

Review exercise 2

1 The equation of the line is:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 8}{6 - 8} = \frac{x - (-2)}{4 - (-2)}$$

$$\frac{y - 8}{-2} = \frac{x + 2}{6}$$

$$3y - 24 = -x - 2$$

$$x + 3y - 22 = 0$$

2 $y - (-4) = \frac{1}{3}(x - 9)$
 $y + 4 = \frac{1}{3}(x - 9)$
 $3y + 12 = x - 9$
 $x - 3y - 21 = 0$
 $a = 1, b = -3, c = -21$

3 Using points A and B :

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 3}{5 - 3} = \frac{x - 0}{k - 0}$$

$$\frac{y - 3}{2} = \frac{x}{k}$$

$$ky - 3k = 2x$$

Substituting point C into the equation:

$$k(2k) - 3k = 2(10)$$

$$2k^2 - 3k - 20 = 0$$

$$(2k + 5)(k - 4) = 0$$

$$k = -\frac{5}{2} \text{ or } k = 4$$

4 a The gradient of l_1 is 3.
 So the gradient of l_2 is $-\frac{1}{3}$.

The equation of line l_2 is:

$$y - 2 = -\frac{1}{3}(x - 6)$$

$$y - 2 = -\frac{1}{3}x + 2$$

$$y = -\frac{1}{3}x + 4$$

b $l_1: y = 3x - 6$
 $l_2: y = -\frac{1}{3}x + 4$

$$3x + \frac{1}{3}x = 4 + 6$$

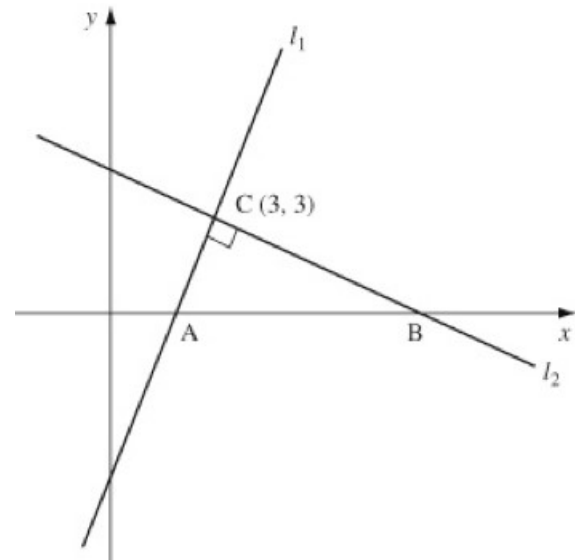
$$\frac{10}{3}x = 10$$

$$x = 3$$

$$y = 3 \times 3 - 6 = 3$$

The point C is $(3, 3)$.

4 c



Where l_1 meets the x -axis, $y = 0$:

$$0 = 3x - 6$$

$$3x = 6$$

$$x = 2$$

The point A is $(2, 0)$.

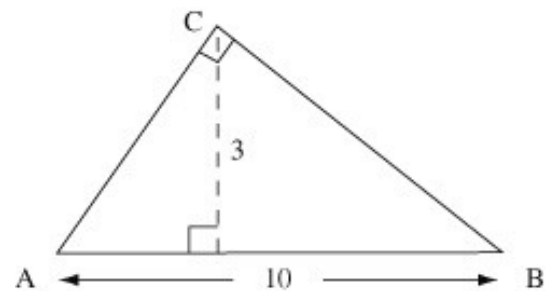
Where l_2 meets the x -axis, $y = 0$:

$$0 = -\frac{1}{3}x + 4$$

$$\frac{1}{3}x = 4$$

$$x = 12$$

The point B is $(12, 0)$.



$$AB = 12 - 2 = 10$$

The perpendicular height, using AB as the base is 3.

$$\begin{aligned} \text{Area of triangle } ABC &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times 10 \times 3 \\ &= 15 \text{ units}^2 \end{aligned}$$

