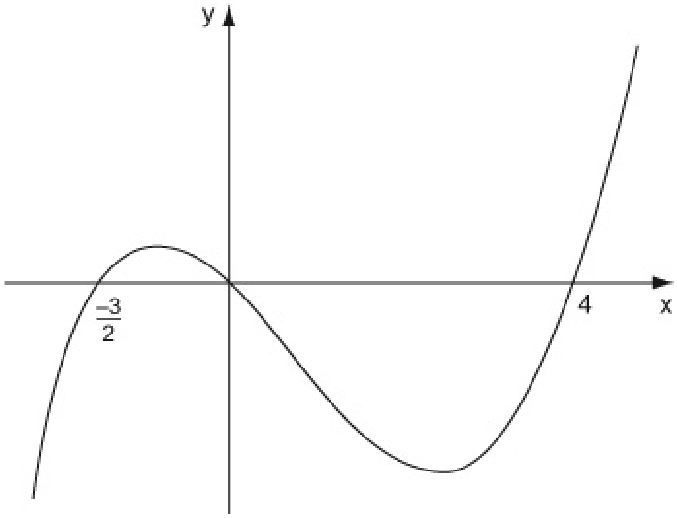
Put $y = 0$ and solve for x

$x = 0, x = -\frac{3}{2}, x = 4$

When $x \to \infty$, $y \to \infty$

When $x \to -\infty$, $y \to -\infty$

Check what happens to y for large positive and negative values of x.



Crosses x-axis at $\left(\frac{-3}{2}, 0 \right)$, $(0, 0)$, $(4, 0)$

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Algebraic fractions
Exercise A, Question 29

Question:

The curve C has equation $y = x^2 \left(x - 6 \right) + \frac{4}{x}$, $x > 0$.

The points P and Q lie on C and have x-coordinates 1 and 2 respectively.

(a) Show that the length of PQ is $\sqrt{170}$.

(b) Show that the tangents to C at P and Q are parallel.

(c) Find an equation for the normal to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

Solution:

(a)

$y = x^2 (x - 6) + \frac{4}{x}$

At P, $x = 1$,

$y = 1 (1 - 6) + \frac{4}{1} = - 1$

P is $(1 , - 1)$
At Q, $x = 2$,

$y = 4 (2 - 6) + \frac{4}{2} = - 14$

Q is $(2 , - 14)$

$PQ = \sqrt{ (2 - 1) ^2 + (- 14 - (- 1)) ^2 }$
$= \sqrt{ (1^2 + (- 13) ^2) }$
$= \sqrt{ (1 + 169) } = \sqrt{170}$

The distance between two points is $\sqrt{ (x_2 - x_1) ^2 + (y_2 - y_1) ^2 }$

(b)

$y = x^3 - 6x^2 + 4x^{-1}$

$\frac{dy}{dx} = 3x^2 - (6 \times 2x') + (4x - 1x^{-2})$

$= 3x^2 - 12x - 4x^{-2}$

At $x = 1$,

$\frac{dy}{dx} = 3 - 12 - 4 = - 13$

The value of $\frac{dy}{dx}$ is the gradient of the tangent.

At $x = 2$,

$\frac{dy}{dx} = (3 \times 4) - (12 \times 2) - (4 \times 2^{-2})$

$= 12 - 24 - \frac{4}{4} = - 13$

At P and also at Q the gradient is −13, so the tangents are parallel (equal gradients).

Give a conclusion

(c)

The gradient of the normal at P is $-\frac{1}{-13}=\frac{1}{13}$

The normal is perpendicular to the tangent, so the gradient is $-\frac{1}{m}$

Equation of the normal:

$y-(-1)=\frac{1}{13}(x-1)$ Use $y-y_1=m(x-x_1)$

$y+1=\frac{x}{13}-\frac{1}{13}$ Multiply by 13

$13y+13=x-1$

$x-13y-14=0$

This is in the required form $ax+by+c=0$, where a, b and c are integers.

Solutionbank C1
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Algebraic fractions
Exercise A, Question 30

Question:

(a) Factorise completely $x^3 - 7x^2 + 12x$.

(b) Sketch the graph of $y = x^3 - 7x^2 + 12x$, showing the coordinates of the points at which the graph crosses the x-axis.

The graph of $y = x^3 - 7x^2 + 12x$ crosses the positive x-axis at the points A and B.

The tangents to the graph at A and B meet at the point P.

(c) Find the coordinates of P.

Solution:

(a)

$x^3 - 7x^2 + 12x$ — x is a common factor

$= x (x^2 - 7x + 12)$

$= x (x - 3) (x - 4)$

(b)

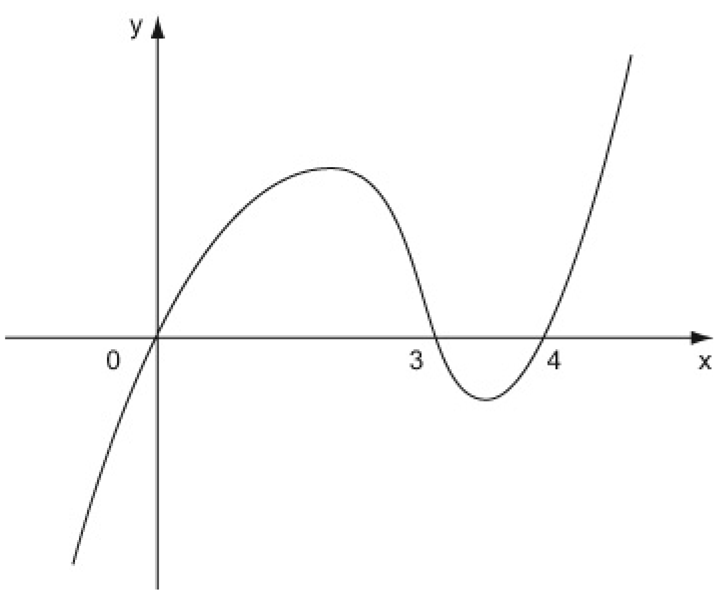
Curve meets x-axis where $y = 0$.

$x (x - 3) (x - 4) = 0$ — Put $y = 0$ and solve for x .

$x = 0 , x = 3 , x = 4$

When $x \to \infty , y \to \infty$ — Check what happens to y for large positive and negative values of x

When $x \to - \infty , y \to - \infty$



Crosses x-axis at $(0 , 0)$, $(3 , 0)$, $(4 , 0)$

(c)

A and B are $(3, 0)$ and $(4, 0)$

$\frac{dy}{dx} = 3x^2 - 14x + 12$

At $x = 3$, (A)

The value of $\frac{dy}{dx}$ is the gradient of the tangent.

$\frac{dy}{dx} = 27 - 42 + 12 = -3$

At $x = 4$ (B)

$\frac{dy}{dx} = 48 - 56 + 12 = 4$

Tangent at A:

Use $y - y_1 = m(x - x_1)$

$y - 0 = -3(x - 3)$

$y = -3x + 9 \quad (\text{i})$

Tangent at B:

$y - 0 = 4(x - 4)$

$y = 4x - 16 \quad (\text{ii})$

Subtract $(\text{ii}) - (\text{i})$:

Solve (i) and (ii) simultaneously to find the intersection point of the tangents

$0 = 7x - 25$

$x = \frac{25}{7}$

Substituting back into (i):

$y = -\frac{75}{7} + 9 = -\frac{12}{7}$

P is the point $\left(\frac{25}{7}, \frac{-12}{7}\right)$