5 Using the sine rule:

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{b}{\sin 45^\circ} = \frac{\sqrt{5}}{\sin 30^\circ}$$

$$b = \frac{\sqrt{5} \sin 45^\circ}{\sin 30^\circ}$$

$$b = \frac{\sqrt{5} \times \frac{\sqrt{2}}{2}}{\frac{1}{2}}$$

$$b = \sqrt{10}$$

$$AC = \sqrt{10} \text{ cm}$$

6 a Using the cosine rule:

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos 60^\circ = \frac{(2x-3)^2 + 5^2 - (x+1)^2}{2(2x-3)(5)}$$

$$\frac{1}{2} = \frac{4x^2 - 12x + 9 + 25 - (x^2 + 2x + 1)}{10(2x-3)}$$

$$5(2x-3) = 3x^2 - 14x + 33$$

$$3x^2 - 24x + 48 = 0$$

$$x^2 - 8x + 16 = 0$$

b $x^2 - 8x + 16 = 0$

$$(x-4)^2 = 0$$

$$x = 4$$

c Area = $\frac{1}{2}ac \sin B$

$$a = 2 \times 4 - 3 = 5$$

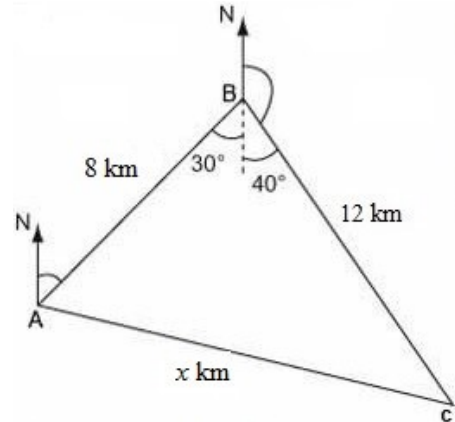
$$c = 5$$

$$\text{Area} = \frac{1}{2} \times 5 \times 5 \times \sin 60^\circ$$

$$= 10.8253\dots$$

$$= 10.8 \text{ cm}^2 \text{ (3 s.f.)}$$

7



a Using the cosine rule

$$x^2 = 8^2 + 12^2 - 2 \times 8 \times 12 \times \cos 70^\circ$$

$$= 142.332\dots$$

$$x = 11.93 \text{ km}$$

The distance of ship C from ship A is 11.93 km.

b Using the sine rule:

$$\frac{\sin 70^\circ}{11.93} = \frac{\sin A}{12}$$

$$\frac{11.93}{12}$$

$$\sin A = 0.94520\dots$$

$$A = 70.9^\circ$$

The bearing of ship C from ship A is 100.9° .

8 a If triangle ABC is isosceles, then two of the sides are equal.

$$AB = \sqrt{(6+2)^2 + (10-4)^2} = \sqrt{100} = 10$$

$$BC = \sqrt{(16-6)^2 + (10-10)^2} = \sqrt{100} = 10$$

$$AC = \sqrt{(16+2)^2 + (10-4)^2} = \sqrt{360} = 6\sqrt{10}$$

$AB = BC$, therefore ABC is isosceles.

b Using the cosine rule:

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$= \frac{10^2 + 10^2 - (\sqrt{360})^2}{2(10)(10)}$$

$$= \frac{100 + 100 - 360}{200}$$

$$= \frac{4}{5}$$

$$= -\frac{4}{5}$$

$$B = 143.13010\dots$$

$$\angle ABC = 143.1^\circ \text{ (1 d.p.)}$$

- 9 Using the sine rule in triangle ABD :

$$\frac{\sin \angle BDA}{4.3} = \frac{\sin 40^\circ}{3.5}$$

$$\sin \angle BDA = \frac{4.3 \sin 40^\circ}{3.5}$$

$$= 0.78971\dots$$

$$\angle BDA = 52.16^\circ$$

Using the angle sum of a triangle:

$$\angle ABD = 180^\circ - (52.16^\circ + 40^\circ)$$

$$= 87.84^\circ$$

Using the sine rule in triangle ABD :

$$\frac{AD}{\sin 87.84^\circ} = \frac{3.5}{\sin 40^\circ}$$

$$AD = 5.44 \text{ cm}$$

$$AC = AD + DC$$

$$= 5.44 + 8.6$$

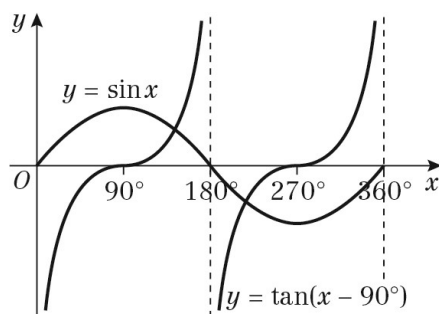
$$= 14.04 \text{ cm}$$

Area of triangle ABC

$$= \frac{1}{2} \times 4.3 \times 14.04 \times \sin 40^\circ$$

$$= 19.4 \text{ cm}^2$$

10 a



- b There are two solutions in the interval $0 \leq x \leq 360^\circ$.

- 11 a The curve $y = \sin x$ crosses the x -axis at $(-360^\circ, 0)$, $(-180^\circ, 0)$, $(0^\circ, 0)$, $(180^\circ, 0)$ and $(360^\circ, 0)$.

$$y = \sin(x + 45^\circ) \text{ is a translation of}$$

$$\begin{pmatrix} -45^\circ \\ 0 \end{pmatrix}$$

so subtract 45° from the x -coordinates.

The curve crosses the x -axis at

$$(-405^\circ, 0), (-225^\circ, 0), (-45^\circ, 0), (135^\circ, 0)$$

$$\text{and } (315^\circ, 0).$$

$(-405^\circ, 0)$ is not in the range, so

$$(-225^\circ, 0), (-45^\circ, 0), (135^\circ, 0) \text{ and } (315^\circ, 0)$$

- b The curve $y = \sin(x + 45^\circ)$ crosses the y -axis when $x = 0$.

$$\sin 45^\circ = \frac{\sqrt{2}}{2}$$

$$\begin{pmatrix} 0, \frac{\sqrt{2}}{2} \end{pmatrix}$$

- 12 Crosses y -axis when $x = 0$ at $\sin \frac{3\pi}{4} = \frac{1}{\sqrt{2}}$

$$\text{Crosses } x\text{-axis when } \sin\left(x + \frac{3\pi}{4}\right) = 0$$

$$x + \frac{3\pi}{4} = -\pi, 0, \pi, 2\pi$$

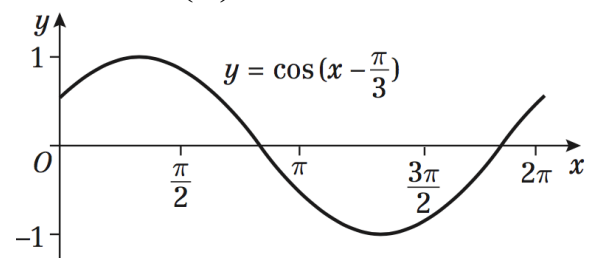
$$x = -\frac{7\pi}{4}, -\frac{3\pi}{4}, \frac{\pi}{4}, \frac{5\pi}{4}$$

So coordinates are

$$\left(0, \frac{1}{\sqrt{2}}\right), \left(-\frac{7\pi}{4}, 0\right), \left(-\frac{3\pi}{4}, 0\right), \left(\frac{\pi}{4}, 0\right), \left(\frac{5\pi}{4}, 0\right)$$

- 13 a $y = \cos\left(x - \frac{\pi}{3}\right)$ is $y = \cos x$ translated by

$$\text{the vector } \begin{pmatrix} \frac{\pi}{3} \\ 0 \end{pmatrix}$$



13 b Crosses y -axis when $y = \cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}$

Crosses x -axis when $\cos\left(x - \frac{\pi}{3}\right) = 0$

$$x - \frac{\pi}{3} = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$x = \frac{5\pi}{6}, \frac{11\pi}{6}$$

So coordinates are

$$\left(0, \frac{1}{2}\right), \left(\frac{5\pi}{6}, 0\right), \left(\frac{11\pi}{6}, 0\right)$$

c $\cos\left(x - \frac{\pi}{3}\right) = -0.27, 0 \leq x \leq 2\pi$

$$\cos^{-1}(-0.27) = 1.844 \text{ (3 d.p.)}$$

$$x - \frac{\pi}{3} \approx 1.844 \text{ and } x - \frac{\pi}{3} \approx 2\pi - 1.844$$

$$x = 2.89, 5.49 \text{ (2 d.p.)}$$

14 a Let C be the midpoint of AB .

Then $AC = 3$ cm, and AOC is a right-angled triangle.

$$\sin\left(\frac{\theta}{2}\right) = \frac{3}{5} = 0.6$$

$$\frac{\theta}{2} = \sin^{-1}(0.6)$$

$$\theta = 2 \times \sin^{-1}(0.6) = 1.29 \text{ rad (3 s.f.)}$$

b Use $l = r\theta$

$$\text{Minor arc } AB = 5 \times 1.29 = 6.45 \text{ cm (3 s.f.)}$$

15 As ABC is equilateral, $BC = AC = 8$ cm

$$BP = AB - AP = 8 - 6 = 2 \text{ cm}$$

$$QC = BP = 2 \text{ cm}$$

$$\angle BAC = \frac{\pi}{3}, PQ = 6 \times \frac{\pi}{3} = 2\pi$$

$$= 6.28 \text{ cm (2 d.p.)}$$

$$\begin{aligned} \text{So perimeter} &= BC + BP + PQ + QC \\ &= 18.28 \text{ cm (2 d.p.)} \end{aligned}$$

Exact answer $12 + 2\pi$ cm

16 a $\frac{1}{2}(r + 10)^2\theta - \frac{1}{2}r^2\theta = 40$

$$20r\theta + 100\theta = 80$$

$$r\theta + 5\theta = 4$$

$$\Rightarrow r = \frac{4}{\theta} - 5$$

b $r = \frac{4}{\theta} - 5 = 6\theta$

$$4 - 5\theta = 6\theta^2$$

$$6\theta^2 + 5\theta - 4 = 0$$

$$(3\theta + 4)(2\theta - 1) = 0$$

$$\Rightarrow \theta = -\frac{4}{3} \text{ or } \frac{1}{2}$$

But θ cannot be negative, so $\theta = \frac{1}{2}, r = 3$

$$\begin{aligned} \text{So perimeter} &= 20 + r\theta + (10 + r)\theta \\ &= 20 + \frac{3}{2} + \frac{13}{2} = 28 \text{ cm} \end{aligned}$$

17 a arc $BD = 10 \times 0.6 = 6$ cm

b Area of triangle $ABC = \frac{1}{2}(13 \times 10) \sin 0.6$
 $= 65 \times 0.567$
 $= 36.7 \text{ cm}^2 \text{ (1 d.p.)}$

$$\begin{aligned} \text{Area of sector } ABD &= \frac{1}{2}10^2 \times 0.6 \\ &= 30 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of shaded area } BCD &= 36.7 - 30 \\ &= 6.7 \text{ cm}^2 \text{ (1 d.p.)} \end{aligned}$$

18 a $\angle OED = 90^\circ$ because BC is parallel to ED

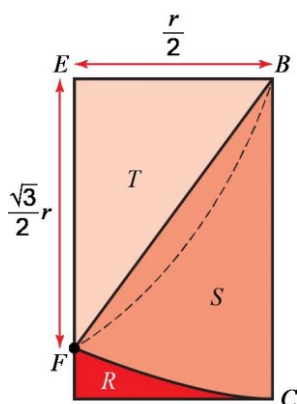
$$\text{So } r = \frac{10}{\cos 0.7} = 13.07 \text{ cm (2 d.p.)}$$

$$\begin{aligned} \text{Area of sector } OAB &= \frac{1}{2}r^2 \times 1.4 \\ &= 119.7 \text{ cm}^2 \text{ (1 d.p.)} \end{aligned}$$

b $BC = AC = r \tan 0.7$

$$\begin{aligned} \text{So perimeter} &= 2r \tan 0.7 + r \times 1.4 \\ &= (2 \times 13.07 \times 0.842) + (13.07 \times 1.4) \\ &= 40.3 \text{ cm} \end{aligned}$$

- 19 Split each half of the rectangle as shown.



EFB is a right-angled triangle, and by

Pythagoras' theorem, $EF = \frac{\sqrt{3}}{2}r$.

Let $\angle EBF = \theta$, so $\tan \theta = \sqrt{3}$, so $\theta = \frac{\pi}{3}$

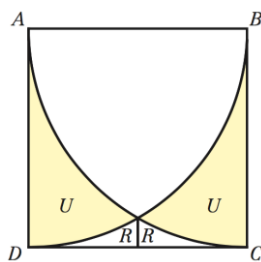
So $\angle FBC = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$

$$\text{Area } S = \frac{1}{2}r^2 \frac{\pi}{6} = \frac{\pi}{12}r^2$$

$$\text{Area } T = \frac{1}{2} \times \frac{\sqrt{3}}{2}r \times \frac{1}{2}r = \frac{\sqrt{3}}{8}r^2$$

$$\Rightarrow \text{Area } R = \frac{1}{2}r^2 - \text{Area } S - \text{Area } T$$

$$= \left(\frac{1}{2} - \frac{\sqrt{3}}{8} - \frac{\pi}{12} \right) r^2$$



$$\text{Area of sector } ACB = \frac{1}{2}r^2 \frac{\pi}{2} = \frac{\pi}{4}r^2$$

$$\text{Area } U = \text{Area } ABCD - \text{Area sector } ACB - 2R$$

$$= r^2 - \frac{\pi}{4}r^2 - 2 \left(\frac{1}{2} - \frac{\sqrt{3}}{8} - \frac{\pi}{12} \right) r^2$$

$$= r^2 \left(\frac{\sqrt{3}}{4} - \frac{\pi}{12} \right)$$

$$\begin{aligned} \text{So area } U &= r^2 - \frac{\pi}{4}r^2 - 2R \\ &= \left(1 - \frac{\pi}{4} - 1 + \frac{\sqrt{3}}{4} + \frac{\pi}{6} \right) r^2 \\ &= r^2 \left(\frac{\sqrt{3}}{4} - \frac{\pi}{12} \right) = \frac{r^2}{12} (3\sqrt{3} - \pi) \end{aligned}$$

$$\text{So shaded area} = 2U = \frac{r^2}{6} (3\sqrt{3} - \pi)$$

$$\text{Thus } U = \frac{r^2}{12} (3\sqrt{3} - \pi)$$

20 $f(x) = 5x^2$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{5(x+h)^2 - 5x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{5x^2 + 10xh + 5h^2 - 5x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{10xh + 5h^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(10x + 5h)}{h} \\ &= \lim_{h \rightarrow 0} (10x + 5h) \end{aligned}$$

As $h \rightarrow 0$, $10x + 5h \rightarrow 10x$, so $f'(x) = 10x$

21 $y = 4x^3 - 1 + 2x^{\frac{1}{2}}$

$$\frac{dy}{dx} = (4 \times 3x^2) + \left(2 \times \frac{1}{2} x^{-\frac{1}{2}} \right)$$

$$\frac{dy}{dx} = 12x^2 + x^{-\frac{1}{2}}$$

Or:

$$\frac{dy}{dx} = 12x^2 + \frac{1}{x^{\frac{1}{2}}}$$

Or:

$$\frac{dy}{dx} = 12x^2 + \frac{1}{\sqrt{x}}$$

$$22 \text{ a } y = 4x + 3x^{\frac{3}{2}} - 2x^2$$

$$\frac{dy}{dx} = (4 \times 1x^0) + \left(3 \times \frac{3}{2}x^{\frac{1}{2}}\right) - (2 \times 2x^1)$$

$$\frac{dy}{dx} = 4 + \frac{9}{2}x^{\frac{1}{2}} - 4x$$

b For $x = 4$,

$$y = (4 \times 4) + \left(3 \times 4^{\frac{3}{2}}\right) - (2 \times 4^2)$$

$$= 16 + (3 \times 8) - 32$$

$$= 16 + 24 - 32$$

$$= 8$$

So $P(4, 8)$ lies on C .

c For $x = 4$,

$$\frac{dy}{dx} = 4 + \left(\frac{9}{2} \times 4^{\frac{1}{2}}\right) - (4 \times 4)$$

$$= 4 + \left(\frac{9}{2} \times 2\right) - 16$$

$$= 4 + 9 - 16$$

$$= -3$$

This is the gradient of the tangent.

The normal is perpendicular to the

tangent, so the gradient is $-\frac{1}{m}$.

The gradient of the normal at P is $\frac{1}{3}$.

Equation of the normal:

$$y - 8 = \frac{1}{3}(x - 4)$$

$$3y - 24 = x - 4$$

$$3y = x + 20$$

d $y = 0$:

$$0 = x + 20$$

$$x = -20$$

Q is the point $(-20, 0)$.

$$PQ = \sqrt{(4 - (-20))^2 + (8 - 0)^2}$$

$$= \sqrt{24^2 + 8^2}$$

$$= \sqrt{576 + 64}$$

$$= \sqrt{640}$$

$$= \sqrt{64} \times \sqrt{10}$$

$$= 8\sqrt{10}$$

$$23 \text{ a } y = 4x^2 + \frac{5-x}{x}$$

$$= 4x^2 + 5x^{-1} - 1$$

$$\frac{dy}{dx} = (4 \times 2x^1) + (5 \times -1x^{-2})$$

$$= 8x - 5x^{-2}$$

At P , $x = 1$, so

$$\frac{dy}{dx} = (8 \times 1) - (5 \times 1^{-2})$$

$$= 8 - 5$$

$$= 3$$

b At $x = 1$, $\frac{dy}{dx} = 3$

The value of $\frac{dy}{dx}$ is the gradient of the tangent.

$$\text{At } x = 1, y = (4 \times 1^2) + \frac{5-1}{1}$$

$$= 4 + 4 = 8$$

Equation of the tangent:

$$y - 8 = 3(x - 1)$$

$$y = 3x + 5$$

c $y = 0: 0 = 3x + 5$

$$3 = -5$$

$$x = -\frac{5}{3}$$

So $k = -\frac{5}{3}$

$$24 \text{ a } f(x) = \frac{(2x+1)(x+4)}{\sqrt{x}}$$

$$= \frac{2x^2 + 9x + 4}{\sqrt{x}}$$

$$= 2x^{\frac{3}{2}} + 9x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}$$

$$P = 2, Q = 9, R = 4$$

$$\text{b } f'(x) = \left(2 \times \frac{3}{2}x^{\frac{1}{2}}\right) + \left(9 \times \frac{1}{2}x^{-\frac{1}{2}}\right) +$$

$$\left(4 \times -\frac{1}{2}x^{-\frac{3}{2}}\right)$$

$$= 3x^{\frac{1}{2}} + \frac{9}{2}x^{-\frac{1}{2}} - 2x^{-\frac{3}{2}}$$

24 c At $x = 1$,

$$\begin{aligned} f'(1) &= \left(3 \times 1^{\frac{1}{2}}\right) + \left(\frac{9}{2} \times 1^{-\frac{1}{2}}\right) - \left(2 \times 1^{-\frac{3}{2}}\right) \\ &= 3 + \frac{9}{2} - 2 \\ &= \frac{11}{2} \end{aligned}$$

The line $2y = 11x + 3$ is

$$y = \frac{11}{2}x + \frac{3}{2}$$

\therefore The gradient is $\frac{11}{2}$.

The tangent to the curve where $x = 1$ is parallel to this line, since the gradients are equal.

25 a $y = 3x^2 + 4\sqrt{x}$

$$= 3x^2 + 4x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \left(3 \times 2x^1\right) + \left(4 \times \frac{1}{2}x^{-\frac{1}{2}}\right)$$

$$\frac{dy}{dx} = 6x + 2x^{-\frac{1}{2}}$$

Or:

$$\frac{dy}{dx} = 6x + \frac{2}{x^{\frac{1}{2}}} = 6x + \frac{2}{\sqrt{x}}$$

b $\frac{dy}{dx} = 6x + 2x^{-\frac{1}{2}}$

$$\frac{d^2y}{dx^2} = 6 + \left(2 \times -\frac{1}{2}x^{-\frac{3}{2}}\right)$$

$$= 6 - x^{-\frac{3}{2}}$$

Or:

$$\frac{d^2y}{dx^2} = 6 - \frac{1}{x^{\frac{3}{2}}}$$

Or:

$$\frac{d^2y}{dx^2} = 6 - \frac{1}{x\sqrt{x}}$$

$$25 \text{ c } \int \left(3x^2 + 4x^{\frac{1}{2}}\right) dx = \frac{3x^3}{3} + \frac{4x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + C$$

$$= x^3 + 4\left(\frac{2}{3}\right)x^{\frac{3}{2}} + C$$

$$= x^3 + \frac{8}{3}x^{\frac{3}{2}} + C$$

$$\left(\text{Or: } x^3 + \frac{8}{3}x\sqrt{x} + C\right)$$

26 a $f'(x) = 6x^2 - 10x - 12$

$$f(x) = \frac{6x^3}{3} - \frac{10x^2}{2} - 12x + C$$

When $x = 5$, $y = 65$, so:

$$65 = \frac{6 \times 125}{3} - \frac{10 \times 25}{2} - 60 + C$$

$$65 = 250 - 125 - 60 + C$$

$$C = 65 + 125 + 60 - 250$$

$$C = 0$$

$$f(x) = 2x^3 - 5x^2 - 12x$$

b $f(x) = x(2x^2 - 5x - 12)$

$$f(x) = x(2x + 3)(x - 4)$$

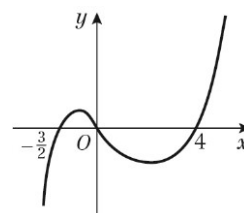
c Curve meets x -axis where $y = 0$

$$x(2x + 3)(x - 4) = 0$$

$$x = 0, x = -\frac{3}{2}, x = 4$$

When $x \rightarrow \infty$, $y \rightarrow \infty$

When $x \rightarrow -\infty$, $y \rightarrow -\infty$



Crosses x -axis at $\left(-\frac{3}{2}, 0\right)$, $(0, 0)$ and $(4, 0)$.

Challenge

- 1 a Finding points B and C using $y = 3x - 12$:

$$\text{When } y = 0, x = 4$$

$$\text{When } x = 0, y = -12$$

The point B is $(4, 0)$ and

the point C is $(0, -12)$.

Using Pythagoras' theorem to find the length of the square:

$$BC = \sqrt{(0-4)^2 + (-12-0)^2} = \sqrt{160}$$

$$\text{Area of square} = (\sqrt{160})^2 = 160$$

- b The point A is $(-8, 4)$ and the point D is $(-12, -8)$.

$$\begin{aligned} \text{The gradient of line } AD &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-8 - 4}{-12 + 8} \\ &= \frac{-12}{-4} \\ &= 3 \end{aligned}$$

The equation of line AD is:

$$y - y_1 = m(x - x_1)$$

$$y - 4 = 3(x + 8)$$

$$y = 3x + 28$$

$$\text{When } y = 0, x = -\frac{28}{3}$$

The point S is $(-\frac{28}{3}, 0)$.

- 2 Angle of minor arc $= \frac{\pi}{2}$ because it is a

quarter circle

Let the chord meet the circle at R and T .

The area of P is the area of sector formed by O, R and T less the area of the triangle ORT .

$$\begin{aligned} \text{So area of } P &= \frac{1}{2}r^2 \frac{\pi}{2} - \frac{1}{2}r^2 \sin \frac{\pi}{2} \\ &= r^2 \left(\frac{\pi}{4} - \frac{1}{2} \right) = \frac{r^2}{4}(\pi - 2) \end{aligned}$$

$$\text{Area of } Q = \pi r^2 - \text{area of } P$$

$$\begin{aligned} &= r^2 \left(\pi - \frac{\pi}{4} + \frac{1}{2} \right) = r^2 \left(\frac{3\pi}{4} + \frac{1}{2} \right) \\ &= \frac{r^2}{4}(3\pi + 2) \end{aligned}$$

$$\text{So ratio} = (\pi - 2) : (3\pi + 2) = \frac{\pi - 2}{3\pi + 2} : 1$$

- 3 a $f'(-3) = k((-3)^2 - 3 - 6) = 0$

$$f'(2) = k(2^2 + 2 - 6) = 0$$

Using the factor theorem, $x + 3$ and $x - 2$ are factors of $f'(x)$.

$$\begin{aligned} \text{So } f'(x) &= k(x + 3)(x - 2) \\ &= k(x^2 + x - 6) \end{aligned}$$

As $f(x)$ is cubic, there are no other factors of $f'(x)$.

$$\begin{aligned} \text{b } \int k(x^2 + x - 6) \, dx &= \int (kx^2 + kx - 6k) \, dx \\ &= \frac{kx^3}{3} + \frac{kx^2}{2} - 6kx + c \end{aligned}$$

At $(-3, 76)$:

$$\begin{aligned} \frac{k(-3)^3}{3} + \frac{k(-3)^2}{2} - 6k(-3) + c &= 76 \\ -9k + \frac{9k}{2} + 18k + c &= 76 \\ \frac{27k}{2} + c &= 76 \end{aligned}$$

At $(2, -49)$:

$$\frac{k(2)^3}{3} + \frac{k(2)^2}{2} - 6k(2) + c = -49$$

$$\begin{aligned} \text{b } \frac{8k}{3} + 2k - 12k + c &= -49 \\ -\frac{22k}{3} + c &= -49 \end{aligned}$$

$$\text{Solving } \frac{27k}{2} + c = 76 \text{ and}$$

$$-\frac{22k}{3} + c = -49 \text{ simultaneously}$$

$$c = 76 - \frac{27k}{2} \text{ and } c = \frac{22k}{3} - 49$$

$$\text{So } 76 - \frac{27k}{2} = \frac{22k}{3} - 49$$

$$456 - 81k = 44k - 294$$

$$125k = 750$$

$$k = 6, c = -5$$

$$\begin{aligned} f(x) &= \frac{kx^3}{3} + \frac{kx^2}{2} - 6kx + c \\ &= \frac{6x^3}{3} + \frac{6x^2}{2} - 6(6)x - 5 \\ &= 2x^3 + 3x^2 - 36x - 5 \end{aligned}$$